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Руководиоцу пројекта ОИ174001, проф. др К. Хедрих

Стручни извештај о учешћу на научном скупу

Поштовани,

У периоду од 24. до 26. јуна 2019. године учествовао сам на Седмом међународном Конгресу механике организованом од стране Српског друштва за механику (СДМ). Конгрес СДМ се одржава сваке друге године, а овог пута био је одржан у Сремским Карловцима, Србија, у просторијама Карловачке гимназије. Службени језик конгреса био је енглески.

На научном скупу учествовао сам са усменим излагањима два рада, од којих су оба настала у оквиру истраживања на пројекту ОИ174001.

На минисимпозијуму "*Waves and diffusion in complex media*" изложио сам коауторски рад под називом:

"***Influence of the attached masses on the dynamic response of a cantilever beam under an impulse support movement***".

На минисимпозијуму "*Nonlinear dynamics*" изложио сам ауторски рад под насловом:

"***Holography in photoelasticity – an overview and a brief review of Professor Vlatko Brčić's contribution to this field***".

На скупу су разматрани проблеми и савремена достигнућа у различитим областима Механике, укључујући и Механику чврстих тела, Механику флуида, Теорију управљања механичким системима, Роботику, Биомеханику и интердисциплинарна истраживања. У оквиру ових области посебна пажња била је посвећена математичком моделирању динамичких система и развоју математичких модела за описивање нелинеарних проблема механике, коришћење рачуна са изводима нецелог реда, примену метода са више редова величина (*multiscale methods*) идр.

Скуп је био организован у три дана, са укупно 7 пленарних предавања по позиву и 24 паралелне секције, које су обухватале главну секцију конгреса и 5 минисимпозијума, и у оквиру којих је изложен различит број радова, тематски одговарајуће разврстаних. Предавања по позиву одржали су: **проф. Валтер Лакарбонара** са Сапијенца Универзитета у Риму, Италија, **проф. Здравко Терзе** са Факултета за машинство и бродоградњу Универзитета у Загребу, Хрватска, **проф. Хонгуанг Сун** са Института за механику меких материјала Хохаи Универзитета у Нанкингу, НР Кина, **проф. Питер Ван** са Института за честице и нуклеарну физику Мађарске академије наука, Будимпешта, Мађарска, **проф. Душан Зориц**а са Математичког института САНУ, огранак у Новом Саду, Србија, **доц. Немања Зорић** са Машинског факултета Универзитета у Београду, Србија**, проф. Ђорђа Каранасиу** са Института за молекуларну биологију и биотехнологију Универзитета у Јањини, Грчка, и **доц. Бојан Међо** са Технолошко-металуршког факултета Универзитета у Београду, Србија. Теме пленарних предавања су биле везане за различите области механике, у складу са експертизама појединих предавача, а укључивале су: пројектовање, конструкцију и примену нелинеарних динамичких апсорбера (*Лакарбонара*), тополошке методе анализе интеракције флуида и конструкције (*Терзе*), моделирање и примену аномалне дифузије уз коришћење фракционог и фракталног рачуна (*Сун*), нове описе термодинамичких феномена у оквиру механике континуума (*Ван*), анализу нелокалних дејстава и наследности при моделирању простирања таласа уз примену фракционог рачуна (*Зорица*), анализа и примена система за контролу вибрација код гипких конструкција (*Зорић*), развој нових модела на више нивоа величине за анализу крвотока и примена на пројектовање, производњу и уградњу стентова код људи (*Каранасоу*), развој микромеханичког модела и анализа критеријума лома и услова течења код метала са применом у оквиру Методе коначних елемената (*Међо*). Паралелне секције су обухватале различите теме, од којих су неке биле моделирање нелинеарних динамичких система и примена рачуна нецелог реда, развој математичких модела за анализу нелинеарних механичких и биомехничких система, управљање и контролу механичких система са временским кашњењем, развој одговарајућим модела материјала и њихова примена у оквиру механике континуума, идр.

С обзиром на начин организовања научног скупа, нисам био у прилици да присуствујем свим секцијама, па сам зато, поред присуствовања свим пленарним предавањима, изабрао да присуствујем оним секцијама које су тематски најближе мојим тренутним истраживањима, тј. онима које су се тицале примене рачуна нецелог реда и анализе вибрација континуалних система, уз могућу примену нелинеарних динамичких система за контролу вибрација конструкција.

