**Spectra of Graphs and Applications**
**(SGA 2016)**

# Book of Abstracts

Editors:
Dragan Stevanović
Tatjana Davidović

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Editors:
Dragan Stevanović
Tatjana Davidović

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## PREFACE

Dear Colleagues,

It is my pleasure to welcome you to Belgrade for the conference on spectra of graphs and their applications. The conference is organized in honor of the 75th birthday of Dragoš Cvetković, a pioneer of spectral graph theory, author of classical monographs in the field, and founder of the Serbian school of spectral graph theory, together with his early PhD students Ivan Gutman and Slobodan Simić.

I sincerely hope that you will enjoy Belgrade's friendly atmosphere and expand your network of spectral graph fellows in the next few days. I also have to mention here that organisation of the conference is supported in part by Ministry of education, science and technological development of the Republic of Serbia and Serbian Academy of Sciences and Arts.

On behalf of the
Scientific and Organizing Committees,

Dragan Stevanović

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D. Cvetković and H. Sachs

D. Stevanović, N. Abreu, and M. Nóvoa

**Dragoš Cvetković - Biography**

Dragoš Cvetković was born on March 6, 1941, in Sremska Mitrovica (Serbia, Former Yugoslavia). He graduated in Electrical Engineering at the University of Belgrade in 1964, and in 1971 the University awarded him a doctorate in Mathematics. Since graduating he has worked in the Department of Mathematics, Faculty of Electrical Engineering, University of Belgrade. In 1986 he became a Full Professor in Mathematics.

*Present and Previous Positions*

* since 2010 Chairman of the Scientific Council of the Mathematical Institute of the Serbian Academy of Sciences and Arts,

* 2012 - 2016 Chairman of the Department of Mathematics, Physics and Geo-Sciences, Serbian Academy of Sciences and Arts,

* 2006-2008 Research Full Professor, Mathematical Institute of the Serbian Academy of Sciences and Arts,

* 1986-2006 Full Professor in Mathematics, Department of Mathematics, Faculty of Electrical Engineering, University of Belgrade, in retirement since 2006,

* 1979-1986 Associate Professor, Department of Mathematics, Faculty of Electrical Engineering, University of Belgrade,
* 1985-1986 Carnegie Research Fellow, Department of Mathematics, University of Stirling, Scotland, UK,
* 1973-1979 Assistant Professor, Department of Mathematics, Faculty of Electrical Engineering, University of Belgrade,
* 1975-1976 Postdoctoral Fellow, Department of Mathematics, Technological University Eindhoven, The Netherlands,
* 1964-1973 Collaborator, Department of Mathematics, Faculty of Electrical Engineering, University of Belgrade.

For shorter periods of time Cvetković was a guest of the University of Manitoba, Winnipeg, Canada, a visiting professor at Technische Hochschule Ilmenau, Ilmenau, Germany, and at many other places.

*Fields of Scholarship*: Discrete mathematics, Graph theory, Combinatorics, Linear algebra, Combinatorial optimization, Artificial intelligence.

He has published over 200 scientific papers and over 70 books (monographs and textbooks, including repeated editions) in graph theory, combinatorics, linear algebra, artificial intelligence, etc.

Dragoš Cvetković's monograph *Spectra of Graphs* (co-authors: M.Doob, University of Manitoba, Winnipeg, and H.Sachs, Technische Hochschule Ilmenau, Ilmenau) had three English editions (1980, 1982, 1995) and a Russian edition (1984).

During the period 1980-1984 he implemented (together with a group of collaborators) the interactive programming system "Graph", an expert system for graph theory.

He has attended numerous international conferences on graph theory and related fields; he was an invited speaker on many occasions, and also an organizer or chairman of the programming committee for some of the conferences.

Cvetković was an invited speaker at the following scientific conferences (a selection):

* Algebraic Methods in Graph Theory, Szeged, Hungary, 1978,
* Algebraic Combinatorics, Stirling, Scotland, 1986,
* Workshop on Algebraic Graph Theory, Edinburgh, Scotland, 1993,
* 27th Australian-Asian Conference for Combinatorial Mathematics and Combinatorial Computing, Newcastle, Australia, 2002,
* Aveiro Workshop on Graph Spectra, Aveiro, Portugal, 2006,
* SGT in Rio, Workshop on Spectral Graph Theory with Applications on Computer Science, Combinatorial Optimization and Chemistry, Rio de Janeiro, Brazil, 2008,
* Applications of Graph Spectra to Computer Science, Barcelona, Spain, 2012.

Cvetković was an organizer of the following conferences:

* Workshop on Algebraic Graph Theory, Edinburgh, Scotland, 1993,
* European Workshop on Algebraic Graph Theory, Edinburgh, Scotland, 2001.

Cvetković was the chairman of programming committees for the following conferences:

* Aveiro Workshop on Graph Spectra, Aveiro, Portugal, 2006,
* SGT in Rio, Workshop on Spectral Graph Theory with Applications on Computer Science, Combinatorial Optimization

and Chemistry, Rio de Janeiro, Brazil, 2008.

*Honours and Awards*

* 1985 Corresponding member of the Serbian Academy of Sciences and Arts,
* 1994 Full member of the Serbian Academy of Sciences and Arts,
* 1991 - 1995 and 1997 - 2001 Honorary Professor at the University of Stirling, Stirling, Scotland,
* 1997 - 2001 The President of Yugoslav Society for Applied and Industrial Mathematics,
* 2005 Member of the International Academy for Mathematical Chemistry,
* 2011 Special Issue of Linear Algebra and Its Applications in honor of Dragoš Cvetković,
* 2012 Member of the Academia Europaea (London).

He has several times been engaged as an advisor to the Serbian Ministry of Science.

Dragoš Cvetković is a member or former member of the Editorial Boards of four Serbian mathematical journals, and Editor-in-Chief for one of them. Cvetković also belongs to the Editorial Board of *Ars Mathematica Contemporanea.* He was one of the Editors of the journal *Linear and Multilinear Algebra* and a member of the Editorial Board of the *Journal of Graph Theory* for some time. Cvetković has been Guest Editor for a special issue of *Linear and Multilinear Algebra* and for three special issues of *Linear Algebra and Its Applications.*

Professor Cvetković has had 8 doctoral students, among them Ivan Gutman, Slobodan Simić and Dragan Stevanović.

