MATHEMATICAL INSTITUTE
SANU - BELGRADE

Treća nacionalna konferencija "Verovatnosne logike i njihove primene"

Beograd, Srbija, 26. septembar 2013.

Knjiga apstrakata

ORGANIZATOR:

Matematički institut, SANU

KONFERENCIJU FINANSIRAJU:

Ministarstvo prosvete i nauke Republike Srbije

Projekat Razvoj novih informaciono-komunikacionih tehnologija, korišćenjem naprednih matematičkih metoda, sa primenama u medicini, telekomunikacijama, energetici, zaštiti nacionalne baštine i obrazovanju, III 044006

Projekat Reprezentacije logičkih struktura i formalnih jezika i njihove primene u računarstvu, ON 174026.

MATHEMATICAL INSTITUTE
SANU - BELGRADE

Treća nacionalna konferencija "Verovatnosne logike i njihove primene"

Beograd, Srbija, 26. septembar 2013.

TEME KONFERENCIJE:
- verovatnosne logike, problemi potpunosti, odlučivosti i složenosti,
- logičke osnove u zasnivanju verovatnoće,
- Bayes-ove mreže i drugi srodni sistemi,
- programski sistemi za podršku odlučivanju u prisustvu neizvesnosti,
- primene verovatnosnog zaključivanja u medicini itd.

ORGANIZATOR:
Matematički institut, SANU

KONFERENCIJU FINANSIRAJU:

Ministarstvo prosvete i nauke Republike Srbije

Projekat Razvoj novih informaciono-komunikacionih tehnologija, korišćenjem naprednih matematičkih metoda, sa primenama u medicini, telekomunikacijama, energetici, zaštiti nacionalne baštine i obrazovanju, III 044006

Projekat Reprezentacije logičkih struktura i formalnih jezika i njihove primene u računarstvu, ON 174026.

Program konferencije:

26. 10. 2013.

10:00 – 10:15	*Otvaranje*
10:15 – 10:45	*Hierarchies of probability logics*, Zoran Ognjanović, Aleksandar Perović, Miodrag Rašković, Nebojša Ikodinović
10:50 – 11:10	*On definition of consistency in a probabilistic sequent calculus*, Marija Boričić
11:15 – 11:35	*Factor analysis in medical research*, Nataša Glišović
11:40 – 12:00	*Adelic estimation of uncertainty*, Angelina Ilić Stepić
12:00 – 12:20	*Pauza*
12:20 – 12:40	*Complex numbers as a model of euclidean geometry*, Saša Rašuo
12:45 – 13:05	*On the SCFG cross-moments computation using the moment generating function*, Velimir M. Ilić, Miroslav D. Ćirić, Miomir S. Stanković
13:10 – 13:30	*Free Boolean vectors and expressions*, Aleksandar Pejović, Žarko Mijajlović
13:30 – 13:50	*Integrating Knowledge and Data in a Medical Decision Support System*, Vladimir Srdanović, Marko Šošić
13:50 – 15:00	*Pauza*
15:00 – 15:20	*Bee-colony optimization for the satisfiability problem in probabilistic logic*, Tatjana Stojanović, Tatjana Davidović, Zoran Ognjanović
15:25 – 15:45	*Deep belief learning and neural networks*, Ivan Čukić
15:50 – 16:10	*One Probabilistic Approach to Classification Problems*, Milica Knežević, Aleksandar Perović
16:15 – 16:35	*On definable operators in certain probability logics*, Zoran Ognjanović, Aleksandar Perović, Miodrag Rašković
16:40 – 17:00	*Probability logic and probability systems of argumentation*, Dragan Doder

Apstrakti

Sadržaj

Hierarchies of probability logics

Zoran Ognjanović
Mathematical Institute SANU

Aleksandar Perović
Faculty of Transportation and Traffic Engineering
University of Belgrade

Miodrag Rašković
Mathematical Institute SANU

Nebojša Ikodinović
Faculty of Mathematics
University of Belgrade

Abstract

Our aim is to present what we call the lower and the upper hierarchies of the real valued probability logics with probability operators of the form $P_{\geqslant s}$ and Q_F, where $s \in [0,1]_{\mathbb{Q}} = [0,1] \cap \mathbb{Q}$ and F is a recursive subset of $[0,1]_{\mathbb{Q}}$. The intended meaning of $P_{\geqslant s}\alpha$ is that the probability of α is at least s, while the intended meaning of $Q_F\alpha$ is that the probability of α is in F.

1 Introduction

The modern probability logics arose from the work of Jerome Keisler on generalized quantifiers and hyperfinite model theory in the mid seventies of the twentieth century [8]. Another branch of research that was involved with automatization of reasoning under uncertainty have led to development of numerous Hilbert style formal systems with modal like probability operators, see for instance [5, 2, 11, 13, 14, 17, 18, 20, 23, 24]. The simplest form of such representation of uncertainty does not allow iteration of probability operators, so formulas are Boolean combinations of the basic probability formulas, i.e. formulas of the form

$$\texttt{ProbOp}(\alpha_1, \ldots, \alpha_n),$$

where $\alpha_1, \ldots, \alpha_n$ are classical (propositional or predicate) formulas and `ProbOp` is an n-ary probability operator. Weighted probability formulas used by Fagin, Halpern and Megiddo in [2] can be treated as n-ary probability operators. For instance,

$$w(\alpha) + 3w(\beta) - 5w(\gamma) \geqslant 1$$

is example of a ternary probability operator.

