Matematička dostignuća Slobodana Simića
Mathematical Achievements of Slobodan Simić

MATEMATIČKA DOSTIGNUĆA SLOBODANA SIMIĆA

MATHEMATICAL ACHIEVEMENTS OF SLOBODAN SIMIĆ

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Akademska misao
Beograd, 2019.

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Izdaje i štampa
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P R E D G O V O R

Profesor Slobodan Simić bio je matematičar izuzetne sposobnosti, talenta, predanosti i vrednoće koji su ga doveli do brojnih značajnih naučnih rezultata. Objavio je preko 160 naučnih radova i više od 40 knjiga. Njegova prerana smrt (15. maja 2019. godine) otrgla ga je od novih rezultata i uskratila novim generacijama mogućnost da uče direktno od njega. Zato smo odlučili da u ovoj knjizi prikažemo naučne rezultate prof. Simića i da na taj način iniciramo njihovo dublje izučavanje kao i da inspirišemo istraživače da nastave rad tamo gde je on stao. Umesto biografije knjiga sadrži originalne autobiografske podatke koje je prof. Simić sastavio za potrebe internet prezentacije Matematičkog instituta u Beogradu. Priložena je i reakcija njegovog dugogodišnjeg kolege i prijatelja profesora Zorana Radosaveljevića. U svom najopširnijem delu, knjiga sadrži spisak naučnih radova prof. Simića, opis najznačajnijih tema kojima se u njima bavio i delimičan spisak apstrakata radova, zatim spisak i prikaz knjiga, te naposletku ekspozitorne članke u kojima su dati detaljniji prikazi nekih aspekata njegovog naučnog rada. Autori ekspozitornih članaka su M. Andjelić, F. Belardo, D. Cvetković, Z. Stanić i D. Stevanović. Urednici prestižnog međunarodnog časopisa *Discussiones Mathematicae Graph Theory* odlučili su pripreme specijalni broj posvećen 70. rođendanu prof. Simića. Iako je dočekao jubilej, specijalni broj časopisa (koji treba da izađe 2020. godine) nažalost nije. U knjizi su date neke napomene gostujućih urednika tog broja.

Knjiga je dvojezična: postoje tekstovi na srpskom i tekstovi na engleskom jeziku. Naslovi tekstova dati su na oba jezika, a u sadržaju knjige pojavljuju se u odgovarajućem redosledu. Jedino je Predgovor napisan na oba jezika. U knjizi se radovi profesora Simića citiraju korišćenjem rednog broja iz spiska njegovih radova uz prefiks S (na primer, [S1], [S2], ...).

Urednici zahvaljuju autorima priloga na njihovom trudu. Zahvalnost dugujemo i Mariji Jerotijević koja je tehnički pripremila knjigu.

Beograd, oktobra 2019. Urednici

P R E F A C E

Professor Slobodan Simić was a mathematician of extraordinary abilities, talents, commitment that resulted in numerous significant scientific results. He published over 160 scientific papers and more than 40 books. His premature death on May 15, 2019, in Belgrade, prevented the appearance of new results. In addition, young students are denied the possibility to learn directly from him. Therefore, the purpose of this book is to present scientific results of Prof. Simić and to initiate a serious study of these results. Instead of a biography the book contains original autobiographic data that are contained in the Internet presentation of the Mathematical Institute in Belgrade. The reaction of Professor Zoran Radosavljević, an old colleague and friend of Professor Simić, is included as well. Most of the space is filled by the list of scientific papers of Professor Simić, the description of most important subjects in his papers and a partial list of abstracts of the papers, followed by the list and review of his books. In addition, the book contains expository articles in which some details of some aspects of his scientific work are presented. The authors of the expository articles are M. Andjelić, F. Belardo, D. Cvetković, Z. Stanić and D. Stevanović. A special issue of the well-known international journal *Discussiones Mathematicae Graph Theory* devoted to 70th anniversary of Prof. Simić, is scheduled to appear in 2020. The remarks by guest editors of that special issue are given in this book.

The book is bilingual: there are texts in Serbian and texts in English. The title of a text is given in the language in which the text is written but the title is immediately translated into the other language. In the table of contents the titles in both languages are given in the order in which they actually appear. Only Preface is given in both languages. Simić's papers are cited by the numbers from the list of his papers with a prefix S (for example, [S1], [S2], ...).

Editors thank the authors of the contributions included into book. We are also grateful to Marija Jerotijević who technically prepared the book.

Belgrade, October 2019 Editors

Ukratko o profesoru Slobodanu Simiću

Na vest o smrti prof. Simića, njegov kolega sa studija Branislav (Bran) Selić, predsednik Malina Software Corp. iz Kanade poslao je sledeću poruku ostalim kolegama:

Ako je vest tačna – a nadam se da nije – onda je naša generacija izgubila svog „tihog genija“ – čoveka koji je bio najobdareniji od svih nas – svetski talenat. Nikad se nije gurao ni nametao, mada je imao čime da se diči. Izuzetno skroman čovek ali dubokog i plemenitog uma. Mada je uvek delovao smireno, u njemu je kuljao nemiran, neobično kreativan i originalan duh. Složene matematičke apstrakcije su za njega bile konkretne i deo stvarnosti. Ako je vest tačna, jako mi je žao; bio mi je uzor pameti i čestitosti. Mada smo se retko sretali, ponosim se što sam mu bio prijatelj. Slava mu.

