CONTRIBUTION TO THE BRACHISTOCHRONIC MECHANICAL SYSTEM MOTION

Brachistochronic motion has been considered in the following cases: brachistochronic rigid body plane and general motion, brachistochronic motion of a nonholonomic rheonomic mechanical system and brachistochronic motion of a variable mass system. The problem of optimal control is solved by applying Pontryagin’s Maximum Principle and the singular optimal control theory. This procedure results in the two-point boundary value problem for the system of ordinary nonlinear differential equations of the first order, with a corresponding number of initial and end conditions. The paper also presents the manner of brachistrochronic motion realization without the action of active control forces. It is realized by subsequent imposition to the system a corresponding number of independent ideal holonomic mechanical constraints. The constraints must be in accordance with the previously determined brachistochronic motion of the system.

The problem of the motion of a rigid body between two given configurations in a plane during the shortest possible time has been solved. The conservation of the mechanical energy is satisfied. In the example the exact analytical solution of the equations of motion has been found. It is shown that brachistochronic motion in a plane is possible to realize with two ideal holonomic constraints without an influence of active forces. One of the possible ways is the rolling of the moving centroid over another fixed centroid, without sliding. These centroids are also shown graphically.

In the problem of brachistochronic rigid body general motion, the generalized coordinates are Cartesian’s coordinates of mass center and the Euler’s angles, that are specified at the initial and the final position. Finite difference method has been applied in order to obtain the solution of the two-point boundary value problem.

The brachistochrone problem of the rheonomic mechanical system whose motion is subject to nonholonomic constraints is solved with nonlinear differential equations of motion. Apart from control forces, the system is influenced by the action of other known potential and nonpotential forces as well. The method is illustrated with a single complex example that represents the first (to-date) known concrete demonstration of brachistochronic motion of the nonholonomic rheonomic mechanical system.

A mechanical system consisting of rigid bodies and material particles, of which some particles are with variable masses, is considered. Laws of variation of the masses of the points and relative velocity of particles separating from the points are well-known. The system is moving in an arbitrary field of known potential and nonpotential forces. A two-point boundary value problem, due to nonlinearity of equations in a general case, is needed to solve using some of the numerical procedures. Here the Shooting method is used, where the missing boundary conditions are chosen so as to be the physical variables (velocity and mass). The field where they are found can be approximately estimated, which is not the case with the conjugate vector coordinates being of purely mathematical nature. The method is illustrated by an example of determining the brachistochronic motion of the system with three degrees of freedom and method of its realization. The system consists of one rigid body to which two points of variable masses are attached, where the system is moving in a vertical plane. Brachistochronic motion is realized by the help of two ideal holonomic constraints.
The Stability of Linear Continuous Singular and Discrete Descriptor Time Delayed Systems Defined over Infinite and Finite Time Interval

CLASSES OF SYSTEMS TO BE CONSIDERED

It should be noticed that in some systems we must consider their character of dynamic and static state at the same time. Singular systems are those the dynamics of which are governed by a mixture of algebraic and differential equations. The complex nature of singular systems causes many difficulties in the analytical and numerical treatment of such systems, particularly when there is a need for their control.

The problem of investigation of time delay systems has been exploited over many years. Time delay is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, etc. The existence of pure time lag, regardless if it is present in the control or and the state, may cause undesirable system transient response, or even instability. These systems are described by differential with time lag argument.

We must emphasize that there are a lot of systems that have the phenomena of time delay and singular simultaneously, we call such systems as the singular differential systems with time delay. These systems have many special characters.

STABILITY CONCEPTS

A numerous significant contributions have been made in last sixty years in the area of Lyapunov stability for different classes of systems.

But in practice one is not only interested in system stability (e.g. in sense of Lyapunov), but also in bounds of system trajectories. A system could be stable but completely useless because it possesses undesirable transient performances.

Thus, it may be useful to consider the stability of such systems with respect to certain sub-sets of state-space, which are a priori defined in a given problem.

Besides that, it is of particular significance to concern the behavior of dynamical systems only over a finite time interval.

CONTRIBUTIONS OF INVITED LECTURE

1. The first part of lecture is devoted to the stability of particular classes of linear continuous singular time delayed systems (LCSTDS) and linear discrete descriptor time delayed systems (LDDTDS).

   A number of new results concerning stability properties of this class of systems in the sense of Lyapunov will be presented. The geometric theory of consistency leads to the natural class of positive definite quadratic forms on the subspace containing all solutions.

   This fact makes possible the construction of Lyapunov stability theory even for the (LCSTDS) and (LDDTDS) in that sense that asymptotic stability is equivalent to the existence of symmetric, positive definite solutions to a weak form of Lyapunov continuous (discrete) algebraic matrix equation, respectively, incorporating condition which refer to time delay term.

2. The second part of this lecture is devoted to presenting quite new results in the area of Non-Lyapunov (finite time stability, practical stability, attractive practical stability, etc.) for the particular classes of (LCSTDS) and (LDDTDS).