На конгресу је учествовало око 100 истраживача из земље и иностранства, од којих је у знатном броју било истакнутих професора и врхунских стрчњака у различитим областима Механике.

Конгрес СДМ је био изванредна прилика за мене да добијем нове информације и увиде везане за области мог тренутног истраживања. Такође, у паузама између секција било је и прилике за упознавање и размену идеја са другим истраживачима што је за мене такође било врло значајно искуство за које верујем да ће ми користити у мом даљем раду и професионалном усавршавању. Напомињем и да сам на овом научном скупу учествовао као део тима са пројекта ОИ174001 којим руководи проф. др Катица (Стевановић) Хедрих.

Радови свих учесника штампани су у облику двостраничних проширених апстраката у књизи апстраката, која има додељен ИСБН број.

Као прилог овом извештају достављам:

- Фотокопију Потврде о учешћу на научном скупу
- Прве три и последњу страницу књиге апстраката
- Објављене и излагане проширене апстракте са којима сам учествовао на скупу

Са поштовањем,

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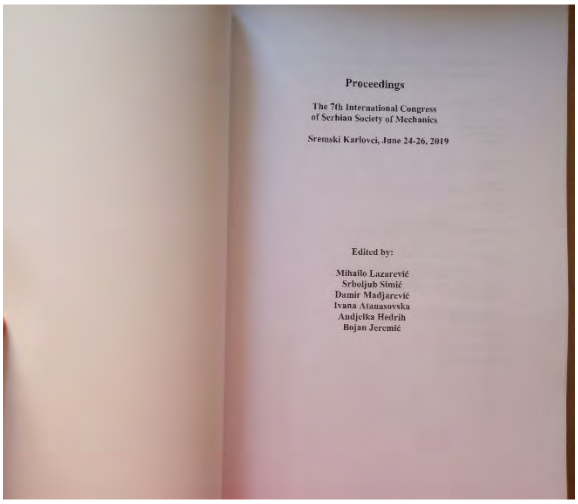
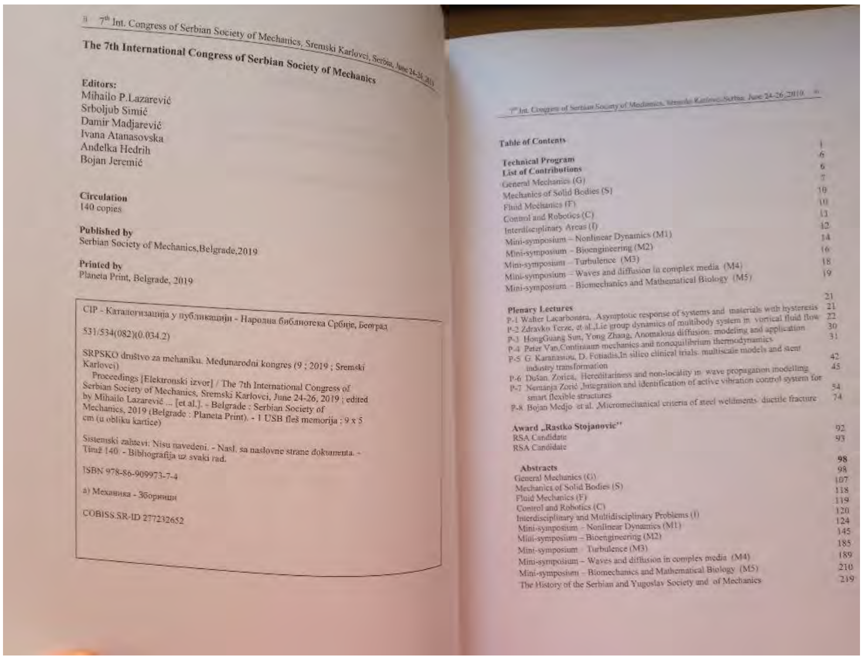
Степа Пауновић,
истраживач приправник

Потврда о учешћу на научном скупу

Прве три, и последња страница књиге апстраката




ii 7th Int. Congress of Serbian Society of Mechanics, Sremski Karlovci, Serbia, June 24-26, 2019

