



# Verovatnosne logike i njihove primene

Beograd, Srbija, 29-30. septembar 2011.

## Knjiga apstrakata

ORGANIZATOR:

Matematički institut, SANU

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Ministarstvo prosvete i nauke Republike Srbije

Projekat Razvoj novih informaciono-komunikacionih tehnologija, korišćenjem naprednih matematičkih metoda, sa primenama u medicini, telekomunikacijama, energetici, zaštiti nacionalne baštine i obrazovanju, III 044006

Projekat Reprezentacije logičkih struktura i formalnih jezika i njihove primene u računarstvu, ON 174026.





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Beograd, Srbija, 29-30. septembar 2011.

## TEME KONFERENCIJE:

- verovatnosne logike, problemi potpunosti, odlučivosti i složenosti,
- logičke osnove u zasnivanju verovatnoće,
- Bayes-ove mreže i drugi srodni sistemi,
- programski sistemi za podršku odlučivanju u prisustvu neizvesnosti,
- primene verovatnosnog zaključivanja u medicini itd.

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Projekat Reprerzentacije logičkih struktura i formalnih jezika i njihove primene u računarstvu, ON 174026.



# Program konferencije:

Dan 1 - 29. 9. 2011.

- Sesija 1      Predsedava Miomir Stanković  
10:00 – 10:15    Otvaranje  
10:20 – 11:20    Zvonimir Šikić: Bayesian probability theory is the probability logic  
11:20 – 12:20    Daniel Romano: Constructive Anti-ideals Theory of Commutative Rings  
12:20 – 13:20    Siniša Crvenković: Konstruktivne polugrupe
- 13:20 – 15:00    Pauza
- Sesija 2      Predsedava Zoran Marković  
15:00 – 15:20    Dragan G. Radojević: Fazi verovatnoće u Bulovom okviru  
15:25 – 15:45    Velimir M. Ilić, Miomir S. Stanković, Branimir T. Todorović: Message Passing Algorithms over the Binomial and the Entropy Semirings  
15:50 – 16:10    Nebojša Ikodinović, Miodrag Rašković, Zoran Marković, Zoran Ognjanović: A first-order probabilistic logic with approximate conditional probabilities  
16:15 – 16:35    Angelina Ilić Stepić, Zoran Ognjanović, Nebojša Ikodinović, and Aleksandar Perović: A p-adic probability logic
- 16:40 – 17:00    Pauza
- Sesija 3      Predsedava Zvonimir Sikić  
17:00 – 17:20    Vladimir Ristić, Radosav Djordjević, Nebojša Ikodinović: Biproability logic with conditional expectation  
17:25 – 17:45    Ivan Živković, Miomir Stanković: Artificial Neural Networks for Decision Support in copper smelting process  
17:50 – 18:10    Edin H. Mulalić, Miomir S. Stanković: Functional Decomposition as an Optimization Technique

Dan 2 - 30. 9. 2011.

- Sesija 1      Predsedava Danijel Romano  
10:00 – 10:20    Nataša Glišović: Analiza DNK mesavine koriscenjem Bjesovih mreza  
10:25 – 10:45    Ivan Čukić: An optimised index structure for static deductonal databases based on fuzzy or probabilistic DLs  
10:50 – 11:10    Tatjana Stojanović: Probability description language P-ALCN  
11:15 – 11:35    Marina Svičević: Primena Bajesovih mreža u medicinskoj dijagnostici
- 11:35 – 12:10    Pauza
- Sesija 2      Predsedava Siniša Crvenković  
12:10 – 12:30    Nataša Glišović, Milan Božić: The System for Random Man not Excluded  
12:35 – 12:55    Saša Rašuo: Monadic Calculus and Constituents  
13:00 – 13:20    Dragan Doder, Zoran Marković, Zoran Ognjanović: A Branching-time Probabilistic Logic  
13:25 – 13:45    Jasmina Fijuljanin, Aleksandar Kartelj, Jelena Kojić: Electromagnetism metaheuristic for probabilistic satisfiability problem
- 13:45 – 14:00    Pauza
- Sesija 3      Predsedava Dragan Radojević  
14:00 – 14:20    Petar Maksimović: Formal Verification of Key Properties for Several Probability Logics in the Proof Assistant Coq  
14:25 – 14:45    Aleksandar Perović: On Archimedean inference rule  
14:50 – 15:10    Djordje Djordjević, Srbislav Nešić: Pseudo-Random Number Generator Using as a Seed Distance (Movement) of a Laboratory Cultured Daphniae  
15:15 – 15:35    Miloš Laban: A solution of the gambling paradox using the principle of dimensionality  
15:40 – 16:00    Vladimir Srdanović: Sistem za podršku u kliničkom odlučivanju koji integriše znanje Eksperta i znanje iz podataka



# Apstrakti





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# A Branching-time Probabilistic Logic

Dragan Doder

Zoran Marković

Zoran Ognjanović

Interest in temporal reasoning came from theoretical and practical points of view. Logicians [3, 4, 17] investigated consequences of different assumptions about the structure of time, while temporal formalisms can be used in computer science to reason about properties of programs [8, 6]. In both cases discrete linear and branching time logics have been extensively studied. Linear temporal logics are suitable for specification and verification of universal properties of all executions of programs. On the other hand, the branching time approach is appropriate to analyze nondeterministic computations described in the form of execution trees. In the later framework a state (a node) may have many successors. Then, it is natural to attach probabilities to the corresponding transitions and to analyze the corresponding discrete time Markov chains as the underlying structures. All this led to probabilistic branching temporal logic [1, 2, 9].