# Conference program

**Wednesday, May 18, Club of SASA, 2nd floor**

Registration, 16 - 19
Welcome cocktail, 17 - 19

**Thursday, May 19, Room 2, 3rd floor**

Registration, 8:30 - 9
Opening, 9 - 9:30

Invited talk, 9:30 - 10:20

- W. Haemers, *Spectral characterizations of graphs*,

**Break, 10:20 - 10:30**

Section 1, 10:30 - 11:50

- D. Stevanović, *Comparing closed walk counts in trees consisting of three paths*,
- M.A. Fiol, *Equivalent characterizations of the spectra of graphs and applications to measures of distance-regularity*,
- M. Fürer, *Locating the eigenvalues for graphs of small clique-width*,
- E.O.D. Andriantiana, *On counting based graph invariants and segment sequences of trees*,

**Coffee break, 11:50 - 12:10**

Section 2, 12:10 - 13:30

- D. Cvetković, *Three examples of a ground-breaking impact of the variable neighborhood search on investigations in graph theory*,

- E. Kaya, *On the Co-PI spectral radius and the Co-PI energy of graphs*,
- A.D. Maden, *Improved bounds for the normalized Laplacian energy of graphs*,
- E. Zogić, *Resolvent energy of graphs*,

**Lunch break, 13:30 - 15**

Invited talk, 15 - 15:50

- J. Koolen, *On graphs with smallest eigenvalue at least -3*,

**Break, 15:50 - 16**

Section 3, 16 - 17:20

- P. Hansen, *Cospectrality of graphs with respect to distance matrices*,
- C. Dalfó, *Cospectral digraphs from locally line digraphs*,
- I. Jovanović, *Constructing graphs with given spectrum and the spectral radius at most 2*,
- N. Abreu, *A Nordhaus-Gaddum upper bound to the second eigenvalue of a graph*,

**Coffee break, 17:20 - 17:40**

Section 4, 17:40 - 19

- M. Mitjana, *The Kirchhoff index of some molecular graphs*,
- M. Bradonjić, *Asymptotic laws for maximum coloring of random geometric graphs*,
- S. Sorgun, *On Laplacian spectrum of the nilpotent graphs over the rings* $\mathbf{Z}_n$,

- A.M. Encinas, *Resistive distances on networks*,

**Conference Dinner, Hotel Palace, at 20**

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- V. Trevisan, *Recent developments on Laplacian eigenvalue distribution*,

**Break, 9:50 - 10**

Section 5, 10 - 11:20

- P. Rowlinson, *Some graphs with just three distinct eigenvalues*,
- F. Belardo, *Star complement technique and the eigenbasis of -2 in signed line graphs*,
- T. Koledin, *Regular graphs with a small number of distinct eigenvalues*,
- I. Sciriha, *A nut graph lies in an extremal singular graph*,

**Coffee break**, 11:20 - 11:40

Section 6, 11:40 - 13

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- R. E. Kooij, *Inconsistencies among spectral robustness metrics*,
- V. Todorčević, *Some remarks on spectral recognition of music melodies*,

- N. Djurdjevac Conrad, *Spectral method for finding dominant graph structures: from undirected to directed graphs*,

**Lunch break**, 13 - 14:30

Invited talk, 14:30 - 15:20

- C. Helmberg, *A semidefinite programming view on spectral properties of graph Laplacians*,

**Break, 15:20 - 15:30**

Section 7, 15:30 - 16:50

- V. Vujčić, *A survey on the complexity Indices for the travelling salesman problem*,
- K. Das, *On (signless) Laplacian eigenvalues of graphs*,
- Lj. Pavlović, *Conjecture for the geometric-arithmetic index with given minimum degree*,
- X. Wang, *Eigenvector matrix of the Laplacian*,

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- D. Cardoso, *Spectral and combinatorial properties of lexicographic powers of graphs*,
- A. Krapež, *Special graphs and quasigroup functional equations*,
- B. Borovićanin, *On the maximum and minimum Zagreb indices of some classes of trees*,
- M. Rašajski, *Some notes on maximal number of cycles in reflexive cacti*,
- B. Mihajlović, *Some transformations that preserve* $\operatorname{sgn}(\lambda_2 - r)$.

# Invited Talks

## Spectral characterizations of graphs

**Willem Haemers**
Tilburg University, The Netherlands

**Abstract.** The generalized spectrum of a graph consists of the adjacency spectrum together with the adjacency spectrum of the complement. The talk will survey some recent results concerning the generalized spectrum.

This includes (1) developments towards a possible proof that almost all graph are determined by the generalized spectrum, and (2) results on Godsil-McKay switching sets for the construction of nonisomorphic graphs which are cospectral with respect to the generalized spectrum.

## A semidefinite programming view on spectral properties of graph Laplacians

**Christoph Helmberg**
Chemnitz University of Technology, Germany

**Abstract.** The Laplace matrix of a graph as well as its eigenvalues and eigenvectors appear in several rather diverse areas such as graph partitioning, Euclidean embedding problems, rigidity and the analysis of mixing rates of Markov chains. Duality in semidefinite optimization allows to develop some intuition on the relation between these applications. Our main focus will be on an appealing geometric embedding interpretation that arises when studying connections between the separator structure of the graph and eigenvectors to optimized extremal eigenvalues of the Laplacian.

## On graphs with smallest eigenvalue at least -3

**Jack Koolen**
University of Science and Technology of China, China

**Abstract.** A result of Cameron et al. states that connected graphs with smallest eigenvalue $-2$ are either generalised line graphs or have at most 36 vertices. Hoffman (1977) and Woo and Neumaier (1995) worked on graphs with smallest eigenvalue slightly smaller than $-2$. In this talk I will discuss recent work on graphs with smallest eigenvalue at least $-3$.

This talk is based on joint work with Q.Q Yang, J.Y. Yang, A. Abiad, H.K. Kim, M. Ur Rehman, Y.R. Li.

## Recent developments on Laplacian eigenvalue distribution

**Vilmar Trevisan**
Universidade Federal do Rio Grande do Sul, Brazil

**Abstract.** A fundamental question in spectral graph theory concerns the *distribution* of the eigenvalues of a graph $G$. Given a graph $G$, its Laplacian matrix $L(G)$ has been extensively studied in the literature and is important in many areas of pure and applied science.

In this talk we discuss some recent results on the spectral distribution of $L(G)$. We show, for example, that the number of Laplacian eigenvalues in $[0, 1)$ is bounded by the domination number. We illustrate how these findings led to advances in some open problems in spectral graph theory. In particular, for the classes of trees and threshold graphs, we find examples of equienergetic graphs, and determine the graph having largest Laplacian energy.

# Regular Talks

## A Nordhaus-Gaddum upper bound to the second eigenvalue of a graph

**Nair Abreu**
Universidade Federal do Rio de Janeiro,
PEP-COPPE-UFRJ Rio de Janeiro,
Brasil

**Abstract.** Since 1956, when Nordhaus and Gaddum [1] determined bounds to the sum and product of the chromatic number of a graph and its complement, several researchers have investigated upper and lower bounds to the sum and/or the product of any invariant of a graph and its referred complement. These bounds are known as Nordhaus-Gaddum upper (lower) bounds, or simply, $NG$-bounds. Recently, Nikiforov and Yan [2] have created a $NG$-bounds concerning the sum of the absolute value of an arbitrary eigenvalue of a graph. In this paper, we find another $NG$-upper bound for every tree and for every graph with girth at least 5 which improves the $NG$-upper bound given by Nikiforov and Yan for the second largest eigenvalue of a graph. Also, we determine the extremal graphs which satisfy our $NG$-upper bound.

