

# On Contribution of Serbian Mathematicians to Set Theory

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## What is not the aim of the paper

In this survey we present the work of mathematicians *living in Serbia and working in set theory or using its typical methods.*

We omit

- The pioneering work of Djuro Kurepa, which was presented by Žarko Mijajlović;
- The contribution of Serbian mathematicians working abroad (Stevo Todorčević, Boban Veličković, Ilijas Farah, Zoran Spasojević, Vojkan Vuksanović, Žikica Perović), which will be presented by Stevo Todorčević;
- The contribution of Serbian mathematicians to the theory of fuzzy sets (which is today a different field, deserving a separated survey).

# “Scientometrics”

## Theorem

**(Surely false)** The contribution to set theory of Serbian mathematicians working in Serbia is contained in 65 papers listed at the end of this survey.

## Proof.

METHODOLOGY: Search the MathSciNet with the following restrictions:

- Institution code: YU\* (Yugoslavia) or SE\* (Serbia)  
(restricts the search to the mathematicians having domestic affiliation)
- Classification: 03E or 04 or 04A  
(restricts the search to the papers classified as set theoretical)
- Classification: not 03E72 and not 04A72  
(in order to avoid fuzzy sets)

# “Scientometrics”

## RESULTS:

|                   | SE | YU |
|-------------------|----|----|
| 03E and not 03E72 | 17 | 45 |
| 04 and not 04A72  | 0  | 36 |

ADDITIONAL RESTRICTIONS: Some of these 98 papers

- Are classified both as 03E and as 04
- Were published by the authors from the other parts of Yugoslavia
- Were published by Professor Kurepa or by the mathematicians working abroad

CONCLUSION: According to these restrictions we found 65 papers satisfying our requirements.

# Why the scientometrics fails?

First, we have the following general result

## Theorem

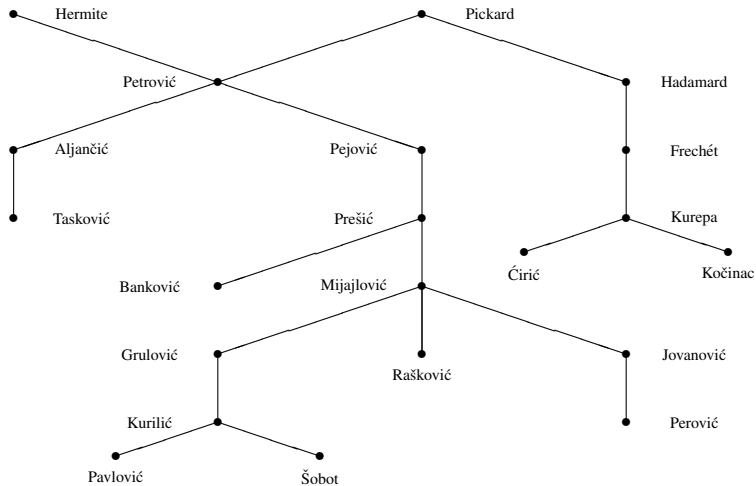
**(Surely true)** The mathematical beauty can not be expressed by numbers, 65 or 0.439 (Thomson's "impact factor" of the Journal of Symbolic Logic in 2008).

## Why the scientometrics fails?

Second, in the previous application of scientometrics the following problems appeared:

- Some of 65 papers in the list are not essentially set theoretical (work with finite sets, algebraic properties of functions and relations)
- Some papers of Serbian authors having substantial set theoretic components are not in the list (for example almost all of 103 papers of Ljubiša Kočinac belong to set-theoretic topology and could have such a classification)
- I discovered (unfortunately, too late) that in the period 1973-1979 set theory was classified by 02K as well (and missed some papers of Aleksandar Jovanović). But at that time the Institution code was not recorded and I was unable to extract the results of Serbian mathematicians with this classification.

