Spectral Characterizations of Graphs; the generalized adjacency spectrum

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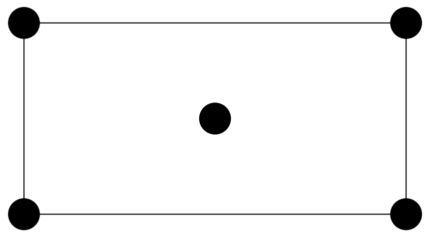
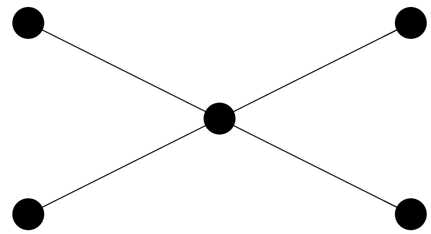
In honour of Dragoš Cvetković





$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

spectrum

$$-2,\ 0^3,\ 2$$

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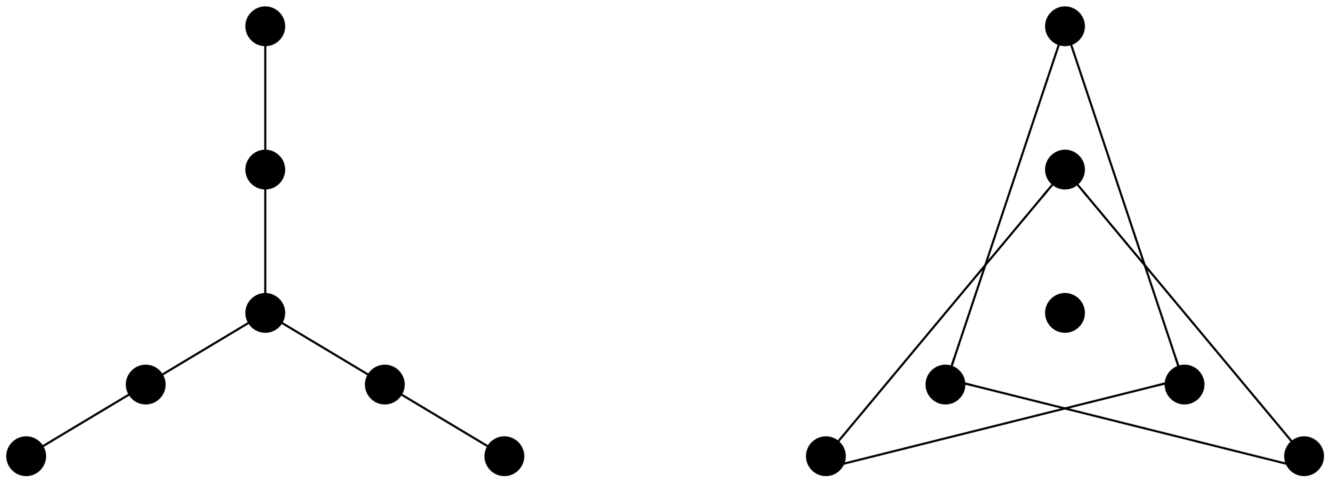
spectra