Ova poruka doslovno odražava mišljenje svih koji su prof. Simića poznavali i stoga je reprodukovana na ovom mestu. Urednici se zahvaljuju Slobodanu Boškoviću koji je poruku prosledio.

Slobodan Simić : Autobiography
Slobodan Simić : Autobiografija[1]

Slobodan K. Simić was born in Belgrade (Serbia) in July 24 1948. When he was in the secondary school, he received three times the first prize on national (ex-yu) competitions in mathematics (in 1966 and 1967) and in physics (in 1966). This explains his early interest in mathematics. But his educational background is in electrical engineering. The B.Sc. degree he received in 1973 from Faculty of Electrical Engineering at University of Belgrade. The next two degrees, both in Applied Mathematics, he received at the same Institution M.Sc. (in 1977) and Ph.D. (in 1979). His supervisor (each time), and later the most important collaborator, was Dragoš Cvetković.

From 1973 up to 2006, he was employed at Faculty of Electrical Engineering (University of Belgrade) at Department of Mathematics, starting as Teaching Assistant. He became Docent in 1982, Associate Professor in 1992 and Full Professor 1996. Two academic years starting from 2001 he was employed at University of Montenegro (in Kotor at Maritime Faculty

[1]Taken from Simić's homepage.

and partly in Podgorica at Mathematical Faculty). From 2003 he was also teaching at Faculty of Computer Sciences in Belgrade. In his career he was the chairman of Department of Mathematics, at Faculty of Electrical Engineering in Belgrade and Maritime Faculty in Kotor. Since 2006 up to retirement in 2014, he was employed at Mathematical Institute of Serbian Academy of Sciences and Arts as Full Research Professor. Next two academic years after retirement, he spent at State University of Novi Pazar (Serbia) as Full Professor, focused only on research. In 2013, he was promoted in Visiting Professor at University of Messina (Italy). Since 2016, he finished his academic career, keeping his interest for mathematics, but also for electrical engineering as was a dominating activity in some perioda when being employed at Faculty of Electrical Engineering.

In his academic career, besides teaching students at various levels (including Ph.D. ones) and preparing them for interstudent competitions, he was introducing them to scientific work. He had one master student and three doctoral students. He had prepared for his students more than ten textbooks and some others for growing attention on mathematics in his country. Besides he participated very much in research activities. So far had prepared more than 160 papers and three books (monographs) in Spectral Graph Theory published by Cambridge University Press. He presented his results as a speaker at many conferences and seminars, or talks as a visitor of various Institutions (over his country or abroad in Chezkoslovakia, England, Germany, Italy, Malta, Poland, Portugal, Slovakia, Slovenia, Scotland etc.). For mathematical community he refereed papers for many international journals (like Applicable Analysis and Discrete Mathematics, Discrete Applied Mathematics, Discrete Mathematics, Discussione Mathematicae - Graph Theory, Graphs and Combinatorics, Journal of Algebraic Graph Theory, Linear Algebra and its Application, Linear Multilinear Algebra, MATCH, etc.). He is (or was) in Editorial Board Discussione Mathematicae - Graph Theory, Publications de l'Institut Mathématique (Belgrade) and Applicable Analysis and Discrete Mathematics. He was a Guest Editor of Linear Algebra and its Application (in honour of Dragoš Cvetković). For many years he was a reviewer for Mathematical Reviews end Zentralblatt für Mathematik. Besides this, he took active part in designing and/or implementing various software packages for research (say, GRAPH, NewGraph, TSP-Solver, NeoGraph), or for practical application: in electronic intelligence (for analysis of radar signals), in power grid networks, in banking (multilateral compensation and cliring), in national railway company (for designing a database), etc.

Slobodanu, u spomen
In the Memory of Slobodan

Zoran Radosavljević

Slobodan Simić se rodio da bude naučnik i matematičar. Verovatno samo manjina ljudi ima sreću da u svom životu pretežno radi posao koji voli, a biće i da je samo deo te manjine svojim sklonostima, talentom i svojim celokupnim bićem istinski za njega predodređen. Sa tim ljudima se najčešće dešava to da, bez obzira kojim obrazovnim i razvojnim putem išli, na kraju dospevaju na svoj neminovni cilj, koji tada postaje snažan oslonac za novi i pravi početak razmaha svih kreativnih potencijala. Slobodan je jedan od tih retkih ljudi. I sad ga vidim kako sedi za svojim stolom u kabinetu, zadubljen u svoje hartije, neverovatno usredsređen i neverovatno otporan na razgovore i uznemiravanja iz najbliže okoline, potpuno u svom svetu, za njega najboljem i najvažnijem. Uvek sam se pitao da li je ta njegova čudesna sposobnost ignorisanja sveta u okruženju bila apsolutno urođena i prirodna, ili je bio potreban dodatni napor volje da se potpuno isključi iz realnosti i eliminiše „buka i bes“ realnog sveta. Kako god bilo, apstraktni svet matematike i konkretan problem „koji napadamo“ u tim trenucima su bili najčistija realnost, a sve ostalo se gubilo u magli apstrakcije, i mislim da ta povremena inverzija realnosti i apstrakcije predstavlja jedno od ključnih obeležja njegove ličnosti. Naravno, može se reći da se ovo, u većoj ili manjoj meri, da primetiti kod svakog pravog matematičara, ali kod njega je to bilo toliko upečatljivo da izvesno potvrđuje da je on bio pravi među pravima.