# Set theoretic Genealogy



# Novi Sad

MILAN GRULOVIĆ defended his PhD thesis entitled “Forsing u teoriji modela” in 1984 at the University of Belgrade, under the supervision of Žarko Mijajlović. Majority of his 23 papers belongs to model theory and one is classified as set theoretical.

MILOŠ KURILIĆ defended his PhD thesis entitled “Redukovani ideal-proizvod topoloških prostora” in 1994 at the University of Novi Sad, under the supervision of Milan Grulović. Majority of his 28 papers belongs to set-theory and general topology and 11 are classified as set theoretical.

SVETOZAR MILIĆ defended his PhD thesis entitled “Prilog teoriji kvazigrupa” in 1972 at the University of Belgrade, under the supervision of Slaviša Prešić. Majority of his 29 papers belongs to group theory and generalizations. His lectures on set theory held in Novi Sad attracted students to this field.

ALEKSANDAR PAVLOVIĆ defended his PhD thesis entitled “Sekvencijalne topologije na Bulovim algebra” in 2009 at the University of Novi Sad, under the supervision of Miloš Kurilić. His 3 papers belong to set-theoretical topology and 2 of them are classified as set theoretical.

BORIS ŠOBOT defended his PhD thesis entitled “Igre na Bulovim algebra” in 2009 at the University of Novi Sad, under the supervision of Miloš Kurilić. His 3 papers are classified as set theoretical.



## Research in NS: Forcing (in)destructibility

### Theorem (K., [39])

- A maximal almost disjoint (MAD) family  $\mathcal{A} \subseteq [\omega]^\omega$  is destructible by the Cohen forcing iff there is a bijection from  $\omega$  to  $\mathbb{Q}$  which maps the sets from  $\mathcal{A}$  onto nowhere dense subsets of  $\mathbb{Q}$ .
- Cohen-indestructible MAD families exist if  $\mathfrak{b} = \mathfrak{c}$  or  $\mathfrak{a} < \text{cov}(\mathcal{K})$ .
- A splitting family  $\mathcal{S} \subseteq [\omega]^\omega$  is destructible by the Cohen forcing iff there is a bijection from  $\omega$  to  $\mathbb{Q}$  which maps the sets from  $\mathcal{S}$  onto the subsets of  $\mathbb{Q}$  having nowhere dense boundaries.

## Research in NS: Games on Boolean algebras

The game  $\mathcal{G}_{\text{ls}}$  is played on a c.B.a  $\mathbb{B}$  in  $\omega$ -many moves: White chooses a non-zero element  $p$  of  $\mathbb{B}$  and, in the  $n$ -th move, a positive  $p_n < p$  and Black responds choosing an  $i_n \in \{0, 1\}$ . White wins the play iff  $\limsup p_n^{i_n} = 0$ .

**Theorem (K., Šobot, [30])**

- White has a winning strategy in the game  $\mathcal{G}_{\text{ls}}$  iff forcing by  $\mathbb{B}$  collapses the continuum to  $\omega$  in some generic extension.
- If  $\mathbb{B}$  carries a strictly positive Maharam submeasure or contains a countable dense subset, then Black has a winning strategy in the game  $\mathcal{G}_{\text{ls}}$  played on  $\mathbb{B}$ .
- $\diamond$  implies that there is a Suslin algebra on which the game  $\mathcal{G}_{\text{ls}}$  is undetermined.

## Research in NS: Topologies on Boolean algebras

The **sequential topology**  $\tau_s$  on a c.B.a.  $\mathbb{B}$  is generated by the algebraic convergence: a sequence  $\langle b_n \rangle$  converges to  $b$  iff  $\liminf b_n = \limsup b_n = b$ .

**Theorem (K., Pavlović, [32])**

- The space  $\langle \mathbb{B}, \tau_s \rangle$  is sequentially compact iff  $\mathbb{B}$  has the property  $(\bar{h})$  and forcing by  $\mathbb{B}$  does not produce independent reals.
- $\mathfrak{s} = \omega_1$  implies that  $P(\omega)$  is the unique sequentially compact c.B.a. in the class of Suslin forcing notions.

