

**MATEMATIČKI INSTITUT SANU , ODELJENJE ZA MEHANIKU**  
**Mathematical Institute SANU, Belgrade, Department for Mechanics**

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**Program of Mechanics Colloquium – OCTOBER 2011**

Start of each lecture is at each Wednesday at 18,00 h in room 301 F at Mathematical Institute SANU, street Knez Mihailova 36/III.

**Sreda (Wednesday), 5 oktobar (October 5) 2011 u 18 sati (18h)**

**Lecture No. 1164**

Assistant Professor dr Natača Trišović, Faculty of Mechanical Engineering University of Belgrade, (Project OI174001)

**Modification of the Dynamics Characteristics in the Structural Dynamic Reanalysis**

Structural dynamic modification (SDM) techniques can be defined as the methods by which dynamic behavior of a structure is improved by predicting the modified behaviour brought about by adding modifications like those of lumped masses, rigid links, dampers, beams etc. or by variations in the configuration parameters of the structure itself. Such methods, especially those with their roots in finite element models, have often been described as REANALYSIS. Most of the techniques imply a dynamic test at some stage of SDM and currently prefer implementation on a personal computer.

The need for SDM arises because of the demands on higher performance capabilities of complex mechanical and structural systems, like machine tools, automobiles, rail vehicles aerospace systems and high speed rotating systems, which require sound dynamic design, i.e. desired dynamic characteristics like vibration levels/response, resonances/eigenvalues, dynamic stability and mode shapes.

**Key words:** Structural dynamic modification, reanalysis, eigenvalues, design variables

**Sreda (Wednesday), 12 oktobar (October 12) 2011 u 18 sati (18h)**

**Lecture No. 1165**

Prof. dr Milutin Marjanov, Mathematical Institute SANU , (Project OI174001)

**THREE-BODY SYSTEM: STABLE AND CHAOTIC ORBITS**

Three bodies moving in the closed orbits, exposed to the gravitational interactions only, enter gradually, as a rule, into the gravitational resonances: periods of their rotations become related as the rational fractions. Because of that, motions of the bodies, for the most part, become harmonized, but some period ratios are causes of the chaotic motions, also.

In this work, investigation of that phenomenon was based on the, so cold, Newton's three-body system consisting of one massive and two considerably smaller bodies ( $m_0 \gg m_1, m_2$ ) turning around it. The same procedure may be used for examination of the correspondent restricted three-body problem ( $m_0 \gg m_1, m_2 \approx 0$ ).

It was revealed that the chaotic motions zones are situated around the  $T_1:T_2 \sim 1:3$  and  $3:1$  resonances and that their extensions mostly depend on the small masses and the large mass ratios.

The adopted model may be applied to the Sun, together with any pair of the (not necessarily mutually closest) heavenly bodies, provided that they do not form the binary system.

Thus, it was possible to determine some of the Solar System regions in which the orbits may become chaotic. Existence of such zones are probably the causes of the fact that numerous meteorodes, comets and asteroides leave their regular orbits and begin to move through the interplanetary space along the paths crossing the planets' orbits.

Concerning the planets, it seems that Mars' and Uranus' orbits lie in the zones of the unstable orbits. This fact, eventually, may produce approach of these bodies towards the Sun.

**Key words:** resonance, stable, unstable orbits, chaotic motions

**Sreda (Wednesday), 19 oktobar (October 19) 2011 u 18 sati (18h)**

**Lecture No. 1166**

Prof. dr Dragomir N. Zeković, University of Belgrade, Faculty of Mechanical Engineering, Kraljice Marije 16, 11120 Beograd, Serbia (Project OI174001)

**Dynamics of mechanical systems with nonlinear nonholonomic constraints – I The history of solving the problem of a material realization of a nonlinear nonholonomic constraint**

The paper brings forth a detailed analysis of the solution of the problem of the material realization of a nonlinear nonholonomic constraint (NNC). The existing models of the NNC are shown that can be classified into two groups: the first group comprises correctly realized physical models, while the second group contains the so-called "quasilinear" nonholonomic

constraints which in fact represent mathematical models. The correctness of the cited models is considered in detail, and the essential nature of such constraints, the basic of which is holonomic, is shown. The second group of models, i.e., the “quasilinear” NC (nonholonomic constraints) in fact represents the given program of motion, while the additional force, which carries out the program, has the analytical form of the reaction of the NNC. That is why are presented the models of the NNC which possess a clear physical sense, on the basis of which certain statements on the method of variation and the reaction of the NNC can be given. With regard to the clear physical sense and the nature of the models cited, the NNC that come out of them are used quite normally in the analysis of motion of such a system.