On svakako nije bio čovek ovog vremena. Kažu da su dve dominantne opšte karakteristike psihe čoveka najnovijeg doba narcizam i površnost. Matematika i površnost nikako ne idu zajedno, matematika je zapravo formalizovani antipod ljudskoj površnosti, takoreći poslednja linija odbrane, pa je prirodno da sa ovom osobinom pravi matematičar nema ništa. Ali on je bio beskonačno daleko i od prve osobine. Izuzetno temeljan i izuzetno skroman, bio je ukorenjen u onom sistemu vrednosti koji su najbolji predstavnici ranijih generacija afirmisali i potvrdili svojim životima i svojim delima. Njegovo delo i njegov život uvek će nas podsećati na onu epizodu iz naše školske istorije (godina je 1886., a mesto događanja Kapetan-Mišino zdanje) kada direktor Prve muške gimnazije Đura Kozarac grdi maturante zbog nekih đačkih nestašluka i kaže: Vi ste ovde došli da se učite nauci, radu, poštenju i rodoljublju. Danas je teško zamislivo da ovako nešto izgovori neki direktor škole ili bilo koji funkcioner u obrazovnom sistemu; u ovo malo reči suviše je mnogo onih koje paraju uši. U postcivilizaciji uloga i ciljevi obrazovnog sistema su drukčiji: oni od prethodno pobrojanog priznaju samo nauku, vaspitna komponenta je izbačena, a onda i nauka, često komercijalizovana i instrumentalizovana, u službi zarade i bez svog moralnog temelja nema više onaj smisao i značenje koje je ranije imala, postaje sklona svakojakim manipulacijama i relativizacijama, kao i sve ostalo i, kako izgleda, umesto naučnim istinama sve više teži „korisnim“ postistinama. Na sreću matematičara, izgleda da je matematika u ovom sumornom razvoju događaja, kao nauka najstarija i ponosna, ne gorda, još i najjači i najotporniji čuvar prava na klasičnu naučnu istinu, najjača tvrđava odbrane naučnog morala. Zato se Slobodan u tom svetu osećao dobro i udobno. Svojim ogromnim talentom bio je snažno usmeren ka pravom i ozbiljnom naučnom radu. Vredan rad, koji je nekada, pre nastanka industrije zabave, za obične, normalne ljude bio metafora života, i za njega je bio podrazumevajući način kako se ostvaruju željeni ciljevi. Apsolutno poštenje – naučno, ali i svako drugo, dakle apsolutno – bilo je svakako blagoslov predaka, kroz genetiku i vaspitanje, i to, zajedno sa onom četvrtom rečju koju spomenu direktor Kozarac, spada u one najfinije komponente nečije ličnosti, o kojima treba govoriti što manje ili ništa i o kojima svako svedoči samo svojim celokupnim životom.

Njegov izuzetan matematički talenat pokazao se rano – već u gimnaziji osvajao je nagrade na takmičenjima svih nivoa, zaključno sa saveznim takmičenjima tadašnje Jugoslavije. Upisivanje studija elektrotehnike moglo je samo za neupućene da zaliči na skretanje sa pravog puta. Tadašnji Elektrotehnički fakultet u Beogradu i njegova Katedra za matematiku, na čijem je čelu počev od 1953. čitave 22 godine bio profesor Dragoslav Mitrinović, razvili su nastavu matematike visokog nivoa i širokog spektra, vodeći pritom posebnu brigu o studentima sa talentom za matematiku kroz organizovanje specijalnih grupa i posebnih namenskih kurseva i predavanja i rano uključivanje u naučni rad putem individualne saradnje sa članovima Katedre. Svršeni studenti elektrotehnike isticali su se svojim odličnim poznavanjem matematike, Katedra je stekla zavidan ugled svojim razvijenim naučnim radom i drugim aktivnostima, a zahvaljujući svemu tome Fakultet je imao matičnost za dodeljivanje diploma magistara i doktora matematike. Prema tome, ovo je izvesno bio jedan od mogućih Slobodanovih prilaznih puteva ka širokoj aveniji njegovih budućih ključnih životnih aktivnosti. Već na prvoj godini odskočio je od ostalih svojim sjajnim talentom za matematiku, pa je svima bilo prirodno kad mu je profesor Mitrinović na ispitu u junu dao specijalne zadatke. Ispite je polagao sa lakoćom, pomagao kome je koliko mogao i, onako briljantan, a tih i skroman, uživao opšte poštovanje i simpatije svojih kolega iz generacije. Sada, kada se podvlači crta i naviru uspomene, upravo je dirljivo sa koliko divljenja i topline ti ljudi govore o njemu, priznajući mu bez rezerve da je bio „najobdareniji među nama“, „uzor pameti i čestitosti“.

