Multidimensional Persistence and Toric Topology, Project financed by the Ministry of Education, Science and Technological Development of the Republic of Serbia and The Scientific and Technological Research Council of Turkey as a part of bilateral cooperation between Serbia and Turkey (2021-2023)

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Institutions:

from Turkey: **Middle East Technical University, Department of Mathematics, Ankara**

from Serbia: **Mathematical Institute SANU, Belgrade**

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Project description:

The joint project “Multidimensional persistence and toric topology” aims to unify expertise of researchers of diverse mathematical skills and backgrounds from Turkey and Serbia in studying multidimensional persistence and its applications. Persistence captures the topology of a filtration, a one-parameter increasing family of spaces. Filtrations arise naturally from many processes such as multiscale analysis of noisy datasets. Given a filtration, persistent homology provides a small description in terms of multiset of intervals called barcode. The intervals correspond to the lifetime of topological attributes. Since features have long lives, while noise is short-lived, an examination of the intervals enables an estimation of the topology of a dataset. This estimation is the key reason for current popularity of persistent homology for solving problems in diverse disciplines, such as shape description, denoising volumetric density data, detecting holes in sensor networks, and analyzing the structure natural images. We often encounter richer structures that are described by multiple parameters. These structures may be modeled with multifiltrations. G. Carlson and A. Zomorodian provided the theoretical foundations for the persistent homology of single parameter filtrations, obtaining a simple classification over fields in terms of the barcode. They showed that the barcode was complete, capturing all the topological information with a filtration. A similar result is unattainable for multidimensional filtrations. There exists no complete description, like the barcode in higher dimension. G. Carlson and A. Zomorodian proposed the rank invariant as a discriminating invariant that enables detection of persistent features in a multifiltration. In dimension one, this invariant is equivalent to the barcode, but unlike the barcode, the rank invariant extends to higher dimensions, where it still captures persistent features making it useful for practical applications. Filtrations arise naturally whenever we attempt to study the topological invariants of a space computationally. Often, our knowledge of a space is limited and imprecise. Consequently, we utilize a multiscale approach to capture the connectivity of the space giving us filtration. Principal idea of the joint proposal is to associate various spaces with toric actions to a filtrated space and instead of the rank invariant for standard homology of the filtration, use the methods of toric topology. We will investigate the algebraic structures of the toric spaces such as the bigraded Betty numbers, cup products in the cohomomology and the Lusternik–Schnirelmann category. These numbers takes into account deeper and intrinsic structure of the cloud in far-off superior way than the standard topology which realistically may result in further and yet undiscovered applications in industry and technique.

Project goals:

The key objectives of the project are:

1. joint research publications between Turkish and Serbian group;

2. fostering greater participation of both groups ininternational projects and networks for exchange of knowledge;
3. Transfer of knowledge in the areas where the teams have compatible expertises with a special attention to new applications of methods of algebraic topology in applied sciences.

Topological data analysis (TDA) is currently one of the most active research areas. TDA has various practical applications and interest for the methods of persistent homology is constantly increasing in last years. Algebraic topology as an abstract mathematical discipline provides a lot invariants which deserves to be studied and our project aims to develop to open new direction of research based on multidimensional persistence and toric topology. Also, our objective is to find first practical applications of our findings. Joint efforts should lead to the first publications and establishment of the bases for a prospective joint projects funded by the European Union. During the project, both teams will pay special attention to improve knowledge and skills of young researchers in applied and toric topology by organizing mini-courses and seminar talks during visits. The lectures opened to other interested participants will be also beneficial for mathematical communities of both countries.