The cited models, together with standard models of nonholonomic Mechanics (sphere, disk, blade) make a group of basic nonholonomic constraints which can be classified, according to the three criteria, into certain types. Finally, it is shown that the cited model can be used for the construction of “nonholonomic chains”, both open and closed ones.

## References

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**Sreda (Wednesday), 26 oktobar (October 26) 2011 u 18 sati (18h)**

**Lecture No. 1167**

Dr Mirjana Filipović, Institute "Mihajlo Pupin", Belgrade

### **Euler-Bernoulli equation in robotic system nonlinear oscillations**

With the aim to exploit the experience of previous research, Meirovitch theory was first analyzed. Meirovitch proposed “modal technique” in 1967. In this paper, Euler Bernoulli equation is formed but “assumed modes technique” is not used in contrast to contemporaries who deal with this issue as well.

Having not found agreement with Meirovitch and his followers, the definition of elastic deformation was made taking into account the first research studies, i.e. the original form of Euler Bernoulli equation. That means that the elastic deformation amplitude and its frequency change depending on the moments (perturbation, inertial moments, Coriolis, centrifugal moments, gravity moments as well as coupling moments between the present modes, and the play of the external forces). It, of course, depends on the mechanism configuration, weight, length of the segments of the reference trajectory choice, dynamic characteristics of the motor motions.

Euler-Bernoulli equation was written in 1750. But, although it was made more than 250 years ago, Euler-Bernoulli equation is still usable and it can be connected logically with the contemporary knowledge from the mechanics and robotics.

Thus, it is very important to connect original Euler-Bernoulli equation and modern robotic knowledge on the principles of classical mechanics. The foundations of classical mechanics are particularly emphasized because synthesis and analysis of kinematics and dynamics of robotic configurations in stiff and elastic elements are based on them. The elasticity of segments on the principles of classical mechanics is implemented in this paper.

*a)* Euler-Bernoulli equation (based on the known laws of dynamics) should be supplemented with all the forces that are participating in the formation of the bending moment of the considered mode. The actuator torque is opposed by the bending moment of the first elastic mode, which comes after the motor, and partly by the bending moments of other modes, which are connected in series after the motor considered. The mathematical model of the actuators is connected to the rest of the mechanism via the equivalent elasticity moment.

***New structure of the mathematical model of actuators appears as a consequence of the coupling between the modes of particular links.***

Due to the strong coupling, there is diversity in the structure of the extended form of Euler Bernoulli equation of each mode. The stiffness matrix is a full matrix as well as the damping matrix not only in the Euler Bernoulli equation but also in equations of a motor. Damping is an integral part of the characteristics of elasticity of real systems and is naturally included in Euler Bernoulli equation. All of these features and this whole discussion is not just related to Euler Bernoulli equation but also to motion equation for any point (and the top point) of the elastic line. This is the case because the motion equation follows directly from Euler Bernoulli equation defining boundary conditions.

It is concluded that the definition of kinematic models is of particular importance. The dynamics of mechanism just over the sizes of elastic deformation is included into its definition, resulting from the dynamics of the motion of mechanism. This makes possible the

process of defining new Denavit-Hartenberg parameters, a new form of matrix transformations and Jacobi matrix and the reference trajectory.

This research has theoretical and practical significance. The purpose is to define as realistically as possible both kinematic and dynamic model of the mechanism with stiff and elastic elements which will describe the real system very well.

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Предавања ће се одржавати средом са почетком у 18.00 часова, у сали 301 F на трећем спрату зграде Математичког института САНУ, Кнез Михаилова 36/III, (зграда преко пута главне зграде САНУ).

Позив научницима и истраживачима да пријаве своја предавања

Пријава потенцијалног предавача треба да садржи апстракт предавања до једне странице на српском језику ћирилицом и превод на енглески језик, као и CV обима до две странице. Пријаву послати на адресу управника Одељења за механику у виду Word DOC на адресу: [khedrih@eunet.rs](mailto:khedrih@eunet.rs)

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Announcement and Invitation

Start of each lecture is at each Wednesday at 18,00 h in room 301 F at Mathematical Institute SANU, street Knez Mihailova 36/III.

All scientists and researchers in area of Mechanics are invited to contribute to the Program of Mechanics Colloquium of Mathematical Institute of Serbian Academy of Sciences and Arts. One page Abstract of proposed Lecture with short CV is necessary to submit in world doc to Head of Department of Mechanics (address: [khedrih@eunet.rs](mailto:khedrih@eunet.rs)), one month before first day in the next moth.



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