In this paper we introduced a propositional discrete probabilistic branching temporal logic (denoted pBTL). We use a logical language which allows us to formulate statements that combine temporal and qualitative probabilistic features. Thus, the statements as “in at least half of paths  $\alpha$  holds in at least a third of states” and “if  $\alpha$  holds in the next moment, then the probability of  $\alpha$  is positive” are expressible in our logic. The language for pBTL is obtained by adding temporal operators  $\bigcirc$  (“next”),  $A$  (universal path operator) and  $U$  (“until”), as well as the two types of probability operators,  $P_r^p$  and  $P_r^s$  ( $r \in \mathbb{Q} \cap [0, 1]$ ), to the classical propositional language. The temporal operators are well known from other formalizations of branching time logics, while the intended meaning of  $P_r^s \alpha$  ( $P_r^p \alpha$ ) is “the probability that  $\alpha$  is true on a randomly chosen branch is at least  $r$ ” (“the probability that  $\alpha$  holds on a particular branch is at least  $r$ ”). The superscript  $s$  in  $P_r^s$  ( $p$  in  $P_r^p$ ) indicates that the probability depends only on a time instant - state (on a chosen branch - path). The formulas are interpreted over models that involve a class of probability measures assigned to states, and a class of probability measures assigned to paths.

We present an infinitary axiomatization for pBTL, for which we prove strong completeness theorem. The proof of the completeness theorem uses ideas (the Henkin construction) presented in [5, 6, 7, 10, 11, 12, 13, 14, 18]. One of the main axiomatization issues for temporal logics with the operators  $\bigcirc$  and  $G$ , and for real valued probability logics is the non-compactness phenomena. The set of formulas  $\{P_{>0}^s \alpha\} \cup \{P_{\frac{1}{n}}^s \alpha \mid n \in \omega\}$  and  $\{G\alpha\} \cup \{\bigcirc^n \neg \alpha \mid n \in \omega\}$  are finitely satisfiable but they are not satisfiable. It is well known that, in the absence of compactness, any finitary axiomatization would be incomplete. Thus, infinitary axiomatic systems are the only way to establish strong completeness.

Up to our knowledge it is the first such result reported in literature.

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# A first-order probabilistic logic with approximate conditional probabilities

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We discuss a first-order probabilistic logic with Keisler-style probabilistic quantifiers allowing nonstandard values. Our main concerns are strongly complete axiomatization and decidability of the proposed logic. Since this logic is clearly undecidable, we proceed to define a quite expressive fragment which is decidable. Among other things, this fragment may be used to model not only the usual defaults but also a generalized version of defaults with several variables.

# A $p$ -adic probability logic

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In this article we present a  $p$ -adic valued probabilistic logic, denoted  $L_{Q_p}$ . The logic allows making statements such as  $K_{r,\rho}\alpha$  with the intended meaning "the probability of  $\alpha$  belongs to the ball with the center  $r$  and the radius  $\rho$ ". We describe the corresponding class of models that combine properties of the usual Kripke models and  $p$ -adic probabilities, and give sound and complete infinite axiomatic system. We prove decidability of  $L_{Q_p}$ .

# An optimised index structure for static deductive databases based on fuzzy or probabilistic DLs

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The standard indexing structures in the common database systems like B-trees and ISAMs are focussed on quick data alteration and querying for specific records. With RDF and semantic databases, the data (knowledge) is not changed often. In these scenarios, B-trees waste too much space, while ISAM wastes resources on keeping the structure sorted and on maintaining the overflow pages.

The second feature of the common indexes that is not needed in querying RDF with SPARQL is locating one specific record. The result of a simple SVO query, where only one variable is known is always a list of records that contain that fixed value.

The structure presented in this paper is optimised specifically for this case, and to have as small memory footprint as possible (one page at any time for one SVO query) that makes it suitable for deployment on the embedded systems.

The additional advantage of this structure is that it provides an efficient way of keeping sorted weights for all stored sentences. This comes handy when the standard RDF inferencing methods are enriched with fuzzy or probabilistic reasoning.

# Analiza DNK mesavine koriscenjem Bjesovih mreza

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Cilj rada je da prikaze koriscenje Bajesovih mreza u forenzickom scenariju DNK mesavine koja potice od vise ucesnika. Pojava ovakvih uzoraka se javlja u slucajevima silovanja i drugih fizickih napada. Za svaku vrstu genetskih markera pojedinac ima najvise dva alela. Dakle, prisustvo vise od dva alela se uzima kao jasna naznaka da uzorka sadrzi bioloski materijal vise od jednog pojedinca. Slozenost procene takvog scenarija se sastoji od pravljenja mogucih kombinacija genotipova koji dolaze u obzir. Pristup koji je ovde koriscen uzima u obzir nezavisnost alela pojedinaca kao i nezavisnost kroz markere tj. odsustvo bilo kakvih efekata podpopulacija. Takodje se pretpostavlja da svi koji su doprineli mrlji DNK mesavine nemaju veze jedan sa drugim. U radu cemo prikazati scenario mesavine tri alela, bez umanjivanja opstosti, model se moze prikazati i na vise ucesnika, a samim tim i broja alela. Prikazani matematicki model ce biti prosiren na Bajesove mreze, pri cemu nece biti uzeto u obzir moguće ispadanje“ alela, kao i visina i površina pika alela. Upravo ovo poslednje navodi na moguca prosirivanja ovog modela i prostor za dalje unapredjivanje i prosirenje matematickog modela.

## 1 Zahvalnost

Rad koji je ovde predstavljen je podrzan od strane srpskog Ministarstva prosvete i nauke (projekat III44006).



# Artificial Neural Networks for Decision Support in copper smelting process

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Considerable development of pyrometallurgical copper smelting process has been recorded in the last few decades. From the traditional process of oxidation roasting - melting in flames furnaces - converting sulfur dioxide ( $SO_2$ ) for the production of sulfuric acid great progress was made by introducing: Outokumpu flash furnace, Mitsubishi smelting concept, Noranda reactor, Peirce-Smith converting, El Teniente converter and others [4]. The purposes of these improvements are: bigger technological exploitations, better environment protection, and reduction of coppers' anode production cost [13]. These improved technological processes led to enlargement of production plants capacities and increase of entire copper production in the world [7] which at the same time led to increasing problems due to environmental pollution [6, 8, 11, 2]. Besides copper minerals, copper ores contain minerals of other metals which during the process of enriching, regardless the attempts of selective separation, are mostly included into the copper concentrate [10].

