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Computer algorithms and software packages

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During the preparation of the *Handbook of Mathematical Functions*, under the direction of Milton Abramowitz at the Bureau of Standards (now the "National Institute of Standards and Technology"), Walter Gautschi, then a young research mathematician, joined this project in 1956. This was the starting point of a period of intense work with special functions. During the 1960s, in addition to theoretical work in several domains of special functions (see Section 6, Vol. 1), Walter developed a number of computer algorithms evaluating special functions: the gamma function and incomplete beta function ratios [GA22], Bessel functions of the first kind [GA23], Legendre functions [GA24], derivatives of e^x/x, $\cos(x)/x$, and $\sin(x)/x$ [GA27], [GA38], regular Coulomb wave functions [GA28], [GA33], the complex error function [GA36], repeated integrals of the coerror function [GA60], and incomplete gamma functions [GA69].

In 1968 Gautschi began to write computer algorithms for Gaussian quadrature formulas, the first being the one in [GA32]. This opened the door for extensive work on orthogonal polynomials and their applications (see Sections 11, 12, 14, 15 in Vol. 2), but also for developing related software. The first major software package, `ORTHPOL`, appeared in 1994 as Algorithm 726 in [GA**141**]. It contains routines, written in FORTRAN, that produce the coefficients in the three-term recurrence relation for arbitrary orthogonal polynomials as well as nodes and weights of Gauss-type quadrature rules. A more specialized package, `GQRAT` [GA159], produced Gauss quadrature rules which are exact for a combination of polynomials and rational functions. They are useful for integrating functions that have poles outside the interval of integration.

The package `ORTHPOL`, as well as the subsequent package `OPQ` of MATLAB routines, both made available on the internet (`http://www.cs.purdue.edu/archives/2002/wxg/codes`), led to a significant boost in the computational use and application of orthogonal polynomials. The companion package `SOPQ`, also available on the internet, contains symbolic versions of some of the more important routines in `OPQ`. They can be used for high-precision work in orthogonal polynomials and Gaussian

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quadrature. A similar package in MATHEMATICA is `OrthogonalPolynomials` [1] (see also [2]).

A very comprehensive account of computational methods and software in MATLAB is provided in [GA**179**]. It illustrates the use of the `OPQ` routines in an elegant, interesting, and methodical way.

References

[1] Aleksandar S. Cvetković and Gradimir V. Milovanović. The Mathematica package "OrthogonalPolynomials". *Facta Univ. Ser. Math. Inform.*, 19:17–36, 2004.

[2] Gradimir V. Milovanović and Aleksandar S. Cvetković. Special classes of orthogonal polynomials and corresponding quadratures of Gaussian type. *Math. Balkanica*, 26(1–2):169–184, 2012.