This is joint work with André Brondani and Carla Oliveira.

**Keywords:** second large eigenvalue, adjacency matrix, Nordhaus and Gaddum inequalities.

**MSC:** 05C50, 05C05, 05C38.

**References**

[1] Nordhaus, E. A. and Gaddum, J., On complemetary graphs. AMM, 63:175-177, 1956.

[2] Nikiforov, V. and Yuan, X., More eigenvalue problems Nordhaus-Gaddum type, LAA, 451:231-245, 2014.

## On counting based graph invariants and segment sequences of trees

**Eric Ould Dadah Andriantiana**
Department of Mathematics (Pure and Applied),
Rhodes University, South Africa

**Abstract.** Given a tree $T$, a segment of $T$ is a path whose end vertices have degrees 1 or at least 3, while all internal vertices have degree 2. The lengths of all the segments of $T$ form its segment sequence.

We study, among all trees with given segment sequence or number of segments, the extremal trees that minimize or maximize various counting based topological indices, including energy and spectral moment.

This is joint work with Stephan Wagner and Hua Wang.

## Star complement technique and the eigenbasis of $-2$ in signed line graphs

**Francesco Belardo**
Department of Mathematics "R. Caccioppoli"
University of Naples "Federico II"

**Abstract.** Recently, several results from the spectral theory of graphs have been generalized to the spectral theory of signed graphs. Here, we discuss about signed line graphs, and we use the star complement technique to construct a basis for $-2$ using their root signed graphs. In other words, we present a generalization of the corresponding results known in the literature for (unsigned) graphs in the context of line graphs and generalized line graphs.

This is joint work with E.M. Li Marzi and S.K. Simić.

**MSC:** 05C50

## On the maximum and minimum Zagreb indices of some classes of trees

**Bojana Borovićanin**
Faculty of Science, University of Kragujevac, Serbia

**Abstract.** For a (molecular) graph G the first Zagreb index M1(G) is defined as the sum of the squares of the vertex degrees, and the second Zagreb index M2(G) is equal to the

sum of the products of the pairs of adjacent vertices vertex degrees. These indices are among the oldest and most studied vertex-degree-based topological indices. They reflect the extent of branching within the molecular carbon-atom skeleton, which allows them to be viewed as molecular structure descriptors. Recently, there has been great interest in studying extremal graphs that minimize (or maximize) Zagreb indices in different classes of graphs.The extremal trees that maximize and minimize the first and second Zagreb indices among the trees with a given number of segments, given number of branching vertices or given number of maximum degree vertices are characterized. In addition, upper bounds on Zagreb indices of trees in terms of domination number are obtained and the extremal trees are characterized. Also, a lower bound on the first Zagreb index of trees with a given domination number is determined and the extremal trees are described as well.

## Asymptotic laws for maximum coloring of random geometric graphs

**Milan Bradonjić**
Bell Labs, Murray Hill, NJ, USA

**Abstract.** We examine maximum vertex coloring of random geometric graphs, in an arbitrary but fixed dimension, with a constant number of colors. Since this problem is neither scale-invariant nor smooth, the usual methodology to obtain limit laws cannot be applied. We therefore leverage different concepts based on subadditivity to establish convergence laws for

the maximum number of vertices that can be colored. For the constants that appear in these results, we provide the exact value in dimension one, and upper and lower bounds in higher dimensions.

This is joint work with Sem Borst.

## Spectral and combinatorial properties of lexicographic powers of graphs

**Domingos M. Cardoso**
Center for Research and Development in Mathematics and Applications, Department of Mathematics, University of Aveiro, Aveiro, Portugal

**Abstract.** The lexicographic product of the graph $H$ by itself $k$ times, $H^k$, is herein called a lexicographic power of $H$. The adjacency spectrum of $H^k$, when $H$ is a regular graph, and the Laplacian spectrum of $G^k$, for arbitrary graphs $G$, are determined for every $k$ and several combinatorial properties of those lexicographic powers are deduced. Such powers can produce huge graphs with big numbers of vertices and edges. As an example, the spectrum of the lexicographic power of the Petersen graph with the googol number (that is, $10^{100}$) of vertices is determined.

This is joint work with Nair Abreu, Paula Carvalho and Cybele Vinagre.

**MSC:** 05C50, 05C76, 15A18.

# Three examples of a ground-breaking impact of the variable neighborhood search on investigations in graph theory

**Dragoš Cvetković**
Mathematical Institute, Serbian Academy of Sciences and Arts, Belgrade, Serbia

**Abstract.** The well known computer package AutoGraphiX (AGX) uses the variable neighborhood search to solve extremal problems in graph theory. We describe how AGX, in its very first application [1], has influenced substantially the study of graph energy. AGX helped very much in creating the spectral graph theory based on the signless Laplacian [2] and created some non-trivial conjectures on the largest eigenvalue of a graph [3].

**Dedicated to Professor Pierre Hansen.**

## References

[1] Caporossi G., Cvetković D., Gutman I., Hansen P., *Variable neighborhood search for extremal graphs*, 2. *Finding graphs with extremal energy*, J. Chem. Inform. Comp. Sci., 39(1999), 984-996.

[2] Cvetković D., Rowlinson P., Simić S.K., *Eigenvalue bounds for the signless Laplacian*, Publ. Inst. Math. (Beograd), 81(95) (2007), 11-27.

[3] Aouchiche M., Bell F.K., Cvetković D., Hansen P., Rowlinson P., Simić S., Stevanović D., *Variable neighborhood search for extremal graphs*, 16. *Some conjectures related to the largest*

*eigenvalue of a graph*, Europ. J. Oper. Res., 191(2008), No. 3, 661-676.

**MSC:** 05C50

## Cospectral digraphs from locally line digraphs

**Cristina Dalfó**
Departament de Matemàtiques,
Universitat Politècnica de Catalunya

**Abstract.** A digraph $\Gamma = (V, E)$ is a line digraph when every pair of vertices $u, v \in V$ have either equal or disjoint in-neighborhoods. When this condition only applies for vertices in a given subset (with at least two elements), we say that $\Gamma$ is a locally line digraph. In this paper we give a new method to obtain a digraph $\Gamma'$ cospectral with a given locally line digraph $\Gamma$ with diameter $D$, where the diameter $D'$ of $\Gamma'$ is in the interval $[D-1, D+1]$.

In particular, when the method is applied to De Bruijn or Kautz digraphs, we obtain cospectral digraphs with the same algebraic properties that characterize the formers.

