Symposium on Analytic Mechanics and Differential Geometry

dedicated to the anniversaries of Anton Bilimovich and Veljko Vujičić

140th anniversary of Professor Bilimovich and 90th anniversary of Professor Vujičić

Mathematical Institute SANU, Belgrade, May 8th, 2019

Mechanical Colloquium and Theoretical and Applied Mechanics

Talks

- Yuri Fedorov, Polytechnic University of Catalonia
- Vladimir Dragović, University of Texas at Dallas, MI SANU
- Teodor Atanacković, SANU, University of Novi Sad
- Siniša Mesarović, Washington State University
- Srdjan Vukmirović, University of Belgrade
- Tijana Šukilović, University of Belgrade
- Darko Milinković, University of Belgrade
- Mihailo Lazarević, University of Belgrade
- Srboljub Simić, University of Novi Sad
- Aleksandar Obradović, University of Belgrade
- Borislav Gajić, Mathematical Institute SANU

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Programme

10:00-10:30 Opening remarks
10:30-11:00 **Vladimir Dragović**, Elliptical billiards in the Minkowski plane and extremal polynomials

Coffee break

11:15-11:45 **Siniša Mesarović**, Lattice continua for polycrystals: dislocations, diffusion, grain boundaries and creep
11:45-12:15 **Mihailo Lazarević**, Advanced modeling and control of dynamic of complex mechanical systems using calculus of general order
12:15-12:45 **Srboljub Simić**, Variational principles of mechanics – from energy to entropy

Lunch break

15:15-15:45 **Darko Milinković**, Spectral numbers in symplectic topology
15:45-16:15 **Srdjan Vukmirović**, Projectively equivalent metrics on Lie groups
16:15-16:45 **Tijana Šukilović**, Four-dimensional metric Lie algebras

Coffee break

17:00-17:30 **Aleksandar Obradović**, Realizing brachistochronic planar motion of a variable mass body by centrodes
17:30-18:00 **Borislav Gajić**, Chaplygin reducing multiplier in nonholonomic mechanics
18:00-18:30 **Teodor Atanacković**, On a Constitutive equation of Heat Conduction with Fractional derivatives of complex order

Cocktail

Mathematical Institute SANU, Belgrade, the first floor, May 8th, 2019
Mechanical Colloquium and Theoretical and Applied Mechanics journal

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Abstracts

Teodor M. Atanacković, Department of Mechanics, University of Novi Sad, Serbia, e-mail: atanackovic@uns.ac.rs

On a Constitutive equation of Heat Conduction with Fractional derivatives of complex order

We study heat conduction with constitutive equation containing fractional derivatives of complex order. Using the entropy inequality in a weak form we derive a sufficient conditions on coefficients in the model that guarantee that the Second law of thermodynamics is satisfied. Several known constitutive equations follow from our model as special cases. For the proposed constitutive equation we present a solution for the problem of a temperature distribution in a semi-infinite rod. For more details, we refer to [1], [2], and [3].

1. T. M. Atanackovic, S. Pilipovic, On a constitutive equation of heat conduction with fractional derivatives of complex order. *Acta Mechanica*, **229**, 1111-1121 (2018).
2. T.M. Atanackovic, S.Pilipovic, B. Stankovic, D. Zorica, Fractional calculus with Application in Mechanics: Vibrations and Diffusion Processes, ISTE, London, John Wiley & Sons, New York, 2014.
3. T.M. Atanackovic, S.Pilipovic, B. Stankovic, D. Zorica, Fractional calculus with Application in Mechanics: Wave Propagation, Impact and Variational Principles, ISTE, London, John Wiley & Sons, New York, 2014.

