MATHEMATICAL INSTITUTE
SANU - BELGRADE

Sedma nacionalna konferencija
“Verovatnosne logike i njihove primene”

Beograd, Srbija, 8. novembar 2017.

Knjiga apstrakata

ORGANIZATOR:

Matematički institut, SANU

KONFERENCIJU FINANSIRAJU:

Projekat Razvoj novih informaciono-komunikacionih tehnologija, korišćenjem naprednih matematičkih metoda, sa primenama u medicini, telekomunikacijama, energetici, zaštiti nacionalne baštine i obrazovanju, III 044006

Projekat Reprezentacije logičkih struktura i formalnih jezika i njihove primene u računarstvu, ON 174026.

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TEME KONFERENCIJE:
Verovatnosne logike, problemi potpunosti, odlučivosti i složenosti; logičke osnove u zasnivanju verovatnoće; bayes-ove mreže i drugi srodni sistemi; programski sistemi za podršku odlučivanju u prisustvu neizvesnosti; primene verovatnosnog zaključivanja u medicini; teorija informacija; mere informacija i složenosti; teorija kodiranja; sigurnost u sajber-prostoru; složeni i dinamički sistemi i donošenje odluka; kauzalnost u kompleksnim sistemima; analiza i simulacija kompleksnih sistema; ali i sve srodne teme koje nisu navedene.

PROGRAMSKI KOMITET:
Miodrag Rašković (Matematički institut SANU), predsednik
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Zoran Ognjanović (Matematički institut SANU)
Nebojša Ikodinović (Univerzitet u Beogradu)
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Program konferencije:

8. 11. 2017.

11:00 *Otvaranje*

11:15 *Polynomial dynamics of human blood genotypes frequencies*, Timur Mradovic Sadikov

12:20 *On a probabilization of intuitionistic sequent calculus*, Marija Boričić

12:40 *Towards probabilistic reasoning about simply typed lambda terms*, Silvia Ghilezan, Jelena Ivetić, Simona Kašterović, Zoran Ognjanović, Nenad Savić

13:00 *Pauza*

14:00 *A propositional logic with metric operators*, Nenad Stojanović, Nebojša Ikodinović, Radosav Djordjević

14:20 *Free Boolean vectors and expressions*, Žarko Mijajlović

14:40 *Note on Serbian–Russian inference rule*, Aleksandar Perović

15:00 *Bisimulations for fuzzy relational systems*, Miroslav Ćirić, Jelena Ignjatović

Apstrakti

Sadržaj

Polynomial dynamics of human blood genotypes frequencies

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The frequencies of human blood genotypes in the ABO and Rh systems differ between populations. Moreover, in a given population, these frequencies typically evolve over time. The possible reasons for the existing and expected differences in these frequencies (such as disease, random genetic drift, founder effects, differences in fitness between the various blood groups etc.) are the focus of intensive research. To understand the effects of historical and evolutionary influences on the blood genotypes frequencies, it is important to know how these frequencies behave if no influences at all are present. Under this assumption the dynamics of the blood genotypes frequencies is described by a polynomial dynamical system defined by a family of quadratic forms on the 17-dimensional projective space. To describe the dynamics of such a polynomial map is a task of substantial computational complexity. We give a complete analytic description of the evolutionary trajectory of an arbitrary distribution of human blood variations frequencies with respect to the clinically most important ABO and RhD antigens. We also show that the attracting algebraic manifold of the polynomial dynamical system in question is defined by a binomial ideal.

On a probabilization of intuitionistic sequent calculus

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We introduce a probabilistic sequent calculus **LJprob**, a modification of the Gentzen's original sequent calculus **LJ** for intuitionistic logic [5], enabling to work with probabilized sequents of the form '$\Gamma \vdash_a^b \Delta$' in the intuitionistic logic. Also, we will point out the differences between this system and the probabilistic sequent calculus **LKprob** for classical propositional logic [1], [2], [3] and [4].