The vast majority of those formal systems have unary or binary probability operators. The unary operators are used for statements about probability of classical formulas: for example we use

$$P_{\geqslant 3/4}(p \vee q)$$

to express "the probability of $p \vee q$ is at least 3/4", while

$$Q_{\{\frac{n}{n+1} \ \mid \ n \in \mathbb{N}\}}(p \vee q)$$

in our notation reads "the probability of $p \vee q$ is an element of the set $\{\frac{n}{n+1} \mid n \in \mathbb{N}\}$". The binary operators are usually used for the expression of conditional probability: for instance, we use

$$CP_{\geqslant 1/3}(p, q)$$

to express that the conditional probability of p given q is at least 1/3.

Over the course of two decades we have developed various probability logics with the mentioned types of probability operators - an extensive survey including a uniform notation for logics is presented in [17]. The aim of this paper is to put the certain class of probability logics into the wider context of mathematical phenomenology - to compare mathematical concepts according to some natural criterion (expressive power, class of models, consistency strength and so on). Here we will focus on the classification of two sorts of probability logics: $LPP_{2,P,Q,O}$ logics introduced in [12] and $LPP_2^{\mathrm{Fr}(n)}$ logics introduced in [3, 13, 17, 20, 24] (L for logic, the first P for propositional, and the second P for probability). Independently, several authors in [4, 6] have developed the fuzzy logics $FP(\text{Ł}_n)$ that extend Łukasiewicz logic. The $LPP_2^{\mathrm{Fr}(n)}$ logics can be embedded into those logics. For the $LPP_{2,P,Q,O}$ logics we introduce the comparison criterion with respect to the classes of models, while the $LPP_2^{\mathrm{Fr}(n)}$ logics we compare in terms of the interpretation method. We show that both criteria can be joined in a single one. Thus we have obtained the hierarchy of probability logics where the lattice of $LPP_{2,P,Q,O}$ logics is the end extension of the lattice of $LPP_2^{\mathrm{Fr}(n)}$ logics.

Acknowledgements

The authors are partially supported by Serbian ministry of education and science through grants III044006, III041103, ON174062 and TR36001.

References

[1] R. Djordjević, M. Rašković, Z. Ognjanović. Completeness theorem for propositional probabilistic models whose measures have only finite ranges. Archive for Mathematical Logic 43, 557 –563, 2004.

[2] R. Fagin, J. Halpern, N. Megiddo. A logic for reasoning about probabilities. Information and Computation 87(1–2), pp 78–128, 1990.

[3] M. Fattorosi-Barnaba and G. Amati. Modal operators with probabilistic interpretations I. Studia Logica 46(4), 383–393, 1989.

[4] T. Flaminio, L. Godo. A logic for reasoning about the probability of fuzzy events. Fuzzy Sets and Systems, 158(6): 625638, 2007.

[5] L. Godo, E. Marchioni. Coherent conditional probability in a fuzzy logic setting. Logic Journal of the IGPL, Vol. 14 No. 3, pp 457–481, 2006.

[6] P. Hajek, L. Godo, F. Esteva, Fuzzy Logic and Probability. In Proc. of UAI95, Morgan-Kaufmann, 237–244, 1995.

[7] N. Ikodinović,, M. Rašković, Z. Marković, Z. ognjanović. Measure logic. ECSQARU 2007: 128–138.

[8] H. J. Keisler. Probability quantifiers. In J. Barwise and S. Feferman, editors, Model–Theoretic Logics, Perspectives in Mathematical Logic, Springer–Verlag 1985.

[9] H. J. Keisler. Elementary calculus. An infinitesimal approach. 2nd edition, Prindle, Weber and Schmidt, Boston, Massachusets, 1986.

[10] D. Lehmann, M. Magidor. What does a conditional knowledge base entail? *Artificial Intelligence*, 55, 1–60, 1992.

[11] N. Nilsson. Probabilistic logic. Artif. Intell. 28, 71–78, 1986.

[12] Z. Ognjanović, M. Rašković. Some probability logics with new types of probability operators, J. Logic Computat., Vol 9 No. 2, pp 181–195, 1999.

[13] Z. Ognjanović, M. Rašković. Some first-order probability logics. Theoretical Computer Science 247(1–2), pp 191–212, 2000.

[14] Z. Ognjanović, Z. Marković, M. Rašković. Completeness Theorem for a Logic with imprecise and conditional probabilities. Publications de L'Institute Matematique (Beograd), ns. 78 (92) 35 - 49, 2005.

[15] Z. Ognjanović. Discrete linear-time probabilistic logics: completeness, decidability and complexity. J. Log. Comput. 16(2), pp 257–285, 2006.

[16] Z. Ognjanović, A. Perović, M. Rašković. Logics with the qualitative probabilty operator. Logic journal of the IGPL 16(2), 105–120, 2008.