Many authors have investigated the application of various Multi-Criteria Decision Making (MCDM) methods in the analysis of problems of air pollution and soil [1, 12, 14, 15, 17]. The multi-variation ranking of copper concentrates from an environmental point of view, based on their content of useful and harmful components, has been used in [16]. The integration of the analysis of environmental with the analysis of economical criteria in further research of this topic is suggested in the same paper. The authors believe that such an approach will give an additional dimension to the ranking of available copper concentrates. Also, as it is acknowledged in most of cases at least technological, economical, environmental and social criteria should be taken into account in a decision process. Impact of the economical criterion in the extension of the ranking process from [16] is considered in the present paper.

In the last two decades, artificial neural network (ANN) has revealed its huge potential in many areas of science and engineering, with the rapid development in its learning algorithms. The neural network has also proven to be a powerful tool in many areas including industrial processes [19], prediction of materials properties such as steel [3, 5, 9] etc. In addition, there are many other reports that the neural network approach has been used in material science based research as discussed by [18]. However, its application to pyrometallurgy industries has not been examined thoroughly.

As such, the ANN approach is adopted in this paper to determining the amount of each of the available raw materials which will be used for obtaining the useful products, in such a way to achieve the greatest difference between profit from useful products sales and costs for obtaining specified quantities of raw materials. It is necessary to take into account interactive relations between the quality of raw materials and the economical criteria as well as the influence of the environmental criteria in the production process.

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# Bayesian probability theory is the probability logic

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The only logic of uncertainty which supplies us with the probability of a proposition A given a proposition B, with no severe restrictions on A and B, is Bayesian probability theory (BPT).

For example, frequentist probability theory supplies us with the probability of a proposition A only if it describes a result of a random experiment. (This restriction is responsible for the emergence of statistics and its many rules of thumb in the 20th century.) Even worse are various AI theories of uncertain reasoning (e. g. so called fuzzy logics) which presuppose the probability functionality of connectors (which is clearly absurd). We will present the main arguments for BPT as the probability logic, including a proof of Cox's theorem.

# Biprobability logic with conditional expectation

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This presentation is devoted to fill the gap in studying logics for biprobability structures. We describe a probability logic with two conditional expectation operators and prove the completeness theorem.

# Constructive Anti-ideals Theory of Commutative Rings \*

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This paper gives an exposition of development of anti-ideal theory of commutative ring with apartness. The setting is Brouwer-Heyting's and Bishop's constructive mathematics. Let  $R = (R, =, \neq, +, 0, \cdot, 1)$  be a commutative ring with apartness. A subset  $S$  of  $R$  is an anti-ideal of  $R$  if

$$0 \times S, -a \in S \implies a \in S, a + b \in S \implies a \in S \vee b \in S, ab \in S \implies a \in S \wedge b \in S.$$

The notion of anti-ideal of ring the first was defined and studied Wim Ruitenburg in his dissertation 1982. Let  $J$  and  $S$  be an ideal and an anti-ideal of a commutative ring  $R$  with apartness. Wim Ruitenburg in 1982 the first stated demand  $J \subseteq \neg S$ . We say that  $J$  and  $S$  are compatible. In this paper we will present some the author's answers on the following questions:

- (1) "If  $J$  is an ideal of  $R$ , is there exists an anti-ideal of  $R$  compatible with  $S$ ." and
- (2) "If  $S$  is an anti-ideal of  $R$ , is there exists an ideal  $J$  of  $R$  compatible with  $S$ ."

After that we will present some details of building of the anti-ideals theory associated by the classical ideal theory of commutative ring done by the author. The anti-ideal  $Q$  is a primary anti-ideal of  $R$  iff  $Q \cdot cr(Q) \subseteq Q$ . In this paper we will be giving two criterions for primariness of anti-ideals and we will be proving that  $cr(Q)$  is the maximal prime anti-ideal under  $Q$ , if  $Q$  is a primary anti-ideal of  $R$ . After that, we will be defining notion of associated prime anti-ideal of a given anti-ideal, and we will be proving two theorems on associated prime anti-ideals of anti-ideal which has irredundant primary decomposition.

In this paper we will be giving definitions of notions of  $S$ -prime,  $S$ -primary and  $S$ -quasi primary anti-ideals. Let  $f : R \rightarrow S$  be a homomorphism of rings. For an anti-ideal  $A$  of  $R$  we say that it is a  $S$ -prime anti-ideal of  $R$  if  $f(A) \cdot AS \subseteq AS$ . An anti-ideal  $B$  of  $R$  is a  $S$ -primary anti-ideal of  $R$  if  $f(B) \cdot cr(BS) \subseteq BS$ . An anti-ideal  $C$  of ring  $R$  is called  $S$ -quasi primary anti-ideal if  $f(cr(C)) \cdot SC \subseteq SC$ .

Beside that, we will be proving some properties of that anti-ideals.

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# Electromagnetism metaheuristic for probabilistic satisfiability problem

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Probabilistic logics are used to reason about uncertainty expressed in terms of probability. In these logics classical propositional language is expanded by expressions that speak about probability, while formulas remain true or false. The problem of verifying satisfiability of a probabilistic formula (PSAT, also called the decision form of probabilistic satisfiability) is NP-complete [2]. PSAT can be reduced to a linear programming problem. The number of variables in the linear system corresponding to the formula is exponential in the number of propositional letters from the formula. In practice, any standard linear system solving procedure is not suitable when scaling up to larger formulas. Few approaches were previously proposed for solving this problem: genetic algorithms [3, 4, 5] and variable neighborhood search [5].

The motivation behind the use of the electromagnetism metaheuristic (EM) originates from the resemblance between the electromagnetism theory of physics, and basic idea is to take the algorithm's search close to the regions with better objective functions. The EM is used for global and combinatorial optimization and categorized as a stochastic population-based algorithm. It is centered on analogy with the attraction-repulsion mechanism of the electromagnetism theory to move the sample solutions to converge to optimality based on their associated charges.

