MATHEMATICAL INSTITUTE
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Četvrta nacionalna konferencija
"Verovatnosne logike i njihove primene"

Beograd, Srbija, 2. oktobar 2014.
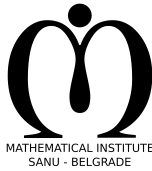
Knjiga apstrakata

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"Verovatnosne logike i njihove primene"

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TEME KONFERENCIJE:
- verovatnosne logike, problemi potpunosti, odlučivosti i složenosti,
- logičke osnove u zasnivanju verovatnoće,
- Bayes-ove mreže i drugi srodni sistemi,
- programski sistemi za podršku odlučivanju u prisustvu neizvesnosti,
- primene verovatnosnog zaključivanja u medicini itd.

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ORGANIZATOR:
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Program konferencije:

2. 10. 2014.

10:00   *Otvaranje*

10:15   *Bisimulation in Interpretability Logic*, Mladen Vuković

10:50   *Computational approach to conjugation*, Silvia Ghilezan

11:10   *Pauza*

11:20   *k-nearest neighbor decision in medical diagnostic*, Nataša Glišović

11:40   *Iterative Process of Group Decision Making using Fuzzy Influence Diagrams*, Aleksandar Janjić, Miomir Stanković, Lazar Velimirović

12:00   *Application of water surplus distribution in assessing meteorological drought*, Milan Gocić, Lazar Velimirović, Miomir Stanković

12:20   *Estimation Methods in Bivariate Autoregressive Time Series of Counts*, Predrag Popović, Miroslav Ristić, Aleksandar Nastić

12:40   *Pauza*

12:55   *Dealing with satisfiability problem in default logic using Bee-colony optimization*, Tatjana Stojanović, Nebojša Ikodinović, Tatjana Davidović, Zoran Ognjanović

13:15   *Possible applications of the probability logic in social sciences*, Miodrag Rašković, Nebojša Ikodinović, Nenad Stojanović

13:35   *Sequent calculus for logic with high probabilities*, Marija Boričić

13:55   *Pauza*

15:00   *Algorithms for Imputation of Missing SNP Genotype Data*, Aleksandar Mihajlović

15:20   *A Propositional Linear Time Logic with Time Flow Isomorphic to ω^2*, Bojan Marinković, Zoran Ognjanović, Dragan Doder, Aleksandar Perović

15:40   *A Probabilistic Extension of Abstract Dialectical Frameworks*, Sylwia Polberg, Dragan Doder

16:00   *Formal Description of the Chord Protocol using Isabelle/HOL Proof Assistant*, Milan Todorović, Bojan Marinković, Aleksandar Zeljić, Paola Glavan, Zoran Ognjanović

16:20   *Pauza*

16:35   *Some applications of nonstandard analysis to functional equations*, Miodrag Rašković, Nebojša Ikodinović, Bojana Lasković

16:55   *Probability measures effect of the use of computers in education*, Natalija Jelenković, Katarina Pfaf-Krstić

17:15   *Infinitary axiomatization of common knowledge*, Siniša Tomović

17:35   *On probable conditionals*, Zvonimir Šikić

Apstrakti

Sadržaj

Iterative Process of Group Decision Making using Fuzzy Influence Diagrams

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In this paper, the use of influence diagrams is extended to the group decision making using fuzzy logic, sequential approach and multi-criteria evaluation. Instead of classical Bayesian networks using conditional probability tables that are often difficult or impossible to obtain, a verbal expression of probabilistic uncertainty, represented by fuzzy sets is used in this approach. This inference engine is illustrated through the assessment of risk caused by improper drug storage in pharmaceutical cold chain by the group of experts in the iterative assessment process.

Unlike standard group decision making procedure where the group consensus relies on the principle of majority, the risk assessment process outlines some behavioral characteristics that are opposite to this principle. Many papers have led to the conclusion that tendency for group decisions to be riskier than the average decision made by individuals exists and is referred to as risky shift. The predicted results were first noted in 1961 and similar results have been obtained since [1]. This shift is an example of a broader result of group decision-making called group polarization. In [2], findings that groups communicating via computer produce more polarized decisions than face-to-face groups are elaborated. This research is focused on influence diagrams and proposes their extension using fuzzy logic and multi-criteria evaluation. The fuzzy logic is introduced in a twofold manner: via fuzzy probability values expressed linguistically, and via fuzzy random variables. Instead of classical