   And finally all these results will be presented, discussed and compared with actual possibilities that are offered by attractive LMI approach.
The paper describes an application of the Computer Aided Development (CADe) in a novel orbiting mechanism invention. Target design is a shaking mechanism aimed for the Allantoic Fluid stirring used in influenza vaccine production. Main innovative goal is to maintain a continual shaking process within a controlled dynamical range to prevent the stratification of ingredients.

Presumed path for an optimal shaking motion is a planar, figure eight closed loop. Carrier platform, loaded with Allantoic Fluid bottles, actually performs a bidirectional reciprocating motion over such a path. Shaking dynamics depend on cycling frequency and on reciprocating displacement. These two parameters are in the focus of preliminary analysis.

Computational work on an optimal path definition is interrelated with the innovative engineering work. The main dispute was how to generate orbital motion of the carrying platform. Instead of a conventional, theoretically established orbiting mechanism, a new one is created in a form of coupled crank mechanism. An academic package of the design program, Solid Works Motion 2010, was employed for parts and assembly definition, as well as for the subsequent mechanism analysis.

The outcome of the CADe work could be evaluated as a successful one, mainly for the attained smooth shaking motion over a closed loop and even more for the uniformly and redundantly distributed acceleration peaks within each cycle. On the basis of this CADe work a novel prototype of the orbit shaker is about to be launched.

Keywords: orbiting mechanism, computer aided development

In this paper the original results of investigation of non-linear dynamics of vibro-impact systems with curvilinear routes and non-linear bonds for the investigation of new information about the mechanical energy components transformation in vibro-impact systems with one, two or three degrees of freedom of motion. The results of the scientific research were presented as:

* Establishment of original methodology of investigation of energy analysis of vibro-impact systems dynamics based on oscillators with free motion on curvilinear paths and non-ideal bonds, with one, two and three degrees of freedom of motion, by combination of analytical expressions for phase trajectories and by application of MathCad as a tool for determination of kinetic system parameters before and after the impact, as well as for graphic visualization of the branches of phase trajectories and energy components curves and total energy of vibro-impact system dynamics as a function of elongation, as well as sliding friction force and the work power of friction force in the intervals between the impacts, collision and kinetic states of motion direction alternations;

* Establishment of original methodology of investigation of energy analysis of vibro-impact system dynamics based on oscillator forced to move by curvilinear paths and non-ideal bonds, with one, two and three degrees of freedom of motion, by combination of numerical method Runge-Kutta for the determination of the branches of phase trajectories of vibro-impact system dynamics exposed to the external one or two frequency force, with application of two software packages MATLAB and Wolfram mathematica for the determinaton of the phase trajectories branches of vibro-impact system dynamics exposed to the action of external one or two frequency force (with the the results of the same order of accuracy with both packages) with graphic presentation of the phase trajectories branches of the forced motion of vibro-impact system in some intervals and sub-intervals between the impacts or the motion direction alternation with approximative analytical expressions as for the phase trajectories branches, but also for for the branches of the curves of the components of mechanical energies, and also for the determination of kinetic parameters and impact time or collision of material particles within the system;
Establishment of original methodology and the procedure of the determination of the time and the position (angular coordinates) of the heavy material particles impacts in their own vibro-impact system dynamics with two and three degrees of freedom of motion, by combination of analytical expressions and MathCad application approach as a tool for obtaining system kinetic parameters before and after the impact, and for the graphic visualization of phase trajectory branches and curves of the component energies and total energy of the dynamics of vibro-impact system as a function of generalized coordinates, and sliding friction force and work power of individual friction force in the intervals between the impacts, or collisions and kinetic states of the motion alternation;

* by extension of the scientific knowledge about the regimes of own, i.e. forced vibro-impact dynamics and the variety of these regimes on the examples of the vibro-impact systems dynamics containing one or more heavy material particles moving along rough circle lines and impact occurrences among them;

* by adaptation of the method of vibro-impact system parameters adjustment to the reduction of the forced motion of heavy material particle along rough circle line with limited elongations application, under the action of one and two frequencies force with resonance fitting parameters, to the periodic motion;

* the estimation of the mass and mechanical characteristics of the material particle moving between two heavy material particles influence to the velocity and the number of impact of other surrounding heavy material particles into elongation limiters, i.e. collisions between material particles, with the analysis of initial motion conditions influence to the establishment to the defined vibro-impact oscillation regimes and the kinetic parameters of the dynamic system.

At the and of this summary of the research results that the entire work presents also an original composition of the combination of other researchers results, which were basis for this research, as well as originaly established methodologies, with significant and original contribution to the research theme of analysis of the vibro-impact system with curvilinear paths and non-ideal relations. The original results are visible through:

* the systematization of previous knowledge and scientific contributions of other researchers needed for the investigation of vibro-impact systems dynamics, for which the literature of high university, science and philosophy content of the mechanical systems kinetics, and the ideas from consultations with the project manager and mentor of this doctor’s thesis. Among others, in the second chapter, based on the scientific results of K. (Stevanović) Hedrih, showed in the consultancies with the author, as well as in the published papers, the differential double equations of motion were presented, and double equations of phase trajectories of phase portraits for the case of material particle movement along rough circle, as well as the analysis of the specific oscillations of heavy material particles along different curves. The comparison between time for the descent of the heavy material particle along rough cycloid line with the features of tautochronosity, isochronosity and brachystochronosity, present during the motion by ideally smooth cycloid line.