The 7th International Congress of Serbian Society of Mechanics

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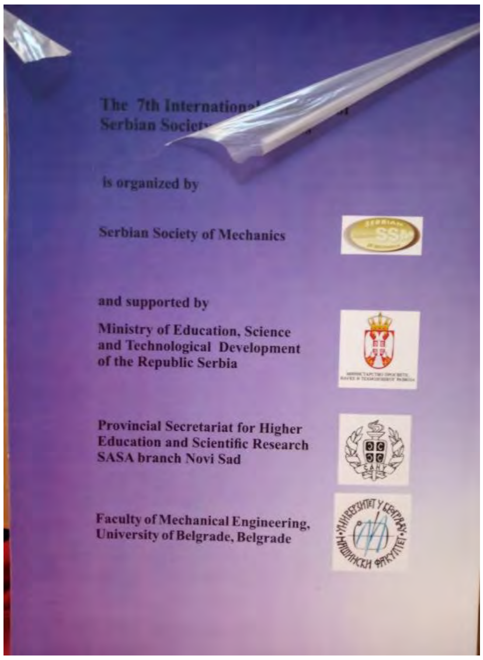

important thing to note is that this kind of approximation is valid only for a finite positive interval. The value for k (number of terms in the approximation) used in this paper is 30, giving the best precision of the approximation for its time of calculation.

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M1r: Stepa M. Paunović[1]

HOLOGRAPHY IN PHOTOELASTICITY – AN OVERVIEW AND A BRIEF REVIEW OF PROF. VLATKO BRČIĆ'S CONTRIBUTION TO THIS FIELD

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Abstract:Here the basic concepts of holographic interferometry and its application to the photoelsastic stress analysis are presented. Schematics of basic experimental setup are shown, and a brief over-view of the development of the method is provided, starting from its original state and basic test-ing procedure, to its contemporary state, pointing out the main improvements in current methods.

Key words: holography, photoelasticity, holographic interferometry, Vlatko Brčić

1. Introduction

Photoelasticity is a method for experimental testing of structures, which utilises the polarisation of light and material optical activity to analyse the stress state in a tested model. However, it provides only the information about the principal stress directions (via *isoclinics*) and the principal stress differences (via *isochromatics*). Thus, some additional information is needed to determine the actual values of principal stresses. One way to overcome this problem is to obtain the set of *isopahics* – contours with constant principal stress sum, and a convenient method for it is the holographic interferometry

2. Holographic interferometry in photoelasticity and prof. Brčić's contribution

The main idea in holographic method is to capture the interferential pattern of two light beams – one original, referential beam, and one distorted by the analysed model. This preserves the information of both the light beams' intensity *and* their phases, so during the hologram reconstruction it is possible to extract the information about the captured object's depth, that is, the reconstructed image appears to be in 3D. For experimental testing applications, quantitative analysis of the hologram is also needed, and it is achieved by holographic interferometry.

Schematics of the basic experimental setup are shown in Figure 1a) [1]. A coherent laser beam *L* passes through a beam splitter, after which one beam arrives at the holographic sheet *H* unchanged (reference beam, *R*), while the other passes through the photoelastic model *M*, and experiences a certain phase shift due to the stress-optical activity of the model. This produces the interferential pattern at *H* that can be reconstructed to obtain a 3D image of the tested model. For a photoelastic analysis, as described in [1], 3 separate states need to be recorded – the unloaded specimen, which provides a reference image, the loaded specimen, which provides the isochro-matics and thus the principal stress differences, and lastly *both the unloaded and the loaded state* recorded *on the same holographic sheet* (double-exposure method). The last hologram provides information on the lateral deformation of the model, which is, for plane stress state, proportional to the principal stress sum. This, combined with the map of isoclinics, is sufficient to determine the stress state in the whole model field, thus successfully solving the plane problem of elasticity.

Prof. Vlatko Brčić (1919-2000) was a prominent researcher, engineer, professor and scientist. He has used photoelastic testing of structures throughout his rich engineering career, and published several scientific papers on this topic (a review of this series of papers can be found in [2]). In his paper [1], Prof Brčić gives a detailed description of the application of holographic interferometry in photoelasticity, and there he gives a contribution in the form of equations for reconstruction of the light fields that form the interferential pattern recorded on holograph. This enables the quantitative analysis of hologram and calculation of the required data for isopachics.