The input of the program, which uses EM for solving PSAT problem, is a formula  $\phi$  in a weight conjunctive form (wcf), with  $L$  weight literals of the form  $t_i \rho c_i$ . Without loss of generality, we demand that classical formulas appearing in weight terms are in disjunctive normal form. Let  $\varphi(f) = p_1, \dots, p_N$  denote the set of all primitive propositions from wcf formula. Our goal is to find a probabilistic model  $M$  such that  $M \models \phi$ .

Each solution point from the solution space is represented by a vector from  $[0, 1]^{L \cdot (N+1)}$ , which describes a probabilistic model. The first  $L \cdot N$  coordinates represent primitive propositions that are rounded to  $\{0, 1\}$  by using some threshold value (in our implementation it is set to 0.5). Rounded value 0 at the position  $i$  denotes negative literal  $\neg p_i$ , while 1 denotes positive literal  $p_i$ . Last  $L$  coordinates correspond to probabilities assigned to each atom, and their values are normalized in such a way that their sum is equal to 1.

We define an objective function  $f$  as a degree of unsatisfiability of EM points, denoted by  $s_k$ ,  $k \in \{1, \dots, m\}$ . The degree is defined as a distance between left side values of the weight literals of the form  $a_1^i w(\alpha_1^i) + \dots + a_{n_i}^i w(\alpha_{n_i}^i) \rho_i c_i$  that are not satisfied in the model described by an EM point  $s_k$ , and the corresponding right side values.

We used local search (LS) to improve the solution in each step, as in [3, 4]. The best and worst formula, according to the objective function, are determined, and their atom probabilities are being exchanged in order to improve it.

In the initialization part, EM points are created and each coordinate from a solution point is randomly selected from  $[0, 1]$ . In each iteration, EM points are being evaluated by calculating their charges, given by the following formula:

$q_i = \exp\left(-|L \cdot (N + 1)| \frac{f(s_i) - f(s_{best})}{\sum_{j=1}^m (f(s_j) - f(s_{best}))}\right)$ , where  $s_{best}$  is the best EM point so far. Power of connection between two points will be proportional to the product of charges and reciprocal to the distance between them. Finally, total force that these charges produce is applied. The resulting force  $F_i$  on the point  $i$  is the sum of force vectors induced by all other

neighbor points on the point  $i$ :  $F_i = \sum_{j=1, j \neq i}^m F_{ij}$ , where  $F_{ij} = \begin{cases} \left(\frac{q_i q_j}{\|s_j - s_i\|^2}\right) \cdot (s_j - s_i), & f(s_j) < f(s_i) \\ \left(\frac{q_i q_j}{\|s_j - s_i\|^2}\right) \cdot (s_i - s_j), & f(s_j) > f(s_i) \end{cases}$ . Calculated

value of  $F_i$  is normalized, so it represents a direction in which a point is going to move. New location of the EM point is calculated as in [1], and it will depend on its objective value and current location so EM point will stay in the solution space. The points with a higher charge will move other points in their direction more strongly. Note that the best EM point will not be moved.

EM based algorithm for PSAT is implemented in C programming language, compiled in Visual Studio 2010, and tested on PRSATX.SAT instances described in [3], which used set of formulas that are known to be satisfiable. For each instance EM was executed with four different seed values of a random number generator. For instances PRSAT1.SAT - PRSAT8.SAT the size of population,  $m$ , is set to 10, and for instances PRSAT9.SAT and PRSAT10.SAT,  $m$  is 12. Maximal number of iterations is 150, and maximal number of LS steps in each iteration is set to 5. All computational results were carried out on Intel 2GHz single processor with 3GB memory.

Table 1: Experimental results on smaller instances

<i>Instance</i>	<i>f</i>	<i>t<sub>tot</sub></i>	<i>t<sub>f</sub></i>	<i>N<sub>f</sub></i>	<i>LSiter<sub>f</sub></i>
PRSAT1.SAT	<i>sat.</i>	2.395	0.042	1	3.25
PRSAT2.SAT	<i>sat.</i>	2.244	0.036	0.75	2.5
PRSAT3.SAT	<i>sat.</i>	7.82	1.13	23.5	5
PRSAT4.SAT	<i>sat.</i>	8.677	0.373	6.75	3
PRSAT5.SAT	<i>sat.</i>	21.13	1.031	5.75	4.75
PRSAT6.SAT	<i>sat.</i>	22.44	2.498	17	3.25
PRSAT7.SAT	<i>sat.</i>	83.13	8.245	15	4
PRSAT8.SAT	<i>sat.</i>	78.66	60.273	115.2	4.25
PRSAT9.SAT	<i>sat.</i>	304.2	20.451	8.5	5
PRSAT10.SAT	<i>sat.</i>	304.6	173.65	83.25	3

The Table 1 shows experimental results of EM for PSAT and is organized as follows: the first column contains the instance name; the second shows the objective function  $f$  (in case when objective value was 0, formula was satisfied and we denoted it by *sat.*); the average total running time  $t_{tot}$  is shown in the third column; the fourth column  $t_f$  shows average running time to finding solution for each instance; the fifth column  $N_f$  contains the average number of iterations needed to find the solution; the average number of LS steps per iteration needed to find the solution is shown in the last column.

Although the previously described procedure is semi decidable, it can be useful when solving medium and large scale instances, where decidable procedures cannot verify the solution existence in an attainable amount of time. As can be seen from the Table 1, for every instance and in every EM run, satisfiability was verified in a reasonable running time.