[17] Z. Ognjanović, M. Rašković, Z. Marković. Probability Logics. Zbornik radova. Logic in Computer Science (edited by Z. Ognjanović), 12(20), 35–111, Mathematical Institute of Serbian Academy of Sciences and Arts, 2009. http://elib.mi.sanu.ac.rs/files/journals/zr/20/n020p035.pdf

[18] Z. Ognjanović, M. Rašković, Z. Marković, and A. Perović, On probability logic, The IPSI BgD Transactions on Advanced Research, 2– 7, Volume 8 Number 1, 2012. (ISSN 1820-4511)

[19] A. Perović, Z. Ognjanović, M. Rašković, Z. Marković. A probabilistic logic with polynomial weight formulas. FoIKS 2008, pp 239–252.

[20] M. Rašković. Classical logic with some probability operators. Publications de l'institut mathematique, Nouvelle série, tome 53(67), 1–3, 1993.

[21] M. Rašković, Z. Ognjanović. A first order probability logic LP_Q. Publications de l'institut mathematique, Nouvelle série, tome 65(79), pp 1–7, 1999.

[22] M. Rašković, Z. Ognjanović, Z. Marković. A logic with Conditional Probabilities. In J. Leite and J. Alferes, editors, 9th European Conference Jelia'04 Logics in Artificial Intelligence, volume 3229 of Lecture notes in computer science, pages 226–238, Springer-Verlag 2004.

[23] M. Rašković, Z. Ognjanović, Z. Marković. A logic with approximate conditional probabilities that can model default reasoning. *Int. J. Approx. Reasoning* 49(1): 52–66, 2008.

[24] W. van der Hoek. Some considerations on the logic P_FD: a logic combining modality and probability. Journal of Applied Non-Classical Logics, 7(3), 287–307, 1997.

On definition of consistency in a probabilistic sequent calculus

Marija Boričić
Faculty of Organizational Sciences
University of Belgrade
marija.boricic@fon.bg.ac.rs

Sequent system **LKprob** presents an extension of Gentzen's calculus for classical propositional logic **LK** based on sequents of the form $\Gamma \vdash_a^b \Delta$, meaning that 'the probability of provability of $\Gamma \vdash \Delta$ is in interval $[a, b]$', where $a, b \in I \subseteq [0, 1]$ and I is a finite set. We give a list of probabilized inference rules (see also [1]) which, with three additional axioms, describes the system. Justification of these inference rules is presented as well, with the most interesting part related to the probabilized version of the cut rule (see also [2] and [5]). One of the main problems is to define the notion of consistency of **LKprob**–theory and we suggest a corresponding definition (see also [3] and [4]). Some propositions concerning consistency and maximal consistent theories are given, too. It is possible to define **LKprob**–models and prove the completeness theorem on this basis.

References

[1] A. M. Frisch, P. Haddawy, Anytime deduction for probabilistic logic, Artificial Intelligence 69 (1993), pp. 93–122.

[2] T. Hailperin, Probability logic, Notre Dame Journal of Formal Logic 25 (1984), pp. 198–212.

[3] Z. Ognjanović, M. Rašković, A logic with higher order probabilities, Publications de l'Institut Mathématique 60 (74) (1996), pp. 1–4.

[4] M. Rašković, Classical logic with some probability operators, Publications de l'Institut Mathématique 53 (67) (1993), pp. 1–3.

[5] C. G. Wagner, Modus tollens probabilized, British Journal for the Philosophy of Science 54(4) (2004), pp. 747-753.

Deep belief learning and neural networks

Ivan Čukić
Faculty of Mathematics
University of Belgrade
ivan@math.rs

Neural networks seemed to have faded out in the active scientific research due to the discovery of Support Vector Machines which outperform the basic form of of feed-forward networks combined with the backpropagation algorithm.

Modern research into deep learning networks proves that ANNs can still be competitive, and that the performance issues are not inherent in all networks, but in the backpropagation algorithm and the feed-forward limitation.

We will cover the Restricted Boltzman Machines and the contrastive divergence learning procedure (Hinton, 2002).

On probabilistic argumentation

Dragan Doder
Faculty of Mechanical Engineering
University of Belgrade
ddoder@mas.bg.ac.rs

Mathematical modelling of abstract argumentation usually consist of directed graphs where vertices are arguments and edges are interpreted as certain conflicts between arguments. This framework is shown to be too simple to model some uncertainty problems in argumentation. As a consequence, in the last three years several proposals are made, in which those graphs are enriched with probabilities. In this talk, I will present the approaches, and I will propose some directions for further research.

Factor analysis in medical research

Nataša Glišović
Department of Mathematical sciences
State University of Novi Pazar
natasaglisovic@gmail.com

Factor analysis is a powerful statistical technique that has broad applications in medical and scientific research. Basically, factor analysis is a method of data reduction whereby a large number of variables are grouped into a smaller number of "factors" without losing essential information. Factor analysis examines the variability and inter-correlations among observed variables to uncover an underlying structure reflected in a smaller number of unobserved or latent factors, thereby reducing the total number of variables in a data set. The most common form of the procedure is "exploratory factor analysis" which seeks to reveal the underlying structure or pattern of the relationship between a large set of variables. "Confirmatory factor analysis" attempts to determine if the factor structure of a set of variables is consistent with what would be expected on the basis of a pre-determined theory or published factor analytic research on the same variables. The major practical applications of factor analysis are data reduction, instrument development and theory development:

One goal of scientific inquiry is parsimony or simplicity of explanation. Factor analysis, by reducing the amount of data, provides a means of creating one or more composite variables (factors) out of a large set of variables.These factors may then be used in subsequent data analysis (e.g. regression or analysis of variance).