Bayesian networks, a verbal expression of probabilistic uncertainty, represented by fuzzy sets is used in this approach. Decision making brings the danger that some solutions will not be well accepted by some experts in the group [3]. To overcome this problem, it is advisable that experts carry out a consensus process, where they discuss and negotiate in order to achieve a sufficient agreement before selecting the best alternative [4]. A comprehensive presentation of the state of the art of all known consensus approaches is given in [5], with the focus on the soft consensus approach. The method of risk assessment is illustrated on an example taken from the supply chain management of pharmaceutical cold chain - pharmaceuticals that must be distributed at temperature between 2 and 8 C [6]. Results presented in case studies proved that this new form of description - fuzzy influence diagram, that is both a formal description of the problem that can be treated by computers and a simple, easily understood representation of the problem can be successfully implemented for various class of risk analysis problems in complex systems.

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Algorithms for Imputation of Missing SNP Genotype Data

Aleksandar Mihajlović

The work presented here was motivated by the urgent need for computational methods to supplement the task of modern genomics research; discovering faulty genes, comparing them and comparing their abnormal effects with the effects of their well-functioning variants. By comparing SNP genotype sequences of healthy with diseased individuals, in what is known as an association study, researchers are able to create a rough map of the location(s) of genomic regions that are unique for the diseased individuals, i.e. which chromosome and where on the chromosome are they located. This information is summarized in Manhattan plot diagrams. This information gives researchers a starting point in their endeavor to understand and cure the disease. Association studies require enormous amount of SNP genotype data from a very large patient and control group of cohorts. Computing machines are utilized in the data analysis procedures of association studies due to the large number of data values considered. In these studies data completeness and data correctness are crucial. The sensitivity of genetic material, and accuracy of the SNP typing machines result in typed SNP genotype data being compromised, leading to missing values in SNP genotype reads. Any missing data values presented within the SNP genotype data set can diffuse the data analysis and cause incoherent Manhattan plots. There are several mechanisms for resolving the missing SNP genotype value problem. The best methods are imputation methods which probabilistically remodel entire data sets, and test estimate values for best fit. Several imputation methods will be analyzed and potential ideas on how to improve their performance will be presented.

A Propositional Linear Time Logic with Time Flow Isomorphic to ω^2

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Primarily guided with the idea to express zero-time transitions by means of temporal propositional language, we have developed a temporal logic where the time flow is isomorphic to ordinal ω^2 (concatenation of ω copies of ω). If we think of ω^2 as lexicographically ordered $\omega \times \omega$, then any particular zero-time transition can be represented by states whose indices are all elements of some $\{n\} \times \omega$. In order to express non-infinitesimal transitions, we have introduced a new unary temporal operator $[\omega]$ (ω-jump), whose effect on the time flow is the same as the effect of $\alpha \mapsto \alpha + \omega$ in ω^2. In terms of lexicographically ordered $\omega \times \omega$, $[\omega]\phi$ is satisfied in $\langle i, j \rangle$-th time instant iff ϕ is satisfied in $\langle i+1, 0 \rangle$-th time instant. Moreover, in order to formally capture the natural semantics of the until operator $\mathtt{U}$, we have introduced a local variant $\mathtt{u}$ of the until operator. More precisely, $\phi \mathtt{u} \psi$ is satisfied in $\langle i, j \rangle$-th time instant iff ψ is satisfied in $\langle i, j+k \rangle$-th time instant for some nonnegative integer k, and ϕ is satisfied in $\langle i, j+l \rangle$-th time instant for all $0 \leqslant l < k$. As in many of our previous publications, the leitmotif is the usage of infinitary inference rules in order to achieve the strong completeness.

Sequent calculus for logic with high probabilities

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An extensive survey of probability logic development is given by Z. Ognjanović, M. Rašković and Z. Marković (2009). The various aspects of the probabilistic versions of inference rules were considered recently by many authors (see T. Hailperin (1984), A. M. Frisch and P. Haddawy (1993), and C. G. Wagner (2004)). A particularly important case is when the probabilities appearing in the rules are close to 1. It seems that an idea introduced by P. Suppes (see P. Suppes (1966), and C. G. Wagner (2004)) may be very fruitful and provides an unexpectedly elegant system. In this work we develop a system of probability logic $\mathbf{LKprob}(\varepsilon)$ based on combination of Gentzen's sequent calculus and Suppes' approach. For any formulae sequences Γ and Δ, and any $n \in \mathbf{N}$, such that $[1 - n\varepsilon, 1] \subseteq [0, 1]$, $\Gamma \vdash^n \Delta$ presents the form of sequent, with the intended meaning that the probability of truthfulness of the sequent $\Gamma \vdash \Delta$ belongs to the interval $[1 - n\varepsilon, 1]$. The system $\mathbf{LKprob}(\varepsilon)$ has the following two forms of the inference rules. The rules which do not change the probability of hypothesis, e.g.

