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Book of Abstracts

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GDIS 2022, 5-11 June 2022, Zlatibor, Serbia

Dedicated to the Memory of Alexey V. Borisov

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ALEXEY VLADIMIROVICH BORISOV
1965–2021

It was with great sadness that we learned that Professor Alexey Vladimirovich Borisov, a member of the Editorial Board of the journal *Theoretical and Applied Mechanics*, had passed away unexpectedly on January 24, 2021. Professor Borisov was a well-known expert in classical mechanics, hydrodynamics, and the theory of dynamical systems, a notable organizer of science in Russia, and a major promoter of scientific collaboration between Russian and Serbian scientists.

Alexey Borisov was born in Moscow on March 27, 1965 to the family of engineers. In 1988, after graduation from the Bauman Moscow State Technical University, he was assigned to the Barmin Design Bureau of Mechanical Engineering, Moscow. Alexey Borisov defended his PhD thesis entitled "Nonintegrability of the Kirchhoff Equations and Related Problems in Rigid Body Dynamics" under the supervision of Professor Valery Vasil'evich Kozlov in 1995. In 2001 he received the Russian Doctor of Sciences degree from the Faculty of Mechanics and Mathematics of Moscow State University with the thesis "Poisson Structures and Lie Algebras in Hamiltonian Mechanics".

In the 1990s, Borisov moved to Izhevsk, as he got affiliated with Udmurt State University. He was a pivotal figure in establishing a new inter-university department, the Faculty of Information Technologies and also in the formation of the Institute of Computer Science in 2001. Today, the Izhevsk Institute of Computer Science is a leading national center dealing with problems of mechanics and nonlinear sciences with a strong international visibility. The Izhevsk team formed by Borisov grew and its range of interests widened: dynamical systems theory including both nonintegrable and integrable systems, Hamiltonian formalism, topological methods for investigating dynamical systems, rigid body dynamics, nonholonomic mechanics, vortex dynamics, etc. In each of these areas, A. V. Borisov and his team succeeded in obtaining remarkable results.

Theoretical and Applied Mechanics **48**(2) (2021), i–iii.

Professor Borisov was the supervisor of thirteen PhD theses. Among his students, I. A. Bizyaev, A. A. Kilin, I. S. Mamaev, S. M. Ramodanov, P. E. Ryabov and S. V. Sokolov have obtained the Russian Doctor of Sciences degree so far.

Professor Borisov established cooperation with many important scientific Russian institutions: the Institute of Machine Science of the Russian Academy of Sciences (RAS) in Moscow where he headed the Laboratory of Computer Simulation, the Institute of Mathematics and Mechanics of the Ural Branch RAS (Yekaterinburg) where he headed the Department of Mathematical Methods of Nonlinear Dynamics, Nizhny Novgorod State University, Saratov State University, Steklov Mathematical Institute RAS, the Moscow Institute of Physics and Technologies (Dolgoprudny) where he headed the Laboratory of Mechatronics and Robotics, and the I. N. Ulyanov Chuvash State University (Cheboksary), where he initiated the establishment of the Laboratory of Artificial Intelligence and Robotics.

Professor Borisov served on the Russian National Committee for Theoretical and Applied Mechanics. He was the representative of mechanical engineers and scientists of Russia in IUTAM. From 2006 on he was a corresponding member of the Russian Academy of Natural Sciences. In 2010, he and the team of the Institute of Computer Science were awarded the so-called "megagrant" (Grant of the Government of the Russian Federation for State Support for Scientific Research Conducted under the Supervision of Leading Scientists at Russian Institutions of Higher Professional Education). In 2012 A. V. Borisov, together with his former student, I. S. Mamaev, was awarded the "Sofya Kovalevskaya Prize" by the Russian Academy of Sciences for a series of works devoted to integrable systems in Hamiltonian mechanics. In 2014 he was granted the honorary title "Honored Scientist of the Udmurt Republic".

In 2012 Borisov became the Editor-in-Chief of the internationally recognized journal *Regular & Chaotic Dynamics*. The journal was a namesake of a publishing center that Borisov had founded in 1998 for publication of literature on physics and mathematics. The small publishing house managed to do what even large Russian publishing houses could not: get the fundamental classical works of A. B. Basset, C. Caratheodory, E. J. Cartan, J. G. Darboux, J. H. Jellett, S. Lie, W. Thomson and of many others translated into Russian and publish them.

Together with Valery V. Kozlov and Vladimir Dragovic, Alexey Borisov organized seven international conferences "Geometry, Dynamics, Integrable Systems" (GDIS 2008 Belgrade, GDIS 2010 Belgrade, GDIS 2011 Lisbon, GDIS 2013 Izhevsk, GDIS 2014 Trieste, GDIS 2016 Izhevsk, GDIS 2018 Moscow), the Mathematical Institute of the Serbian Academy of Sciences and Arts, the Steklov Mathematical Institute of RAS, and the Izhevsk Institute of Computer Science being the main organizing institutions. In 2019, Borisov organized a large international conference in Cheboksary which was dedicated to the memory of S. A. Chaplygin.

We should also mention one of the remarkable hobbies of Alexey Borisov. He was an outstanding musician on the accordion. Many will remember him as both a professor and a musician, with his numerous concerts, masterly play, creative charisma and charm.

It should be especially stressed that Alexey Vladimirovich was a great friend of Serbian scientists and put a lot of effort into enhancing the scientific ties between the Russian Federation and Serbia. His achievements in this direction were formally recognized and ratified by the Mathematical Institute of the Serbian Academy of Sciences and Arts in 2016 when Alexey Borisov was awarded "Захвалница Математичког института САНУ поводом 70 година од оснивања" (the Certificate of Gratitude of the Mathematical Institute of the Serbian Academy of Sciences and Arts on the occasion of its 70^{th} anniversary).