# Belgrade

DUŠAN ĆIRIĆ defended his PhD thesis entitled “Direktne granice funktora u nekim kategorijama” in 1981 at the University of Belgrade, under the supervision of Djuro Kurepa. Majority of his 9 papers belongs to general topology and 3 of them are classified as set theoretical.

ALEKSANDAR JOVANOVIĆ defended his PhD thesis entitled “Prilog teoriji ultraproizvoda” in 1982 at the University of Belgrade, under the supervision of Žarko Mijajlović. Majority of his 19 papers belongs to mathematical logic and foundations of mathematics and 16 of them are classified as set theoretical.

ŽARKO MIJAJLOVIĆ defended his PhD thesis entitled “Prilog teoriji modela i Bulovih algebri” in 1977 at the University of Belgrade, under the supervision of Slaviša Prešić. Majority of his 46 papers belongs to mathematical logic and foundations of mathematics and 8 of them are classified as set theoretical.

ALEKSANDAR PEROVIĆ defended his PhD thesis entitled “—” in ——— at the University of Belgrade, under the supervision of Aleksandar Jovanović. Majority of his 7 papers belongs to mathematical logic and foundations of mathematics and 2 of them are classified as set theoretical.

SLAVIŠA PREŠIĆ defended his PhD thesis entitled “Prilog teoriji algebarskih struktura” in 1963 at the University of Belgrade, under the supervision of Tadija Pejović. Majority of his 50 papers belongs to difference and functional equations and 3 of them are classified as set theoretical.

MILAN TASKOVIĆ defended his PhD thesis entitled “Banahova preslikavanja na prostorima i uredjenim skupovima” in 1978 at the University of Belgrade, under the supervision of Slobodan Aljančić. Majority of his 69 papers belongs to general topology and operator theory and 5 of them are classified as set theoretical.

## Research in BG: Generalized selectivity for ultrafilters

If  $S$  is a set of cardinals, an ultrafilter  $D$  over  $\lambda$  is  $S$ -selective if for every  $f \in {}^\lambda \lambda$  there is  $A \in D$  such that  $|f[A]| \notin S$ .

**Theorem (A. Jovanović, Ž. Mijajlović, [19])**

- An ultrafilter  $D$  over  $\lambda$  is  $S$ -selective iff every ultrafilter  $E$  over  $\lambda$  which is less than  $D$  in the Rudin-Keisler order satisfies  $\min\{|A| : A \in E\} \notin S$ .
- If  $\kappa$  is strongly compact cardinal and  $\text{cf}(\lambda) \geq \kappa$ , then some ultrafilter  $D$  over  $\lambda$  is both  $[\omega, \kappa)$ -selective and  $\{\alpha\}$ -selective for all  $\alpha$  such that  $\text{cf}(\alpha) < \kappa$ .

## Research in BG: Real-valued measures on cardinals

### Theorem (A. Jovanović [22])

The consistency of ZFC + “there is a supercompact cardinal” implies the consistency of ZFC + “there exist a uniform measure  $\mu$  on the cardinal  $2^\omega$  and a set  $X \subseteq 2^\omega$  of positive  $\mu$ -measure such that for every  $y \in X$  there is a uniform measure on  $y$  which is  $|y|$ -additive.”

# Research in BG: Equivalents of the AC

Theorem (M. Tasković [61])

AC  $\Leftrightarrow$  If  $P$  is any poset in which each chain has an upper bound, then any mapping  $f : P \rightarrow P$  such that  $x \leq f(x)$  for all  $x \in \text{Sub } f[P]$  has a fixed point.

Here  $\text{Sub } f[P]$  is the union of  $f[P]$  and the set of upper bounds of chains in  $f[P]$ .