$$\frac{\Gamma A \vdash^n B \Delta}{\Gamma \vdash^n A \to B \Delta} (\vdash \to)$$

and the rules enlarging the interval of hypotheses, e.g.

$$\frac{\Gamma \vdash^n A \Delta \quad \Pi B \vdash^m \Lambda}{\Gamma \Pi A \to B \vdash^{m+n} \Delta \Lambda} (\to \vdash)$$

Our system $\mathbf{LKprob}(\varepsilon)$ can be considered a modification of Gentzen's original sequent calculus $\mathbf{LK}$ for the classical propositional logic making possible to manipulate with sequents labelled with probability intervals.

AMS 2000 Mathematics Subject Classification: 03B48, 03B50, 03B05, 03B55.

Key words: consistency; cut–elimination; probability; soundness; completeness.

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Application of water surplus distribution in assessing meteorological drought

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The main objective of this study is to apply distributions of water surplus in assessing meteorological drought and to determine the best fitting distribution to standardize the Water Surplus Variability Index (WSVI) that is used to characterized drought. Five distributions which are considered are 3-parameter gamma, 3-parameter log-logistic, logistic, 3-parameter lognormal and general extreme values distributions. The estimation of parameters of these distributions is determined using the L-moment. The adequacy of the distributions based on parameter estimates are evaluated using goodness-of-fit tests. When the goodness-of-fit results for these distributions are compared, it is found that, the performance of the logistic distribution is better than the performance of other distributions. For that reason, the logistic distribution is selected for standardizing the water surplus series to obtain the WSVI.

Drought is a natural hazard, which is characterized by the lack of precipitation and classified as meteorological, agricultural, hydrological and socio-economic drought [1, 2]. It is represented using drought indices that facilitate identification of drought intensity, duration and spatial extent. Adequate estimation of drought characteristics can be helpful for planning the efficient use of water resources, hydroelectric and agricultural production. Therefore, a variety of indices for detecting and monitoring droughts has been developed such as SPI (Standardized Precipitation Index) [3], PDSI (Palmer Drought

Severity Index) [4], RDI (Reconnaissance Drought Index) [5], SPEI (Standardized Precipitation Evapotranspiration Index) [6]. The WSVI (Water Surplus Variability Index) [7] is the novelty drought index, which is used to describe meteorological drought. It has clear and simple calculation procedure based on monthly values of precipitation and reference evapotranspiration. The main objective of present research is to use probability distributions of water surplus in assessing meteorological drought. The obtained results suggested the selection of the logistic distribution for standardizing the water surplus series to obtain the WSVI. Parameters of the logistic distribution were obtained following the L-moment procedure [8].

1 Acknowledgment

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Formal Description of the Chord Protocol using Isabelle/HOL Proof Assistant

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A decentralized Peer-to-Peer system (P2P) involves many peers (nodes) which execute the same software, participate in the system having equal rights and might join or leave the system continuously. In such a framework processes are dynamically distributed to peers, and there is no centralized control. P2P systems have no inherent bottlenecks and can potentially scale very well. Moreover, since there are no dedicated nodes that are critical for functioning of the systems, those systems are resilient to failures, attacks, etc.

P2P systems are frequently implemented in a form of overlay networks, a structure that is totally independent of the underlying network that is actually connecting devices. Overlay network represents a logical look on organization of the resources. usually, the nodes of an overlay network form a well defined structure and one of the most importan aspects for proper functioning of that system is to maintain the correct organization of the resources in the

desired structure (structural correctness). Some of the overlay networks are realized in the form of Distributed Hash Tables (DHT) that provide a lookup service similar to a hash table; $(key, value)$ pairs are stored in a DHT, and any participating peer can efficiently retrieve the value associated with a given key. Responsibility for maintaining the mapping from keys to values is distributed among the peers, in such a way that any change in the set of participants causes a minimal amount of disruption. It allows a DHT to scale to extremely large number of peers and to handle continual node arrivals, departures, and failures. The Chord protocol is one of the first, simplest and most popular DHTs.

Our aim is to verify correctness of the Chord protocol using Isabelle/HOL proof assistant. This is motivated by the obvious fact that it is difficult to reproduce errors in concurrent systems or just by program testing.