The international conference GDIS 2022, devoted to the memory of A. V. Borisov is going to be organized on the Zlatibor mountain in Serbia, June 5–9, 2022 (http://www.mi.sanu.ac.rs/~gdis2022/index.html). Before that, the Regular and Chaotic Dynamics Conference devoted to the memory of A. V. Borisov was organized by the Steklov Mathematical Institute in Moscow, Nov. 22–Dec. 3, 2021, with the participation of scientists from all over the world, including four from Serbia: http://www.mathnet.ru/php/conference.phtml?confid=1952&option_lang=eng.

Professor Borisov's professional and creative career got prematurely interrupted. As it was observed multiple times after he passed away, this charismatic, surprisingly energetic, and multi-faceted talented intellectual managed to accomplish more than enough for several lives.

We will be missing Alexey Vladimrovich. Dear Alexey, rest in peace and thank you.

Vladimir Dragović
Borislav Gajić
Božidar Jovanović

Abstracts

THE 3-BODY PROBLEM IN R^4: STABLE RELATIVE EQUILIBRIA, CRITICAL POINTS AT INFINITY

Alain Albouy

Abstract. The Newtonian 3-body problem is essentially considered in a space of dimension 3. The simplest motions are the relative equilibria. They are equilibria of the reduced dynamics with a given angular momentum. These equilibria are expected to be all Lyapunov unstable, even if for example the Lagrange relative equilibrium with a dominant mass is linearly stable. Indeed, its shape remains nearly equilateral for very long times.
The same equations define motions in dimension 4, and this is the highest possible dimension for 3 bodies in the Galilean frame of their center of mass. We prove that for any choice of angular momentum corresponding to truly 4-dimensional motions, there is a relative equilibrium which is Lyapunov stable. This is the minimum of the energy (a related result is published in [**2**]).
Richard Montgomery informed us of a gap in our published article [**1**]. We will present an interesting lemma which finishes the proof: a critical point at infinity of the energy restricted to a level set of the angular momentum cannot correspond to an infimum (with Holger Dullin).

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VISUALIZATION OF GEODESIC SPHERES OF COMPLEX HYPERBOLIC PLANE WITH RESPECT TO VARIOUS LEFT-INVARIANT METRICS

Marijana Babić

Abstract. Complex hyperbolic space is a symmetric space of negative sectional curvature; therefore, it can be viewed as a connected solvable real Lie group with a left-invariant metric [**1**]. All possible left-invariant Riemannian metrics on this Lie group have been classified recently [**2**]. We consider geodesics in a special 4-dimensional case of the Complex hyperbolic plane with different left-invariant metrics. Using the Euler–Arnold equation one can simplify the system of 2nd order differential equations of geodesics on a Lie group to a system of 1st order differential equations on its Lie algebra. By solving these equations numerically, we can visualize geodesic spheres.

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Mathematical Institute SANU, Belgrade, Serbia.

DYNAMICS NEAR THE HOMOCLINIC SET OF A SLOW-FAST HAMILTONIAN SYSTEM

Sergey Bolotin

Abstract. In slow-fast systems, fast variables change with the rate of order one, and slow variables with the rate of order $\epsilon \ll 1$. The system obtained for $\epsilon = 0$ is called frozen. If the frozen Hamiltonian system has one DOF, then in the region where the level curves of the frozen Hamiltonian are closed, there is an adiabatic invariant. A. Neishtadt showed that when the fast variable crosses a separatrix of the frozen system, the adiabatic invariant has quasirandom jumps of order ϵ. We partially extend Neishtadt's result to slow-fast Hamiltonian systems with many DOF such that the frozen system has a hyperbolic critical point possessing transverse homoclinics. We show that for small ϵ there are local analogs of adiabatic invariants for trajectories in a neighborhood of the homoclinic set. The slow variables evolve in a quasirandom way, shadowing trajectories of systems whose Hamiltonians are these adiabatic invariants. This extends the work of V. Gelfreich and D. Turaev who considered similar phenomena away from critical points of the frozen Hamiltonian.

Steklov Mathematical Institute, Moscow, Russia.

ISOPERIODIC PENCILS OF CONFOCAL CONICS AND PAINLEVE VI EQUATIONS

Vladimir Dragović

Abstract. We study Poncelet polygons inscribed in a circle and circumscribed about conics from a confocal family, which is a question that naturally arose in the analysis of the numerical range and Blaschke products. We examine the behaviour of the rotation numbers and discover confocal families of conics with the property that each conic from the family is inscribed in k-Poncelet polygons inscribed in the circle, with the same k. Characterization of all such families is given and it is proved that they always correspond to $k = 4$. A relationship with solutions to Painleve VI equations is established.

The talk is based on a joint work with Milena Radnović.

University of Texas at Dallas, USA.
Mathematical Institute SANU, Serbia.

A TALE OF TWO POLYTOPES RELATED TO GEODESIC FLOWS ON SPHERES

Holger Dullin

Abstract. Separation of variables for the geodesic flows on round spheres leads to a large family of integrable systems whose integrals are defined through the separation constants. Reduction by the periodic geodesic flow leads to integrable systems on Grassmanians. Specifically for the geodesic flown on the round S^3 the reduced system defines a family of integrable systems on $S^2 \times S^2$. We show that the image of these systems under a continuous momentum map defined through the action variables has a triangle as its image. The image is rigid and does not change when the integrable system is changed within the family. Each member of the family can be identified with a point inside a Stasheff polytope. Corners of the polytope correspond to toric systems (possibly with degenerations), edges correspond semi-toric systems (in various meanings of the word), and the face corresponds to "generic" integrable systems. A fundamental difference of this momentum map to that of a toric or semi-toric system is that the number of tori in the preimage of a non-critical point may be 1, 2, or 4. The momentum map is continuous but not smooth along the images of hyperbolic singularities. The corresponding quantum problem and generalisations to higher dimensional spheres will be discussed.