Factor analysis can play a significant role in the creation of new measurement tools and instruments. For instance, items on a psychometric test may be evaluated by factor analysis to determine which items are central to the construct being measured and which are redundant or irrelevant.

Scientific research is based on the building of theories to explain phenomena. Factor analysis can play a crucial role in exploratory levels of research which lead to theory development by identifying constructs which unite a set of elements, explore relation-ships between variables, build systems of classification and test hypotheses.

The goal of this research is demonstrated how factor analysis can used to address research problems in medicine.

Adelic estimation of uncertainty

Angelina Ilić Stepić

An adele α is an infinite sequence $\alpha = (\alpha_\infty, \alpha_2, \ldots, \alpha_p, \ldots,)$, where $\alpha_\infty \in \mathbf{R}$, $\alpha_p \in \mathbf{Q}_p$, and for all but a finite set $\mathcal{P}$ of primes p, $\alpha_p \in \mathbf{Z}_p$. In this article we present two logics to formalize reasoning with adelic-valued function μ, such that for every event A, $(\mu(A))_1$ is a real valued probability, while for $i \geq 2$ each coordinate $(\mu(A))_i$ represents a probability in an appropriate field $\mathbf{Q}_p$. We describe the corresponding class of models that combine properties of the usual Kripke models and p-adic probabilities, and give sound and complete infinite axiomatic system. First logic, denoted by $L_{\mathbf{A}_{Z_p}}$ allows only finite conjunctions and disjunctions which implies some syntactical constrains, but decidability of this logic is proved. On the other hand, the language of the second logics second $L_{w_1,\mathbf{A}_{Z_p}}$, contains some countable conjunctions such that set of formulas remains countable.

On the SCFG cross-moments computation using the moment generating function

Velimir M. Ilić
Mathematical Institute SANU
velimir.ilic@gmail.com

Miroslav D. Ćirić
Faculty of Sciences and Mathematics
University of Niš
miroslav.ciric@pmf.edu.rs

Miomir S. Stanković
Faculty of Ocupational Safety
University of Niš
miomir.stankovic@gmail.com

The cross-moments of random variables modeled with stochastic context-free grammars(*SCFG*) are important quantities for the *SCFG* parameter estimation [8]. They are defined as expected value of the product of integer powers of the entries of random vector variable, which can represent string or derivation length, the number of rule occurrences in derivation or uncertainty associated with the occurring rule. The expectation can be taken either with respect to the sample space of all *SCFG* derivations, or with respect to the sample space of all derivations which generate a string belonging to the language of the grammar. Throughout this paper, the name *cross-moments* is usually used in the former case, while in the latter case we talk about *conditional cross-moments*.

The computation of cross-moments may become demanding if the sample space is large. In the past, this problem has been widely studied, but mainly for the cross-moments of scalar variables (called simply *moments*) and up to the second order. The first order moments computation, such as expected length of derivations and expected string length, are given in [19]. The computation of *SCFG* entropy is considered in [12]. The procedure for computing the moments of string and derivation length is given in [8], where the explicit formulas for the moments up to the second order are derived. First order conditional cross-moments are considered in [9], where the algorithm for conditional *SCFG* entropy is derived. A more general algorithm for computing the conditional cross-moments of a vector variable of the second order is derived in [11].

We present the recursive formulas for computing the cross-moments and the conditional cross-moments of a vector variable of an arbitrary order. The formulas are derived by differentiating the recursive equations for the moment generating function [16], which are obtained from the algorithms for computing the partition function of a *SCFG* [13] for the cross-moments and with the inside algorithm [10], [5] for the conditional cross-moments.

References

[1] T. L. Booth, R. A. Thompson, Applying probability measures to abstract languages, IEEE Transactions on Computers 22 (1973) 442–450.

[2] S. B. Cohen, R. J. Simmomns, N. A. Smith, Products of weighted logic programs, Theory and Practice of Logic Programming 11 (2011) 263–296.

[3] C. Cortes, M. Mohri, A. Rastogi, M. Riley, On the computation of the relative entropy of probabilistic automata, International Journal of Foundations of Computer Science 19 (2008) 219–242.

[4] L. E. Fraenkel, Formulae for high derivatives of composite functions, Mathematical Proceedings of the Cambridge Philosophical Society 83 (1978) 159–165.

[5] J. Goodman, Semiring parsing, Computational Linguistics 25 (1999) 573–605.

[6] T. E. Harris, The Theory of Branching Processes, Dover Books on Advanced Mathematics, Dover Pubns, 1963.

[7] R. A. Horn, C. R. Johnson, Matrix analysis, Cambridge University Press, New York, NY, USA, 1985.

[8] S. E. Hutchins, Moments of string and derivation lengths of stochastic context-free grammars, Information Sciences 4 (1972) 179–191.

[9] R. Hwa, Sample selection for statistical grammar induction, Proceedings of the 2000 Joint SIGDAT conference on Empirical methods in natural language processing and very large corpora: held in conjunction with the 38th Annual Meeting of the Association for Computational Linguistics - Volume 13, pp. 45–52, Morristown, NJ, USA, 2000. Association for Computational Linguistics.

[10] K. Lari, S. J. Young, The estimation of stochastic context-free grammars using the inside-outside algorithm, Computer Speech & Language 4 (1990) 35–56.