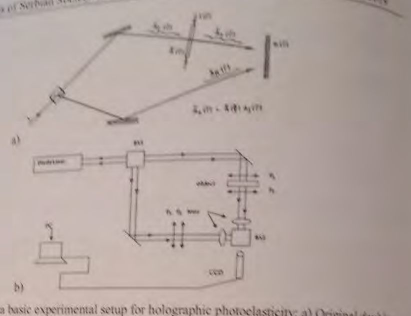

Fig. 1. Schematics of a basic experimental setup for holographic photoelasticity; a) Original double-expo-sure setup as presented in [1], b) Contemporary Digital holographic photoelasticity setup as reported in [3]

3. Contemporary holographic interferometry methods

The main drawback of the previously described method is the need for a constant reference beam, which makes the experiment very sensitive to surrounding's vibrations. Digital holographic photoelasticity reported in [3] overcomes this problem by mathematically extracting the reference beam from the holograph and then simulating this beam virtually, for each of the subsequent images, producing the interference pattern digitally and showing the results in real-time. The setup is presented in Fig. 1b). There are also other methods derived from the basic holographic photoelasticity, such as the Speckle Holographic Interferometry, Thermal Stress Analysis (TSA), or Digital Image Correlation (DIC), which are more robust and automated, digital and precise, thus complementing photoelasticite stress analysis even better and broadening its application.

Acknowledgments

This research was funded by the Ministry of Education, Science and Technological development of the Republic of Serbia, through the Project grant 174001 "Dynamics of hybrid systems of complex structures. Mechanics of materials".

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M4k: Stepa Paunović[1], Milan Cajić[1], Danilo Karličić[1, 2]

INFLUENCE OF THE ATTACHED MASSES ON THE DYNAMIC RESPONSE OF A CANTILEVER BEAM UNDER AN IMPULSE SUPPORT MOVEMENT

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Abstract:Herein, forced transversal vibrations of a fractional viscoelastic cantilever carrying multiple concentrated masses are analysed by the Galerkin method. The response due to the unit impulse movement of the support is determined, and the influence of the position of the attached masses on the beam behaviour and propagation of the excitation wave along the beam is investigated.

Key words: base excitation, attached masses, fractional order, cantilever beam, Galerkin method

1. Introduction and formulation of the problem

Exact analytical methods for forced vibrations of cantilever beams become cumbersome and practically inapplicable with the increasing number of the attached masses. When dealing with fractional viscoelastic beams, common numerical solutions also become difficult to apply. There are certain analytical methods for solving these problems [1], but they deal mostly with discrete systems, and seldom give the explicit solution for *displacements in* forced vibrations. Moreover, there is an obscure literature on the fractional viscoelastic beams under the base excitation. An exact solution to this problem for a cantilever carrying a tip mass has recently been reported [2], but this solution is hardly applicable to the case of a beam carrying multiple masses. However, this problem can be solved by the Galerkin method, in combination with the Fourier analysis.

In this extended abstract, the dynamic response of a fractional viscoelastic beam carrying multiple attached masses and exposed to an impulse support movement (Fig.1) is determined, and the influence of the position of the attached masses on the beam dynamic behaviour is analysed.

2. Determining the beam response by using the Galerkin method

If the N_m attached masses m_p are introduced via the Dirak δ function, equation of motion is:

$$EI\left(1+\tau^{\alpha}D^{\alpha}\right)w''''(x,t)+\left(\rho A+\sum_{p=1}^{Nm}m_p\delta(x-x_p)\right)\ddot{w}(x,t)=0, \qquad (1)$$