Joint work with Diana Nguyen and Sean Dawson

University of Sydney, Sydney, Australia.

SPECTRAL ANALYSIS OF A SYSTEM OF PENDULA HANGING FROM A VISCOELASTIC STRING AND OF THEIR SYNCHRONIZATION

Francesco Fasso

Abstract. The talk reports the study of the spectrum of the linearization of a hybrid (discrete-continuous) mechanical system formed by a number of identical pendula hanging from a (Kelvin–Voigt) viscoelastic string, with an analysis of the ensuing synchronization patterns in the pendula's motion.

Joint work with S. Galasso and A. Ponno.

University of Padova, Padova, Italy.

BI-HAMILTONIAN STRUCTURES OF SPIN SUTHERLAND MODELS FROM POISSON REDUCTION

László Fehér

Abstract. We review our results on bi-Hamiltonian structures of spin Sutherland models built on collective spin variables. Our basic observation was that the holomorphic cotangent bundle $T^*\mathrm{GL}(n,\mathbb{C})$ and its real form $T^*\mathrm{U}(n)$, as well as $T^*\mathrm{GL}(n,\mathbb{C})_{\mathbb{R}}$, carry a natural quadratic Poisson bracket, which is compatible with the canonical one. The quadratic bracket arises by change of variables and analytic continuation from an associated Heisenberg double. Then the reductions of $T^*\mathrm{GL}(n,\mathbb{C})$ and $T^*\mathrm{U}(n)$ by the conjugation actions of the corresponding groups lead to the holomorphic and real trigonometric spin Sutherland models, respectively, equipped with a bi-Hamiltonian structure. The reduction of $T^*\mathrm{GL}(n,\mathbb{C})_{\mathbb{R}}$ by the group $\mathrm{U}(n)\times\mathrm{U}(n)$ gives a generalized Sutherland model coupled to two $\mathfrak{u}(n)^*$-valued spins. We also show that a bi-Hamiltonian structure on the associative algebra $\mathfrak{gl}(n,\mathbb{R})$ that appeared in the context of Toda models can be interpreted as the quotient of compatible Poisson brackets on $T^*\mathrm{GL}(n,\mathbb{R})$. All these reductions were studied previously using the canonical Poisson structures of the cotangent bundles, without realizing the bi-Hamiltonian aspect.

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Department of Theoretical Physics, Wigner RCP, Budapest, Hungary.

SPHERICAL BALL BEARINGS

Borislav Gajić

Abstract. We construct nonholonomic systems of n homogeneous balls with the same radius r that are rolling without slipping around a fixed sphere with center O and radius R. In addition, it is assumed that a dynamically nonsymmetric sphere of radius $R + 2r$ and the center that coincides with the center O of the fixed sphere rolls without slipping over the moving balls. We prove that these systems possess an invariant measure. Also, we consider the limit, when the radius R tends to infinity. We obtain a corresponding planar problem consisting of n homogeneous balls with the same radius r that are rolling without slipping over a fixed plane, and a moving plane that moves without slipping over the homogeneous balls. We prove that this system possesses an invariant measure and that it is integrable in quadratures according to the Euler–Jacobi theorem.

This is joint work with Vladimir Dragović and Božidar Jovanović

Mathematical Institute SANU, Belgrade, Serbia.

ON RATIONALLY INTEGRABLE PLANAR DUAL AND PROJECTIVE BILLIARDS

Alexey Glutsyuk

Abstract. A *caustic* of a strictly convex planar bounded billiard is a smooth curve whose tangent lines are reflected from the billiard boundary to its tangent lines. The famous Birkhoff Conjecture states that if the billiard boundary has an inner neighborhood foliated by closed caustics, then the billiard is an ellipse. It was studied by many mathematicians, including H. Poritsky, M. Bialy, S. Bolotin, A. Mironov, V. Kaloshin, A. Sorrentino and others.

We study its following generalized *dual* version stated by S. Tabachnikov. Consider a closed smooth strictly convex curve $\gamma \subset \mathbb{RP}^2$ equipped with a *dual billiard structure:* a family of non-trivial projective involutions acting on its projective tangent lines and fixing the tangency points. *Suppose that its outer neighborhood admits a foliation by closed curves* (*including* γ) *such that the involution of each tangent line permutes its intersection points with every leaf. Then* γ *and the leaves are conics forming a pencil.*

We prove positive answer in the case, when the curve γ is C^4-smooth and the foliation admits a rational first integral. To this end, we show that each C^4-smooth germ γ of planar curve carrying a rationally integrable dual billiard structure is a conic and classify all the rationally integrable dual billiards on conics. They include the dual billiards induced by pencils of conics, two infinite series of exotic dual billiards and five more exotic ones.

ENS de Lyon, Lyon, France.

THE PAINLEVÉ AND PETROVIĆ THEOREMS AND PUISEUX SERIES SOLUTIONS OF ALGEBRAIC ODEs OF THE FIRST ORDER

Renat Gontsov

Abstract. According to the celebrated Painlevé theorem every solution of an ordinary differential equation (ODE) of the first order can only have a singularity of an algebraic type at any point $x = x_0$, with the exception of points of some fixed finite set Σ at most, which is determined by the equation. In other words, non-algebraic singular points of the solutions of a first order algebraic ODE cannot fill domains in $\mathbb{C}$. This theorem completely describes a possible behavior of solutions near each point x_0 of the set $\mathbb{C} \setminus \Sigma$: solutions are presented by convergent Puiseux series in $(x - x_0)$ with a finite principal part. On other hand, Petrović's theorems allow to determine the leading term of such series.
These results partially help to study the question of convergence of formal Puiseux series solutions of an algebraic ODE. We also study this question of convergence paying attention to the cases where the Painlevé theorem cannot be applied explicitly.