The specification of the Chord presented in this paper has been written following the implementation of the high level C++-like pseudo code from, and the Abstract State Machine specification given in.

Bisimulation in Interpretability Logic

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The interpretability logic **IL** results from the provability logic **GL** (Gödel–Löb), by adding the binary modal operator $\triangleright$. The language of the interpretability logic contains propositional letters $p_0, p_1, \ldots$, the logical connectives $\wedge, \vee, \rightarrow$ and $\neg$, and the unary modal operator $\square$ and the binary modal operator $\triangleright$. The axioms of the interpretability logic **IL** are: all tautologies of the propositional calculus, $\square(A \rightarrow B) \rightarrow (\square A \rightarrow \square B)$, $\square A \rightarrow \square\square A$, $\square(\square A \rightarrow A) \rightarrow \square A$, $\square(A \rightarrow B) \rightarrow (A \triangleright B)$, $(A \triangleright B \wedge B \triangleright C) \rightarrow (A \triangleright C)$, $((A \triangleright C) \wedge (B \triangleright C)) \rightarrow ((A \vee B) \triangleright C)$, $(A \triangleright B) \rightarrow (\Diamond A \rightarrow \Diamond B)$, and $\Diamond A \triangleright A$, where $\Diamond$ stands for $\neg\square\neg$ and $\triangleright$ has the same priority as $\rightarrow$. The deduction rules of **IL** are modus ponens and necessitation.

There are several kinds of semantics for the interpretability logic. The basic semantics are Veltman models. D. de Jongh and F. Veltman prove the completeness of **IL** w.r.t. Veltman models (see [4]). We think that there are two main reasons for other semantics. First one is a complexity of the proofs of arithmetical completeness of **IL**. Second, the characteristic classes Veltman frames of some principles of interpretability are equal. Generalized Veltman models were defined by D. de Jongh. We use generalized Veltman models in [12] and prove independences between principles of interpretability.

If we want to study a correspondence between Kripke models we can consider an isomorphism or an elementarily equivalence. If we want to study a "weaker" correspondence we can consider a bisimulation. J. Van Benthem defines bisimulations of Kripke models. A. Visser defines in [8] a notion of bismulation between two Veltman models. A. Berarducci in [1] uses bisimulation for proving arithmetical completeness. We define a notion of bisimulation between two generalized Veltman models in [13], and prove Hennessy–Milner theorem for generalized Veltman semantics. We study various kinds of bisimulations of Veltman models in [11]. In [10] bisimulation quotients of generalized Veltman models are considered. We prove in [14] that there is a bisimulation between Veltman model and generalized Veltman model. The existence of a bisimulation in general setting is an open problem.

The correspondence theory is the systematic study of the relationship between modal and classical logic. Bisimulations and the standard translation

are two of the tools we need to understand modal expressivity. J. van Benthem's characterization theorem (cf. [7]) shows that modal languages are the bisimulation invariant fragment of first–order languages, and it is established by classical methods of first-order model theory. The preservation theorems (cf. [5]) characterize a correspondence between semantic conditions of a class of models and logical formulas, too. However, the preservation property is usually much less significant than the corresponding expressive completeness property that any formula satisfying the semantic invariance condition is equivalent to one of the restricted syntactic form. D. Janin and I. Walukiewicz prove that a formula of monadic second–order logic is invariant under bisimulations if, and only if, it is logically equivalent to a formula of the μ–calculus. E. Rosen prove that the characterization theorem holds even in restriction to finite structures. A. Dawar and M. Otto in [3] investigate ramifications of van Benthem's characterization theorem for specific classes of Kripke structures. They study in particular Kripke modal classes defined through conditions on the underlying frames. Classical model theoretic arguments as saturated models and ultrafilter extensions do not apply to many of the most interesting classes. In the proofs the game–based analysis is used. V. Čačić and D. Vrgoč define in [2] a bisimulation game between Veltman models and prove the basic properties. T. Perkov and M. Vuković prove in [6] the following modal invariance theorem for **IL**: *A first–order formula is equivalent to standard translation of some formula of interpretability logic with respect to Veltman models if and only if it is invariant under bisimulations between Veltman models.* In the proof we use a finite approximation of bisimulation game.

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Possible applications of the probability logic in social sciences

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Some sentences could be allocated to number from interval $[0, 1]$ thus representing the degree of probability of their truthfulness. Such allocation can sometimes be inconsistent. If inconsistent, it is necessary to determine the distance of that allocation from being consistent. In this manner a new method of drawing conclusions can be developed, which we will try to illustrate with an example from Serbian history.