# Research in BG: Topologies on classes

Theorem (D. Ćirić, Ž. Mijajlović)



LJUBIŠA KOČINAC defended his PhD thesis entitled “Neke osobine kardinalnih funkcija” in 1983 at the University of Belgrade, under the supervision of Djuro Kurepa. Majority of his 103 papers belongs to set-theoretic and general topology and 11 of them are classified as set theoretical.

ŽIKICA PEROVIĆ defended his PhD thesis entitled “Cardinalities of Algebraic Structures Satisfying Completeness and Saturation Conditions” in 1987 at the University of Minesota-Mineapolis, under the supervision of Karel Prikry. Majority of his 13 papers belongs to mathematical logic and order theory and 5 of them are classified as set theoretical.

## Research in NI: Combinatorial properties of open covers

If  $X$  is a topological space then  $C_p(X)$  denotes the space of real-valued continuous functions on  $X$ , with the topology of pointwise convergence.

**Theorem (Lj. Kočinac [28])**

For any  $T_{3\frac{1}{2}}$  space  $X$  the following conditions are equivalent:

- $C_p(X)$  has countable fan tightness and the Reznichenko property;
- All finite powers of  $X$  have the Hurewicz property.

# Research in NI: Possible cardinalities of $\eta_1$ -fields

Theorem (Ž. Perović [50])

- (GCH) A cardinal  $\kappa$  is the size of an  $\eta_1$ -field iff  $\kappa^\omega = \kappa$ .
- For maximally valued  $\eta_1$ -fields the same is true without using the GCH.

# Kragujevac, Čačak

DRAGIĆ BANKOVIĆ defended his PhD thesis entitled “Reproduktivna rešenja jednačina” in 1980 at the University of Belgrade, under the supervision of Slaviša Prešić. Majority of his 43 papers belongs to the theory of order and 6 of them are classified as set theoretical.

DRAGAN DJURČIĆ defended his PhD thesis entitled “–” in ——— at the University of ———, under the supervision of ———. Majority of his 26 papers belongs to real functions and 2 of them are classified as set theoretical.

MIODRAG RAŠKOVIĆ defended his PhD thesis entitled “Logike sa merom u Lajbnicovom univerzumu” in 1983 at the University of Belgrade, under the supervision of Žarko Mijajlović. Majority of his 42 papers belongs to mathematical logic and foundations of mathematics and 2 of them are classified as set theoretical.

MALIŠA ŽIŽOVIĆ defended his PhD thesis entitled “Prilog izučavanju odnosa topoloških struktura i algebarskih sistema” in 1980 at the University of Belgrade, under the supervision of Dušan Adamović. Majority of his 51 papers belongs to Group theory and generalizations and 4 of them are classified as set theoretical.

# Research in KG and ČA: Nonclassical and second-order set theories

AST denotes the alternative set theory and  $\lambda$  the Lebesgue measure.

**Theorem (M. Rašković [55])**

The following theorem of Luzin is a theorem of AST:

A function  $F [0, 1] \rightarrow \mathbf{R}$  is measurable iff for each  $\varepsilon > 0$  there is a closed set  $K$  such that  $\lambda([0, 1] \setminus K) < \varepsilon$  and  $F$  is continuous on  $K$ .

# Research in KG and ČA: Selection principles and games in divergent processes

Theorem (D. Djurčić, Lj. Kočinac , M. Žižović, [12])

# Research in KG and ČA: Solutions of finite equations

S. Prešić introduced and studied equations over a finite set  $Q$  such that  $Q \cup \{0, 1\}$  is endowed with two binary operations  $+$  and  $\cdot$  extending the disjunction and conjunction over  $\{0, 1\}$ , respectively, and a third operation  $x^y = 1$  or  $0$  according as  $x = y$  or  $x \neq y$ .

Theorem (D. Banković [3])

## More contributions

The following mathematicians working in other areas of mathematics also made a contribution to set theory:

- Ivana Berković
- Radoslav Dimitrić
- Mirko Jovanović
- Branko Malešević
- Žarko Mitrović
- Marica Prešić
- Jadran Stojanović
- Branimir Šešelja
- Janez Ušan



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