The talk will be based on the joint work with Vladimir Dragović and Irina Goryuchkina.

Institute for Information Transmission Problems RAS, Moscow, Russia.

THE SMALL DIVISORS PHENOMENON IN THE PROBLEM OF CONVERGENCE OF FORMAL SOLUTIONS TO q-DIFFERENCE EQUATIONS

Irina Goryuchkina

Abstract. A sufficient condition for the convergence of a generalized formal power series solution to an algebraic q-difference equation is provided. The result uses a geometric property related to the semi-group of (complex) power exponents of such a series. This semi-group is finitely generated and there are two different situations depending on whether its generators are placed in some open half-plane in $\mathbb{C}$ or not. In the second situation the small divisors phenomenon arises and the study of convergence of generalized formal power series solutions to an algebraic q-difference equation resembles the study of the problem of linearization of diffeomorphisms of $(\mathbb{C}^n,\ 0)$.

Keldysh Institute of Applied Mathematics RAS, Moscow, Russia.

BIER SPHERES - STARSHAPEDNESS AND POLYTOPALITY

Filip Jevtić

Abstract. Bier sphere $Bier(K) = K *_\Delta K^\circ$ is a simplicial complex defined as join of a simplicial complex K and its Alexander dual. It has no a priori geometric realization and, while combinatorialy a sphare, in general it is not polytopal. We examine the questions of polytopality and non-polytopal geometric realizations. In particual, we show that all Bier spheres are starshaped and that Bier sphere associated to treshhold complexes are polytopal.

Joint work with M. Timotijević and R. Živaljević.

Mathematical Institute SANU, Belgrade, Serbia.

INTEGRABLE EULER EQUATONS RELATED TO CHAINS OF SUBALGEBRAS

Božidar Jovanović [a], Tijana Šukilović [b], and Srdjan Vukmirović [b]

Abstract. In 1983 Bogoyavlenski conjectured that if the Euler equations on a Lie algebra $\mathfrak{g}_0$ are integrable, then their certain extensions to semisimple lie algebras $\mathfrak{g}$ related to the filtrations of Lie algebras

$$\mathfrak{g}_0 \subset \mathfrak{g}_1 \subset \mathfrak{g}_2 \cdots \subset \mathfrak{g}_{n-1} \subset \mathfrak{g}_n = \mathfrak{g}$$

are integrable as well. In particular, by taking $\mathfrak{g}_0 = \{0\}$ and natural filtrations of $so(n)$ and $u(n)$, we have Gel'fand–Cetlin integrable systems. We proved the conjecture for filtrations of compact Lie algebras $\mathfrak{g}$: the system are integrable in a noncommutative sense by means of polynomial integrals. Various constructions of complete commutative polynomial integrals for the system are also given.
In addition, related to commutative polynomial integrability, we classify almost multiplicity free subgroups of compact simple Lie groups.

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PROBLEMS AND EXAMPLES OF EXPERIMENTAL APPLICATION OF NONHOLONOMIC MODELS IN MECHANICS AND ROBOTICS

Yury L. Karavaev

Abstract. My report discusses mathematical models describing the kinematics and dynamics of various mobile robots. These models are based on the assumption that there is no slipping, which is illustrated by examples of several experiments of rolling bodies. The simulation results are compared with the results of experimental studies.

Kalashnikov Izhevsk State Technical University, Izhevsk, Russia.

HAMILTONIAN AND RIEMANNIAN GEOMETRY BEHIND COMPRESSIBLE FLUIDS

Boris Khesin

Abstract. We describe a geometric framework to study Newton's equations on infinite-dimensional configuration spaces of diffeomorphisms and smooth probability densities. It turns out that several important PDEs of hydrodynamical origin can be described in this framework in a natural way. In particular, the so-called Madelung transform between the Schrödinger-type equations on wave functions and Newton's equations on densities turns out to be a Kähler map between the corresponding phase spaces, equipped with the Fubini–Study and Fisher–Rao information metrics.

This is a joint work with G. Misiolek and K. Modin.

University of Toronto, Toronto, Canada.

FREE AND CONTROLLED ROLLING MOTION OF A BALL ON A VIBRATING PLANE

Alexander Kilin

Abstract. In this work we consider two problems on the motion of spherical bodies on a vibrating plane. The first problem is related to the motion of a Chaplygin sphere rolling without slipping on a plane performing horizontal periodic oscillations. For this problem we show that in the system under consideration the projections of the angular momentum onto the axes of the fixed coordinate system remain unchanged. The investigation of the reduced system on a fixed level set of first integrals reduces to analyzing a three-dimensional period advance map on SO(3). The analysis of this map suggests that in the general case the problem considered is nonintegrable. We find partial solutions to the system which are a generalization of permanent rotations and correspond to nonuniform rotations about a body- and space-fixed axis. We also find a particular integrable case which, after time is rescaled, reduces to the classical Chaplygin sphere rolling problem on the zero level set of the area integral. The second part of the work addresses the problem of a spherical robot having an axisymmetric pendulum drive and rolling without slipping on a vertically vibrating plane. It is shown that this system admits partial solutions (steady rotations) for which the pendulum rotates about its vertical symmetry axis. Special attention is given to problems of stability and stabilization of these solutions. An analysis of the constraint reaction is performed, and parameter regions are identified in which a stabilization of the spherical robot is possible without it losing contact with the plane. It is shown that the partial solutions can be stabilized by varying the angular velocity of rotation of the pendulum about its symmetry axis, and that the rotation of the pendulum is a necessary condition for stabilization without the robot losing contact with the plane.