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# Fazi verovatnoće u Bulovom okviru

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Fazi verovatnoće [1] su pojam uveden od strane Lotfi Zadea, oca fazi logike u širem smislu (teorija fazi skupova, fazi logika u užem smislu i fazi relacije). Motiv za uvoenje fazi verovatnoće je činjenica da se slučajni događaj u opštem slučaju ne može adekvatno opisati klasičnim skupom. Tako je, umesto klasičnog pristupa zasnovanog na klasičnoj teoriji skupova, predložena nova teorija verovatnoće, zasnovana na teoriji fazi skupova. Kao što je poznato, konvencionalna fazi logika je preuzela princip istinitosne funkcionalnosti isto tako kao što je to prethodno uračeno u slučaju više-vrednosnih logika. Ovo ima za posledicu da se generalizacije – viševrednosne realizacije, za razliku od polaznih dvo-vrednosnih teorija ne nalaze se u Bulovom okviru i/ili da ne predstavljaju konzistentnu generalizaciju klasičnog slučaja. Tako da teorija fazi verovatnoće, zasnovana na konvencionalnoj fazi logici, nije konzistentna generalizacija klasične teorije verovatnoće. Ono što se čini da ide u prilog konvencionalnog pristupa je činjenica da je npr. aksiom isključenja trećeg definisan ne algebarski (iskaz je tačan ili nije tačan; analizirani element pripada ili ne pripada posmatranom skupu). Tako da sam Zade kaže za fazi logiku da nije fazi nego precizna logika koja tretira gradaciju  $a$  u kojoj ne važi princip isključenja trećeg. Informacija o posedovanju i/ili ne posedovanju osobine u klasičnom slučaju čuva svojstva imanentno samim osobinama pa je stoga dovoljna za sva izračunavanja. U opštem slučaju to nije tako. U slučaju delimičnog posedovanja osobine intenzitet posedovanja ne može da zameni samu osobinu kao što je to u klasičnom slučaju, pa je istinitosna funkcionalnost u opštem slučaju neopravdana. U novom pristupu se polazi se od toga da svakoj osobini jednoznačno odgovara njoj komplementarna osobina (npr. u logici komplementarna osobina istinitosti je neistinitost). Komplementarna osobina je određena isključenjem trećeg: sadrži sve ono što ne sadrži analizirana osobina; i nekontradikcijom: nema ničeg zajedničkog sa analiziranom osobinom. Ovako sagledani problem dovodi do konzistentne generalizacije isključenja trećeg u više-vrednosnom kao i u realno-vrednosnom slučaju.

U ovom radu se daje Bulovski konzistentna generalizacija teorije verovatnoće zasnovana na konzistentnoj teoriji fazi skupova [2]. Konzistentna teorija fazi skupova je zasnovana na realno-vrednosnoj realizaciji konačne Bulove algebre [3]. Kao što je poznato, osobina generiše skup u analiziranom univerzumu. Bulova algebra definiše zakone koji se odnose u ovom slučaju na same osobine. Teorija fazi verovatnoće čuva sve aksiome klasične verovatnoće sa bogatijom interpretacijom. Klasična verovatnoća je samo posebni slučaj ovako generalizovane verovatnoće kao što je i teorija klasičnih skupova samo posebni slučaj konzistentne teorije fazi skupova zasnovane na realno-vrednosnoj realizaciji konačne Bulove algebre.

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# Formal Verification of Key Properties for Several Probability Logics in the Proof Assistant Coq

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Here, we present an encoding of four probability logics in the proof assistant Coq. These logics allow for reasoning on the probability of events, each of them with its own specificities:

- The logic  $LPP_1^{\mathbb{Q}}$ , in which the probability of events can be any rational number from the unit interval, with iterations of probability operators,
- The logic  $LPP_2^{\mathbb{Q}}$ , in which the probability of events can be any rational number from the unit interval, without iterations of probability operators,
- The logic  $LPP_1^{Fr(n)}$ , in which the probability of events is restricted to the set  $\{0, \frac{1}{n}, \dots, \frac{n-1}{n}, 1\}$ , for a given positive integer  $n$ , with iterations of probability operators, and
- The logic  $LPP_2^{Fr(n)}$ , in which the probability of events is restricted to the set  $\{0, \frac{1}{n}, \dots, \frac{n-1}{n}, 1\}$ , for a given positive integer  $n$ , without iterations of probability operators.

For each of these logics, we encode their syntax, semantics, and axiom systems, and provide formal proofs of several important meta-theorems, notably soundness, strong and simple completeness, and compactness or non-compactness, with the strong completeness theorem being of particular importance, as it gives formal justification for the use of probabilistic SAT-checkers for problems such as:

- determining whether probability estimates placed on certain events are consistent,
- calculating, given probability estimates of certain assumptions, the probability of the conclusion,

which could arise in various expert systems applying any one of these logics to fields such as game theory, economy and medicine.

# Functional Decomposition as an Optimization Technique \*

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## 1 Introduction

A basic operation such as calculating the value of a function is in the heart of any problem solving process. In specialized systems where the speed of calculation is of great importance, various optimization techniques are applied. There is no general recipe for successful optimization, it usually requires problem dependant heuristic. One can express a function to be calculated in different ways, use various decomposition techniques, use hardware implementation (or a combination of software and hardware implementation), improve speed of data structures used in the algorithm, use precomputed values, use approximative solutions etc. In this paper, we will explore the possibility of taking advantage of functional decomposition combined with using underlying probability distribution of input variables for determining which values should be used for pre computation.

## 2 Preliminaries

Let's suppose that we have given a finite commutative ring  $(R, +, *)$ , where  $R = \{r_1, r_2, \dots, r_K\}, K \in \mathbb{N}$ . We want to evaluate function  $f : R^N \rightarrow R, N \in \mathbb{N}$  ( $f(\mathbf{x}) = f(x_1, x_2, \dots, x_N)$  where  $x_i \in R$  for  $1 \leq i \leq N$ .) Computing the function value for a specific input requires time  $T_c(f)$ . If we have a memory of limited size  $M$ , we would be able to pre compute and store function value for up to  $M$  values of input parameter combinations (input vectors). Assuming that reading a value from the memory requires time  $T_M$  and that  $T_M$  is significantly less than  $T_c(f)$ , with this approach we can cut down the average time of evaluating the function  $f$ . Note that the term *memory* in this context can denote a physical memory or a convenient data structure. Let's assume that there is an underlying probability distribution of input variables, so that probability of  $x_i = r_j$  is denoted as  $p_{ij}$ , where  $\sum_{k=1}^{K} p_{ik} = 1, 1 \leq i \leq N$ . We will also assume that input parameters have independent distributions. The information about the distribution can be used to find  $M$  most probable combinations of input parameters and use them for precomputed and stored values. Obviously, that will minimize the expected time of evaluating the function which is given by formula:

$$E_f[T] = T_c(f) - P(X_M)(T_c(f) - T_M) \quad (1)$$

where  $X_M$  is set of all input vectors used for pre computation and  $P(X_M)$  is probability that an input vector is used for pre computation.