The axioms of the system are $\Gamma \vdash_0^1 \Delta$, $\vdash^0$, and $A \vdash_1 A$, for any words Γ and Δ, and any formula A. In both systems we have structural and logical rules. For instance, the following rules are for introducing negation and implication in the consequence in **LJprob**:

$$\frac{\Gamma A \vdash_a^b}{\Gamma \vdash_a^b \neg A}(\vdash_a^b \neg) \quad \frac{\Gamma A \vdash_a^b B}{\Gamma \vdash_a^b A \to B}(\vdash_a^b \to)$$

while the same rules in **LKprob** are as follows:

$$\frac{\Gamma A \vdash_a^b \Delta}{\Gamma \vdash_a^b \neg A \Delta}(\vdash_a^b \neg) \quad \frac{\Gamma A \vdash_a^b B\Delta}{\Gamma \vdash_a^b A \to B\Delta}(\vdash_a^b \to)$$

There are also some specific structural rules for this system.

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Towards probabilistic reasoning about simply typed lambda terms

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Keywords: simply typed lambda calculus, probabilistic logic, soundness, strong completeness

We present a formal model $\mathsf{P\Lambda_{\to}}$ for probabilitic reasoning about simply typed lambda terms, which is a combination of lambda calculus [1] and probabilistic logic [3]. We propose its syntax, Kripke-style semantics and an infinitary axiomatization. We use simple type assignment which is sound and complete ([2]) with respect to the simple semantics (based on the concept of a term model).

We define two sets of formulas basic formulas and probabilistic formulas. *Basic formulas*, denoted by $\mathsf{For_B}$, are lambda statements and Boolean combinations of lambda statements. Basic probabilistic formula is any formula of the form $P_{\geq s}\alpha$, such that $\alpha \in \mathsf{For_B}$ and $s \in [0,1] \cap \mathbb{Q}$. The set of all *probabilistic formulas*, denoted by $\mathsf{For_P}$, is the smallest set containing all basic probabilistic formulas which is closed under Boolean connectives. The language of $\mathsf{P\Lambda_{\to}}$ consist of both basic formulas and probabilistic formulas

$$\mathsf{For_{P\Lambda_{\to}}} = \mathsf{For_B} \cup \mathsf{For_P}.$$

Neither mixing of basic formulas and probabilistic formulas, nor nested probability operators is allowed. The following two expressions are *not* (well defined) formulas of the logic $\mathsf{P\Lambda_{\to}}$:

$$\alpha \wedge P_{\geq \frac{1}{2}}\beta, \qquad P_{\geq \frac{1}{3}}P_{\geq \frac{1}{2}}\alpha.$$

An axiomatic system for the logic $\mathsf{P\Lambda_{\to}}$ is introduced. It consists of axiom schemes and two groups of inference rules, such that rules from the first group can be applied only on lambda statements.

The semantics for $\mathsf{P}\Lambda_{\rightarrow}$ is a Kripke-style semantics based on the possible-world approach. The main results are soundness and strong completeness of $\mathsf{P}\Lambda_{\rightarrow}$ with respect to the proposed model. We proved strong completeness using the fact that simple type assignment is sound and complete with respect to the simple semantics and property that every consistent set can be extended to a maximal consistent set. The crucial part in the proof of strong completenes is the construction of the canonical model using a maximal consistent set.

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A propositional logic with metric operators

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We investigate metric logic with the binary metric operators. This logic, denoted $LFAM$, allows making statements such as $D_{\geqslant s}(\alpha, \beta)$ with the intended meaning "distance between formulas α and β is greater than or equal to s". Infinitary axiomatic system for our logic which is sound and complete with respect to the mentioned of models is given.

Implicit Programming

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The goal of the presentation is to show solutions of certain problems using methods of mathematical logic and some specialized software **SpecS** such as Prover9-Mace4 and Wolfram's Mathematica. Problems are from various fields, ranging from finite combinatorics, to problems from algebra and geometry. The idea is to think of first order logic as of a programming language and use it to obtain a description D of a problem P and leave D to **SpecS** to solve it. The code D is called the implicit program for the problem P. We consider two types of solutions:

1. Building a (finite) model for the set of formulas in D (Mace4).
2. Elimination of quantifiers in formulas from D (Wolfram's Mathematica).