This is joint work with Miguel A. Fiol.

# On (Signless) Laplacian eigenvalues of graphs

**Kinkar Ch. Das**
Department of Mathematics, Sungkyunkwan University,
Suwon 440-746, Republic of Korea

**Abstract.** Let $G = (V, E)$ be a simple graph. Denote by $D(G)$ the diagonal matrix of its vertex degrees and by $A(G)$ its adjacency matrix. Then the Laplacian matrix of $G$ is $L(G) = D(G) - A(G)$ and the signless Laplacian matrix of $G$ is $Q(G) = D(G) + A(G)$. Denote the spectrum of $A(G)$ by $S(A(G)) = (\lambda_1, \lambda_2, \ldots, \lambda_n)$, where we assume the eigenvalues to be arranged in nonincreasing order: $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{n-1} \geq \lambda_n$. The largest eigenvalue of $A(G)$, $\lambda_1$ is called the index of $G$. Denote the spectrum of $L(G)$ by $S(L(G)) = (\mu_1, \mu_2, \ldots, \mu_n)$, where we assume the eigenvalues to be arranged in nonincreasing order: $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_{n-1} \geq \mu_n = 0$. Let $a$ be the algebraic connectivity of graph $G$. Then $a = \mu_{n-1}$. Denote the spectrum of $Q(G)$ by $S(Q(G)) = (q_1, q_2, \ldots, q_n)$, where we assume the eigenvalues to be arranged in nonincreasing order: $q_1 \geq q_2 \geq \cdots \geq q_{n-1} \geq q_n \geq 0$. In this talk we discuss the relation between eigenvalues of different matrices of graphs.

# Spectral method for finding dominant graph structures: from undirected to directed graphs

**Nataša Djurdjevac Conrad**
Zuse Institute Berlin, Germany

**Abstract.** Finding dominant graph structures is a very important problem in graph theory, but also in applied mathematics where these are shown to correspond to building blocks of complex systems. For example, in graphs describing biological systems, dominant structures are often functional units like protein complexes and bio-chemical pathways.

In the case of undirected graphs, one is interested in finding modules, i.e. densely inter-connected substructures sparsely connected to the rest of a graph. Module identification is often done by spectral methods which use a relation between modules and real-valued eigenvalues. In particular, with random-walk-based approaches modules can be found by looking at real-valued eigenvalues of the transition matrix $P$ which are close to $\lambda = 1$ and the corresponding eigenvectors. However, when a graph is directed, this problem is not very well understood, because a spectrum of $P$ can become complex and dominant structures can include both modules and dominant cycles.

In this talk we will show how an undirected modular graph can be perturbed into a directed graph with dominant cycles. To this end, we will investigate how real-valued eigenvalues can be driven into the complex plane away from the real line. Finally, we will present our new algorithmic approach for finding

dominant structures (modules, dominant cycles and mixture of both) for undirected as well as directed graphs.

This is joint work with Marcus Weber and Christof Schütte.

**MSC:** 05C50

## Resistive distances on networks

**Andrés M. Encinas**
Departament de Matemàtiques,
Universitat Politècnica de Catalunya

**Abstract.** The *resistive distance* has become an useful tool to analyze structural properties of networks. Unlike the *geodesic distance*, the resistive distance takes into account all paths between vertices. The high sensibility of this metric with respect to small perturbations, makes it suitable to compare different network structures. This is one of the main reason for which effective resistances and the corresponding Kirchhoff Index, have emerged as structure-descriptors in Organic Chemistry, where the topology of chemical compounds is represented by a molecular network where edge weights correspond to bond properties. Effective resistances can be expressed in terms of the group inverse of the Laplacian matrix and the corresponding Kirchhoff index is nothing else but its trace.

Other distances on networks have been studied with similar purposes. In this work, we pay attention on the so–called *forest metric* introduced by P. Chebotarev and E. Shamis at late 90's,

or more specifically on its reformulation as *adjusted forest metric*, that they interpret as a measure of the *accessibility*. The adjusted forests metrics form a one–parametric family, where the parameter determines the proportion of taking into account long and short routes between vertices. Moreover, the usual resistive distance is the asymptotic value of the metric when the parameter goes to infinite. In this communication we show that each distance of the one–parametric family is a resistive distance corresponding to an Schrödinger operator on the network. We also study the effect of some perturbation on the operator on its resistive distance.

This is joint work with Ángeles Carmona and Margarida Mitjana.

**MSC:** 05C12,94C15

## Equivalent characterizations of the spectra of graphs and applications to measures of distance-regularity

**Miguel Angel Fiol**
Universitat Politècnica de Catalunya

**Abstract.** As it is well-known, the spectrum $\operatorname{sp} G$ (of the adjacency matrix $A$) of a graph $G$, with $d$ distinct eigenvalues other than its spectral radius $\lambda_0$, usually provides a lot of information about the structure of $G$. Moreover, from $\operatorname{sp} G$ we can define the so-called predistance polynomials $p_0, \ldots, p_d \in \mathbb{R}_d[x]$,

with $\operatorname{dgr} p_i = i$, for $i = 0, \ldots, d$, which are orthogonal with respect to the scalar product $\langle f, g \rangle_G = \frac{1}{n} \operatorname{tr}(f(A)g(A))$ and normalized in such a way that $\|p_i\|_G^2 = p_i(\lambda_0)$. They can be seen as a generalization for any graph of the distance polynomials of a distance-regular graph. Going further, we consider the preintersection numbers $\xi_{ij}^h$ for $i, j, h \in \{0, \ldots, d\}$, which generalize the intersection numbers of a distance-regular graph, and they are the Fourier coefficients of $p_i p_j$ in terms of the basis $\{p_h\}_{0 \le h \le d}$. The aim of this talk is to show that, for any graph $G$, the information contained in its spectrum, preintersection polynomials, and preintersection numbers is equivalent. Also, we give some characterizations of distance-regularity which are based on the above concepts. For instance, we comment upon the so-called spectral excess theorem stating that a connected regular graph $G$ is distance-regular if and only if its spectral excess, which is the value of $p_d$ at $\lambda_0$, equals the average excess, that is, the mean of the numbers of vertices at extremal distance $d$ from every vertex.

This is joint work with V. Diego and J. Fàbrega.

MSC: 05E30, 05C50

## Locating the eigenvalues for graphs of small clique-width

**Martin Fürer**
Institut für Theoretische Informatik ETH Zürich
Visiting from Pennsylvania State University

**Abstract.** Diagonalizing the matrix $A - cI$ for constants $c$, where $A$ is the adjacency matrix of a graph $G$ allows to quickly tell the number of eigenvalues of $G$ in a given interval. If $G$ has tree-width $k$ and a corresponding tree decomposition is known, then diagonalization can be accomplished in time $O(k^2 n)$ where $n$ is the number of vertices. We show that the same result hold for clique-width. The new algorithm can be much more efficient, because the tree-width can be linear in $n$, even when the clique-width is a small constant.