Some applications of nonstandard analysis to functional equations

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In this talk we will present and discuss some nonstandard methods applied for solving certain functional equations. In particular, we will prove that all measurable solutions of the considered equations are continuous. Some applications will be given.

Probability measures effect of the use of computers in education

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Associative thinking and recognition in humans is based on a comparison of previously experienced and remembered a situation with the currently ongoing. In this case you are thinking how to get new, recognized as similar evokes some of the previously known situations and thus to conclude that before in such a situation verified. Of primary importance in the study of associative thinking is form-pattern. It represents a set of data describing a situation, event or occurrence. Principles of associative inference can be implemented using pattern recognition methods that have been developed in an attempt to get all mathematical and logical formulated and calculated. The use of computers in education is a situation that was very interesting for our work and research.

Many people think and conclude as follows: A computer is smart and he knows everything. The computer knows how to think and for a short time has many correct answers to a single question. Are those the correct statements? What happens when accuracy is determined? How do we conclude with the presence of uncertainty?

Through further practical work with students, we explored and sought answers to the following questions: Is the amount of time that students spent with the computer, dedicated to learning the school curriculum? How many different types and methods of teaching contribute to facilitate learning, memorizing and mastering the process?

Initial questions launched many other of whom we stayed with those who have become hypotheses: Individual differences have a crucial importance in the choice of learning methods and success of learning. There is no universal method good (quality) learning. A large number of students do not use computers and information technology for the purpose of school learning (studying). Achieved success in education does not differ significantly

with respect to the use of computers and information technology in learning. Students believe that computer and information technology are more applicable in the teaching of subjects that belong "natural" sciences, as well as the visualization of mathematics. Students believe that computer and information technology have always be performed at systematization of the teaching material.

At the end, when probability measures of feasibility hypothesis are certain, the effects (conclusions) of using computers in education have discussed.

k Nearest Neighbor decision in medical diagnostic

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Abstract. The aim of this study is to examine the efficiency of k-nearest neighbor classifier in the diagnosis of systemic disease. The used technique is aims that on the basis of so-called test patients classified into three groups, patients suffering from LE (Lupus), Sjogren (systemic Sjogren) and PSS (progressive systemic sclerosis), determines which class of diseases newly registered patient belongs to. The efficiency of the algorithm is tested on the basis of the patients of the Clinical Centre in Belgrade.

I Introduction

Developing of new techniques the better decision-making, especially in medicine, is considered to be important because they contribute to the development of science and practice, especially the models that are implemented to support decision making in medical diagnosis, which increases the speed of decision-making and reduce the cost of testing. The objective is to find a model that optimizes the input data to a small number of data needed for the right decision.

When detecting certain diseases a physician must perform as many analyzes as possible on the patient to come to the conclusion of which illness the patient is suffering [1]. A larger number of test parameters increases the material costs and the time duration until the final diagnosis. The model, to optimize the parameters of conclusion and by reducing them reduces the time and cost, but not to reduce the accuracy of diagnosis, is of great importance [2].

The goal of this research is to show the k-nearest neighbor classifier implemented in the C # programming language that aims to perform this optimization. The model was tested on the data from the Clinical Centre in Belgrade.

The work is divided into several sections. In Section II an overview of the mathematical model is provided. Its success in the diagnosis of systemic diseases is presented in section III results of data processing. The conclusion
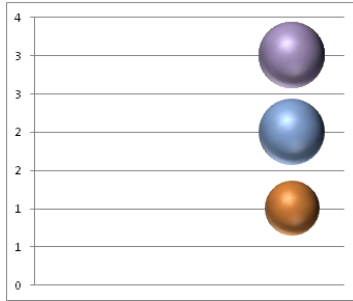
28

Figure 1: Display of base patients divided into three groups on the basis of systemic diseases: 1-patients with a diagnosis of LE, 2-patients with the diagnosis of Sjogren, 3 patients diagnosed with PSS

and further research in order to improve the proposed model are given in section IV.

II Mathematical model

kNN (K-nearest neighbor algorithm) is one of the simplest and most commonly used classification methods. kNN is a fast classification method because it does not have any training stages unlike most others. In this method, k training samples are obtained for each test sample using a distance measure like Euclidean distance (1). The class of the test sample is classified using majority voting within these k samples obtained [3]. Euclidean distance can be given as follows:

$$d = \sqrt{\Sigma_{i=1}^{N}(x_i - y_i)^2} \quad (1)$$

where x and y are test and training samples respectively, i is the index of the feature within x and y, and N is the number of the features [4].