Vladimir Dragović, University of Texas at Dallas, MI SANU, , e-mail: vladad@mi.sanu.ac.rs

Elliptical billiards in the Minkowski plane and extremal polynomials

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Borislav Gajić, Mathematical Institute SANU, e-mail: gajab@mi.sanu.ac.rs

Chaplygin reducing multiplier in nonholonomic mechanics

We compare various approaches in geometrical formulation of nonolonomic systems by using affine connections, including the Chaplygin reduction. Although mentioned objects are very well studied, some natural relationships between them are pointed out. We consider the Newton type equations on a Riemannian manifold *(M, g)* and look for a conformal metric $g^* = f^2 g$ such that solutions of the Newton equations, after a time reparametrization,

become the geodesic lines of *g*. This is a generalization of the Chaplygin multiplier method for Hamiltonization of G-Chaplygin systems. The results are joint work with Božidar Jovanović.

Mihailo Lazarević, Faculty of Mechanical Engineering, University of Belgrade e-mail: : mlazarevic@mas.bg.ac.rs

Advanced modeling and control of dynamic of complex mechanical systems using calculus of general order

The investigation into the dynamics of robotic and complex mechanical systems has been an active topic of research for many years. The modelling complex rigid multibody systems (RBS) using symbolic equations can provide many advantages over the more widely-used numerical methods of modeling these systems. In this talk, we propose using procedure for recursive symbolic form computation of the complete dynamics of robotic systems with the open kinematic chain structures using Rodriquez approach for matrices of coordinate transformations. Dynamic equations are given as Lagrange equations of the second kind in the covariant form with external generalized forces of the gravity, motor-torque, viscous and spring. Also, a model of a mechanical system with a structure of a topological tree will be discussed. Dynamics of mechanical systems with closed kinematic chains will be also considered. On the other side, the use of adaptive (hereditary/actuator: viscoelastic element with an actuator, piezo-viscoelastic,thermo-viscoelastic and magneto-rheologic) elements in complex RBS can be significant for the additional control of these systems and for reducing undesirable vibrations. Recently, fractional calculus (FC) i.e calculus of general order has attracted an increased attention of scientific society. The fractional integro-differential operators are a generalization of integration and derivation to fractional operators where fractional derivatives (FD) are often used to describe viscoelastic, rheological properties of advanced materials and dissipative forces in structural dynamics. Here, modeling of dynamics of multibody systems involving generalized forces of a spring/spring-pot/actuator (SSPA) and MR elements modeled with fractional order derivatives, are done. Generalized forces of an element are derived by using the principle of virtual work and force–displacement relation of the fractional order Kelvin–Voigt/Zener model.

Siniša Mesarović, Washington State University, e-mail: smesarovic@wsu.edu

Lattice continua for polycrystals: dislocations, diffusion, grain boundaries and creep

At high temperatures, the interior of each grain in a polycrystal suffers: (1) dislocation glide, (2) climb, and (3) diffusion of vacancies. Grain boundaries undergo: (4) growth/disappearance, as a result of vacancy diffusion, and, (5) crystallographic reorientation/mismatch, as a result of dislocations arriving to the boundaries either by glide or by climb. All the above deformation mechanisms are naturally described in the lattice continuum framework, whereby the lattice represents the material. Climbing edge dislocations are lattice sink/source which must be reflected in the continuity equation and the transport theorem. The interacting kinematics of dislocation glide and climb requires

dual definition of crystallographic slip fields: the true slip and the apparent slip. The transport theorem for grains with lattice growing or disappearing lattice at different grain boundary faces results in the direct formulation of the boundary condition for vacancy diffusion flux in terms of the boundary velocity (different from the lattice velocity). The field equations for each grain are derived by means of the principle of virtual power. Additional boundary conditions result from the relative motion of the adjacent crystal faces: Change of tilt and twist angle and surface elastic mismatch are derived from the geometry of dislocations arriving to the boundary. The resulting polycrystal initial/boundary value problem consists of elasticity-plasticity-diffusion field equations in each crystalline domain with moving boundaries, coupled through the boundary conditions.