[11] Z. Li, J. Eisner, First- and second-order expectation semirings with applications to minimum-risk training on translation forests, In: Proceedings of the 2009 Conference on Empirical Methods in Natural Language Processing: Volume 1 - Volume 1, EMNLP '09, pp. 40–51, Morristown, NJ, USA, 2009. Association for Computational Linguistics.

[12] M.-J. Nederhof, G. Satta, Kullback-leibler distance between probabilistic context-free grammars and probabilistic finite automata, In: Proceedings of the 20th international conference on Computational Linguistics, COLING '04, Stroudsburg, PA, USA, 2004. Association for Computational Linguistics.

[13] M.-J. Nederhof, G. Satta, Computing partition functions of pcfgs, Research on Language and Computation, 6 (2008) 139–162.

[14] M.-J. Nederhof, G. Satta, Probabilistic parsing, In: New Developments in Formal Languages and Applications, volume 113 of Studies in Computational Intelligence, pp. 229–258, Springer Berlin / Heidelberg, 2008.

[15] X. S. Raymond, Elementary Introduction to the Theory of Pseudodifferential Operators, Studies in Advanced Mathematics, CRC Press, Boca Raton, 1991.

[16] R. B. Lund, Elementary probability theory with stochastic processes and an introduction to mathematical finance, The American Statistician 58 (2004) 173–174.

[17] F. Tendeau, Computing abstract decorations of parse forests using dynamic programming and algebraic power series, Theoretical Computer Science 199 (1998) 145–166.

[18] A. B. Thaheema, A. Laradjia, Classroom note: A generalization of Leibniz rule for higher derivatives, International Journal of Mathematical Education in Science and Technology 34 (2001) 905–907.

[19] C. S. Wetherell, Probabilistic languages: A review and some open questions, ACM Computing Surveys 12 (1980) 361–379.

One Probabilistic Approach to Classification Problems

Milica Knežević
Mathematical Institute SANU

Aleksandar Perović
Faculty of Transportation and Traffic Engineering
University of Belgrade

Let f be any $[0,1]$-valued evaluation of the set of propositional letters. Then, f can be uniquely extended to finitely additive product and Gödel's measures on the set of formulas of classical propositional logic. Those measures satisfy the following condition, the measure of any conjunction of distinct propositional letters is equal to the product, or to the minimum, of the measures of the propositional letters, respectively. Product measures correspond to the one extreme, stochastic or probability independence of elementary events (propositional letters), while Gödel's measure corresponds to the other extreme, logical dependence of elementary events. Any linear convex combination of a product measure and a Gödel measure is also a finitely additive probability measure. In that way, infinitely many intermediate measures that corresponds to various degrees of dependence of propositional letters can be generated. Those measures give certain truth-functional flavor to probability, enabling applications to preferential problems, in particular, classification problems. We will discuss one possible application of this framework in areas of medical research.

References

[1] A. Perović, Z. Ognjanović, M. Rašković, D. Radojević, Finitely additive probability measures on classical propositional formulas definable by Gödel's t-norm and product t-norm, Fuzzy Sets and Systems 169 (1) (2011) 65-90.

[2] R. Fagin, J. Halpern, N. Megiddo, A logic for reasoning about probabilities, Information and Computation 87 (12) (1990) 78-128.

[3] T. Flaminio, Strong non-standard completeness for fuzzy logic, Soft Computing 12 (4) (2008) 321-333.

[4] L. Godo, E. Marchioni, Coherent conditional probability in a fuzzy logic setting, Logic Journal of the IGPL 14 (3) (2006) 457-481.

[5] P. Hajek, F. Esteva, L. Godo, Fuzzy logic and probability. Uncertainty in AI, in: P. Besnard, S. Hanks (Eds.), Proceedings of 11th Conference, Montreal, Canada, 1995, pp. 237-244.

[6] P. Hajek, Metamathematics of Fuzzy Logic, Kluwer Academic Publishers, 1998.

[7] J. Halpern, Reasoning about Uncertainty, The MIT Press, 2003.

[8] Z. Ognjanović, M. Rašković, Z. Marković, Probability logics, in: Z. Ognjanović (Ed.), Zbornik Radova, Subseries Logic in Computer Science, Matematički institut, Beograd, vol. 12, issue 20, 2009, pp. 35-111.

[9] D. Radojević, [0,1]-valued logic: a natural generalization of Boolean logic, Yugoslav Journal on Operations Research 10 (2) (2000) 185-216.

[10] D. Radojević, Interpolative realization of Boolean algebra, in: NEUREL 2006, Eight Seminar on Neural Network Applications in Electrical Engineering, 2006, pp. 201-206.

[11] D. Radojević, Interpolative realization of Boolean algebra as a consistent frame for gradation and/or fuzziness. Forging new frontiers: fuzzy pioneers II, in: M. Nikravesh, J. Kacprzyk, L.A. Zadeh (Eds.), Studies in Fuzziness and Soft Computing, Springer, 2007, pp. 326-351.

[12] D. Radojević, Logical aggregation based on interpolative realization of Boolean algebra, Mathware and soft computing 15 (1) (2008) 125-141.

[13] D. Radojević, A. Perović, Z. Ognjanović, M. Rašković, Interpolative Boolean logic, in: AIMSA 2008, 2008, pp. 209-219.