This is joint work with David P. Jacobs an Vilmar Trevisan.

**MSC:** 05C50

## Cospectrality of graphs with respect to distance matrices

**Pierre Hansen**
GERAD and HEC Montreal, Canada

**Abstract.** The distance, distance Laplacian and distance signless Laplacian spectra of a connected graph $G$ are the spectra of the distance, distance Laplacian and distance signless

Laplacian matrices of $G$. Two graphs are said to be cospectral with respect to the distance (resp. distance Laplacian or distance signless Laplacian) matrix if they share the same distance (resp. distance Laplacian or distance signless Laplacian) spectrum. If a graph $G$ does not share its spectrum with any other graph, we say $G$ is defined by its spectrum. In this talk we are interested in the cospectrality with respect to the three distance matrices. First, we report on a numerical study in which we looked into the spectra of the distance, distance Laplacian and distance signless Laplacian matrices of all the connected graphs on up to 10 vertices. Then, we prove some theoretical results about what we can deduce about a graph from these spectra. Among other results we identify some of the graphs defined by their distance Laplacian or distance signless Laplacian spectra.

This is joint work with Mustapha Aouchiche (GERAD and HEC Montreal).

**MSC:** 05C12; 05C50; 05C31; 05C30

## Constructing graphs with given spectrum and the spectral radius at most 2

**Irena M. Jovanović**
School of Computing, Union University, Belgrade, Serbia

**Abstract.** Two graphs $G_1$ and $G_2$ are *cospectral*, denoted by $G_1 \sim G_2$, if their spectra coincide. The set of all graphs that are

cospectral to a given graph $G$ we call the *cospectral equivalence class* and denote by $[G]$. Such a set includes $G$ due to the reflexivity of the relation $\sim$. We say that a graph is determined by its spectrum, or that it is a *DS-graph*, if it is a unique graph having that spectrum.

The problem of determining graphs by spectral means is one of the oldest problems in the spectral graph theory. This problem is studied in the literature for various kinds of graph spectra (i.e. based on different types of graph matrices), while in this talk we have in mind the adjacency matrix. In that sense, two challenging tasks have been posed - identification of DS-graphs as well as determination of the cospectral equivalence class of a given graph.

We threat these problems on the so called Smith graphs, i.e. graphs whose spectral radius is at most 2, by looking for the solutions of a system of linear Diophantine equations, previously exposed by D. Cvetković and I. Gutman. By solving such a system, we proved that $Z_n + P_1$, for $n \geq 9$ and $C_{2n} + P_1$, for $n \geq 4$ are DS-graphs, where $Z_n$, $P_n$ and $C_n$ are $(n+2)$-vertex snake, and $n$-vertex path and cycle, respectively. The cospectral equivalence class of graphs $T_5 + T_6$, $W_1 + T_4$, $W_1 + T_5$ and $Z_n + W_n$, where $T_4$, $T_5$, $T_6$ and $W_n$ are connected graphs with index equal to 2, will also be presented.

This is joint work with: Dragoš Cvetković, Mathematical Institute SANU, Belgrade, Serbia.

**MSC:** 05C50

## On the Co-PI spectral radius and the Co-PI energy of graphs

**Ezgi Kaya**
Departmant of Mathematics-Computer, Faculty of Science and Literature, Igdir University, Igdir, Turkey

**Abstract.** The Co-PI eigenvalues of a connected graph G are the eigenvalues of its Co-PI matrix. In this study, Co-PI energy of a graph is defined as the sum of the absolute values of Co-PI eigenvalues of G. We also give some bounds for the Co-PI spectral radius and the Co-PI energy of graphs.

This is joint work with Ayse Dilek Maden.

**MSC:** 05C22, 05C50

## Regular graphs with a small number of distinct eigenvalues

**Tamara Koledin**
Faculty of Electrical Engineering, University of Belgrade, Serbia

**Abstract.** We report our results concerning spectra of the adjacency matrix or the distance matrix of regular (and, in particular cases, semiregular bipartite) graphs with a common property that they have a comparatively small number of distinct eigenvalues (of the corresponding spectrum).

Considering regular bipartite graphs with three distinct nonnegative eigenvalues (of the adjacency matrix), we explore their relations with two-class symmetric partially balanced incomplete block designs and derive some structural and spectral properties of these graphs. In particular, we identify a number of situations in which they are incidence graphs of these designs. As a consequence, we present a sequence of constructions of connected regular bipartite graphs with six distinct eigenvalues.

Next, we put forward the analogous consideration regarding distance-regular graphs with small diameter and at most four distinct eigenvalues of their distance matrix ($D$-eigenvalues for short). Precisely, we focus on a detailed analysis of cases when the diameter is three and the corresponding graphs have also three distinct $D$-eigenvalues or the diameter is four and the corresponding graphs are bipartite and have four of less distinct $D$-eigenvalues. In this context, we also present an infinite family of semiregular bipartite graphs with diameter four and also four distinct $D$-eigenvalues (in this way we answer a question concerning the existence of such graphs). Finally, we demonstrate an application by constructing pairs of regular bipartite distance equienergetic graphs.

This is joint work with Zoran Stanić.

**MSC:** 05C50

# Inconsistencies among spectral robustness metrics

**Robert E. Kooij**
Faculty of Electrical Engineering, Mathematics and Computer Science, Delft University of Technology, The Netherlands;
TNO (Netherlands Organization for Applied Scientific Research), Den Haag, The Netherlands

**Abstract.** Our society nowadays depends critically on the proper functioning of a variety of networks, such as the Internet, the power grid, water management networks and mobile communication networks. Therefore, being able to quantify the robustness of such networks is of the uttermost importance. The robustness reflects the extent to which networks are able to cope with perturbations imposed upon them. It is common practice to quantify the robustness of a network by means of spectral graph metrics, such as the algebraic connectivity or the spectral radius. In this talk we will consider eight different spectral metrics that have been proposed in literature for the quantification of the robustness of networks. Four of the metrics are derived from the adjacency matrix, the other follow from the Laplacian spectrum. The aim of this talk is to show that there is no consistency among the eight spectral metrics w.r.t. the quantification of robustness. In fact, for every pair of spectral metrics we consider here, we will show that a pair of graphs, say G1 and G2, exist such that the spectral robustness metrics point in opposite direction, i.e. according to one metric G1 is more robust, but according to the other metric G2 is more robust. Given that we

consider eight different spectral metrics, we need to construct inconsistencies among 28 pairs of metrics. We will realise this number of inconsistencies with the help of 10 graphs, all having 7 nodes and 10 links. Finally, we calculate and compare the spectral metrics for a number of real-life networks.

This is joint work with Xiangrong Wang, Delft University of Technology.