* By establishing two original methodologies in combination of analytical and numerical and approximation analytical methods with application of software tools in software packages MathCad, MATLAB and Wolfram mathematica having obvious originality and also theoretical and practical importance.

* By original application and using software programs MathCad, MATLAB, Wolfram mathematica and analytical expressions for the phase trajectories branches in intervals between impact (collisions) and by graphical determination of the kinnetic state parameters, in the proces of momental impact and velocities alternations, the vibro-impact dynamics visualization was given. Through different visualizations, the basic, easy applicable methodology in engineers practice was set for the analysis of dynamics of real vibro-impact systems, and whose abstraction leads toward some od vibro-impact dynamics models. Although this methodology was presented in several examples, it gets more significant, as well as the algorithm, that enables the analysis of the kinetic parameters of vibro-impact system dynamics with one, two or three degrees of freedom of motion.

New methodology was applied to 21 numeric examples, dedicated to the energy analysis of vibro-impact system dynamics with curvilinear routes and non-ideal connections. The expression vibro-impact system, as non-linear system is explained, not only because of its non-linearity of the basic system with ideal bonds without impacts and collisions, but for its strong non-linear nature of impacts, collisions and non-ideal bonds caused by dry Coulomb’s type friction. The analysis of vibro-impact system motion with one, two and three degrees of freedom of motion were done by using phase plane method, analytical method of adjusting, as accurate methods, and numerical method. After these methods, the corresponding energy analysis of vibro-impact system dynamics based on oscillator moving naturally or forced along curvilinear paths and non-ideal bonds, by using software tools from packages MathCad; MATLAB; Wolfram mathematica, graphic software CorelDraw. Curvilinear routes were in a shape of: parabolic, cycloid and circle. Non-ideal bond originates from the sliding Coulomb’s type friction force.

There is a need for special emphasis of third and fourth chapter containing new original contribution, obtained and published in cooperation with project leader of the project ON144002, and mentor, or are in the process of revision. It is particularly noted the visualization of the kinetic energy of the system and total mechanical energy of the system presented for each interval during kinetics of vibro-impact process, as well as the alternation of the forces and components of non-ideal bond resistances, or friction force as a function of generalized coordinates.
For the conduction this work, and achieving research experience, I was supported by the Ministry for Science and Technology of Republic of Serbia, as a researcher on the basic science projects: Mathematics and mechanics, noted as: Project 1616 “Real mechanical problems”, (2000-2005), 1828 “Dynamics and managing of active constructions”, (2000-2005), ON144002 "Problems of theory and technical mechanics of rigid and solid bodies. Materials mechanics", (2006-2010), and 144042 "Application of computers in mechanics", (2008-2010). Project leader for the projects 11616, 111828, 144002 is prof. dr Katica R. (Stevanović) Hedrih, and project leader of the project 144042 is prof. dr Nikola Maričić.

*Key words:* vibro-impact system, heavy material particle, rough circle line, impact elongation limiter, new methodology, parabolic line, cycloid line, circle line, sliding Coulomb’s type friction force, vibro-impact, incoming and outgoing velocity, equilibrium position bifurcation, force direction alternation, discontinuity, singular point, phase trajectory, angular elongation limiter, total mechanical energy, kinetic and potential energy, friction force working power, analytical expression, numerical integration, examples, graphic visualization, representative point, approximation, interpolation polynomial, MathCad, MATLAB, Wolfram Mathematica.

Предавања ће се одржавати средом са почетком у 18.00 часова, у сали 301 F на трећем спрату зграде Математичког института САНУ, Кнез Михаилова 36/III, (зграда преко пата главне зграде САНУ).

Позив научницима и истраживачима да пријаве своја предавања
Пријава потенцијалног предавача треба да садржи апстракт предавања до једне странице на српско језику ћирилицом и превод на енглески језик, као и ЦВ обима до две странице. Пријаву послати на адресу управника одељења механику у виду Word DOC на адресу: khedrih@eunet.rs

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*Announcement and Invitation*

Start of each lecture is at each Wednesday at 18,00 h in room 301 F at Mathematical Institute SANU, street Knez Mihailova 36/III.

All scientists and researchers in area of Mechanics are invited to contribute to the Program of Mechanics Colloquium of Mathematical Institute of Serbian Academy of Sciences and Arts. One page Abstract of proposed Lecture with short CV is necessary to submit in world doc to Head of Department of Mechanics (address: khedrih@eunet.rs), one month before first day in the next moth.

Katica R. (Stevanovic) Hedrih
Head of Department of Mechanics