$-1^2,\ 1,\ \frac{1}{2} \pm \frac{1}{2}\sqrt{17}$ $\qquad$ $-1^3,\ 0,\ 3$



spectrum

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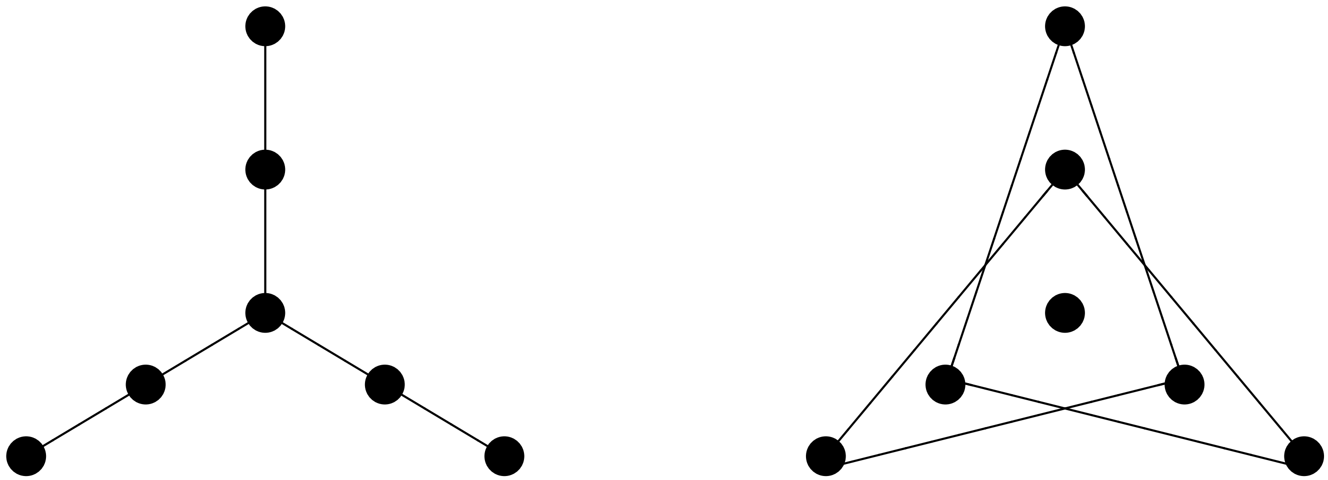

spectrum

$$0,\ \pm 1^2,\ \pm 2$$

spectrum complement

$$-2^2,\ 1,\ 0^2,\ \frac{3}{2} \pm \frac{1}{2}\sqrt{33}$$

the **generalized adjacency matrix**

$$A(\alpha,\beta,\gamma) = \alpha A + \beta J + \gamma I \quad (\alpha \neq 0)$$

$$A(\alpha,\beta,\gamma)_{i,j} = \begin{cases} \alpha+\beta & \text{if} \quad \{i,j\} \in E \\ \beta & \text{if} \quad \{i,j\} \notin E,\ i \neq j \\ \beta+\gamma & \text{if} \quad i = j \end{cases}$$

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$$A(1, 0, 0) = A, \quad A(-1, 1, -1) = \overline{A}, \quad A(-2, 1, -1) = S$$

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$$A(1,0,0) = A, \quad A(-1,1,-1) = \overline{A}, \quad A(-2,1,-1) = S$$

$$A(\alpha,\beta,\gamma) \text{ cospectral with } A'(\alpha,\beta,\gamma)$$

$$\leftrightarrow$$

$$A + \frac{\beta}{\alpha} J \text{ cospectral with } A' + \frac{\beta}{\alpha} J$$

THEOREM (Johnson and Newman 1980)

The following are equivalent:

- $A + yJ$ is cospectral with $A' + yJ$ for all $y \in \mathbb{R}$,
- $A + yJ$ and $A' + yJ$ are cospectral for two distinct values of y,
- There exists a regular orthogonal matrix Q such that $Q^\top AQ = A'$.

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Two graphs are cospectral with respect to every generalized adjacency matrix

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The *generalized spectrum* of a graph G is the adjacency spectrum of G together with the adjacency spectrum of the complement of G.

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Almost all graphs are determined by their adjacency spectrum (DAS)

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CONJECTURE 2
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For regular graphs DGS is the same as DAS

Ways to prove that a given graph is DAS or DGS?

• Find structural properties from the spectrum and hope that it determines the graph.

• Generate all graphs with the same number of vertices (and edges and triangles) and check.

• (Wang and Xu) Find all regular orthogonal matrices Q, such that $Q^\top AQ$ is a $(0,1)$ matrix.

Method of Wang and Xu

Graph G with adjacency matrix A is *controllable* if the walk matrix

$$W = \left[\mathbf{1}\ \ A\mathbf{1}\ \ A^2\mathbf{1}\ \ \ldots\ \ A^{n-1}\mathbf{1}\right]$$

is nonsingular.

Method of Wang and Xu

Graph G with adjacency matrix A is *controllable* if the walk matrix

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is nonsingular.

Suppose G is controllable and $Q^\top AQ$ is a $(0,1)$-matrix A' (say), for some regular orthogonal matrix Q, then

- Q is unique (for fixed A'),
- Q is rational,
- SNF(W) gives an integer ℓ such that ℓQ is integral,
- In many cases $\ell = 2$ in which case Q is characterized.