1. Mesarovic, S.Dj. 2019 Physical foundations of mesoscale continua. In Mesoscale models: From micro-physics to macro-interpretation. CISM International Centre for Mechanical Sciences Courses and Lectures 587. Eds. S.Dj. Mesarovic, S. Forest & H.M. Zbib. Springer.
2. Mesarovic, S.Dj. 2017 Dislocation creep: Climb and glide in the lattice continuum. Crystals 7(8), 243.
3. Mesarovic, S.Dj. 2016 Lattice continuum and diffusional creep. Proc. R. Soc. A 472, 20160039.

Darko Milinković, Mathematical Faculty, University of Belgrade, e-mail: milinko@matf.bg.ac.rs

Spectral numbers in symplectic topology

Spectral numbers are the minimax values of Hamiltonian action functional, constructed by filtered Floer homology. They can be used to detect periodic orbits of Hamiltonian systems, to measure the symplectic size of a set, to estimate the minimal energy needed to achieve given configuration, to construct symplectic quasi-states of a system, to study the geometry of the group of Hamiltonian difeomorphisms etc. We will sketch some of these motivations and (if time permits) give the construction (based on recent joint work with J. Katić and J. Nikolić) of spectral numbers associated to the action functional subject to some singular Lagrangian boundary conditions.

Aleksandar Obradović, Faculty of Mechanical Engineering, University of Belgrade, e-mail: aobradovic@mas.bg.ac.rs

Realizing brachistochronic planar motion of a variable mass body by centrodes

The paper considers realization of the brachistochronic planar motion of a mechanical system composed of one body and variable mass material points, by means of an ideal constraint in the form of the centroides. The laws of the time-rate of mass variation of the material points, as well as relative velocities of the expelled particles are known. Applying Pontryagin's maximum principle and singular optimal control theory, the problem of brachistochronic motion is solved as a two-point boundary value problem (TPBVP). The conditions are examined under which such motion is feasible for the case of unilateral constraint subjected to the Coulomb friction laws. The considerations are illustrated via an

example, where it is examined how change in the initial energy of the system affects the constraint reactions.

Srboljub Simić, Department of Mathematics and Informatics, Faculty of Sciences, University of Novi Sad, e-mail: ssimic@uns.ac.rs

Variational principles of mechanics – from energy to entropy

Calculus of variations and variational principles are in the heart of analytical mechanics. They provide a description of the mechanical problem in terms of extremal values of certain functional. They also form a basis for approximate analytical methods and numerical methods of solution of ordinary and partial differential equations in mechanics. This talk presents an overview of different applications of variational methods of mechanics. It starts with Hamilton's principle and the principle of minimum of the potential energy, continues with application of the Ritz method to the shock structure problem, and ends up with the maximum entropy principle and its application in the kinetic theory of gases.

Tijana Šukilović, Mathematical Faculty, University of Belgrade, e-mail: tijana@matf.bg.ac.rs

Four-dimensional metric Lie algebras

The pair $(\mathfrak{g}, \langle\ ,\ \rangle)$ where $\mathfrak{g}$ is a Lie algebra and $\langle\ ,\ \rangle$ an inner product of arbitrary signature is called a metric Lie algebra. Note that the inner product on the Lie algebra extends uniquely to a left invariant metric *g* on the corresponding simply connected Lie group *G*. In this talk the main focus will be on the difference between Riemannian and pseudo-Riemannian setting. While the Reimannian case is thoroughly studied and well-understood, there are several open questions in the pseudo-Riemannian case. Those questions will be addressed and partially answered in some special cases.

Srdjan Vukmirović, Mathematical Faculty, University of Belgrade e-mail: vsrdjan@matf.bg.ac.rs

Projectively equivalent metrics on Lie groups

Two pseudo-Riemannian manifolds are said to be projectively equivalent if they have the same set of unparameterized geodesics. We show that if two left-invariant metrics on a Lie group G are projectively equivalent than they are also affinely equivalent, i.e. they have the same Levi-Civita connection. From results of Eisenhart it follows that in the Riemmanian case such two metric are proportional. We are interested in families of pseudo-Riemannian left invariant metrics that are affinely equivalent, but not proportional. This is joint work with N. Bokan and T. Sukilovic.