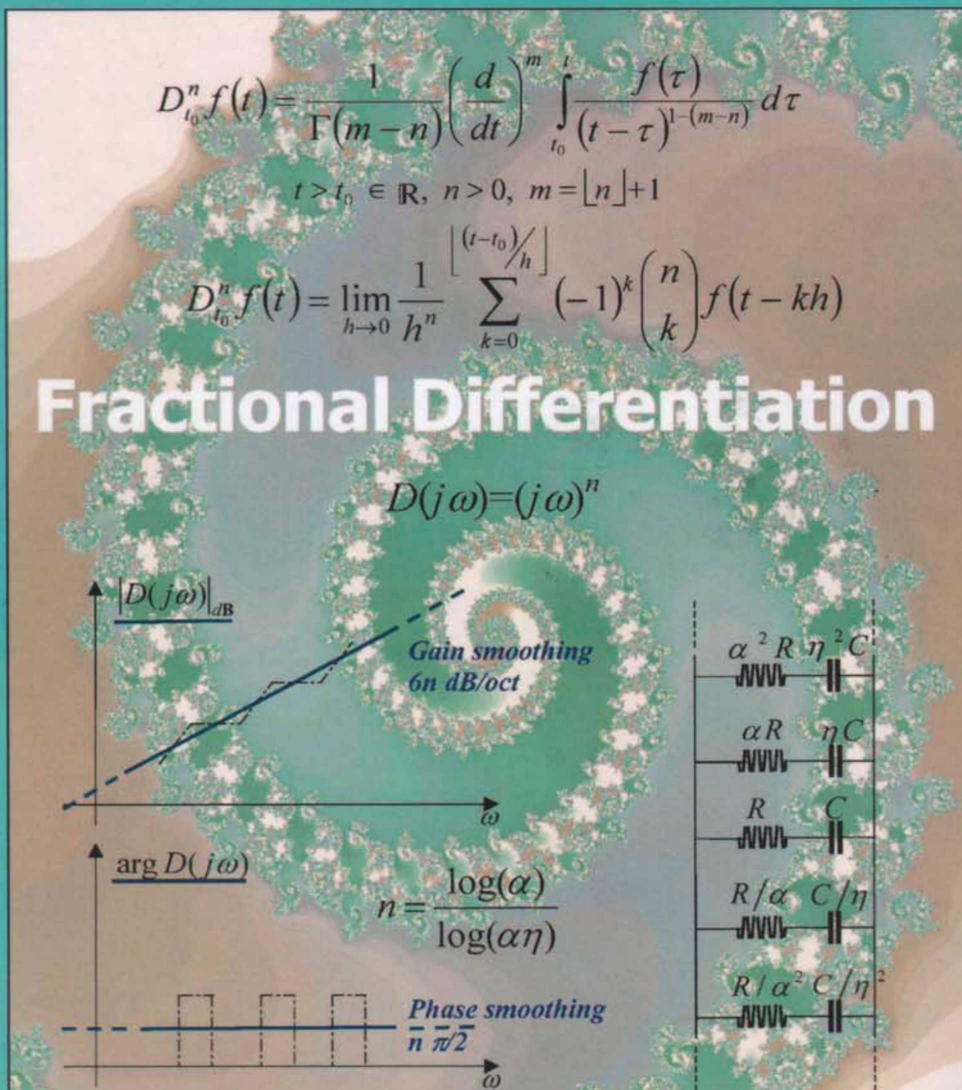
Fractional differentiation and its applications



A. Le Mehauté, J. A. Tenreiro Machado, J. C. Trigeassou, J. Sabatier (Eds)

Partial Fractional Order Differential Equations of Transversal Vibrations of Creep-connected Double Plate Systems