Of course, this is not the only way of using the memory resource. Depending on the usage of the system, this might be satisfying solution. But there are some issues in this approach. First, is it possible to use memory resource in a different way to reduce average evaluation time even more? And second, how can we affect more than  $M$  input vectors? One way to approach to these two problems is functional decomposition. A decomposition  $\Delta(f)$  of a function  $f$  is set of functions  $\Delta(f) = \{F, f_1, f_2, \dots, f_D\}$ , such that

$$f(\mathbf{x}) = F(f_1(\mathbf{x}_1), f_2(\mathbf{x}_2), \dots, f_D(\mathbf{x}_D)) \quad (2)$$

where  $\mathbf{x}_i(1 \leq i \leq D)$  are vectors formed as sub vectors of the initial vector  $\mathbf{x}$ . In this paper, from now on, we are considering decompositions of form

$$f(\mathbf{x}) = f_1(\mathbf{x}_1) \oplus_1 f_2(\mathbf{x}_2) \oplus_2 \dots \oplus_{D-1} f_D(\mathbf{x}_D) \quad (3)$$

where  $\oplus_i \in \{+, *\}, 1 \leq i \leq D-1$ . Basically, that means that we are not examining hierarchical decompositions of form  $g(h(\mathbf{y}))$ .

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The natural question here is how to find decomposition which minimizes the average time of calculation for function  $f$ . But before that, we have to find a way to efficiently find the average time for one particular decomposition  $\Delta(f)$ . The main issue is how to divide the available memory resource among functions  $f_i$  in the optimal way. The next section gives answer to that question.

### 3 Optimal resource distribution

The average time for evaluating the function  $f$  is given by formula

$$E_{f,\Delta}[T] = T_c(\Delta(f)) - \sum_{j=1}^D P(X_M^{(j,m_j)})(T_c(f_j(\mathbf{x}_j)) - T_M) \quad (4)$$

We are looking to maximise  $\omega(D)$ , where

$$\omega(q) = \sum_{j=1}^q P(X_M^{(j,m_j)})(T_c(f_j(\mathbf{x}_j)) - T_M) \quad (5)$$

Each function  $f_j$  from a decomposition  $\Delta(f)$  can count on  $m_j$  memory locations and  $\sum_{j=1}^D m_j M, 0m_j M$ . Since  $P(X_M^{(j,m_j)})$  depends on  $m_j$ , finding minimal average time  $E_{f,\Delta}[T]$  requires optimal configuration  $[m_1, m_2, \dots, m_D]$ . One way to solve this is by brute force, but the problem with this approach is exponential complexity. Here, we will propose the algorithm based on dynamic programming which solves the problem in polynomial time.

**Step 1.** For each  $1 \leq j \leq D$  calculate  $P_{ij} = P(X_M^{(j,i)})$  where  $m_j = i, 0 \leq i \leq l_j$  and  $l_j = \min\{M, \text{length of vector } \mathbf{x}_j\}$ . Once again, this can be done by brute force, but exponential complexity can be avoided by reducing this problem to finding  $M + 1$  best paths in a trellis. Therefore, the complexity of this procedure is  $O(DMK^2N)$ .

**Step 2.** Let's define  $Q_{ij} = P_{ij}(T_c(f_j(\mathbf{x}_j)) - T_M)$ . Let's define matrix  $\Omega$  and its element  $\Omega[i, j, k]$  as  $\max\{\omega(j)\}$  such that  $m_j = i$  and  $\sum_{a=1}^j m_a = k$ . Now we can find  $\max\{\omega(D)\}$  by the following procedure with complexity  $O(M^3D)$ .

*Initialization.* For  $0 \leq i \leq l_1, 0 \leq k \leq M$

$$\Omega[i, 1, k] = \begin{cases} Q_{i1} & \text{if } i = k \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

*Recursion.* For  $2 \leq j \leq D, 0 \leq i \leq l_j, 0 \leq k \leq M$

$$\Omega[i, j, k] = Q_{ij} + \max_{0 \leq i' \leq i} \Omega[i', j-1, k-i] \quad (7)$$

$$\Psi[i, j, k] = \operatorname{argmax}_{i', 0 \leq i' \leq i} \Omega[i', j-1, k-i] \quad (8)$$

*Stopping.* Let's define  $M'$  as  $M' = \min\{M, \sum_{a=1}^D l_a\}$ . Then,  $\max\{\omega(D)\} = \max_{0 \leq i \leq l_D} \Omega[i, D, M'], m_D = \max_{i, 0 \leq i \leq l_D} \Omega[i, D, M'], k_D = M'$

*Reconstruction.* For  $D-1 \geq j \geq 1$

$$m_j = \Psi[m_{j+1}, j+1, k_{j+1}], k_j = k_{j+1} - m_{j+1} \quad (9)$$

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# Konstruktivne polugrupe

*Siniša Crvenković*

Konstruktivne polugrupe su nastale kao prirodno uopštenje konstruktivnih grupa. Klasična teorija polugrupa razvila se zahvaljujući alatima specifičnim za polugrupe. Najznačajniji polugrupni pojmovi su Grinove relacije. Pomoću Grinovih relacija određujemo strukturalna svojstva polugrupa. U konstruktivnim polugrupama nije moguće definisati Grinove relacije na klasičan način, tj. pomoću egzistencijalnih kvantifikatora. U izlaganju će biti reči o analogonima klasičnih algebarskih pojmova u teoriji konstruktivnih polugrupa.

