Workshop on even-hole-free graphs

Belgrade, April 15th–19th 2019
Organised by Kristina Vušković and Nicolas Trotignon

Schedule of talks

All talks are held at Room 102, in Annex of the Serbian Academy of Sciences and Arts (SANU), Kneza Mihaila 36, first floor.

Monday April 15th

10:00 – 11:00	Kristina Vušković	Even-hole-free graphs
11:00 – 12:00	Eun Jung Kim	Graph decompositions and width measures
12:00 – 14:00	Lunch break	
14:00 – 15:00	Nicolas Trotignon	Truemper configurations
15:00 – 15:30	Ni Luh Dewi Sintiari	Layered wheels
15:30 – 16:00	Marko Radovanović	Induced paths and cycles in (theta,wheel)-free graphs

Tuesday April 16th

10:00 – 11:00	Édouard Bonnet	When Maximum Independent Set is solvable in poly- or FPT-time
11:00 – 12:00	Chính Hoàng	Graph coloring problems for even hole-free graphs
12:00 – 14:00	Lunch break	
14:00 – 15:00	Kathie Cameron	(Cap, Even Hole)-free Graphs: Structure, Colouring and Hadwiger's Conjecture
15:00 – 15:30	Jake Horsfield	β-perfect graphs
15:30 – 16:00	Coffee break	
16:00 – 16:30	Irena Penev	Computing the chromatic number of a ring
16:30 – 17:00	Cléophée Robin	Coloring problem in classes of graphs defined by forbidden induced subgraphs

Abstracts

When Maximum Independent Set is solvable in poly- or FPT-time

Édouard Bonnet

Maximum Independent Set (MIS) is NP-hard and W[1]-hard in general. However, in some classes of graphs, this problem becomes more tractable, FPT or even polytime solvable. We will survey some tools and results, in an incomplete and biased fashion. In the first part of the talk, we will present potential maximal cliques and their basic algorithmic implications. In the second part, we will focus on the classical and parameterized "dichotomy conjectures" for MIS on H-free graphs. As for the relevant FPT techniques there, we will try to cover the so-called iterative expansion and Ramsey-based regularization processes. We will use the former to design FPT algorithms in classes that can be seen as cographs with some parameterized noise, and eventually, showcase how this turns out useful for H-free graphs.

(Cap, Even Hole)-free Graphs: Structure, Colouring and Hadwiger's Conjecture

Kathie Cameron

A *hole* is a chordless cycle with at least 4 vertices and is *even* if it has an even number of vertices. A *cap* is a hole together with an additional vertex whose neighbours are two adjacent vertices of the hole. A graph is *(cap, even hole)-free* if it has no induced subgraph isomorphic to a cap or to an even hole.

A *minor* of a graph G is obtained from a subgraph of G by contracting edges. In 1943, Hadwiger made his famous conjecture: For every integer $t \geq 0$, every graph with no K_{t+1} minor is t-colourable. Hadwiger proved the conjecture for $t = 3$. For $t = 4$, it is equivalent to the Four Colour Theorem. Robertson, Seymour and Thomas proved it for t=5, using the Four Colour Theorem. For $t \geq 6$, it remains open.

We give an explicit construction of a superclass of (cap, even hole)-free graphs called (cap, 4-hole)-free odd-signable graphs. We use our construction and other structural results to prove that Hadwiger's Conjecture holds these graphs. We also use the construction to prove that they satisfy $\chi(G) \leq \frac{3}{2}\omega(G)$, where $\omega(G)$ denotes the size of a largest clique in G and $\chi(G)$ denotes the chromatic number of G.

We give an $O(nm)$ algorithm for q-coloring (cap, 4-hole)-free odd-signable graphs for fixed q and an $O(nm)$ algorithm for maximum weight stable set, where n is the number of vertices and m is the number of edges of the input graph. We also give a polynomial-time algorithm for minimum coloring.