Free Boolean vectors and expressions

Aleksandar Pejović
Mathematical Institute SASA
pejovica@mi.sanu.ac.rs

Zarko Mijajlović
Faculty of Mathematics
Univ. of Belgrade
zarkom@matf.bg.ac.rs

We propose a method for computing Boolean expressions using the parallel structure of standard computer processors. Examples of this kind are logical operations which can be computed bitwise, i.e. by use of all register bits in one processor cycle. The background of our approach is based on the properties of finite free Boolean algebras. A program implementation of the method is described. We also show how various combinatorial objects can be codded in the formalism of Boolean algebras and counted by the method. The idea of parallelization of computing logical operations in this way is indicated in [1]. The basic idea is as follows.

Let $f(x_1, x_2, \ldots, x_n)$ be a Boolean expression in n variables $x_1, x_2, \ldots, x_n$. We give a construction of n Boolean vectors $b_1, b_2, \ldots, b_n$ of the size 2^n with the following property:

($\mathcal{P}$) $f(b_1, b_2, \ldots, b_n)$ is a Boolean vector that codes the perfect DNF of f.

It appears that the vectors $b_1, b_2, \ldots, b_n$ are exactly the free generators of a free Boolean algebra having n free generators.

Using a translation procedure from first order predicate formulas to propositional formulas, we give a method for constructing and counting various combinatorial objects. This idea is formally developed in [2], but it was used in studies problems in the infinitary combinatorics, particularly in finding their complexity in the Borel hierarchy. Related combinatorial problems are considered, for example the number of automorphisms of finite structures and various partition problems over finite sets. We also give an implementation of the method that can be run on multi-core CPUs as well as on highly parallel GPUs such as Nvidia Tesla C2075.

References

[1] Ž. Mijajlović, *On free Boolean vectors*, Publl. Inst. Math, 64 (78), 1998, 2–8.

[2] Ž. Mijajlović, D. Doder, A. Ilić-Stepić, *Borel sets and countable models*, Publ. Inst. Math, 90 (104), 2011, 1–11.

[3] Ž. Mijajlović, *Model Theory*, Novi Sad, 1985.

[4] R. Sikorski, *Boolean Algebras*, Springer-Verlag, Berlin, 1969.

[5] A. Dow, P. Nyikos, *Representing free Boolean algebras*, Fundamenta Mathematicae, 141 (1992), 1992, 21–30.

[6] Chang, Keisler, *Model theory*, North Holland, 1973.

On definable operators in certain probability logics

Zoran Ognjanović
Mathematical Institute SANU

Aleksandar Perović
Faculty of Transport and Traffic Engineering
University of Belgrade

Miodrag Rašković
Mathematical Institute SANU

In this talk we will give characterization of definable predicates of the form “probability of α is in the set X” in class of $LP_{P,Q,O}$ probability logics. Namely, we will show that the above predicate is definable in $LP_{P,Q,O}$ iff the set X can be obtained by finite application of union, intersection, set difference and quasi complement $1-$ $(1-F = \{1-a \ : \ a \in F\})$ on some sets from O.

Complex numbers as a model of euclidean geometry

Saša Rašuo

The second part of the Karamata's book "Complex numbers" refers to the application of the theory of complex numbers in geometry. A more detailed analysis of this book shows that the essence of this application is that the field of complex numbers is algebraically closed and that this is good enough for the foundation of elementary geometry. Namely, the algebraic closure can replace continuity in building a Pythagorean plane, the basis of elementary geometry. Pythagorean plane is constructed cumulatively by cross-sections of two lines, a line and a circle and two circles . The theory of algebraically closed fields suffices for the proof of the existence of these cross-sections. In other words, the continuity , i.e. the supremum axiom for real numbers is unnecessary. Thus, it immediately follows that for the foundation of elementary geometry and construction of Pythagorean plane, the field of algebraic numbers, or field of real algebraic numbers if the order is included, is sufficient. Since the theory of these structures is complete and decidable, immediately follows Tarski's theorem on the decidability of elementary geometry. These facts, of course, are not mentioned explicitly in the Karamata's book, but we can say that he anticipated it and approved it by geometrical constructions and proofs of famous theorems of elementary geometry: the Ptolemy's theorem , Pappus - Pascal theorem, Desargues' theorem, constructions of regular polygons, and other examples. The goal of our presentation is to highlight some of these aspects of Karamata's book. We also give an almost formal system of axioms for Pythagorean plane. Along this line, we note that the elementary geometry based on Euclid's axioms, provides only positive geometrical constructions. Namely, the elementary geometry cannot prove that some geometric problems are not solvable with ruler and compass, or at least we are not aware of such proofs. On the other hand, the theory of algebraic fields eliminates this "shortage" of elementary geometry. It provides simple and elegant methods for proving that there are no such constructions for particular problems. Famous examples of this kind are Delian problems (trisection of an angle, doubling the cube and squaring the circle) and the Gauss theorem on the possibility of constructions of regular polygons. In this regard, this communication has the methodological and methodical purpose as well and points out the close relationship between geometry, algebra and logic.