**MSC:** 68M10

## A survey on the complexity indices for the travelling salesman problem

**Vera Kovačević-Vujčić**
Faculty of Organizational Sciences, University of Belgrade

**Abstract.** We present a survey on complexity indices for the travelling salesman problem according to [1], [2] and report on related recent computational experiments.

Let $A$ be an (exact) algorithm for solving an NP-hard combinatorial optimization problem $C$ and let $I$ be an instance of $C$ of dimension $n$. A complexity index of $I$ for $C$ with respect to $A$ is a real $r$, computable in polynomial time from $I$, by which we can predict (in a well defined statistical sense) the execution time of $A$ for $I$.

We consider the symmetric travelling salesman problem with instances $I$ represented by complete graphs $G$ with distances between vertices (cities) as edge weights (lengths). Intuitively,

the hardness of an instance $G$ depends on the distribution of short edges within $G$. Therefore we consider some short edge subgraphs of $G$ (minimal spanning tree, critical connected subgraph, and several others) as non-weighted graphs and several their invariants as potential complexity indices. Here spectral invariants (e.g. spectral radius of the adjacency matrix) play an important role since, in general, there are intimate relations between eigenvalues and the structure of a graph.

The results of computational experiments along these ideas will be presented.

**References**

[1] Cvetković D., Čangalović M., Kovačević-Vujčić V., *Optimization and highly informative graph invariants*, Two Topics in Mathematics, ed. B.Stanković, Zbornik radova 10(18), Matematički institut SANU, Beograd 2004, 5-39.

[2] Cvetković D., *Complexity indices for the travelling salesman problem and data mining*, Transactions of Combinatorics, **1**(2012), No. 1, 35-43.

This is joint work with Dragoš Cvetković, Mathematical Institute, Serbian Academy of Sciences and Arts, Belgrade, Serbia, Mirjana Čangalović, Faculty of Organizational Sciences, University of Belgrade, and Zorica Dražić, Faculty of Mathematics, University of Belgrade

**MSC:** 05C50

## Special graphs and quasigroup functional equations

**Aleksandar Krapež**
Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade, Serbia

**Abstract.** In his PhD thesis, Sava Krstić defined a particular class of (multi)graphs which are (3-edge-)connected and cubic. Such graphs are called Krstić graphs by A. Krapež and M. A. Taylor. They are related to generalized quadratic quasigroup functional equations via the following:

**Theorem** (S. Krstić). Two generalized quadratic quasigroup functional equations are parastrophically equivalent iff their Krstić graphs are isomorphic.

We define *special graphs* as Krstić graphs with colored vertices and bidirected edges, one of which is also designated. They give us the following refinement of the Krstić Theorem:

**Theorem**. Two generalized quadratic quasigroup functional equations are logically equivalent iff their special graphs are isomorphic.

This is joint work with Slobodan Simić and Dejan Živković.

**MSC:** 05C25

## Improved bounds for the normalized Laplacian energy of graphs

**Ayşe Dilek Maden**
Department of Mathematics, Faculty of Science,
Selçuk University, Campus, 42075, Konya, Turkey

**Abstract.** The normalized Laplacian eigenvalues of a network play an important role in its structural and dynamical aspects associated with the network. In this paper, we consider the energy of a simple graph with respect to its normalized Laplacian eigenvalues, which we call the normalized Laplacian energy. We present new bounds which improve some results was obtained on this energy. As a result of this bounds, we also obtain some bounds for normalized Laplacian energy of bipartite and complete multipartite graphs.

**MSC:** 05C12, 05C90

## Some transformations that preserve $\mathbf{sgn}(\lambda_2 - r)$

**Bojana Mihailović**
Faculty of Electrical Engineering, University of Belgrade,
Serbia

**Abstract.** We consider simple, nonoriented graphs without loops or multiple edges. Their eigenvalues, which are real numbers, are the roots of the characteristic polynomial of their adjacency matrix. As usual, the roots are labeled in nonincreasing

order $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n$. We describe some transformations within certain families of graphs or between different families that preserve the sign of $\lambda_2 - r$, where $r > 0$ and by applying them we describe some families of graphs whose second largest eigenvalue does not exceed $r$.

This is joint work with Marija Rašajski.

**MSC:** 05C50

## The Kirchhoff index of some molecular graphs

**Margarida Mitjana**
Departament de Matemàtiques Universitat Politècnica de Catalunya

**Abstract.** A molecular graph is a representation of a chemical compound in terms of a graph. The Kirchhoff Index was introduced as an alternative to other parameters used to discriminate among different molecules with similar shapes and structures. The Kirchhoff index is defined as the sum of the effective resistances of all the vertices of the (molecular) graph conventionally used to represent the topology of a chemical compound, where edge weights correspond to bond properties. It is known that the Kirchhoff index coincides with the sum of the inverses of the (non null) eigenvalues of the generalized inverse of the Laplacian matrix of the graph.

In this work we consider a class of generalized linear chains obtained from a $2n$–path, $P$, by the addition of weighted edges

between appropriate vertices in order to model different chemical compounds. A generalized linear chain is seen as a perturbation of a path and we obtain its Kirchhoff index as non trivial functions of the corresponding expression for the path $P$. To this end, we deal with some $(s \times s)$–resistance matrix that holds all the information considered as the perturbation that makes a path to become a generalized chain. This approach requires the computation of the inverse of a tridiagonal $M$–matrix that leads to solve second order difference equations.

This is joint work with Ángeles Carmona and Andrés M. Encinas.

**MSC:** 05C12, 15A09, 92E10

## Conjecture for the geometric-arithmetic index with given minimum degree

**Ljiljana Pavlović**
Faculty of Science, University of Kragujevac, Serbia

**Abstract.** The geometric-arithmetic index GA of a graph is defined as sum of weights of all edges of graph. The weight of one edge is quotient of the geometric and arithmetic mean of degrees of its end vertices. The predictive power of GA for physico-chemical properties is somewhat better than the predictive power of other connectivity indices. Let $G(k, n)$ be the set of connected simple $n$-vertex graphs with minimum vertex deg-

ree $k$. We give a conjecture about structure of extremal graphs of this index for $n$-vertex graphs with given minimum degree. This is joint work with Milica Milivojević.

## Some notes on maximal number of cycles in reflexive cacti

**Marija Rašajski**
Faculty of Electrical Engineering, University of Belgrade, Serbia

**Abstract.** Reflexive graphs are graphs whose second largest eigenvalue $\lambda_2$ does not exceed 2. The eigenvalues of a graph $G$ are the roots of the characteristic polynomial of its adjacency matrix $P_G(\lambda) = \det(\lambda I - A)$. Previously, some classes of multicyclic reflexive cacti have been investigated. They have been considered under certain conditions among which was the condition that their cycles do not form a bundle. It has been shown that under these conditions such graphs have at most five cycles. Multicyclic reflexive cacti whose cycles do form a bundle are considered here in general. We find the maximal number of cycles in those cacti if this number is finite.