Our algorithms are based on our results that triangle-free odd-signable graphs have treewidth at most 5 and thus have clique-width at most 48, and that (cap, 4-hole)-free odd-signable graphs G without clique cutsets have treewidth at most $6\omega(G) - 1$ and clique-width at most 48.

This is joint work with Kristina Vušković, and some is joint work Murilo V. G. da Silva and Shenwei Huang.

Graph coloring problems for even hole-free graphs

Chính Hoàng

A hole is a chordless cycle with at least four vertices. An even hole is a hole with an even number of vertices. It is known that the chromatic number of an even hole-free graph G is at most 2 times the number of vertices in a largest clique of G. However, the complexity of computing the chromatic number of an even hole-free graph is not yet determined. The coloring problem is polynomial time solvable for some restricted classes of graphs such as (diamond, even hole)- free, or (pan, even hole)-free graphs. There are relationships between even hole- free graphs and tree widths, and circular arc graphs. In this talk, we survey the results in this area.

β-perfect graphs

Jake Horsfield

Let G be a graph, and define $\beta(G) = \max\{\delta(G') + 1 \mid G' \text{ is an induced subgraph of } G\}$, where $\delta(G)$ denotes the minimum degree of a vertex in G. The value $\beta(G)$ is an upper bound on the chromatic number of G, i.e. $\chi(G) \leq \beta(G)$.

In 1996, Markossian, Gasparian and Reed introduced the class of β-perfect graphs. A graph G is *β-perfect* if $\chi(G') = \beta(G')$ for every induced subgraph G' of G. Observe that for an even hole H, $\chi(H) = 2$ while $\beta(H) = 3$. Therefore β-perfect graphs form a subclass of even-hole-free graphs. Colouring greedily on an easily constructible ordering of vertices gives a polynomial time colouring algorithm for β-perfect graphs.

In this talk I will survey what is known about β-perfect graphs, present a proof of the β-perfectness of (even hole, twin wheel, cap)-free graphs, and will conclude by mentioning some open problems related to this class.

Graph decompositions and width measures

Eun Jung Kim

Trees are presumably the simplest structure in graph theory. Prominent aspects of a tree is that every edge is a cut edge, and every vertex pair is connected by a unique path. Thanks to this extreme simplicity of 'information flow' over trees, many properties can be readily proven and algorithms are easy to design on trees.

In this talk, we survey various decomposition scheme and width measures which try to portray graph structures via tree. The allowed quantity of information flow over edge/vertex would indicate the thickness of a tree. Depending on how to measure the information flow, various width measures can be defined. We overview some of the well-known width measures such as treewidth, branchwidth, rank/clique width and tree-cut width, as well as modular decomposition.

Computing the chromatic number of a ring

Irena Penev

A *ring* is a graph R whose vertex set can be partitioned into $k \geq 4$ nonempty sets $X_1, \ldots, X_k$ such that for all $i \in \{1, \ldots, k\}$ the set X_i can be ordered as $X_i = \{u_i^1, \ldots, u_i^{|X_i|}\}$ so that

$$X_i \subseteq N_R[u_i^{|X_i|}] \subseteq \cdots \subseteq N_R[u_i^1] = X_{i-1} \cup X_i \cup X_{i+1},$$

with subscripts taken modulo k. Under such circumstances, we say that the ring R is of *length* k. An *odd* (resp. *even*) ring is a ring of odd (resp. even) length.

Truemper configurations are prisms, pyramids, thetas, and wheels. Rings have played an important role in the study of a couple of classes defined by excluding certain Truemper configurations as induced subgraphs. A maximum clique and a maximum stable set of a ring can be computed in polynomial time, as can an optimal vertex-coloring of an even ring. However, odd rings present obstacles for coloring.

Our main result is that every ring R satisfies

$$\chi(R) = \max\{\chi(H) \mid H \text{ is a hyperhole in } R\}.$$

We present several corollaries of this result. One corollary is that the chromatic number of a ring can be computed in polynomial time. (However, we still do not know whether it is possible to find an optimal coloring of an odd ring in polynomial time.)