III Results and data

The model was tested on a database of 33 subjects with a history of 3 systemic diseases: LE (11 patients), Sjogren (14 patients) and PSS (8 patients) (Figure 1). The algorithm was studied on 29 patients for whom we know which illness they are suffering from (data about the disease was not used during testing, only the data of the performed test analyses have been used). As the test, patients, 4 patients were used for whom the system was supposed to give the diagnosis. The diagnoses given by the system were compared with the diagnoses that given by an expert (doctor).

The implemented algorithm randomly selects 10 patients LE disease, Sjogren 13 patients and 7 patients PSS as a base from which to learn. The remaining three patients, one for each diagnosis, are treated as "new" pa-

tients for whom it is necessary to establish the diagnosis. The system has demonstrated success of providing a diagnosis in 98

The testing of the success of diagnosis when the patient data are reduce and the testing is performed on a less set of data (from the initial 25 test data was also performed reduced to 12). In this case, the system gave the success of the predictions in 95% of cases.

IV Conclusion and future research

With the advancement of technology, decision support systems leads to significant reliable and practical results. This paper aims to justify the application of the proposed algorithm in the diagnosis. The results show a high degree of reliability while making the diagnosis. While reducing of the parameters of the analysis that level does not decrease rapidly. Some of the future research in the improvement of the system will go in the direction of applying Bayesian inference an upgrading of KNN-a.

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Estimation Methods in Bivariate Autoregressive Time Series of Counts

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We present a general structure of a model for bivariate autoregressive time series of counts. The structure of the model is similar to the standard autoregressive (AR) models of order one. AR-recursion is achieved by using the thinning operator. Also, dependency between processes is introduced through AR components. The estimation methods for unknown parameters of the model are presented in details where we investigate conditional least square, Yule-Walker, generalized method of moments and conditional maximum likelihood method. Asymptotical properties of the methods are discussed. Efficiency of all methods are tested on a simulated data sets.

Time series of counts appear in many fields of science, so their modelling is an interesting topic for many researchers. Since the classic ARMA models appear inappropriate, a different approach based on thinning operator is investigated. The first models of this type were introduced by [McKenzie (1985)] and [Al-Osh and Alzaid (1987)]. The process with these models is dependent only on its previous values. When there is dependency between two processes bivariate models prove to be a better solution. The bivariate models that we are going to discuss are composed of two components: survival and innovation. The survival component defines autoregressive dependance while the innovation component represents the influence to the process from external factors. The dependance between the two processes is introduced through the survival part.

We discuss BINAR models of the form

$$\boldsymbol{Z}_n = \boldsymbol{A}_n \star \boldsymbol{Z}_{n-1} + \boldsymbol{e}_n \tag{1}$$

where $\boldsymbol{Z}_n$ and $\boldsymbol{e}_n$ are two-dimensional random vectors and $\boldsymbol{A}_n$ is 2×2 matrix whose elements are random variables. The thinning operator, $\star$, might be binomial or negative binomial one. Binomial thinning operator, denoted by $\circ$, is defined as $\alpha \circ X = \sum_{i=1}^{X} B_i$ where $\{B_i\}$ is a sequence of iid Bernoulli random variables with parameter α. Negative binomial thinning operator, denoted with $*$, is defined as $\alpha * X = \sum_{i=1}^{X} G_i$ where $\{G_i\}$ is a sequence of iid geometric random variables with parameter $\frac{\alpha}{1+\alpha}$. The models of the form (1) are investigated in [Ristić et al.(2012)] and [Nastić et al.(2014)].

Our aim is to present different methods for the estimation of unknown parameters. First, we discuss the conditional maximum likelihood method and derive a close form solution. Using the results from [Tjøstheim(1986)] we prove asymptotic properties of the estimates. Further, we derive Yule-Walker estimates and prove asymptotic equivalence between these two estimates. We discuss in details the generalized method of moments and also investigate asymptotic behavior of the estimates. Finally, we give directions for derivation of the conditional probability mass function of the process (1) and state the conditional maximum likelihood method. The tests on a simulated data sets are conducted in order to show efficiency of these methods. The methods are tested on the samples of different size in order to prove convergence of the estimates with the growth of the sample size. Different tests with respect to parameter values are conducted.