THEOREM
O'Rourke and Touri (arχiv 2015)

Almost all graphs are controllable

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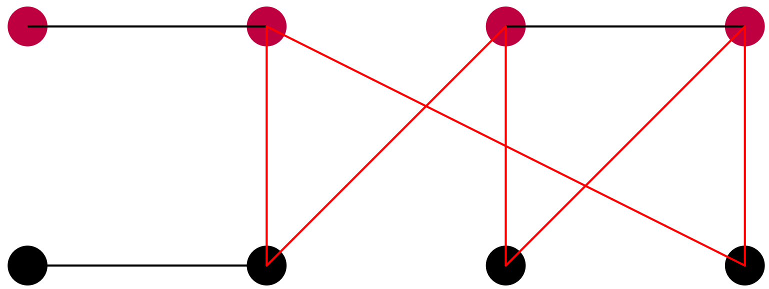
Wang (arχiv 2014)

If $\det(W)/\lfloor 2^{n/2} \rfloor$ is odd and square free, then G is DGS.

Fractions of graphs on n vertices which are not DAS, and not DGS.

n	number of graphs	not DAS	not DGS	reference
1	1	0	0	
2	2	0	0	
3	4	0	0	
4	11	0	0	
5	34	0.059	0	
6	156	0.064	0	
7	1044	0.105	0.038	
8	12346	0.139	0.094	
9	274668	0.186	0.160	Godsil, McKay (1976)
10	12005168	0.213	0.201	H, Spence (2004)
11	1018997864	0.211	0.208	H, Spence (2004)
12	165091172592	0.188		Brouwer, Spence (2009)

Godsil-McKay switching produces graphs which are cospectral with respect to the generalized adjacency matrix.



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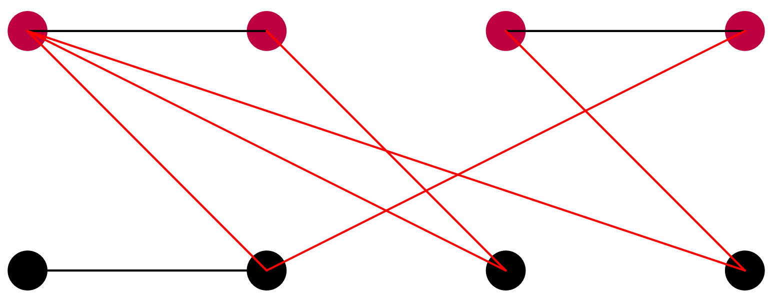
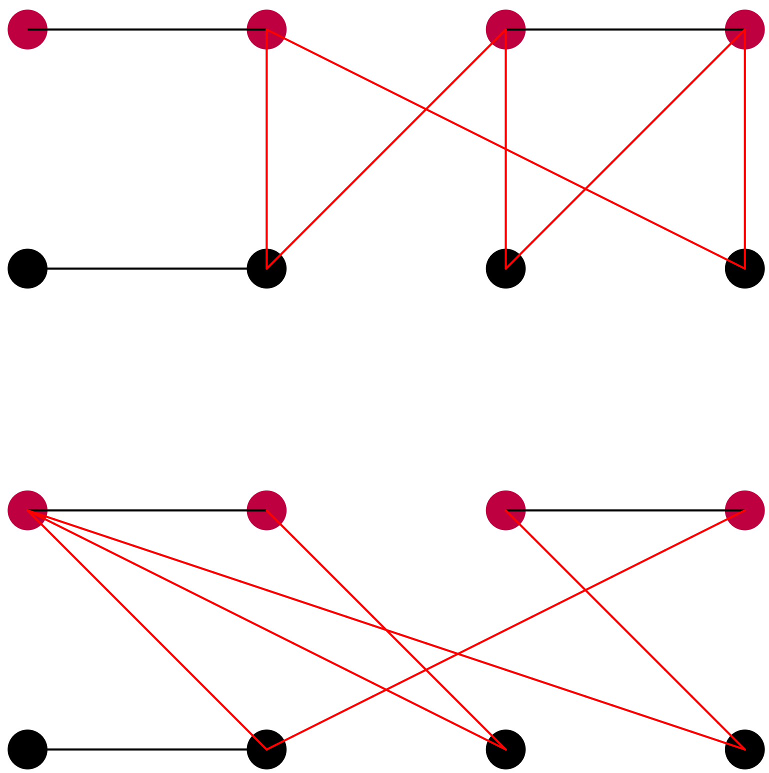
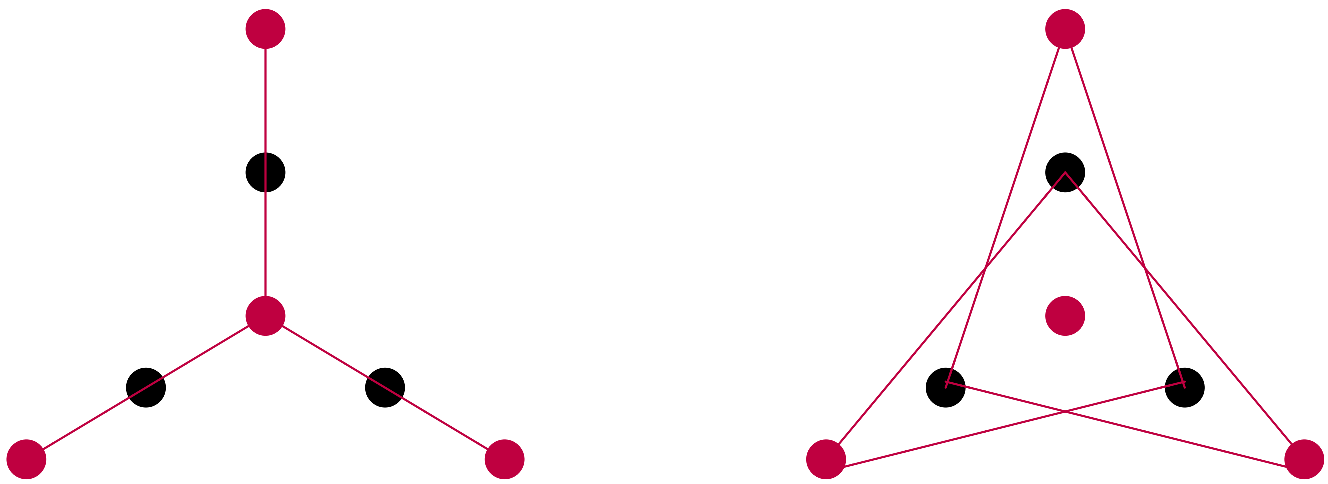