Pokazalo se da su studije elektrotehnike bile ne samo moguć, nego verovatno i najbolji put ka njegovom prirodnom cilju – matematici i naučnom radu. One su ga, pored ostalog, usmerile i na diskretnu matematiku i neke njene oblasti koje su se slabo proučavale na drugim fakultetima (uključujući i studije matematike). Otvorile su mu vidike ka širokim poljima primene matematike u drugim, naročito inženjerskim naukama. Omogućile su mu vrlo rano uključivanje u naučni rad, u čemu je odlučujuću ulogu odigralo rano poznanstvo i saradnja sa dr Dragošem Cvetkovićem, sada akademikom, a tada docentom i njegovim budućim mentorom, čiji su tadašnji rad i njegova kruna – doktorska disertacija „Grafovi i njihovi spektri“ (1971) izvršili raznolike i snažne uticaje, kako globalne, u pravcu konstituisanja spektralne teorije grafova, tako i na domaćem terenu, otkrivanjem oblasti matematike o kojima se znalo vrlo malo. Sve je to otvorilo Slobodanu puteve i mogućnosti da se afirmiše na nov način – objavljivanjem prvih naučnih radova i postavljanjem za asistenta pripravnika na Katedri za matematiku. Dalje je sve išlo prirodno i očekivanom, optimalnom brzinom: magistarski rad sa temom iz oblasti grafovskih jednačina, pa odlučnije usmeravanje ka spektralnoj teoriji grafova, doktorska disertacija, odgovarajući izbori u viša zvanja, naučno osamostaljivanje i potpuna naučna afirmacija na međunarodnom planu. Da li uopšte treba podsećati da iza ovog kratkog pregleda i nekoliko rečenica leži višegodišnji, veliki, vredan i istrajan rad, snažna motivacija i oduševljenje, sve ono bez čega ni veliki talenat ne može da zablista punim sjajem? Bespredmetna su sva ona licitiranja koliki je u nečijem uspehu udeo talenta, odnosno rada. I zar ne postoji, pored ostalog, i međusobni unutrašnji uticaj talenta i rada? Talenat i motivacija prirodno generišu rad; istrajan rad podiže i oplemenjuje talenat. Sve smo to videli na Slobodanovom primeru.

Na Elektrotehničkom fakultetu proveo je najveći deo svog radnog veka. Inženjer na Katedri za matematiku i sjajan matematičar među inženjerima (i pritom ne jedini), to je za Fakultet bila idealna kombinacija. Široka matematička znanja i velika erudicija, do koje se dolazi kako kroz naučni rad tako i dodatnim zadovoljavanjem mnogih interesovanja, a sa druge strane i odlično razumevanje jezika i naučnih i praktičnih problema kolega sa Fakulteta činili su ga dragocenim kolegom i saradnikom. Zato je on povremeno uzimao učešće i na nekim inženjerskim projektima, ali ipak nije dozvolio da to utiče na njegovu osnovnu aktivnost – rad na polju matematike. To je pre svega značilo rešavanje naučnih problema i objavljivanje rezultata, što mu je pričinjavalo najveće zadovoljstvo – pretežno u oblasti spektralne teorije, ali i u drugim oblastima teorije grafova, a povremeno je imao i šira interesovanja. Pisanje knjiga kretalo se u dijapazonu od vrhunskih monografija najboljih svetskih izdavača do udžbenika i zbirki zadataka za naše studente. Kroz dugi niz godina učestvovao je na razne načine u uređivanju „Publikacija Elektrotehničkog fakulteta – serija matematika“ i njihovog naslednika „Applicable Analysis and Descrete Mathematics“. Bez nabrajanja svega što radi dobar univerzitetski nastavnik, neka ovde bude pomenut još samo onaj očekivani karakterističan detalj: naravno da je kao inženjer - matematičar bio često i rado pozivan u komisije za ocenu i odbranu diplomskih, magistarskih i doktorskih radova. Što se tiče nastave, i to je radio apsolutno korektno i bez zamerke, ali je povremeno, u zavisnosti od sadržaja predavanja i auditorijuma, pokazivao i izvesnu distancu prema ovom poslu, zbog čega bi se moglo reči da je njegov odnos prema nastavi bio u izvesnoj meri ambivalentan. Slušao sam ga i na međunarodnim naučnim konferencijama, kada je držao predavanja po pozivu: motivisan, pun oduševljenja i pred istinski zainteresovanom publikom, činio je to briljantno. Na Fakultetu, na nižim godinama studija, studenti su ponekad očekivali od njega i ono što on nije bio spreman da čini: da ponešto što im se čini teškim uprosti ili preskoči. On je smatrao da matematika nije guma koja se rasteže i skuplja i nije odstupao od svog koncepta.