**MSC:** 05C50

## Some graphs with just three distinct eigenvalues

**Peter Rowlinson**
Mathematics and Statistics Group Institute of Computing Science and Mathematics University of Stirling Scotland

**Abstract.** Let $G$ be a connected non-regular non-bipartite graph whose adjacency matrix has spectrum $\rho, \mu^{(k)}, \lambda^{(l)}$, where $k, l \in \mathbb{N}$ and $\rho > \mu > \lambda$. We investigate those graphs $G$ for which $\mu$ is non-main and $\delta(G) = 1 + \mu - \lambda\mu$. There are three infinite families of graphs which have these two properties.

**MSC:** 05C50

## A nut graph lies in an extremal singular graph

**Irene Sciriha**
University of Malta

**Abstract.** A graph G is singular if the nullspace of its adjacency matrix is non–trivial. Such a graph contains induced subgraphs called singular configurations of nullity one. Singular configurations may be considered as the 'atoms' of a singular graph. We explore how, for a given order of a graph, the nullity controls the size of the singular substructures. An algorithm is presented for the construction of a nut graph, a graph of nullity one whose null vector has no zero entries. It is shown that an

extremal singular graph of a given order, with maximal nullity and support, has a nut graph induced as a maximal singular configuration.

**MSC:** 05C50, 05C60, 05B20

## On Laplacian spectrum of the nilpotent graphs over the rings $\mathbb{Z}_n$

**Sezer Sorgun**
Department of Mathematics, Nevşehir Hacı Bektaş Veli University, Turkey

**Abstract.** Let $R$ be a ring with unity. The nilpotent graph of $R$, denoted by $\Gamma_N(R)$, is a graph with vertex set $V_N(R)^*$ $=\{0 \neq x \in R : xy \in N(R) \text{ for some } 0 \neq y \in R\}$; and two distinct vertices $x$ and $y$ are adjacent if and only if $xy \in N(R)$, where $N(R)$ is the set of all nilpotent elements of $R$. In this talk we present the Laplacian spectral properties of the nilpotent graph over the ring $\mathbb{Z}_n$.

**MSC:** 05C50, 05C25

# Comparing closed walk counts in trees consisting of three paths

**Dragan Stevanović**
Mathematical Institute, Serbian Academy of Science and Arts, Belgrade Serbia

**Abstract.** Spectral moments of adjacency matrix of a graph, which also represent counts of its closed walks, are useful objects in dealing with spectral radius and Estrada indices of graphs. For $k \geq 0$, let $M_k(G)$ denote the number of closed walks in $G$ of length $k$. Let $G \preceq H$ denote that $M_k(G) \leq M_k(H)$ for all $k \geq 0$. For example, if $G \preceq H$ then the spectral radius and the Estrada index of $G$ are smaller than or equal to the spectral radius and the Estrada index of $H$, respectively.

Let $P_{a,b,c}$ denote the tree obtained from the union of paths $P_{a+1}$, $P_{b+1}$ and $P_{c+1}$ by identifying one endvertex from each of the three paths. We show that for any $a, b, c, d, e, f$ holds either

$$P_{a,b,c} \preceq P_{d,e,f} \quad \text{or} \quad P_{a,b,c} \succeq P_{d,e,f}.$$

This is well known in the case that $\min\{a, b, c\} = \min\{d, e, f\}$, but its proof needs to play with both characteristic polynomials and walk embeddings when $\min\{a, b, c\} + 1 = \min\{d, e, f\}$.

This is joint work with A. Kanso and M. Ghebleh.

**MSC:** 05C50

# Some remarks on spectral recognition of music melodies

**Vesna Todorčević**
Faculty of Organizational Sciences, University of Belgrade, and Mathematical Institute, Serbian Academy of Sciences and Arts, Belgrade, Serbia

**Abstract.** A spectral graph theory approach is described for representing melodies as graphs, based on intervals between the notes they are composed of. These graphs are then indexed using eigenvalues of some graph matrices. The eigenvalues are used to define a spectral distance between graphs. Two graphs are considered as similar if their spectral distance is small. This makes it possible to find melodies similar to a given melody [1], [2].

Our contribution is related to the selection of graph matrices which are used in indexing melodies. The paper [2] considers spectra of the adjacency matrix and of the Laplacian of graphs for indexing them. After presenting some shortcomings of the procedure from [2], we have suggested using singular values of the adjacency matrix of digraphs considered [1]. Singular values of the adjacency matrix of digraphs have been studied in [3].

We present some additional remarks on these problems.

## References

[1] Cvetković D., Manojlović V., *Spectral recognition of music melodies*, SYM-OP-IS 2013, 269-271.

[2] Pinto A., Leuken R.H. van, Demirci M.F., Wiering F., Veltkamp R.C., Indexing music collections through graph spectra, Proc. 8th Internat. Conf. Music Information Retrivial, ISMIR 2007, Vienna, September 23 - 27, 2007, 153-156.

[3] Jovanović I., Spectral recognition of graphs and networks (Serbian), Doctoral Thesis, Faculty of Mathematics, University of Belgrade, Belgrade, 2015.

This is joint work with Dragoš Cvetković, Mathematical Institute, Serbian Academy of Sciences and Arts, Belgrade, Serbia.

**MSC:** 05C50

## The algebraic connectivity of an interdependent network

**Piet Van Mieghem**
Faculty of Electrical Engineering, Mathematics and Computer Science, Delft University of Technology, The Netherlands

**Abstract.** An interdependent network, also called interconnected or multi-layer network or network of networks, is a network consisting of different types of networks, that depend upon each other for their functioning. For example, a power grid is steered by a computer network, that in turn needs electricity to function. We focus on a general two-layer network, where the interconnection pattern between the two networks $G_1$ and $G_2$ is specified by a real interconnectivity matrix $B$. Many dynamic processes on networks are described by the Laplacian matrix $Q$.

A regular topological structure of the interconnectivity matrix $B$ (constant row and column sum), associated to equitable partitions, enables the computation of a nontrivial eigenmode (eigenvector and eigenvalue) of $Q$. The latter eigenmode is independent from $G_1$ and $G_2$. We briefly overview earlier work, where for the special case $B = wI$, an exact coupling strength $w^*$ is found so that, for all $w < w^*$, the eigenvalue of the "regular eigenmode" corresponds to the algebraic connectivity. Moreover, at $w^*$, two dynamic regimes are separated: for $w > w^*$, the interdependent network acts as a whole, whereas below $w^*$, the network operates as if separated into $G_1$ and $G_2$.