Godsil-McKay switching



Godsil-McKay switching



Fractions of graphs on n vertices which are not DAS,

not DGS, and with a proper GM switching set.

n	number of graphs	not DAS	not DGS	GM
1	1	0	0	0
2	2	0	0	0
3	4	0	0	0
4	11	0	0	0
5	34	0.059	0	0
6	156	0.064	0	0
7	1044	0.105	0.038	0.038
8	12346	0.139	0.094	0.085
9	274668	0.186	0.160	0.139
10	12005168	0.213	0.201	0.171
11	1018997864	0.211	0.208	0.174
12	165091172592	0.188		

$$
\left[\begin{array}{cccc|cccccc}
 & & & & \mathbf{1} & & \mathbf{1} & & \mathbf{1} & \\
 & & & & \mathbf{1} & & & \mathbf{1} & & \mathbf{1} \\
 & & & & & \mathbf{1} & \mathbf{1} & & & \mathbf{1} \\
 & & & & & \mathbf{1} & & \mathbf{1} & \mathbf{1} & \\
\hline
\mathbf{1} & \mathbf{1} & & & & 1 & & & & \\
 & & \mathbf{1} & \mathbf{1} & 1 & & & & & \\
\mathbf{1} & & \mathbf{1} & & & & & 1 & & \\
 & \mathbf{1} & & \mathbf{1} & & & 1 & & & \\
\mathbf{1} & & & \mathbf{1} & & & & & & 1 \\
 & \mathbf{1} & \mathbf{1} & & & & & & 1 &
\end{array}\right]
\quad \text{GM-switch} \quad
\left[\begin{array}{cccc|cccccc}
 & & & & & \mathbf{1} & & \mathbf{1} & & \mathbf{1} \\
 & & & & & \mathbf{1} & \mathbf{1} & & \mathbf{1} & \\
 & & & & \mathbf{1} & & & \mathbf{1} & \mathbf{1} & \\
 & & & & \mathbf{1} & & \mathbf{1} & & & \mathbf{1} \\
\hline
 & & \mathbf{1} & \mathbf{1} & & 1 & & & & \\
\mathbf{1} & \mathbf{1} & & & 1 & & & & & \\
 & \mathbf{1} & & \mathbf{1} & & & & 1 & & \\
\mathbf{1} & & \mathbf{1} & & & & 1 & & & \\
 & \mathbf{1} & \mathbf{1} & & & & & & & 1 \\
\mathbf{1} & & & \mathbf{1} & & & & & 1 &
\end{array}\right]
$$