Bio je miran i tih, bez neodmerenih reakcija; onda kad nije bio usredsređen na rad, ne samo što je izgledalo da je ravnodušan i bez nekog jačeg emotivnog odnosa prema zbivanjima oko sebe, nego je često i sam imao potrebu da to naglasi i stavi do znanja drugima. Trebalo ga je dugo i dobro poznavati da bi se razumelo da to nije verna slika njegovog emocionalnog sveta, da se radi o obuzdavanju osećanja prema ljudima i događajima i da su u njegovom slučaju putevi stvaranja stabilnog emocionalnog otiska pojava, stvari i ljudi često bili procesi dugog trajanja. U tom svetlu treba gledati i na njegove povremene potrebe za promenom u životu i radu i, s tim u vezi, njegove odlaske sa Fakulteta. Prvi put se to dogodilo krajem osamdesetih godina: afirmisan i cenjen, pravi čovek na pravom mestu, bez stvarnog razloga za nezadovoljstvo, ipak je imao neku nagomilanu potrebu da izađe iz šeme svakodnevnog fakultetskog rada. Proveo je jedan semestar u Australiji i tamo se uverio da je čovek trajno prognan iz raja i da mora da prihvati zemaljski život. Zatim su došle teške devedesete, koje smo podnosili, zajednički i solidarno, iako nismo nazirali svetlo na kraju tunela, da bismo na kraju upali u još gušći mrak. Zloglasni Zakon o univerzitetu iz 1998. i prinudna uprava na Fakultetu, a onda proganjanje i zabrana držanja nastave, iznošenje profesora iz učionica, straže i zakovana vrata po hodnicima i razne druge vrste nasilja uskomešali su i najtvrđe duše. Mnogi su otišli, za dve godine oko šezdeset ljudi, među njima i Slobodan (na Fakultet za pomorstvo u Kotoru). Fakultet čine pre svega ljudi i ovaj najstrašniji period poniženja i sramote naneo je nenadoknadivu štetu. Posle normalizacije stanja neki su se vratili, a neki nisu; on se vratio. Najzad, posle više od trideset godina rada na Fakultetu otišao je treći i poslednji put i, oslobodivši se nastavnih i drugih fakultetskih obaveza, prelaskom u Matematički institut SANU omogućio sebi da sve svoje vreme posveti isključivo naučnom radu, svom prioritetnom i pravom životnom opredeljenju. Sada, u tom životnom dobu, u šestoj deceniji života, ta odluka je bila prava, donela mu je mnogo zadovoljstva u potpunoj posvećenosti nauci, uvela ga na vrhuncu naučne zrelosti u verovatno najproduktivniji period karijere i plodnu saradnju sa grupom vrlo talentovanih mlađih saradnika i doktoranata. Formalni kraj aktivne karijere dočekao je sa briljantnim rezultatima.

Ars longa, vita brevis – govorili su stari Latini. Nije drukčije ni sa naukom; uostalom, vrhunska nauka bliska je rođaka umetnosti. Verujući ljudi podrazumevaju besmrtnost ljudske duše, oni drugi to odriču. Ali ono što bilo ko teško može da ospori jeste nepostojanje granica stvaralačkih mogućnosti bogate, obdarene i nadahnute ljudske ličnosti, beskonačnost ljudskog uma, duha i kreativnosti, kao što je beskonačan i fizički svet čiji smo deo; uostalom, i matematika, najsavršenija tvorevina ljudskog uma, operiše beskonačnostima temeljno i slobodno. A ono što je izvesnost jeste da te beskonačne sfere stvaralaštva i duha obitavaju u krhkom i slabom telu, kratkog trajanja. *Plamen božestveni u ništavom hramu*, kaže Njegoš.

Uprkos svemu, zemlja jesmo. *Svet je ovaj groblje naših predaka, koje je otvoreno i čeka na nas*, pisao je sveti vladika Nikolaj Žički. Još samo malo, i mi ćemo biti preci. I eto, unuk prote Radoslava Simića i sin inženjera Koste Simića sada je i sam predak. A šta sve može da obuhvati pojam relacije pretka i potomka, to matematičari najbolje znaju; to je još jedna beskonačnost!

Ostavio nam je značajno i impresivno naučno delo – monografije vrhunskih svetskih izdavača i veliki broj radova u naučnim časopisima, i to je njegovo pisano nasleđe koje, sa jedne strane, kao celina, zaslužuje dublji uvid i ocenu naučnog doprinosa, a sa druge strane izvesno predstavlja i predstavljaće mnogostruki oslonac i motivaciju nastavljačima i naučnim potomcima. Tom nasleđu, koje je dostupno svima, treba dodati i knjige na srpskom jeziku, koje ostaju trajno dobro domaće matematičke literature. Ali postoji i ono nasleđe koje nije beleženo slovima i naslovima, koje je dostupno samo ljudima koji su ga poznavali i koje je njihova velika sreća i privilegija. Čovek u čoveku ostavlja svoju projekciju, otisak – različitog značaja i intenziteta u zavisnosti od više faktora, a najviše od snage i bogatstva ličnosti. Te mnogobrojne projekcije, zahvaljujući našem pamćenju i uz odgovarajuću selekciju, ostaju utisnute u naša čula i razum i kao takve postaju deo naše misaone i duhovne realnosti. Naš intelektualni, emocionalni i svaki drugi razvoj ugrađuje i taloži te važne uticaje ljudi koje poznajemo ili smo poznavali, a oni najbolji i oni koji su ostavili najdublji trag utiču odlučujuće na naše formiranje. Trag koji je za sobom ostavio Slobodan, najbolji ili jedan od najboljih mnogo puta i u mnogo čemu, a pritom beskrajno skroman i potpuno nezainteresovan za bilo kakve počasti i priznanja, nije samo dubok; on je duboko konstruktivan, duboko usmeravajući – ka pravim putevima i pravim vrednostima, i duboko plemenit. Takav je njegov trag u nauci, takav trag je ostavio na Elektrotehničkom fakultetu, gde će ga pamtiti generacije kolega i studenata, tako je i na drugim mestima gde je radio, a najlepšu i najplemenitiju uspomenu ostavio je u ljudima. Nastanio se u pamćenju ljudi koji su ga dobro poznavali, onih sa kojima je radio i onih sa kojima je sarađivao, mlađih saradnika koje je uvodio u naučni rad, a kasnije im bio i ostao učitelj i uzor, drugova iz generacije, kolega i prijatelja i svih koji su ga poštovali i voleli. Taj bogati i lepi kaleidoskop sećanja, to prisustvo u duhovnoj realnosti tolikog broja takvih ljudi, to nije fikcija i pre bi se moglo nazvati nekom vrstom njegovog transcendentnog života, koji će trajati dok živi i poslednji među nama koji ga pamtimo. To nisu elektronski zapisi koji se uklanjaju jednim klikom; ovo je mnogo nadmoćnija tehnologija, stara koliko i ljudski rod. Takvo moje pamćenje, vizuelno i zvučno, probrano i kompleksno, taloženo decenijama u intelektu i čulima, može biti uklonjeno samo onim klikom posle koga ću ja otići tamo gde je on sada.