Udmurt State University, Izhevsk, Russia.

COMMENTS ON THE INTERPLAY BETWEEN VORTICES AND HARMONIC FIELDS

Jair Koiller

Abstract. Let M a compact manifold without boundary with metric g. When on writes Arnold's geodesic equations [**1**] in $SDiff(M)$ in terms of 1-forms ν instead of vectorfields, it is well known that a Poisson structure results ie in the dual of the Lie algebra. The Hodge decomposition $\nu = df + \delta\psi + \eta$, with η harmonic will not contain the term df, because one wants $\nu^\sharp$ to be divergence free. Moreover, $\delta\phi$ can be uniquely recovered from the vorticity 2-form $\omega = d\delta\psi$, and one could write the equations of motion in terms of (ω, η) [**2**]. In the literature it is usually assumed that the ambient is simply connected. However, when $H_1(M) \neq \emptyset$, there is an interplay between ω and η. When $M = \Sigma$ is a Riemann surface, this coupling has been made explicit very recently by Björn Gustaffson [**3**] and checked by Clodoaldo Ragazzo [**4**] with additional results. In this talk I will comment on these developments and the impact that I imagine these works should provoke in the vorticitists community.

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Federal University of Juiz de Fora, Brazil.

DISCRETE SYMMETRIES OF DYNAMICAL EQUATIONS WITH POLYNOMIAL INTEGRALS OF HIGHER DEGREES

Valery Kozlov

Abstract. An analysis is made of systems with toral configuration space and the kinetic energy in the form of a "flat" Riemannian metric on a torus. The potential energy V is a smooth function on the configuration torus. The dynamics of such systems is described by "natural" Hamiltonian systems of differential equations. If one replaces V with εV, where ε is a small parameter, then the investigation of such Hamiltonian systems at small values of ε is the "main problem of dynamics" in the sense of Poincaré. A discussion is given of the hypothesis of single-valued polynomial (in momenta) integrals of the equations of motion: if there is an integral of degree m polynomial in momenta, then there is necessarily a first integral linear or quadratic in momenta. This hypothesis is completely proved for $m = 3$ and $m = 4$. Also discussed are cases of "higher" degrees where $m = 5$ and $m = 6$. Following the theory of perturbation of Hamiltonian systems, resonant straight lines on the plane of momenta are introduced. If the system admits a polynomial integral, then the number of these lines is finite. Symmetries of the set of resonant lines are found, which gives, in particular, necessary conditions for integrability. Some new criteria for the existence of single-valued polynomial integrals are obtained.

Steklov Mathematical Institute, Moscow, Russia.

NON-INTEGRABILITY OF THE PLANAR ELLIPTIC RESTRICTED THREE BODY PROBLEM

Andrzej Maciejewski

Abstract. The planar restricted elliptic three body problem is considered. It is proved that for non-zero values of the eccentricity and the mass ratio the system is not integrable. The proof is based on the analysis of the variational equations for the triangular libration point. The same statement is also proved for the parameters generalization of the system, namely for the planar restricted elliptic photo-gravitational three body problem.

University of Zielona Gora, Poland.

PERMANENT ROTATIONS IN NONHOLONOMIC MECHANICS. OMNIROTATIONAL ELLIPSOID

Ivan Mamaev

Abstract. This paper is concerned with the study of permanent rotations of a rigid body rolling without slipping on a horizontal plane (i.e., the velocity of the point of contact of the ellipsoid with the plane is zero). A more detailed analysis is made of permanent rotations of the omnirotational ellipsoid whose characteristic feature is that permanent rotations are possible about any point of its surface.

Izhevsk State Technical University, Izhevsk, Russia.

RATIONAL FUNCTIONS ON SPECTRAL CURVES AND COMMUTING DIFFERENCE OPERATORS

Andrey Mironov

Abstract. We shall discuss a connection between commuting ordinary differential operators and commuting difference operators. In particular, we construct a discretization of the Lamé operator that preserves the spectral curve.

Sobolev Institute of Mathematics, Novosibirsk, Russia.

SMALL OSCILLATIONS AND STABLE RELATIVE EQUILIBRIA OF THE N-VORTEX PROBLEM ON THE SPHERE

Luis García Naranjo

Abstract. We consider the N-vortex problem on the sphere assuming that all vortices have equal strength. In the first part of the talk I will present a theoretical framework to analyse solutions of the equations of motion with prescribed symmetries which relies on the discrete reduction of the system by twisted subgroups of the full symmetry group that rotates and permutes the vortices. This approach allows us to prove the existence of several 1-parameter families of periodic orbits and in particular it shows the existence of small nonlinear oscillations emanating from some equilibrium configurations including the Platonic solids for $N = 4, 6, 8, 12, 20$. In the second part of the talk I will present ongoing work on the existence and stability of relative equilibria based on an approach that combines techniques of symmetric Hamiltonian systems with computer assisted proofs.

This is joint work with K. Constantineau, C. García-Azpeitia and J. P. Lessard.

University of Padua, Padua, Italy.

