# Groups in Geometry A and B

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The primary aim of these lectures is to introduce students to the theory of Lie groups and Lie algebras, with the ultimate goal of working out the finite-dimensional representations of the classical groups. The topics covered in this course:

- Lie groups and Lie algebras.
- Lie subgroups and homomorphisms.
- Coverings of Lie groups.
- Exponential Map.
- Adjoint representation.
- Automorphisms.  Killing form.
- Complexification.
- Important examples of Lie groups (Abel groups, orthogonal groups, unitary groups, symplectic groups).
- Nilpotent and Solvable Lie algebras.
- Semisimple Lie algebras .
- Compact Lie algebras.
- Left-invariant and bi-invariant metrics on Lie groups.
- Maximal Torus and Weyl group.
- Cartan subalgebra and roots.
- Dynkin diagram and classification.
- Weyl group.
- Compact forms.
- Maximal root.
- Representation Theory – General Definitions.
- Representations of semisimple Lie algebras.
- Representations of classical Lie algebras.
- Real Representations of Real Lie Groups.
- Symmetric Spaces – Basic geometric properties and examples.
- Cartan involutions.
- Geodesics and Curvature.
- Symmetric Spaces of non-compact type.
- Hermitian Symmetric Spaces.
- Topology of Symmetric Spaces.

References:

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2. B. Hall, Lie Groups, Lie Algebras and Representations, An Elementary Introduction, Springer-Verlag, New York, (2003).
3. S. Helgason , Differential Geometry, Lie groups and symmetric spaces, Academic Press, New York, (1978).
4. J. Humphreys, Introduction to Lie Algebras and Representation Theory, Springer-Verlag, New York, (1980).
5. H. Samelson, Notes on Lie algebras, Springer-Verlag, New York, (1990).
6. W. Ziller, Lie Groups. Representation Theory and Symmetric Spaces, University of Pennsylvania, (2010).