Joint work with Frédéric Maffray and Kristina Vušković

Induced paths and cycles in (theta,wheel)-free graphs

Marko Radovanović

A *theta* is a graph formed by three internally vertex-disjoint paths of length at least 2 between the same pair of distinct vertices so that the union of any two induces a hole. A *wheel* is a graph formed by a hole and a vertex that has at least 3 neighbors in the hole.

In this talk, using the decomposition theorem for the class $\mathcal{C}$ of (theta,wheel)-free graphs that we obtained earlier, we prove that the problem k-in-a-Cycle is fixed parameter tractable for graphs in $\mathcal{C}$ (when parameterized on k), and provide an $O(n^{2k+6})$-time algorithm for the k-Induced Disjoint Paths problem for graphs in $\mathcal{C}$ (where k is fixed).

Joint work with Nicolas Trotignon and Kristina Vušković.

Coloring problem in classes of graphs defined by forbidden induced subgraphs

Cléophée Robin

The vertex coloring problem is known to be NP-complete in the general case but it can become polynomial in some restricted classes of graphs. Those that we are interested in, are the classes defined by forbidden induced subgraphs. Given a set of graphs $\mathscr{H}$ we denote by Free$\{\mathscr{H}\}$ the class of graphs that do not contains any graph from $\mathscr{H}$ as an induced subgraph.

Král et al. gave a complete complexity classification for vertex coloring when $\mathscr{H}$ has cardinality one.

Lozin and Malyshev studied the cases where $\mathscr{H}$ has cardinality two and contains only graphs with four vertices. They highlight the three cases where the complexity of the coloring problem remains unknown. These cases are the classes: Free$\{C_4; O_4\}$, Free$\{K_{1;3}; 2P_1 + P_2\}$ and Free$\{K_{1;3}; O_4\}$ (O_4 is the stable set of size four). They also proved a polynomial equivalence of the coloring problem between the class Free$\{K_{1;3}; 2P_1 + P_2\}$ and Free$\{K_{1;3}; 2P_1 + P_2; O_4\}$.

We will present these results and some of our partial structural results on Free$\{C_4; O_4\}$ and on Free$\{K_{1;3}; 2P_1 + P_2; O_4\}$ also called the class of anti-prismatic graphs.

Joint work with Frédéric Maffray, Myriam Preissmann and Nicolas Trotignon

Layered wheels

Ni Luh Dewi Sintiari

We present a construction called layered wheel. Layered wheels are graphs of arbitrarily large treewidth and girth. They might be an outcome for a possible theorem characterizing graphs with large treewidth in term of their induced subgraphs (while such a characterization is well understood in term of minors). They also provide examples of graphs of large treewidth in well studied classes, such as (theta, triangle)-free graphs and even-hole-free graphs with no K_4 (where a hole is a chordless cycle of length at least 4, a theta is a graph made of three internally vertex disjoint paths of length at least 2 linking two vertices, and K_4 is the complete graph on 4 vertices). We believe that layered wheels could be important in a structural description of these classes, and to support this idea we prove that some classes excluding them have bounded treewidth, namely graphs with no theta, triangle, and k-wheel, and graphs with no even hole, K_4, pyramid, and k-wheel.

Joint work with Nicolas Trotignon

Tremper configurations

Nicolas Trotignon

A *3-path-configuration* in a graph is an induced subgraph made of three internally vertex disjoint paths $P_1 = a_1 \dots b_1$, $P_2 = a_2 \dots b_2$, and $P_3 = a_3 \dots b_3$ of length at least 1, such that :

- $a_1 = a_2 = a_3$ or $a_1a_2a_3$ is a triangle;
- $b_1 = b_2 = b_3$ or $b_1b_2b_3$ is a triangle;
- for every $1 \leq i < j \leq 3$, $V(P_i) \cup V(P_j)$ induces a hole.