OPTIMAL DYNAMIC ABSORBERS WITH PIEZOELECTRIC PROPERTIES FOR FRACTIONALLY DAMPED BEAMS

Stepa Paunović

Abstract. In this contribution the dynamic properties of a system composed of a beam with multiple dynamic absorbers attached to it are investigated. The beam is modelled as a continuous system with infinite number of degrees of freedom and the absorbers are modelled as point masses connected viscoelastically to the beam. Structural damping is modelled with the use of non-integer order derivatives. To each absorber a piezoelectric bimorph beam is attached, which adds piezoelectric properties to the absorbers and enables energy harvesting from beam vibrations. In this study the optimal mass, number, disposition and electrical properties of these compound absorbers are sought, for the case of a harmonic excitation force acting on the beam. The governing equations are derived through the use of Hamilton's variational principle, and solved by applying the Galerkin spatial discretisation. Frequency response functions are determined analytically, while in the time domain the equations are solved in quadratures, by the use of Newmark's iterative procedure. The presented methodology has a potential for application to a wide variety of engineering problems, including energy harvesting from bridge vibrations, which example is provided in this study.

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CHAOS IN COUPLED HETEROCLINIC CYCLES AND ITS PIECEWISE-CONSTANT REPRESENTATION

Arkady Pikovsky [a] and Alexander Nepomnyashchy [b]

Abstract. We consider two stable heteroclinic cycles rotating in opposite directions, coupled via diffusive terms. A complete synchronization in this system is impossible, and numerical exploration shows that chaos is abundant at low coupling levels. With the increase of coupling strength, several symmetry-changing transitions are observed, and finally, a stable periodic orbit appears via an inverse period-doubling cascade. To reveal the behavior at extremely small couplings, a piecewise-constant model for the dynamics is suggested. Within this model we construct a Poincaré map for a chaotic state numerically, it appears to be an expanding non-invertable circle map thus confirming the abundance of chaos in the small coupling limit. We also show that within the piecewise-constant description, there is a set of periodic solutions with different phase shifts between subsystems, due to dead zones in the coupling.

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CONTROLLED MOTION OF AN INHOMOGENEOUS BALL ON A HORIZONTALLY VIBRATING PLANE

Elena Pivovarova

Abstract. In this work we investigate the controlled motion of a balanced inhomogeneous ball (the Chaplygin sphere) on a plane performing horizontal periodic oscillations. The motion of the ball is controlled by the controlled rotation of the internal noncoplanar gyrostats. The work addresses two control problems concerning the construction of controls which generate motion along a trajectory given either on a moving plane or in a fixed frame of reference. It is shown that, using a control torque constant in the fixed frame of reference, the general problem can be reduced to the problem of control on the zero level set of the angular momentum integral. It is proved that, on the zero level set of the angular momentum integral, the system under consideration is completely controllable according to the Rashevsky-Chow theorem. Control algorithms for the motion of the sphere along an arbitrary prescribed trajectory are constructed. Examples are given of controls for the ball rolling in a straight line in an arbitrary direction and in a circle, and for the ball turning so that the position of the center of mass, both relative to the moving plane and relative to the fixed frame of reference, remains unchanged.

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INTEGRABILITY OF HAMILTONIAN SYSTEMS WITH GYROSCOPIC TERM

Maria Przybylska

Abstract. We study the integrability of 2D Hamiltonian systems $H_\mu = \frac{1}{2}(p_1^2 + p_2^2) + \omega(p_1q_2 - p_2q_1) - \frac{\mu}{r} + V(q_1, q_2)$, where $r^2 = q_1^2 + q_2^2$, and potential $V(q_1, q_2)$ is a homogeneous rational function of degree k. The main result states that under very general assumptions: $\mu\omega \neq 0$, $|k| > 2$ and $V(1, \mathrm{i}) \neq 0$ or $V(-1, \mathrm{i}) \neq 0$, the system is not integrable. It was obtained by combining the Levi-Civita regularization, differential Galois methods and the so-called coupling constant metamorphosis transformation. The proof that the regularized version of the problem is not integrable contains the most important theoretical results, which can be applied to study integrability of other problems.

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CONFOCAL CONICS AND BILLIARDS ON 1-SHEETED HYPERBOLOID IN THE MINKOWSKI SPACE

Milena Radnović

Abstract. We will present geometrical properties of confocal families in this setting and analyse dynamics of billiards within compact domains bounded by such conics.

This is joint work with Sean Gasiorek.

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ON SOME MODELS OF NON-LOCAL MODIFIED GRAVITY

Zoran Rakić

Abstract. Despite to all significant gravitational phenomena discovered and predicted by general theory of relativity, it is not a complete theory. One of actual approaches towards more complete theory of gravity is its nonlocal modification.
We consider nonlocal modification of the Einstein theory of gravity in framework of the pseudo-Riemannian geometry. The nonlocal term has the form $\mathcal{H}(R)\mathcal{F}(\Box)\mathcal{G}(R)$, where $\mathcal{H}$ and $\mathcal{G}$ are differentiable functions of the scalar curvature R, and $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of the d'Alambert operator $\Box$. Our motivation to modify gravity, in an analytic nonlocal way, comes mainly from string theory, in particular from string field theory and p-adic string theory.
Using the calculus of variations we derived the corresponding equations of motion. The variation of action is induced by variation of the metric tensor $g_{\mu\nu}$. Firstly, we consider several models of the above-mentioned type, as well as the case when the scalar curvature is constant.
Recently, we deal with the cases where: (1) $\mathcal{H}(R) = \mathcal{G}(R) = R - 4\Lambda$, and (2) $\mathcal{H}(R) = \mathcal{G}(R) = \sqrt{R - 2\Lambda}$. Specially, we are paid our attention to the case (2) with scaling factor of the form $a(t) = At^{\frac{2}{3}} e^{\frac{\Lambda}{14} t^2}$, and we find some new cosmological solutions, and we test validity of obtained solutions with experimental data.