# Message Passing Algorithms over the Binomial and the Entropy Semirings

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The efficient computation of the partition function of a multivariate function is important in many areas including information theory, artificial intelligence, and statistical physics. When a cycle-free factor graph representation of the function is available, then partition function can exactly be computed by sum-product message passing in the factor graph [1, 3, 12, 14].

The "sum" and the "product" in sum-product message passing may belong to an arbitrary commutative semiring [1]. In this talk, we propose to use it with the binomial semiring and the resulting algorithm will be called the "binomial semiring message passing" (*BSMP*) [10]. The similar idea appears in [8] where we have introduced the binomial semiring and applied it to inside algorithm [4], which computes the partition function of the stochastic context free grammar. In this paper, we translate the ideas of [8] into the language of factor graphs and message passing algorithms.

The *BSMP* can compute cross-moments of an arbitrary order, for functions which admit the cycle free factor graph representation. Thus, the *BSMP* generalizes the prior algorithms for the computation of the first order moments [11], the second order moments [13] and the algorithms for higher order moments [2, 5] which are applicable only for the functions whose factor graph has a chain structure. Its time and memory complexity are the same as for the ordinary sum-product algorithm [12] up to the multiplicative factor corresponding to the order of the cross-moment.

We devote the special attention to the algorithm for the first order moments computation, called "entropy message passing". The primary use of *EMP* is to compute the entropy of a model with a cycle-free factor graph for fixed observations [7]. The main prior work on this subject is by Hernando et al. [6]. A main point of the talk is to clarify and to generalize this prior work by reformulating it in terms of sum-product message passing. Also, we show how the *EMP* can be applied to low memory computation of probabilistic model gradients [9].

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# Monadic Calculus and Constituents

*Saša Rasuo*

In this paper we propose a decidability procedure for the first order monadic calculus. The method is based on a combinatorial algorithm resting on constituents. First we show how to reduce a first order predicate formula which involves only unary predicates to an equivalent boolean combination of constituents. Computing the logical value of this boolean combination, we obtain the answer if the starting monadic sentence is a theorem of predicate logic, or not. We also show that this procedure is in fact a kind of quantifier elimination. Several examples are given which illustrates the method. We see the possible applications of the presented method in data base for computing formal queries and filters.

# On Archimedean inference rule

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The Archimedean inference rule is an infinitary inference rule designed to overcome the compactness issue in non restricted real valued probability logics. The key reason behind incompleteness lies in possibility to construct finitely satisfiable theories that express something like “the probability of  $\alpha$  is infinitely close but different from  $s$ ”, where  $s$  is some rational number from the real unit interval  $[0, 1]$ . Since  $\langle \mathbb{R}, \rangle$  omits the type of the infinitely small element, any such theory is unsatisfiable - hence we get both incompleteness and non-compactness.

Intuitively, the Archimedean rule works as follows: if probability of  $\alpha$  is infinitely close to  $a$ , than it must be equal to  $a$ . The exact formulation will be presented at the talk. I will also discuss the application of Archimedean-like inference rules in real valued formalization of other types of many valued logics such as possibility and fuzzy logic.



# Primena Bajesovih mreža u medicinskoj dijagnostici

*Svičević Marina*

Bajesove mreže su reprezentacije zavisnosti izmedju promenljivih, koje daju detaljniju specifikaciju zajedničke raspodele verovatnoća i koriste Bajesovo pravilo zaključivanja. Bajesovo učenje je vezano za algoritme učenja koji koriste verovatnoću i statistiku kao model.

Pri dijagnostikovanju neke bolesti potrebno je doneti ispravnu odluku. Glavna prednost verovatnosnog razmišljanja nad logičkim je dozvola donošenja racionalnih odluka čak i kada ne postoji dovoljno informacija da se dokaže da će bilo koja akcija biti izvršena. U izlaganju biće prikazani i najpovoljniji alati za izgradnju sistema koji bi pomogao lekarima pri dijagnostikovanju bolesti i njihova konkretna primena.

# Probability description language $P - \mathcal{ALCN}$

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Descriptive logics (DL) are commonly used languages for representing ontologies. Classical DL are a part of first-order logic, and as such do not allow representation of uncertainty of any kind. Some authors introduced a possibility in DL [3, 7]. Others work with statistical probability in DL [3, 4]. Last year Lutz and Schröder [5] published their result introducing subjective probability in descriptive language  $\mathcal{ALC}$  and some weaker languages. They used a Halpern's approaches for first-order logic [2] and they obtained probabilistic DLs with two-dimensional semantics similar to popular combinations of DL with temporal logic.

Our main task is to expand this language with more DL concept operators. We will start with classical  $\mathcal{ALCN}$  descriptive language, i.e. with sets of atomic concepts  $N_C = \{C_0, C_1, \dots\}$ , atomic roles  $N_R = \{R_0, R_1, \dots\}$  and individuals  $N_O = \{a_0, a_1, \dots\}$ , and define concepts, using classical concept constructors ( $\sqcap, \sqcup, \neg, \exists, \forall, n, n$ ), and formulas,  $C = D$ ,  $a : C$ ,  $aRb$ . Probability formulas are going to be  $P_s a : C$  and  $P_s aRb$  ( $s \in \mathbb{Q} \cap [0, 1]$ ). In that way we will create formulas  $\neg\phi$  and  $\phi \wedge \psi$  ( $\phi$  and  $\psi$  are both probability formulas or both classical DL formulas).

Main result of that work will be to prove that this formal system is sound and complete, and then to describe algorithm for checking consistency of a set of formulas. For all of that we will use the same technique as used in [6] for first-order language. Since, for the classical DL language  $\mathcal{ALCN}$  checking consistency is EXPTIME-complete, we expect that this formal system will have same complexity.