Acknowledgment

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Computational approach to conjugation

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Lambek's production grammar is a simple computational method for generating conjugational forms (inflected forms) of simple tenses step by step. The mathematical structure involved is the *finitely generated partially ordered semi-group*, also called "*semi-Thue system*" in mathematics, "*rewriting system*" in computer science and "*production grammar*" or Chomsky's Type zero language ([1]) in linguistics.

With each verb V, there is associated a $p \times n \times m$ matrix of conjugational verb-forms, $C_{ij}^k(V)$. The index $i = 1, ... n$ represents the (simple) tense, and the index $j = 1, ..., m$ represents the person-number and the index $k = 1, ..., p$ represents the pattern. Only simple tenses are considered here, whereas participles and compound tenses are disregarded. A production grammar, in general, provides a method for calculating $C_{ij}^k(V)$ for a given (i, j, k, V).

This method has been applied to English, French ([3]), Latin ([4]), Serbian and Croatian ([2]), Hebrew (Biblical), Turkish, Arabic, and Japanese, so far.

We shall present a simple production grammar developed in [2] for generating 24 verb forms of Serbian and Croatian verb. In Serbian, as in Latin, each conjugational verb form can be regarded as one-word sentence. Hence, this production grammar is extended to sentence generation.

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A Probabilistic Extension of Abstract Dialectical Frameworks

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Within the last decade, argumentation has emerged as a central field in Artificial Intelligence. One of its subfields is the abstract argumentation, in which only the relations between arguments are taken into account when evaluating a certain scenario. Consequently, the actual contents of the arguments do not play a role. The most simple tools for abstract argumentation are Dung's argumentation frameworks (AFs). Although they are quite powerful, for many applications Dung's AFs appear too simple in order to conveniently model all aspects of an argumentation problem. This has led to the development of a variety of their enrichments. One of AFs shortcomings is the lack of handling levels of uncertainty, an aspect which typically occurs in domains, where diverging opinions are raised. This calls for augmenting simple AFs with probabilities and among the possible solutions are the probabilistic frameworks due to Li,Oren and Norman and Hunter. A probabilistic argumentation framework (PrAF) enriches the Dung's AFs by probabilities assigned to both arguments and conflicts (in the first case) or just arguments (in the latter). They are then used to calculate probabilities of AF–subgraphs of the given PrAF via the *independency* assumption. The probability for a given set S to be a PrAF extension (with respect to a particular semantics) is obtained from the sum of the probabilities of the subgraphs for which S is such an AF extension. Consequently, in many ways the uncertainty layer is independent of the underlying semantics and the framework itself, which is considered as one of its biggest strengths.

Including argument uncertainties had proved to be useful concept, therefore, it is not surprising that a generalization of the Dung's framework should also consider incorporating them. Adding probabilities to conflicts allowed us to analyze cases where we were, for example, not sure that a conflict really occurs due to arguments imprecision, incompletion, or we had doubt if the party would really like to carry out the attack. Due to the independency of

the probability layer, it was claimed that it can be easily shifted to any of the Dung's extensions, such as bipolar frameworks. However, it is somehow natural to expect that the probability of a relation in case of support can be interpreted in further ways. While we can doubt if e.g. an argument a will commit to support b, we can at the same time ask ourselves whether b really requires a (or only a) to hold. A simple situation is when e.g. a student goes to a conference relying on the financial support of his university, but also has to take into account that the received amount might not be sufficient, in case of which he will have to cover certain costs from his own pocket. Thus, the conditions for accepting an argument might be uncertain. Those two interpretations of dependencies are modelled in exactly opposite ways in a probabilistic setting– assuming that the support relation does not occur, in the first case b would not be acceptable, while in the latter there would be no contraindications. Consequently, generating the subgraphs in the PrAF manner would allow us to model only one of the scenarios at a time. Thus, new relations pose new challenges also in the case of probabilities and this research should not be dismissed so easily.

Another major issue of AFs is the fact that they permit only binary conflict. In order to address this problem, the abstract dialectical frameworks (ADFs for short) were developed. They make use of so–called acceptance conditions in order to be able to express arbitrary relations. In this paper we will show that the freedom they give us make ADFs a good base for a probabilistic framework that would allow us to model various interpretations of uncertain relations, not just limited to attack or support. We create a framework joining both uncertainty and relation research, which properly generalizes both ADFs and the existing probabilistic extensions of AFs. Moreover, we will also provide an example in which it permits us to partially relax the independency assumptions made in PrAFs. Finally, we will explain various other methods of incorporating uncertainties that we had considered and give pointers for future work.