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ERGODIC PROPERTIES OF THE ANGLE-EXPANDING BILLIARDS

Georgii Sechkin

Abstract. We consider a variation of classic mathematical billiard problem where the angle of rebound is determined from the angle of incidence via linearly expanding map. Informally, if φ_{in} is the angle between the normal and the incoming segment of trajectory then $\varphi_{\text{out}} = \lambda\varphi_{\text{in}}$ where λ is some constant.
For $\lambda = 1$ the above mapping corresponds to the classical case, while for $\lambda = 0$ one recover the so-called slap-map.
We will describe some local ergodic properties of the above mapping for $\lambda > 1$. We present complete analysis for the case of circular domain and provide some partial results for the general case for the curves with bounded curvature.

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N-SYMMETRIC INTERACTION OF *N* HETONS

M. A. Sokolovskiy [a,b], K. V. Koshel [c,d], D. G. Dritschel [e], and J. N. Reinaud [e]

Abstract. This work is devoted to the search for N-symmetric solutions of the equations of dynamics of two-layer geostrophic eddies with zero total intensity and the analysis of these solutions. Families of new stationary solutions are obtained and their properties, including asymptotic ones, are investigated. From the point of view of geophysical applications, the results can be useful in explaining the propagation of thermal anomalies in the oceans.

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GEOMETRY OF COTANGENT BUNDLE OF HEISENBERG GROUP

Tijana Šukilović and Srđan Vukmirović

Abstract. In this talk the classification of left-invariant Riemannian metrics, up to the action of the automorphism group, on cotangent bundle of $(2n+1)$-dimensional Heisenberg group is presented. Also, it is proved that the complex structure on that group is unique and the corresponding pseudo-Kähler metrics are described and shown to be Ricci flat. It is well known that this algebra admits an ad-invariant metric of neutral signature. Here, the uniqueness of such metric is proved. Finally, the classification of sub-Riemannian structures, under the Lie group automorphisms, is obtained.

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INTEGRABLE TRANSFORMATIONS ON CENTROAFFINE POLYGONS

Sergei Tabachnikov

Abstract. One can interpret the famous Korteweg–de Vries equation as a completely integrable evolution on centroaffine curves. Accordingly, symmetries (the Backlund transformation) of the KdV equation also can be realized as transformations of centroaffine curves. I shall discuss a discrete version of these transformations, where the curves are replaced by polygons. The focus will be on the geometrical aspects of the problem.
"The small divisors phenomenon in the problem of convergence of formal solutions to q-difference equations"
A sufficient condition for the convergence of a generalized formal power series solution to an algebraic q-difference equation is provided. The result uses a geometric property related to the semi-group of (complex) power exponents of such a series. This semi-group is finitely generated and there are two different situations depending on whether its generators are placed in some open half-plane in $\mathbb{C}$ or not. In the second situation the small divisors phenomenon arises and the study of convergence of generalized formal power series solutions to an algebraic q-difference equation resembles the study of the problem of linearization of diffeomorphisms of $(\mathbb{C}^n, 0)$.

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UNIVERSAL SPACE OF PARAMETERS $\mathcal{F}_n$ AND THE MODULI SPACE $\bar{\mathcal{M}}(0,n)$

Svjetlana Terzić

Abstract. The canonical action of the compact torus T^n on the complex Grassmann manifold $G_{n,2}$ of two-dimensional complex subspaces in $\mathbb{C}^n$ is an important and widely known example of a Hamiltonian action on a symplectic manifold. The study of this action naturally leads to problems of dynamical systems theory and the study of the orbit space $G_{n,2}/T^n$ is closely related to the problems on integrals of the correspinding dynamical systems. In the focus of our talk is going to be a space of parameters for T^n-action on $G_{n,2}$ and the structure of the orbit space of this action.
The moduli space $\bar{\mathcal{M}}(0,n)$ of stable n-pointed genus zero curves is the Deligne–Mumford–Grothendieck–Knudsen compactification of the moduli space $\mathcal{M}(0,n)$ of n-pointed genus zero curves. The space $\bar{\mathcal{M}}(0,n)$ is proved by Kapranov to coincide with the Chow quotient $G_{n,2}//(\mathbb{C}^*)^n$.
In this talk we present an another approach to the compactification of $\mathcal{M}_{0,n}$ from the point of view of T^n-equivariant topology of $G_{n,2}$ for the canonical compact torus T^n-action. In describing the orbit space $G_{n,2}/T^n$, the main stratum $W_n \subset G_{n,2}$ given by those points from $G_{n,2}$ whose all Plücker coordinates are non-zero, plays a crucial role, as it is an open dense set in $G_{n,2}$ and it belongs to any Plücker chart for $G_{n,2}$. In addition, we earlier proved that $W_n \cong \overset{\circ}{\Delta}_{n,2} \times F_n$, where $\Delta_{n,2}$ is the hypersimplex and $F_n = W_n/(\mathbb{C}^*)^n \subset \mathbb{C}P^N, N = \binom{n-2}{2}$ is an open algebraic manifold.
In order to construct a model for the orbit space $G_{n,2}/T^n$ it turns out to be important to look for a compactification $\mathcal{F}_n$ for F_n such that any automorphism of F_n induced by the transition maps between the Plücker charts extends to the automorphism of $\mathcal{F}_n$. Such compactification $\mathcal{F}_n$ we call the universal space of parameters. We obtain the space $\mathcal{F}_n$ by resolving singularities arising in the context of required extensions and by making use of the construction from algebraic geometry known as the wonderful compactification. Finally, we prove that the space $\mathcal{F}_n$ coincides with $\bar{\mathcal{M}}(0,n)$.

The talk is based on the results jointly obtained with Victor M. Buchstaber.