$$
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 & & & & \mathbf{1} & & \mathbf{1} & & \mathbf{1} & \\
 & & & & \mathbf{1} & & & \mathbf{1} & & \mathbf{1} \\
 & & & & & \mathbf{1} & \mathbf{1} & & & \mathbf{1} \\
 & & & & & \mathbf{1} & & \mathbf{1} & \mathbf{1} & \\
\hline
\mathbf{1} & \mathbf{1} & & & & 1 & & & & \\
 & & \mathbf{1} & \mathbf{1} & 1 & & & & & \\
\mathbf{1} & & \mathbf{1} & & & & & 1 & & \\
 & \mathbf{1} & & \mathbf{1} & & & 1 & & & \\
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\end{array}\right]
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 & & & & & \mathbf{1} & \mathbf{1} & & \mathbf{1} & \\
 & & & & \mathbf{1} & & & \mathbf{1} & \mathbf{1} & \\
 & & & & \mathbf{1} & & \mathbf{1} & & & \mathbf{1} \\
\hline
 & & \mathbf{1} & \mathbf{1} & & 1 & & & & \\
\mathbf{1} & \mathbf{1} & & & 1 & & & & & \\
 & \mathbf{1} & & \mathbf{1} & & & & 1 & & \\
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 & \mathbf{1} & \mathbf{1} & & & & & & & 1 \\
\mathbf{1} & & & \mathbf{1} & & & & & 1 &
\end{array}\right]
$$

Isomorphic after GM-switching: Petersen graph

$$A = \left[\begin{array}{c|c} {\color{purple}X} & N^\top \\ \hline N & B \end{array}\right] \quad \text{GM-switch} \quad \left[\begin{array}{c|c} {\color{purple}X} & N'^\top \\ \hline N' & B \end{array}\right] = A'$$

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$$N = \begin{bmatrix} H \\ O \\ J \end{bmatrix}, \quad N' = \begin{bmatrix} J-H \\ O \\ J \end{bmatrix}, \quad H\mathbf{1} = (J-H)\mathbf{1} = \frac{|X|}{2}\mathbf{1}.$$

$$A = \left[\begin{array}{c|c} X & N^\top \\ \hline N & B \end{array}\right] \quad \text{GM-switch} \quad \left[\begin{array}{c|c} X & {N'}^\top \\ \hline N' & B \end{array}\right] = A'$$

Suppose there exist permutation matrices P_1 and P_2 such that

$$P_1^\top X P_1 = X, \qquad P_2^\top B P_2 = B, \qquad P_2^\top N P_1 = N',$$

then $\qquad P^\top A P = A' \qquad$ with $A = \begin{bmatrix} P_1 & O \\ O & P_2 \end{bmatrix}$.

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EXAMPLES: case $|X| = 2$, Petersen graph, grid.

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EXAMPLES: case $|X| = 2$, Petersen graph, grid.

THEOREM (Abiad, Brouwer, H)
The converse is not true.

Useful easy necessary conditions for isomorphism after GH-switching.

- Same degree sequence.

- Same numbers of 3-vertex configurations.

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Does not work for regular graphs.

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Does not work for strongly regular graphs.

Symplectic graph $\mathrm{Sp}(2, 2\nu)$. Vertex set $V = \mathbb{F}_2^{2\nu} \setminus \{\mathbf{0}\}$.

$\mathbf{v} = [v_1 \ldots v_{2\nu}]^\top$ adjacent to $\mathbf{w} = [w_1 \ldots w_{2\nu}]^\top$ whenever

$(v_1 w_2 + v_2 w_1) + (v_3 w_4 + v_4 w_3) + \cdots + (v_{2\nu-1} w_{2\nu} + v_{2\nu} w_{2\nu-1}) = 1.$

Equivalently, $\quad \mathbf{v}^\top K \mathbf{w} = 1 \quad$ with

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$(v_1 w_2 + v_2 w_1) + (v_3 w_4 + v_4 w_3) + \cdots + (v_{2\nu-1} w_{2\nu} + v_{2\nu} w_{2\nu-1}) = 1.$