If the absolute beam displacements $w(x,t)$ are presented as a sum of the rigid body motion, equal to the support movement $w_s(t)$, and the displacements relative to the base of the beam $v(x,t)$, the boundary conditions are homogenised, and inertial forces $F(x,t)$ arise. After introducing the n-term Galerkin approximation for the relative displacements, while using the mode shape functions of the bare beam as the trial functions, and by requiring that the weighted residuals vanish, a system of n coupled non-homogeneous fractional order differential equations is obtained, and inertial forces become $Q_i(t)$. This system can be solved by first determining the unit impulse response through applying the Fourier transformation of the equations, and then taking the convolution with the actual inertial forces. If the

proportional damping is assumed, the system can be decoupled by using the matrix of eigenvectors Φ of the equivalent elastic system [3]. Using the Theory of the residuum and neglecting small terms, the impulse responses $G_i(t)$ in terms of the complex roots of the characteristic equation $s_i=-\sigma_i+i\Omega_i$ can be obtained as in [4]. Finally, the approximate transversal beam displacements are determined through the convolution:

$$w(x,t)=w_s(t)+v(x,t)=w_s+\sum_{i=1}^{n}\phi_i(x)q_i(t)=w_s+\sum_{i=1}^{n}\phi_i(x)\int_0^t G_i(t-\tau)Q_i(\tau)d\tau, \qquad (1)$$

3. Influence of the position of the attached masses

The described methodology was used to investigate the influence of the position of the attached masses on the initial behaviour of a cantilever under an impulse base excitation $w_s(t)$ (*grey dashed line in Fig.1*). The aim was to determine whether the periodic mass formation influences the propagation of the excitation wave along the beam, and to conclude if the response of a beam carrying a heavy tip mass can be mitigated by the appropriate placement of the patch of additional, much smaller masses. It was concluded that the central patch position leads to the slowest excitation propagation, but also to the largest displacements, while the patch in the first third of the beam had almost no effect. Positioning the patch the furthest from the support is opti-mal – it leads to a considerably slower wave propagation and also to the smallest displacements.

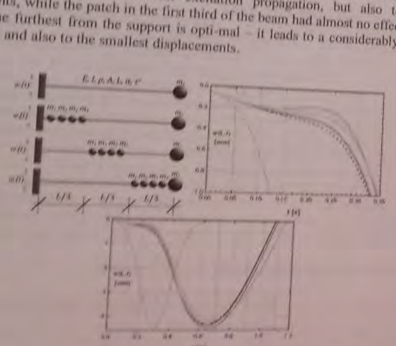

Fig. 1 The analysed systems and displacements of the free end; blue line – beam carrying only a tip mass; red, orange and green lines – additional masses added to the first, second and last third of the beam.

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M4l: Nikola Nešić[1], Milan Cajić[2], Danilo Karličić[2,3]

FRACTIONALLY DAMPED NONLINEAR PARAMETRIC VIBRATION OF A FUNCTIONALLY GRADED NONLOCAL BEAM

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Abstract The nonlinear dynamic behavior of functionally graded (FG) structures occurs in many engineering applications on macro as well as on micro-/nano-scales. This paper investigates the parametric vibration of a functionally graded (FG) nonlocal beam with geometric nonlinearity, which is embedded in the fractional Kelvin-Voigt viscoelastic medium. The material properties of FG nanobeam vary continuously through the thickness direction based on the power-low distribution. Nonlinear partial differential equations of motion are derived using d'Alembert principle. For analyzing instability regions of FG nanobeam, incremental harmonic balance method is used to obtain iterative relationships between frequency and amplitude of time-varying axial load.

Key words: Dynamic stability, Functional graded nanobeam, Incremental harmonic balance method, Fractional damping

1. Introduction

Structures made of FG materials are composed of at last two-phase inhomogeneous particulate composite and are synthesized in such a way that volume fractions of the constituents vary continuously along any desired spatial direction. The incremental harmonic balance (IHB) method is a semi-analytical method based on the Newton – Raphson iterative procedure that was proposed by Lau et al. [1]. Karličić et al. [2] determined instability regions of an axially loaded single-walled carbon nanotube embedded in a viscoelastic medium by using the IHB method. Shen [3] has shown the procedure of solving the equation for the fractional-order Duffing oscillator using the IHB method.

The model suggested in this paper includes the effects of thermal environment, parametric excitation, small-scale effect and fractional viscoelastic damping. Periodic solutions are found using the IHB method and corresponding formulas for fractional derivatives and then validated against multiple-scales approximate solution. The results predicted by IHB method are also verified by using the numerical integration.

2. Mathematical model and IHB solution