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LINEARIZATION BY MEANS OF A FUNCTIONAL PARAMETER

Dmitry Treschev

Abstract. Consider a Hamiltonian system near an equilibrium point or a symplectic map near a fixed point in a phase space of dimention $2n$. Suppose the system depends on a functional parameter, a function of n variables. We study the possibility to choose the functional parameter to make the system conjugated to a linear system on an open set.

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BIANCHI-VII GEODESIC PROBLEM, MATHIEU EQUATION AND ARITHMETIC

Alexander Veselov

Abstract. I will discuss classical and quantum integrability of the geodesic flows on 3D manifolds with the Riemannian metric of Bianchi-VII type. In the quantum case the eigenfunctions can be expressed in terms of the classical Mathieu functions with parameters determined by the arithmetic of binary quadratic forms.

The talk is based on a joint work with Yiru Ye.

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OBSERVATIONS OF WAVE BREAKING STATISTICS

Teodor Vrećica [a,b], Nick Pizzo [a], and Luc Lenain [a]

Abstract. Wave breaking modulates fluxes of mass, momentum, heat and energy between the atmosphere and the ocean. Despite its importance, progress in better understanding this phenomena is impeded by the fact that breaking is a two-phase turbulent unsteady process that is intermittent in both space and time, making it difficult to observe. Traditionally, breaking statistics have been defined as the fraction of the ocean surface covered by whitecaps, estimated from still photos or video imagery. A more general framework proposed by [**1**] is to characterize wave breaking kinematics by defining the wave breaking distribution, $\Lambda(\mathbf{c_b})$: the average length of breaking crests with speed $\mathbf{c_b}$, per unit surface area. The utility of Phillips's approach is that the spectral moments of the distribution yields important physical variables, such as the wave breaking induced mean currents, air entrainment and the energy dissipation by breaking. In this work, we review the state of the art of observations and applications of wave breaking statistics over a broad range of environmental conditions, including near an ocean submesoscale front, where breaking effects may be particularly pronounced. Technical challenges (e.g. georeferencing, wave breaking kinematics characterization through optical flow and other methods etc.) are discussed; we present a novel open-source code capable of computing wave breaking distribution, $\Lambda(\mathbf{c_b})$, for a broad range of environmental conditions.

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PSEUDO-RIEMANNIAN METRICS ON COTANGENT BUNDLE OF HEISENBERG GROUP AND THEIR RELATION TO THE HYPERBOLIC CONICS

Srđan Vukmirović, Tijana Šukilović, and Neda Bokan

Abstract. The cotangent bundle of Heisenberg group is 6-dimensional, 2-step nilpotent group with 3-dimensional center. In this work we classify, up to automorphisms, left-invariant pseudo Riemannian metrics on the cotangent bundle of Heisenberg group. Particularly interesting is case of metrics with Lorentzian central restriction. The classification of such metrics is directly related to the classification of conics in the hyperbolic plane.

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PROJECTIVE AND AFFINE EQUIVALENCE OF SUB-RIEMANNIAN METRICS: INTEGRABILITY, GENERIC RIGIDITY, THE WEYL TYPE THEOREMS, AND SEPARATION OF VARIABLES CONJECTURE

Igor Zelenko

Abstract. Sub-Riemannian metrics are defined by a distribution (a subbundle of the tangent bundle) together with an Euclidean structure on each fiber. The Riemannian metrics correspond to the case when the distribution is the who;e tangent bundle. Two sub-Riemannian metrics are called projectively equivalent if they have the same geodesics up to a reparameterization and affinely equivalent if they have the same geodesics up to affine reparameterization. In the Riemannian case, both equivalence problems are classical: local classifications of projectively and affinely equivalent Riemannian metrics were established by Levi-Civita in 1898 and Eisenhart in 1923, respectively. In particular, a Riemannian metric admitting a nontrivial (i.e. non-constant proportional) affinely equivalent metric must be a product of two Riemannian metrics i.e. separation of variables (the de Rham decomposition) occur, while for the analogous property in the projectively equivalent case a more involved ("twisted") product structure is necessary. The latter is also related to the existence of sufficiently many commuting nontrivial integrals quadratic with respect to velocities for the corresponding geodesic flow. We will describe the recent progress toward the generalization of these classical results to sub-Riemannian metrics. In particular, we will discuss the genericity of metrics that do not admit non-constantly proportional affinely/projectively equivalent metrics and the separation of variables on the level of linearization of geodesic flows (i.e. on the level of the Jacobi equations) for metrics that admit non-constantly proportional affinely equivalent metrics. We also describe the sub-Riemannian analog of the Weyl theorem that all metrics that are simultaneously projectively equivalent and conformal are constantly proportional.

The talk is based on the collaboration with Frederic Jean (ENSTA, Paris), Sofya Maslovskaya (INRIA, Sophya Antipolis), and Zaifeng Lin (Texas A&M University).

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GENERALIZED TONNETZ AND DISCRETE ABEL–JACOBI MAP

Rade T. Živaljević

Abstract. Motivated by classical Euler's *Tonnetz*, we introduce and study the combinatorics and topology of more general simplicial complexes $Tonn^{n,k}(L)$ of *Tonnetz type*. Out main result is that for a sufficiently generic choice of parameters the generalized tonnetz $Tonn^{n,k}(L)$ is a triangulation of a $(k-1)$-dimensional torus T^{k-1}. In the proof we construct and use the properties of a *discrete Abel–Jacobi map*, which takes values in the torus $T^{k-1} \cong \mathbb{R}^{k-1}/\Lambda$ where $\Lambda \cong \mathbb{A}^*_{k-1}$ is the permutohedral lattice.

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