Zadužio je matematiku, zadužio je mnoge, mnogi ga čuvaju u najlepšem sećanju, mnogi su mu zahvalni. Sećanje i zahvalnost, zahvalnost i lepo i toplo sećanje lebde nad večnim mirom u kome počiva.

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Master Thesis

S. Simić, *Graph equations*, (Serbian), Master Thesis, University of Belgrade, 1977.

Doctoral Dissertation

S. Simić, *Contributions to investigations of graph operations*, (Serbian), Ph.D Thesis, University of Belgrade, 1979.

Subjects of Scientific Papers of Slobodan Simić
Teme naučnih radova Slobodana Simića

Dragoš Cvetković

The list of published scientific papers of Slobodan Simić contains 167 items. These papers have been published jointly with 74 different coauthors.

Most frequent coauthors are the following ones (the number of joint papers is indicated): D.M. Cvetković 56, F. Belardo 20, P. Rowlinson 19, M. Andjelić 17, E.M. Li Marzi 7, K.T. Balińska 13, Z. Radosavljević 12, Z. Stanić 11, D.V. Tošić 11, K.T. Zwierzyński 9, C.M. da Fonseca 9, F.K. Bell 8, D. Stevanović 6, M. Lepović 5, D.M. Cardoso 5, D.Živković 5, V.Lj. Kocić 4, J. Wang 4, A. Al-Azemi 3, V. Baltić 3, P. Hansen 3, M. Kupczyk 3, M. Petrović 3, L. Kraus 3, I. Sciriha 3, G. Caporossi 2, I.B. Lacković 2, S. Li 2, P. Rama 2 , I. Gutman 2, M. Doob 2.

The following coauthors published each just one joint paper with Simić (they are given in order as they appear in the list of the papers): J. Akiyama, K. Kaneko, M. Syslo, J. Topp, P. Hotomski, I. Pevac, M. Čangalović, V. Dimitrijević, M. Milosavljević, A. Jovanović, V. Milovanović, Zs. Tuza, G. Stojanovski, V. Tintor, V. Brankov, M.C. Marino, S.M. Perovich, S.I. Bauk, B. Zhou, M. Aouchiche, T. Biyikoğlu, M. Ćirić, Q. Zhao, A. Krapež, X. Geng, T. Davidović, A. Ilić, Z. Huang, H. Deng, Q.X. Huang, E.M. V. de Filippo, T. Aleksić, B. Arsić, M. Škarić, N.M.M. de Abreu, I. Barbedo, E. Andrade, T. Pisanski, P. Carvalho, E. Ć. Dolićanin, Z. Du, E. Ghorbani, F. Ashraf.

The work of Simić belongs mainly to mathematics and partly to Computer Science and Electrical Engineering. Mathematical work is related to graph theory and some other fields of Dicrete Mathematics.

In general graph theory Simić has published several papers on

graph equations.

Great majority of papers on graph theory is related to spectral graph theory, in particular, to the following subjects:

- integral graphs,
- largest eigenvalue,
- second largest eigenvalue,
- least eigenvalue (e.g. graphs with least eigenvalue at least -2),
- eigenspaces of graphs,
- star complements,
- eigenvalues of the signless Laplacian,
- eigenvalues of signed graphs.

Other works can be classified as

- applications to computer science,
- computer aided research,
- computer implementations,
- miscellaneous.

Miscellaneous results include convex functions [S4], radar signal analysis [S24], [S30], data classification [S25], multilateral compensation [S40], [S41], optimization [S46], chemistry [S61], transcendental equations [S95], eLearning [S111] and quasygroups [S127].

Abstracts of Scientific Papers of S.K. Simić
Apstrakti naučnih radova S.K. Simića

1. *On enumeration of certain types of sequences*

   U jednom svom radu L. Carlitz je odredio broj n-torki $(a_1, a_2, \ldots, a_n)$ takvih da je $|a_i - a_{i+1}| = 1 \quad (i = 1, \ldots, n-1)$ i $a_1 = j$ i rešio neke druge srodne probleme. U ovom radu su svi Carlitzovi problemi rešeni sredstvima teorije grafova a dobijeni izrazi su jednostavniji od Carlitzovih. Upoređivanjem dobijenih izraza sa Carlitzovim dobijena su dva kombinatorna identiteta, inače nezabeležena u literaturi.

2. *A note on faces and cycles*

   In this paper we shall discuss necessary and sufficient conditions for the existence of an embedding of a planar graph in the plane, in which a given cycle is a face. This problem has been already treated in literature but, it seems, there is a certain vagueness.