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# Pseudo-Random Number Generator Using as a Seed Distance (Movement) of a Laboratory Cultured Daphniae

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Laboratory cultured Daphniae (*Daphnia Magna*) are used for a scale of experiments, in particular for water quality assessment. Their distance (position and movement) can be used as a seed for random number generator. Such a number, using bio-seed, can be used in different simulations, especially in the chaos-based models.<sup>1</sup>

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# The System for Random Man not Excluded

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The aim of this paper is to present a system for calculating Random Man not Excluded (RMNE). RMNE is a type of measure in population genetics to estimate the probability that an individual randomly picked out of the general population would not be excluded from matching a given piece of genetic data. RMNE is frequently employed in cases where other types of tests such as random match possibility are not possible because the sample in question is degraded or contaminated with multiple sources of DNA.

## I INTRODUCTION

This paper explains a methodology for analyzing the problem of identification of DNA profiles. Assume that it is necessary to calculate the percentage of random men in the population to be excluded from paternity of the child. The probability of excluding a random man from paternity is another quantity that may be used by some scientists and paternity laboratories. The exclusion probability can be calculated even before the collection of the genotype of the alleled father. [1]

Part II of the paper presents some genetic background material. System for random man not excluded is presented in part III. Part IV illustrates the model through the real forensic casework examples and shows this in implemented system in C#. Finally, part V discusses further work.

## II Genetic background

An individual's DNA profile comprises a collection of genotypes, one for each marker. Each genotype consists of an unordered pair of alleles, one inherited from the father and one from the mother (though one can not distinguish which is which). When both alleles are identical the individual is homozygous at that marker, and only a single allele value is observed; else the individual is heterozygous.

Databases have been gathered from calculated allele frequency distributions. These allele frequency distributions can be calculated for various populations and can be estimated for each forensic marker. Calculations presented in this paper are gathered from self developed software.

## III Statistical model

Random Man Not Excluded (RMNE) is a type of measure in population genetics to estimate the probability that an individual randomly picked out of the general population would not be excluded from matching a given piece of genetic data. RMNE is frequently employed in cases where other types of tests such as random match possibility are not possible because the sample in question is degraded or contaminated with multiple sources of DNA.

Suppose that the mother and the child have genotypes  $M = A_iA_j$  and  $C = A_iA_k$  at a particular locus  $l$ , respectively. A man without allele  $A_k$  (i.e. both of his alleles are not  $A_k$ ) will be excluded as the true father of the child. Thus, the probability that a random individual in the male population is excluded from paternity in this case is  $(1-p_k)^2$ . This exclusion probability at locus  $l$  is

$$EP_l = (1-p_k)^2,$$

and so the inclusion probability or the 'random man not excluded' probability would be  $IPl = 1 - EP_l$ . The  $EP_l$ 's for various combinations of genotypes of the mother and child can be derived similarly and are given in Table 1.

<b>M</b>	<b>C</b>	<b>EP<sub>l</sub></b>
$A_iA_i$	$A_iA_i$	$(1-p_i)^2$
$A_iA_i$	$A_iA_j$	$(1-p_j)^2$
$A_iA_j$	$A_iA_i$	$(1-p_i)^2$
$A_iA_j$	$A_iA_j$	$(1-p_i-p_j)^2$
$A_iA_j$	$A_iA_k$	$(1-p_k)^2$

Table 1. Probability ( $EP_l$ ) that a random man is excluded from paternity of the child, given genotypes of the mother  $M$  and child  $C$  at a particular locus  $l$ .

## IV System for random man not excluded to forensic casework

A mixed DNA profile has been collected and the genotypes of one or more unknown individuals who have contributed to the mixture are desired, for example with the purpose of searching for a potential perpetrator among an existing database of DNA profiles.

For a two-person mixture, the easiest case to consider is clearly that of separation of a single unknown profile, i.e. when the genotype of one of the contributors to the mixture is known. The case when both contributors are unknown is more difficult. In the latter situation this is only possible to a reasonable accuracy when the contributions to the DNA mixture have taken place in quite different proportions.

In the system for random match probability to forensic casework is implemented the above-mentioned model. First are calculated the frequency of certain alleles in the corresponding locus. The database used does not bring up reference as people who are related and contain 4756 people. As for the number of loci for which allele frequencies are calculated-no. System from the input file base and locus with alleles, calculated for each locus allele frequencies  $p_i$ .

Random man not excluded used .txt file that contains data about the mother and child, their loci and allele values??. The output also given in .txt file.

#### V DISCUSSION

Further research is planned upgrade the system to calculate other probability forensic methods.

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# A solution of the gambling paradox using the principle of dimensionality

*Miloš Laban*

Not distinguishing between a logical and a statistical approach to a probability, leads to a situation well known as The gambling paradox. For example, if someone repeats throwing a coin hundred times, and gets a result of 33 heads and 67 tails, which is quite possible, the question is what we should expect in the next throwing. According to the logical approach, it is equal to expect a head as well as a tail. But from the statistical point of view, the number of heads and tails in the sequence of throwing a coin should be approximately equal. Hence, it is reasonable to expect that a head is a result of next throw, rather than a tail.

This work suggests that such a contradiction in expectations can be overcome observing the problem in two-dimensional surrounding instead of looking at it in one dimension (only the probability of a very step). Analogously, the way out from the plane circle is not possible in that plane without crossing the circle line, but is possible if you add a third dimension and raise the pencil in the air. This idea is formulated in this article in a general form as The principle of dimensionality and consequently applied in order to mathematically overcome the gambling paradox.

# Sistem za podršku u kliničkom odlučivanju koji integriše znanje eksperta i znanje iz podataka

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Jedan od osnovnih problema prilikom konstrukcije sistema za podršku u odlučivanju je da se eksplicitno formuliše znanje relevantno za specifičan domen. Značajan deo ovog znanja, međutim, često implicitno sadrže podaci vezani za taj domen. Kada je u pitanju medicina, posebno, kliničko odlučivanje, ovakvi podaci su relativno dobro sistematizovani i lako dostupni u obliku istorija bolesti pacijenata. U radu se diskutuju neke strategije za integrisanje znanja medicinskog eksperta sa podacima iz kliničke prakse, čime se ukazuje na mogući pristup u rešavanju problema. Ove strategije ugrađene su u sistem BELART, koji je namenjen pružanju podrške u kliničkom odlučivanju.