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Abstract — *Two coupled partial fractional order differential equations of transversal vibrations of a creeping connected double plate system have been derived. The constitutive relations of stress-strain state of plates' material and creep layer are expressed through members of the fractional order derivatives. The analytical solutions of system coupled partial fractional differential equations are obtained by using method of Bernoulli particular integral and Laplace transform method. It is shown that two-frequency-like regime for free vibrations induced by initial conditions corresponds to one mode vibrations. It is shown that time functions of different* mn*-family vibration modes* $n,m = 1,2,3,4,\ldots\infty$ *are uncoupled.*

Keywords: *Double plates system, creep, fractional order derivative, vibrations, partial fractional-differential equation.*

1 Introduction

Plates have been extensively used as structural elements in many industrial applications. Investigation of the vibrations of plates dates back to the 19th century. There has been a great amount of research and literature presented over the last century. The problem of free vibration of a circular plate was first investigated by Poisson [23]. Rayleigh (see Refs. [26], reprint 1945) presented a well known general method of solution to determine the resonant frequencies of vibrating structures. The method was improved by Ritz by assuming a set of admissible trial functions. This approach is one of the most popular approximate methods for vibration analysis, as well as asymptotic methods [10],[11]. Extensive studies of the vibrations of plates for various shapes, boundary and loading conditions have been conducted for nearly two centuries. The interested readers are refereed to the excellent reviews of Leissa [18] and Liew et all. [19] of this class of problems and lists of references.

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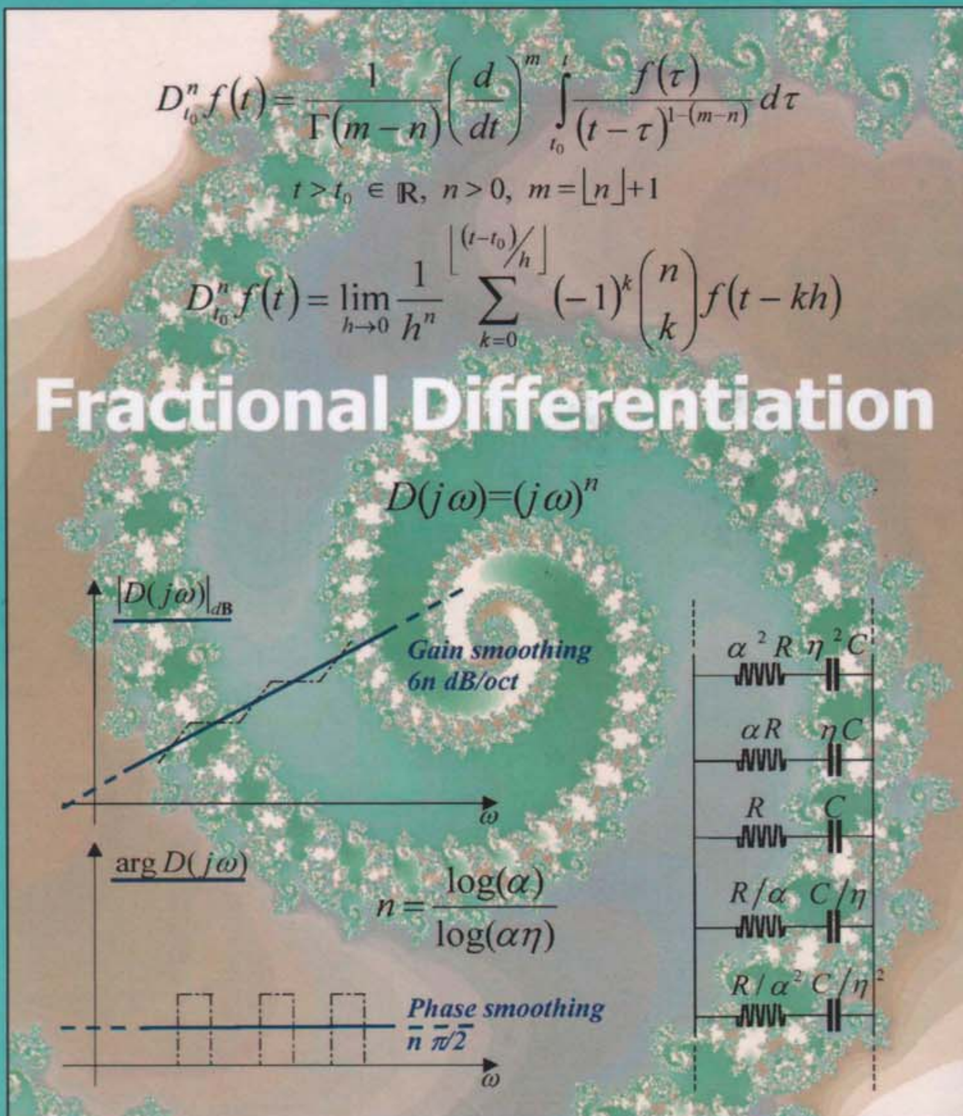
Katica (Stevanović) Hedrih received a Master`s degree in engineering sciences – nonlinear and deformable body mechanics, and a doktoral sciences degree in technical sciences both from University of Niš, Yugoslavia, and also a kandidat mininum in mathematical physics, nonlinear differential equations and nonlinear ocillations at National Institute of Mathematics NANU Kiev, Ukraine, in the years 1972, 1975 and 1971, respectively. Her main professors on mechanics and mathematics was Draginja Nikolić, Danilo Rašković, academician Tatomir Andjelić, Dragoslav Mitrinović and academician Yuri Alekseevich Mitropolskiy. Her research interest focuses on nonlinear problems in mechanics, and include theory of oscillations, elastodynamcs, photoelasticity, piezoelectricity, ultrasonic, electromechanical systems, mechanical engineering, applied mathematics, physical mathematics; strength of materials, machine dynamics, computational mechanics, control motions and nonlinear dynamical systems, analytical mechanics, damage and fracture dynamics, active structure, fractional calculus and applications, history and philosophy of sciences, multidisciplinary problems and phenomenological mapping and mathematical analogy.