Integrating Knowledge and Data in a Medical Decision Support System

Vladimir Srdanović
Institute for Multidsciplinary Research
University of Belgrade
vladats@eunet.rs

Marko Šošić
Mathematical Institute SANU
sosic.marko@gmail.com

Acquiring domain knowledge in knowledge based systems is known to be a difficult and time consuming process. Medical domain, unlike many others, provide large amounts of relatively homogeneous medical knowledge being implicitly contained in patient case histories.

As noted by Quinlan in his seminal paper, experts can articulate the framework for their knowledge, and systems based on induction can produce useful knowledge from such frameworks supplied with examples. The same approach can sometimes work even when there is no human expert.

A medical decision support system integrating medical expert knowledge with the knowledge extracted from patients case histories (data mining) is presented. System's basic features have been taken over from an earlier system, which was successfully tested in several medical domains.

For the system to allow adequate representation and easy manipulation of data and metadata, it was most convenient to choose object oriented programming paradigm. The system is implemented in .NET framework, in C# language because of the maturity of the platform and good programming environment, which enable more productive work. Currently, the system is tested with local data only, and implementation of interface to stand-alone databases is planned for the near future.

Expert clinicians begin with defining medical domain - in this case a group of systemic diseases of connective tissue. They select manifestations/attributes that are relevant for the domain, define their types, values they could take on, their formats and constraints, their "costs" (of establishing whether a specific manifestation is present, i.e. establishing a specific value for an attribute; they range from those requiring invasive and/or expensive and lengthy procedures in order to be established, i.e. "most costly", to those readily observable, i.e.

"least costly"), etc. Medical experts are also invited to provide additional knowledge about the domain in the form of production rules.

Domain definition allows for a clinical data base formation, to which the system is linked. Analyses performed on the data base provide the knowledge about associations among entities in the knowledge base and the estimates of their strength. This new knowledge is in the form of simple rules with manifestations as their IF parts, and diagnoses as their THEN parts. Corresponding diagnostic strengths are estimated by relative frequencies of occurrence of particular manifestations with the given diagnosis; they can take on values from 0 to 1, with 0.1 increments (0 meaning that manifestation does not occur with the particular disease, and 1 that it is pathognomonic for the disease).

Clearly, knowledge base consisting of the rules entered by expert, and, particularly, those generated by the system in the way just described can potentially be very large. To resolve that, the system employs several heuristic procedures to restructure its knowledge base additionally, and prepare it for the consultation process.

The heuristics for extracting key attributes presents core of the inductive inference algorithm employed by the system. The algorithm can easily cope with attributes having differing number of values, concurrent values, and missing values, as well as with noise. It is being used to restructure system's knowledge base, so it can be adequately applied to solving problems within its domain. There are two main directions the system makes use of the heuristics: a) to direct consultation process adaptively, and eventually suggest possible diagnosis, and b) to automatically derive criteria for diagnosing (suspecting) a disease from a given domain.

Testing of the present system is under way in the domain of systemic diseases of the connective tissue. Present research is directed mainly toward determining the system's diagnostic accuracy, comparing of the key attributes extraction heuristic to standard statistical methods, as well as comparison of the diagnostic criteria automatically generated by the system to those used by medical community as a standard.

Clearly, knowledge base consisting of the rules entered by expert, and, particularly, those generated by the system in the way just described can potentially be very large. To resolve that, the system employs several heuristic procedures to restructure its knowledge base additionally, and prepare it for the consultation process.

The heuristics for extracting key attributes presents core of the inductive inference algorithm employed by the system. The algorithm can easily cope with attributes having differing number of values, concurrent values, and missing values, as well as with noise. It is being used to restructure system's knowledge base, so it can be adequately applied to solving problems within its

domain. There are two main directions the system makes use of the heuristics: a) to direct consultation process adaptively, and eventually suggest possible diagnosis, and b) to automatically derive criteria for diagnosing (suspecting) a disease from a given domain.

Testing of the present system is under way in the domain of systemic diseases of the connective tissue. Present research is directed mainly toward determining the system's diagnostic accuracy, comparing of the key attributes extraction heuristic to standard statistical methods, as well as comparison of the diagnostic criteria automatically generated by the system to those used by medical community as a standard (1).

References

[1] Petri, M. et al. Derivation and Validation of the Systemic Lupus International Collaborating Clinics Classification Criteria for Systemic Lupus Erythematosus, ARTHRITIS & RHEUMATISM, Vol. 64, No. 8, August 2012, pp 2677-2686 DOI 10.1002/art.34473 © 2012, American College of Rheumatology.

[2] Quinlan, J. R. 1986. Induction of decision trees. Machine Learning 1:1.

[3] Srdanović, V., BELART - A Consultation System Integrating Knowledge and Data, MEDINFO-86, (Salamon/Blum/Jorgensen, eds.), North-Holland, 1986, 228-231.

Bee-colony optimization for the satisfiability problem in probabilistic logic

Tatjana Stojanović
Faculty of Science
University of Kragujevac

Tatjana Davidović
Mathematical Institute SANU

Zoran Ognjanović
Mathematical Institute SANU

This paper presents a new method for solving the satisfiability problem in the logic with approximate conditional probability, based on the bee-colony optimization meta-heuristic. The solution space consists of variables, which are arrays of 0 and 1 and the associated probabilities, belonging to recursive non-archimedean Hardy field which contains all rational functions of a fixed positive infinitesimal. We apply the improvement variant of the bee colony optimization algorithm which chooses variables from the solution space and determines their probabilities combining some other fast heuristic for solving the obtained linear system of inequalities. Experimental evaluation showed a high percentage of success in proving the satisfiability of randomly generated formulas.