If both $a_1a_2a_3$ and $b_1b_2b_3$ are triangles, then the 3-path configuration is a *prism*. If exactly one of them is a triangle, then the 3-path configuration is a *pyramid*. If none of them is a triangle, then the 3-path configuration is a *theta*. Observe that the condition that two of the paths form a hole imply that:

- The three paths of a prism have length at least 1.
- The three paths of a pyramid have length at least 2, except one that may have length 1.
- The three paths of a theta have length at least 2.

A *wheel* is a graph formed by a hole and a vertex that has at least 3 neighbors in the hole. A *Truemper configuration* is a graph that is either a 3-path-configuration or a wheel.

In the talk is about:

1. The original theorem of Truemper about signing the edges of a graph.
2. The role of Truemper configurations in the study of several classes of graphs, including perfect graphs and even-hole-free graphs.
3. The recent progress on the systematic study of classes of graphs defined by excluding some Truemper configurations.
4. Recent open questions about how Truemper configuration might help to understand graphs of large tree-width in terms of their Truemper configuration (analogous to how grids help to understand graphs of large tree width in terms or their minors).

Even-hole-free graphs

Kristina Vušković

In this talk we will describe known decomposition theorem for even-hole-free graphs and show how it is used to obtain a polynomial-time recognition algorithm for even-hole-free graphs. We will also describe the fastest known algorithm for maximum weight clique problem for even-hole-free graphs. We also discuss a number of related classes.

Participants

Édouard Bonnet (CNRS, ENS Lyon, France)
Kathie Cameron (Wilfrid Laurier, Canada)
Chính Hoàng (Wilfrid Laurier, Canada)
Jake Horsfield (Leeds, UK)
Eun Jung Kim (CNRS, LAMSADE, Paris, France)
Irena Penev (Charles University, Prague, Czech Republic)
Myriam Preissmann (CNRS, G-Scop, Grenoble, France)
Marko Radovanovi (University of Belgrade, Serbia)
Cléophée Robin (G-Scop, Grenoble, France)
Ni Luh Dewi Sintiari (ENS Lyon, France)
Nicolas Trotignon (CNRS, ENS Lyon, France)
Kristina Vušković (Leeds, UK)

Workshop on even-hole-free graphs
Belgrade, 15-19 April, 2019

Organizers: Kristina Vušković and Nicolas Trotignon

Even-hole-free graphs have been studied for the past 30 years, initially due to their relationship to perfect graphs. Indeed, it was in the study of even-hole-free graphs that crucial techniques were developed that lead to the resolution of the famous Strong Perfect Graph Conjecture. The two classes are structurally quite similar, and present a good playing ground for developing algorithmic techniques for hard problems, understanding properties of hereditary graph classes, and seeking for boundaries of polynomial solvability.

The workshop will consist of 2 days of talks (April 15 and 16) followed by 3 days of research. There will be survey/tutorial talks and research talks.
The survey talks will include structure of graph classes, width parameters, techniques for coloring and stable set problems and Hadwiger's Conjecture.

List of participants and speakers:

Edouard Bonnet (CNRS, ENS Lyon, France)
Kathie Cameron (Wilfrid Laurier, Canada)
Chinh Hoang (Wilfrid Laurier, Canada)
Jake Horsfield (Leeds, UK)
Eun Jung Kim (CNRS, LAMSADE, Paris, France)
Irena Penev (Charles University, Prague, Czech Republic)
Myriam Preissmann (CNRS, G-Scop, Grenoble, France)
Marko Radovanović (University of Belgrade, Serbia)
Cléophée Robin (G-Scop, Grenoble, France)
Dewi Sintiari (ENS Lyon, France)
Nicolas Trotignon (CNRS, ENS Lyon, France)
Kristina Vušković (RAF, Serbia/Leeds, UK)

The talks are open to all, but please register your attendance by emailing the organizers at k.vuskovic@leeds.ac.uk.