While the first method is essentially based on the skolemization and building of Herbrand universe for formulas in D, the second one is based on the application of quantifier elimination for algebraically closed and real closed fields. We present examples and solutions of a puzzle, Sudoku, $n \times n$ Queen problem and the problem of Apollonian circles.

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Note on Serbian–Russian inference rule

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In this talk we shall attempt to identify necessary technical steps for propositional coding of certain fragments of ZFC. More precisely, we are interested in propositional representation of the constructible universe L. The leitmotif of the talk will be the the variations on the well known Serbian–Russian inference rule.

Bisimulations for fuzzy relational systems

Miroslav Ćirić   *Jelena Ignjatović*

Bisimulations have been introduced by Milner [22] and Park [24] in computer science, more precisely in concurrency theory. Roughly at the same time they have been also discovered in some areas of mathematics, e.g., in modal logic and set theory. They are widely used to model equivalence between various systems, as well as to reduce the number of states of these systems, and are employed today in many areas of computer science, such as functional languages, object-oriented languages, types, data types, domains, databases, compiler optimizations, program analysis, verification tools, etc.

Bisimulations have been studied on various relational systems, the most commonly on labelled transition systems, but also in the context of deterministic, nondeterministic, weighted, probabilistic, timed and hybrid automata. Bisimulations have been mostly used to model equivalence between states of the same system, and to reduce the number of states of this system. Bisimulations between states of two distinct systems have been much less studied, probably due to the lack of an appropriate concept of a relation between two distinct sets which would behave like an equivalence relation.

Recently, bisimulations have been also studied in the context of fuzzy automata, and the main purpose of this talk is to present the most important results from these studies. One can distinguish two general approaches to the concept of bisimulation for fuzzy automata. The first approach, which we encounter, for example, in [1, 25, 31], uses ordinary crisp relations and functions. Another approach, proposed in [5, 6, 7, 8, 19, 29], is based on the use of fuzzy relations, which have been shown to provide better results both in the state reduction and the modeling of equivalence of fuzzy automata. In [7, 8] the state reduction of fuzzy automata was carried out by means of certain fuzzy equivalences, which are exactly bisimulation fuzzy equivalences, and in [29] it was proved that even better results can be achieved by using suitable fuzzy quasi-orders, which are nothing but simulation fuzzy quasi-orders. Moreover, it turned out in [7, 8, 29] that the state reduction problem for fuzzy automata is closely related to the problem of finding solutions to certain systems of fuzzy relation equations. This enabled not only to study fuzzy automata using very powerful tools of the theory of fuzzy sets, but also, it gave a great contribution to the theory of fuzzy relational equations and has led to the development of the general theory of weakly linear fuzzy relation equations and inequalities in [15, 16, 17].

The same approach has been used in [5], in the study of simulations and bisimulations between fuzzy automata, where simulations and bisimulations have also been defined as fuzzy relations. There have been introduced two types of simulations (forward and backward simulations) and four types of bisimulations (forward, backward, forwardbackward, and backwardforward bisimulations). There has been proved that if there is at least one simulation/bisimulation of some of these types between the given fuzzy automata, then there is the greatest simulation/bisimulation of this kind. In [6], for any of these types of simulations/bisimulations efficient algorithms have been provided for deciding whether there is a simulation/bisimulation of this type between the given fuzzy automata, and for computing the greatest one, whenever it exists. The algorithms use the method developed in [16, 17], which comes down to the computing of the greatest post-fixed point, contained in a given fuzzy relation, of an isotone function on the lattice of fuzzy relations, and is based on the well-known Knaster-Tarski and Kleene fixed point theorems. Similar method has also been used in [9] in the study of simulations and bisimulations for weighted automata over additively idempotent semirings.

We will also present certain recent results concerning simulations and bisimulations for some other types of ordinary and fuzzy relational systems, such as fuzzy social networks (cf. [12, 13, 14, 18, 21, 30]), fuzzy modal logics (cf. [10, 11, 32]), coalgebras (cf. [26, 27, 28]), etc.

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