3. *Some remarks on the complement of the line graphs*

   Glavni rezultat rada glasi: *Graf G bez trouglova je komplement nekog grafa grana H ako i samo ako je G indukovani podgraf jednog od sledećih grafova : $K_1 \cup (K_{m,n} - pK_2), K_{3,n} - C_6,$ Petersenov graf.* Okarakterisani su i pripadnici nekih drugih klasa grafova (stabla, bihromatski grafovi) koji su komplementi grafova grana.

4. *On weighted arithmetic means which are invariant with respect to k-th order convexity*

   Let (a_n) be a real $k-$th order convex sequence, let (P_n) be a positive sequence, and let us define the sequence (A_n) by (1). In this paper we give the form of (P_n) which is necessary and sufficient to ensure the k-th order convexity of (An).

5. *Graph equation* $L^n(G) = \overline{G}$

   In this paper we shall consider only finite, undirected graphs without loops or multiple edges, or shortly, according to Harary[1], only graphs, For all definitions and notation the reader is referred to [1]. Here, we shall mention only the following definitions.

6. *Graph equations for line and total graphs*

   U radu su rešene grafovske jednačine $L(G) = T(H)$ i $L(G) = \overline{T(H)}$, gde je $L(G)$ graf grana grafa G a $T(H)$ totalni graf grafa H. U dokazima se koristi karakterizacija grafa grana pomoću zabranjenih podgrafova.

7. *Graph equations, graph inequalities and a fixed point theorm*

   Već u ranijim radovima termin "grafovska jednačina" je korišćen neformalno. U ovam radu se striktno definišu grafovska jednačina i grafovska nejednačina. Daje se inverzija jedne teoreme o nepokretnoj tački za parcijalno uređene skupove i ona se primenjuje na rešavanje grafovskih nejednačina $L^n(G) \subseteq G$ i $L^n(G) \subseteq \overline{G}$.

8. *Graph equations*

   U radu je prikazano oko 100 grafovskih jednačina uključujući i neke nove rezultate. Predlaže se klaslfikacija grafovskih jednačina prema broju nepoznatih. Opisuju se metodi rešavanja grafovskih jednačina.

9. *Graph equations line and n-th power graphs I*

10. *Graphs which are switching equivalent to their line graphs*

    U radu se određuju svi grafovi koji su prekiclački ekvivalentni svojim grafovima grana. To su regularni grafovi stepena 2 i 10 izuzetnih grafova malog formata.

11. *On self pseudo-inverse graphs*

    Proučava se grafovska jednačina $P(G) = G$, gde je $P(G)$ graf definisan na istom skupu čvorova kao i G i u kome su čvorovi x i y susedni ako i samo ako $G-x-y$ ima 1-faktor. Dobijena su rešenja ove jednačine koja pripadaju skupu grafova čija je najkraća kontura duža od 4. Opisuju se neke primene dobijenih rezultata u hemiji.

12. *On the decomposition of the line (total) graphs with respect to some binary operations*

    It is proved that line and total graphs (and their complements too) are nearly always prime, i.e. excluding some exceptions, they could not be decomposed with respect to some binary operations (sum, product and strong product). In other words, the corresponding graph equations have not too many solutions.

13. *A bibliography on graph equations*

    Rad je napisan po pozivu prof. F. Harary-a, glavnog urednika časopisa J. Graph Theory. Daje se klasifikacija grafovskih jednačina, opisuju metodi njihovog rešavanja i navodi potpuni spisak literature o grafovskim jednačinama.

14. *Some results on generalized line graphs*

    Bez dokaza se navode rezultati iz rada [16] o generalisanim grafovima grana.

15. *Graphs which are switching equivalent to their complementary line graphs I*

    In this paper we will find all connected graphs which are switching equivalent to their complementary line graphs. The notion of switching equivalency is taken here in Seidel's sense, while for some facilities, we also introduce some conventions from an earlier paper.

16. *Generalized line graphs*

    Za generalisane grafove grana dokazuju se analogoni poznatih teorema koje važe za grafove grana: teorema o izomorfizmu dva generalisana grafa grana, karakterizacija generalisanih grafova grana pomoću kolekcije od trideset jednog zabranjenog podgrafa, teorema o grupi automorfizama generalisanog grafa grana i dr.

17. *Graphs having planar complementary line (total) graphs*

    The planarity of graphs (see [1] for basic definitions and notation) obtained by some graph operations (or graph valued functions) has bean considered in many papers by different authors. Here we shall mention only a few results. In [2] J. Sedlaček settled, historically the most famous result, which characterizes the graphs having planar line graphs. The same problem was treated by D. L. Greenwell and R. L. Hemminger [3] but with forbidden subgraphs involved. As far as the total graphs are concerned, the analogous problems were solved by M. Behzad [4] and J. Akiyama [5]. The purpose of this paper is to consider the planarity of the complements of line (total) graphs. The main feature of the latter problem is that planarity occurs now in the very restricted cases.

18. *Discussing graph theory with a computer I, Implementation of graph theoretic algorithms*

    Ovo je prvi iz serije radova "Discussing graph theorv with a computer" u kojima se opisuje implementacija programskog sistema "Graph". Ovde je opisan deo sistema koji sadrži grafovske algoritme. Opisana je interna reprezentacija grafa, klasifikacija grafovskih algoritama, interaktivna grafika, komuniciranje sa korisnikom i dr.