Fractional differentiation and its applications



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Fractional differentiation, also called non-integer differentiation, is a concept that dates back to the beginning of differential calculus when it came to the attention of Leibniz and L'Hospital (1695) who exchanged letters about the half-order derivative. Since then many famous mathematicians and physicists have studied fractional integrals and derivatives mainly from a theoretical point of view, the main ideas being related to the names of Abel, Liouville, Riemann, Grunwald and Letnikov. By the beginning of the twentieth century only a few applications had been proposed, namely by O. Heaviside (1880) and A. Gemant (1936) who developed fractional models for electrical and mechanical engineering applications. In spite of these contributions this area of research remained almost unknown for many applied scientists until twenty or thirty years ago, when a considerable attention started to be paid to systems governed by fractional differential equations (now commonly called fractional systems). All over the world a spreading scientific community has indeed brought to light that many real physical systems (systems with long memory or hereditary behavior) are well characterized by fractional differential equations. Fractional differentiation and fractional systems play now a very important role in various fields such as biology, bio-physics, control theory, economics, electrical engineering, electronics, electromagnetism, electrochemistry, image and signal processing, mechanics, mechatronics, physics, rheology, material modeling and thermal engineering.

In this context, the first IFAC workshop on Fractional Differentiation and is Applications, FDA'04, was held in Bordeaux, France, in 2004. This workshop aimed at bringing together experts in the field of fractional differentiation and its applications and all interested researchers, from universities and industries, to look at the state of the art and current research lines in theory, methodology, applications and tools.

This book integrates three parts gathering a selection of articles presented during FDA'04. Its attempts to give to the reader a presentation of current research and the latest industrial applications of fractional differentiation. The first part is dedicated to mathematical tools and geometrical and physical aspects. The second part presents applications in the domains of econophysics, mechanics, material modeling, thermal systems, electronics and electrical systems. Finally, the third part presents applications in systems analysis, implementation and simulation, system identification and system control.

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