Infinitary axiomatization of common knowledge

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An infinitary axiomatization of common knowledge will be proposed and the proof of its extended completeness.

Dealing with satisfiability problem in default logic using Bee-colony optimization

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This paper presents a new method for default reasoning, as a form of non-monotonic reasoning. Such a reasoning was first presented by Reiter in his papers [9,10]. In these papers term "default reasoning" is used to denote the process of arriving to conclusions based upon patterns of inference of the form "In absence of any information to the contrary, assume...". Benferhat, Saffiotti and Smets in [2] say that default reasoning is based on drawing conclusions from a set of general rules which may have exceptions, and a set of facts representing the available information, which is often incomplete. Since 1980 various systems have been developed for reasoning with default logics and a detailed overview of these systems is given in the paper [2]. Our main attention will be directed to the system, denoted by **P**, the basics of which is given by Kraus, Lehmann and Magidor in the paper [6].

In this paper we discus default reasoning in the system **P** using results published by Rašković, Marković and Ognjanović in [8]. In their work they used the logic with approximate conditional probabilities to model default reasoning. This logic enriches the propositional calculus with probabilistic operators which are applied to propositional formulas: $CP_{\geqslant s}(\alpha, \beta)$, $CP_{\leqslant s}(\alpha, \beta)$ and $CP_{\approx s}(\alpha, \beta)$, with the intended meaning "the conditional probability of α given β is at least s", "at most s" and "approximately s", respectively. They showed that, if we restrict attention only to formulas of type $CP_{\approx 1}(\alpha, \beta)$, the resulting system coincides with the system **P** when we work only with the finite sets of assumptions. Satisfiability problem in logic with approximate

conditional probabilities can be reduced to the linear programming problem, as showed in [8] and for this we have developed a solver and the results obtained are shown in [11]. For the application of this solver to a default reasoning we have to make some significant adjustments.

Since the main objective of default reasoning is to examine whether it is possible to derive B from assumptions $A_1, A_2, \ldots, A_n$, we reduced this to the problem of satisfiability of the following tree sets of formulas $\Delta = \{A_1, A_2, \ldots, A_n\}$, $\Phi_1 = \Delta \cup \{B\}$ and $\Phi_2 = \Delta \cup \{\neg B\}$, where $A_1, A_2, \ldots, A_n, B$ are default formulas. Satisfiability of set Δ will only verify consistency of the base. Satisfiability of set Φ_1 will show that default B is not in contradiction to the base. Unsatisfiability of the set Φ_2 implies that B is a consequence of the coresponding set of assumptions. To solve the obtained linear programming problem we used Fourier-Motzkin elimination procedure and Bee-colony optimization (BCO) as a meta-heuristics approach, since Fourier-Motzkin elimination procedures did not give us satisfiable results.

Bee-colony optimization (BCO) is a stochastic, random-search technique that belongs to the class of population-based algorithms. This technique showed very good performance in solving hard combinatorial optimization problems [7,4,3]. BCO uses an analogy between the way in which bees in nature search for food, and the way in which optimization algorithms search for an optimum of the given combinatorial optimization problems. In our implementation, we used the improved variant of BCO, denoted by BCOi and proposed in [5].

For testing purpose we selected 18 examples from literature [2,6,1], which gave us 60 sets whose satisfiability we tested. In these examples, there are two types: one in which $\Delta |\!\!\sim \alpha \rightarrowtail \beta$ and other where $\Delta \not|\!\!\sim \alpha \rightarrowtail \beta$ and $\Delta \not|\!\!\sim \alpha \rightarrowtail \neg\beta$.

All probabilities and constants in the obtained systems of linear inequalities are from Hardy field $\mathbb{Q}(\varepsilon, K)$. This means that they are represented as rational functions depending of ε and K with double precision real coefficients, i.e,

$$p = \frac{\sum_{j=0}^{e} \left(\sum_{k=0}^{l_j} h_{jk} K^k \right) \varepsilon^j}{1 + \sum_{j=1}^{e} \left(\sum_{k=0}^{l_j} h_{jk} K^k \right) \varepsilon^j}$$

The Fourier-Motzkin method in most cases can not complete the calculation, and the method BCOi is successful with significantly lower running time. Reason for this failure of Fourier-Motzkin method is the exponential growth in the number of inequalities during elimination of variables from the system.

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On probable conditionals

Zvonimir Šikić

It is tempting to transfer the properties of conditionals to probable conditionals. We prove that such transfers would be inappropriate.