In addition to the criterion "If the algebraic connectivity is positive, then the graph is connected", we found an alternative condition: "If $\min_{1 \le k \le N} \left(\lambda_k^2(A)\right) = d_{\min}$, then the graph is disconnected."

This is joint work with Xiangrong Wang, Delft University of Technology.

**MSC:** 68M10

## Eigenvector matrix of the Laplacian

**Xiangrong Wang**
Faculty of Electrical Engineering, Mathematics and Computer Science, Delft University of Technology, The Netherlands

**Abstract.** Eigenvalues and eigenvectors of the Laplacian matrix characterize the structural and dynamical properties of a

graph, such as node/link connectivity, graph partitioning and robustness against cascading failures. Since there are few results on eigenvectors, we focus on the orthogonal eigenvector matrix $Z$ of the Laplacian matrix of a graph with $N$ nodes, whose $k$-th column is the eigenvector $x_k$ belonging to the $k$-largest eigenvalue $\mu_k$. We study the fundamental weight vector $w$ and its dual $\varphi$, which are the row sum and column sum of an orthogonal matrix, respectively. In particular, the column sum vector $w$ of $Z$ is, for a connected graph, proportional to the basic vector $e_N = (0, 0, \ldots, 1)$, so that we can confine to its dual $\varphi$ to deduce specific graph properties. Basic properties and bounds for the dual fundamental weight vector $\varphi$ are derived.

As an application, we found that, for the particular class of Erdős-Rényi random graphs, a product of a Gaussian and a super-Gaussian distribution approximates accurately the distribution of $\varphi_U$, a uniformly at random chosen component of the dual fundamental weight vector of $Z$.

This is joint work with Robert E. Kooij and Piet Van Mieghem, Delft University of Technology. Robert E. Kooij is also in TNO (Netherlands Organization for Applied Scientific Research), Den Haag, The Netherlands.

**MSC:** 68M10

## Resolvent energy of graphs

**Emir Zogić**
State University of Novi Pazar, Novi Pazar, Serbia

**Abstract.** The resolvent energy of a graph $G$ of order $n$ is defined as $ER = \sum_{i=1}^{n} (n - \lambda_i)^{-1}$, where $\lambda_1, \lambda_2, ..., \lambda_n$ are the eigenvalues of $G$. We establish a number of properties of $ER$. In particular, we establish lower and upper bounds for $ER$ and examine the trees, unicyclic, bicyclic and tricyclic graphs with smallest and greatest $ER$.

This is joint work with Ivan Gutman, Boris Furtula, and Edin Glogić.

## SGA 2006 photos

D. Cardoso, T. Pisanski, S.K. Simić, and I. Gutman

Conference dinner

## Author Index

Wang, Xiangrong (x.wang-2@tudelft.nl), 43

Zogić, Emir (ezogic@np.ac.rs), 45

## Other Participants

Davidović, Tatjana (tanjad@mi.sanu.ac.rs)

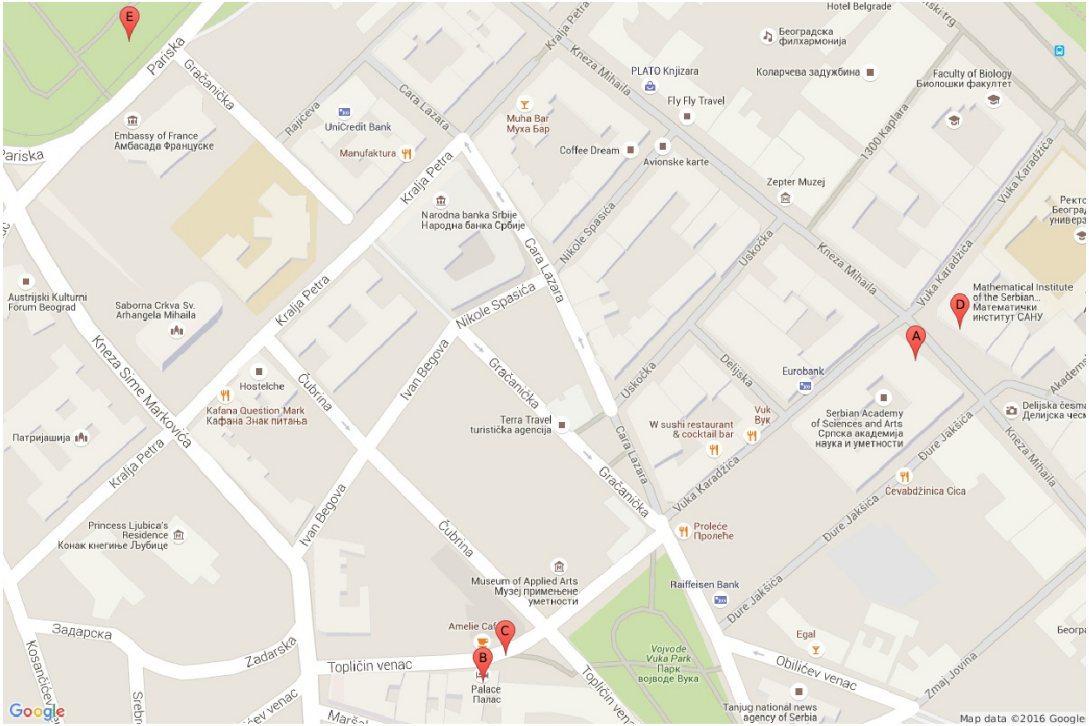
Furtula, Boris (furtula@kg.ac.rs)

Glogić, Edin (edinglogic@np.ac.rs)

Jurišić, Aleksandar (aj@fri.uni-lj.si)

Tulu, Tomas (gebriel1919@yahoo.com)

Vidali, Janoš (janos.vidali@fmf.uni-lj.si)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## Practical information

The Conference location:
Serbian Academy of Sciences and Arts, Knez Mihajlova 35

The Academy is located in downtown Belgrade, very close to main bus and railway stations.

On a Google map the relevant locations are marked:
A - Serbian Academy of Sciences and Arts;
B - Hotel Palace, Topličin venac 23, the place where the conference dinner will be held;
C - The station of the bus for excursion on Saturday, May 21, 2016 (in front of Hotel Palace);
D - Mathematical Institute of the Serbian Academy of Sciences and Arts, located across the street with respect to the Academy (Knez Mihailova 36).

Mathematical Institute of the Serbian Academy of Sciences and Arts is celebrating its 70th anniversary this year. The main activities are held in May at the Institute and at the Academy. You are welcome to take a part in the events scheduled for the period of your stay in Belgrade. Weekly schedules are available at http://www.mi.sanu.ac.rs/main_pages/weekly.htm

A lot of interesting places are located in the neighborhood of Academy, museums, galleries, churches, restaurants, etc. The most interesting place is a park, Kalemegdan (marked by E on the map). There are a lot of monuments, sport fields, coffee shops in it. The zoo is also located in this park.