Equivalently, $\quad \mathbf{v}^\top K \mathbf{w} = 1 \quad$ with

$$K = \begin{bmatrix} & 1 & & & & & & \\ 1 & & & & & & & \\ & & & 1 & & & & \\ & & 1 & & & & & \\ & & & & \cdot & & & \\ & & & & & \cdot & & \\ & & & & & & \cdot & \\ & & & & & & & 1 \\ & & & & & & 1 & \end{bmatrix}.$$

The symplectic graph $\mathrm{Sp}(2, 2\nu)$ is strongly regular with parameters

$$n = 2^{2\nu} - 1.\ k = 2^{2\nu-1},\ \ \lambda = 2^{2\nu-2},\ \ \mu = 2^{2\nu-2}.$$

and adjacency matrix $\quad A = MKM^\top$ (over $\mathbb{F}_2$), where M consists of all nonzero vectors in $\mathbb{F}_2^{2\nu}$.

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Note that 2-rank$(A) = 2\nu$.

Any linear combination (over $\mathbb{F}_2$) of columns of A is again a column of A, or $\mathbf{0}$.

Godsil-McKay switching set in $\mathrm{Sp}(2, 2\nu)$ $\quad (\nu \geq 3, \quad \mathbf{z} \in \mathbb{F}_2^{2\nu-6})$:

$$
\begin{array}{cccc}
\mathbf{v} & \mathbf{w} & \mathbf{x} & \mathbf{y} \\
\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ \mathbf{z} \end{bmatrix} &
\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ \mathbf{z} \end{bmatrix} &
\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ \mathbf{z} \end{bmatrix} &
\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ \mathbf{z} \end{bmatrix}
\end{array}
$$

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$$\begin{array}{cccc} \mathbf{v} & \mathbf{w} & \mathbf{x} & \mathbf{y} \\ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ \mathbf{z} \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ \mathbf{z} \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ \mathbf{z} \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ \mathbf{z} \end{bmatrix} \end{array}$$

$\mathbf{v}$, $\mathbf{w}$, $\mathbf{x}$, $\mathbf{y}$ are mutually nonadjacent, and for $\mathbf{u} \notin \{\mathbf{v}, \mathbf{w}, \mathbf{x}, \mathbf{y}\}$

$$\mathbf{u}^\top K\mathbf{v} + \mathbf{u}^\top K\mathbf{w} + \mathbf{u}^\top K\mathbf{x} + \mathbf{u}^\top K\mathbf{y} = \mathbf{u}^\top K(\mathbf{v} + \mathbf{w} + \mathbf{x} + \mathbf{y}) = \mathbf{u}^\top \mathbf{0} = 0.$$

$$
A = \left[\begin{array}{cccc|c}
 & & & & \\
 & & & & \\
\hline
1 & 1 & 0 & 0 & \text{row } 1 \\
1 & 0 & 1 & 0 & \text{row } 2 \\
 & & & & \\
 & & & &
\end{array}\right]
$$

$$A = \left[\begin{array}{cccc|c} & & & & \\ \hline 1 & 1 & 0 & 0 & \text{row } 1 \\ 1 & 0 & 1 & 0 & \text{row } 2 \\ 0 & 1 & 1 & 0 & \text{row } 1 + \text{ row } 2 \\ & & & & \end{array}\right]$$

The sum of these three rows equals $[\ 0\ 0\ 0\ 0 \mid 0\ \ldots\ 0\ ]$.

$$A' = \left[\begin{array}{cccc|c} & & & & \\ & & & & \\ \hline 0 & 0 & 1 & 1 & \text{row } 1 \\ 0 & 1 & 0 & 1 & \text{row } 2 \\ 1 & 0 & 0 & 1 & \text{row } 1 + \text{ row } 2 \\ & & & & \\ & & & & \end{array}\right]$$

The sum of these three rows equals $[\ 1\ 1\ 1\ 1 \mid 0\ \ldots\ 0\ ]$.

A is not isomorphic to A'. $\quad$ 2-rank$(A') = 2\nu + 2$.

THEOREM Abiad and H (2015)

There exist strongly regular graphs with the parameters of $\mathrm{Sp}(2, 2\nu)$ and 2-rank

$$2\nu,\ 2\nu+2,\ \ldots,\ 2\nu+12\lfloor\frac{\nu}{3}\rfloor$$

CONGRATULATIONS DRAGOŠ