Working with uncertain knowledge has been a big and well documented problem in mathematical logic and computer science, since the first works of Leibnitz and Bool. Many of the formalisms for representing and reasoning with uncertainty are based on probabilistic logic ([1, 2, 3, 4]). These logics are extensions of classical logic with probabilistic operators. Satisfiability problem in these logics (PSAT) can be reduced to the linear programming problem. However, solving it by any standard linear solver is inapplicable in practice. Therefore, the application of some other techniques for solving this problem, such as different types of meta-heuristics, could prove very useful. Using meta-heuristics for solving PSAT problem is not a new idea. For example, in the logics described by Fagin, Halpern and Megiddio [1] or by Ognjanović and Rašković [2], Genetic algorithm [5] and Variable neighborhood search [6] are used for solving PSAT problem.

In this paper we discus the satisfiability problem in approximate conditional probabilities logic described by Rašković, Marković and Ognjanović in

[4]. We denote this version of satisfiability problem with CPSAT-ϵ. The main differences between PSAT and CPSAT-ϵ are:

- CPSAT-ϵ involves conditional probability operator on the contrary to PSAT
- probabilities of formulas in CPSAT-ϵ may take infinitesimal values, and not only real-values as in PSAT

The logic, described in [4], enriches the propositional calculus with probabilistic operators which are applied to propositional formulas: $CP_{\geqslant s}(\alpha, \beta)$, $CP_{\leqslant s}(\alpha, \beta)$ and $CP_{\approx s}(\alpha, \beta)$, with the intended meaning "the conditional probability of α given β is at least s", "at most s" and "approximately s", respectively. Further more, formulas of the form $CP_{\approx 1}(\alpha, \beta)$ from this logic may be used to model defaults. Since CPSAT-ϵ problem for the logic described in [4] has no automated solver, our main effort was to apply meta-heuristics, in particular Bee-colony optimization, for solving CPSAT-ϵ problem in this logic.

Bee-colony optimization (BCO) is a meta-heuristic technique, which showed very good performance in solving hard combinatorial optimization problems ([7, 8, 9, 10]). It is a stochastic, random-search technique that belongs to the class of population-based algorithms. This technique uses an analogy between the way in which bees in nature search for food, and the way in which optimization algorithms search for an optimum of the given combinatorial optimization problems. Until now, BCO has not been applied to a class of problems involving search for a solution, only to the problems that already have some feasible solutions that one wants to improve. We used, the so called improvement variant of BCO, denoted by BCOi and proposed in [11] to address CPSAT-ϵ. The experimental results obtained by testing CPSAT-ϵ were compared with those obtained using Fourier-Motzkin elimination procedure and thus demonstrated the superiority of BCOi method.

References

[1] R. Fagin, J. Y. Halpern, and N. Megiddo, "A logic for reasoning about probabilities," *Information and computation*, vol. 87, no. 1, pp. 78–128, 1990.

[2] Z. Ognjanović and M. Rašković, "Some first-order probability logics," *Theoretical Computer Science*, vol. 247, no. 1, pp. 191–212, 2000.

[3] M. Rašković, "Classical logic with some probability operators," *Publ. Inst. Math., Nouv. Sér*, vol. 53, no. 67, pp. 1–3, 1993.

[4] M. Rašković, Z. Marković, and Z. Ognjanović, “A logic with approximate conditional probabilities that can model default reasoning,” *International Journal of Approximate Reasoning*, vol. 49, no. 1, pp. 52 – 66, 2008.

[5] Z. Ognjanović, U. Midić, and J. Kratica, “A genetic algorithm for probabilistic sat problem,” *Artificial Intelligence and Soft Computing-ICAISC 2004*, pp. 462–467, 2004.

[6] D. Jovanović, N. Mladenović, and Z. Ognjanović, “Variable neighborhood search for the probabilistic satisfiability problem,” *Metaheuristics*, pp. 173–188, 2007.

[7] P. Lučić and D. Teodorović, “Bee system: modeling combinatorial optimization transportation engineering problems by swarm intelligence,” in *Preprints of the TRISTAN IV Triennial Symposium on Transportation Analysis*, pp. 441–445, Sao Miguel, Azores Islands, 2001.

[8] P. Lučić and D. Teodorović, “Transportation modeling: an artificial life approach,” in *Proceedings of the 14th IEEE International Conference on Tools with Artificial Intelligence*, (Washington, DC), pp. 216–223, 2002.

[9] T. Davidović, M. Šelmić, and D. Teodorović, “Bee colony optimization for scheduling independent tasks,” in *Proc. Symp. on information technology, YUINFO 2009, (on CD 116.pdf)*, (Kopaonik), 2009.

[10] T. Davidović, D. Ramljak, M. Šelmić, and D. Teodorović, “Bee colony optimization for the p-center problem,” *Computers and Operations Research*, vol. 38, no. 10, pp. 1367–1376, 2011.

[11] T. Davidović, M. Šelmić, D. Teodorović, and D. Ramljak, “Bee colony optimization for scheduling independent tasks to identical processors,” *J. Heur.*, vol. 18, no. 4, pp. 549–569, 2012. DOI:10.1007/s10732-012-9197-3.