19. *Graphs which are switching equivalent to their complementary line graphs II*

    All connected graphs which are switching equivalent to their complementary line graphs have been found in [15]. Here, we will find the corresponding disconnected graphs.

20. *Graph equations for line graphs and n-th distance graphs*

21. *A note on generalized line graphs*

    In this paper we will find all graphs G such that G and its complement (denoted by $\overline{G}$) are generalized line graphs. We consider only finite undirected graphs without loops or multiple lines. The theorem we are going to prove as a generalization of a result of L. W. Beineke, who found all graphs G such that G and $\overline{G}$ are line graphs. In a series of papers F. Harary et al. considered problems about graphs and their complements sharing a given property; our problem fits in their investigation.

22. *There are just thirteen connected nonregular nonbipartite integral graphs having maximum vertex degree four (shortend report)*

23. *On the largest eigenvalue of some homeomorphic graphs*

    Two particular classes of mutually homeomorphic graphs are considered. For any two graphs of the same class, the relationship between the structure and the largest eigenvalue is discussed. Some relevant applications are outlined.

24. *An algoritam for radar signal filtering (in Serbian)*

25. *Combinatorial algorithm for data classification (in Serbian)*

26. *On some computer-aided investigations in graph theory*

27. *On the largest eigenvalue of unicyclic graphs*

    We first establish some relations between the graph structure and its largest eigenvalue. Applying these results to unicyclic graphs (with a fixed number of points), we explain some facts about the λ_1-ordering of these graphs. Most of these facts were suggested by the experiments conducted on the expert system "GRAPH", which has been developed and implemented at the Faculty of Electrical Engineering, University of Belgrade.

28. *Computer aided search for all graphs such that both graph and its complement have its spectrum bounded from below by* -2

    Let $\bar{G}$ denote the complement of G. We determine graphs described in the title splitting the procedure into two parts: the case when both G and $\bar{G}$ are generalized line graphs (the result of [13]) and the situation when at least one of them is not a generalized line graph (solved by the aid of a computer and described in this paper). The result is presented by means of maximal graphs having the requested property.

29. *Some results on the largest eigenvalue of a graph*

    The relation between the eigenvalues of a graph and its structure can be most easily visualised by ordering the graphs according to some spectral invariants. The λ_1-ordering is the ordering of the graphs according to the largest eigenvalue. In this paper we summarize some of our results on the λ_1-ordering. Most of them were suggested by computer experiments, and then proved by other means.

30. *An algorithm for radar signal analysis (in Serbian)*

31. *A note on the graph equation* $C(L(G) = L(C(G))$

    We find all solutions to the graph equation from the title. The same equation was already treated in the literature, but solved only partially.

32. *On the largest eigenvalue of bicyclic graphs*

    Among bicyclic graphs (connected graphs with two independent cycles) we find those graphs whose largest eigenvalue (index, for short) is minimal.

33. *Some experiences in using a programming system in graph theoretical investigations (in Serbian)*

    U ovom radu opisuju se iskustva u korišćenju ekspertnog sistema "Graph" u istraživanjima na polju teorije grafova. U odeljku 1 daju se opšti podaci o ekspertnom sistemu "Graph". Odeljak 2 sadrži opis podsistema za procesiranje grafova (ALGOR). U 3. se daju neke opšte a u 4. neke specijalne karakteristike podsistema ALGOR značajne za istraživanja.

34. *Ten years of the development and usage of the expert system "Graph"*

    Novembra 1989. godine navršava se 10 godina odkako je na Elektrotehničkom fakultetu u Beogradu počeo razvoj interaktivnog ekspertnog sistema za teoriju grafova "Graph". Tim povodom u ovom članku se izlaže hronologija rada na sistemu, daju se podaci o saradnicima, o postignutim rezultatima, o objavljenim publikacijama, o održanim predavanjima i dr. Ovaj pregledni rad upotpunjuje podatke objavljene u ranijim radovima, o aktivnostima u vezi sistema "Graph". Takođe se izlažu neka iskustva iz rada na sistemu "Graph" koja su, možda, od opštijeg značaja na polju istraživanja veštačke inteligencije.

35. *An algorithm to recognize a generalized line graphs and output its root graph*

    We present an efficient algorithm of complexity $O(m)$ (m being the number of lines) to recognize a generalized line graph giving an output its root graph.

36. *A note on reconstructing the characteristic polynomial of a graph*

    It is well known that the characteristic polynomial of any graph is determined up to an additive constant from its polynomial deck, i.e. the collection of the characteristic polynomials of the point deleted subgraphs. Here we prove that these constants are equal for any two connected graphs with the same polynomial decks, whenever the spectra of all subgraphs are bounded from below by -2.

37. *TSP-SOLVER - A Programming package for the traveling salesman problem*

    We report on the implementation of a programming package, called TSP–SOLVER, for the travelling salesman problem (TSP). Various variants of TSP can be treated by TSP–SOLVER: both symmetric and asymmetric cases, one- or multiple-TSP, one or first k best solutions, bandwidth limited distance matrix and others special cases, algorithms and heuristics. The system is user-friendly and offers the user, among other things, some possibilities to intervene during the solving a problem.

38. *Best suboptimal solutions in combinatorial optimization problems*

    We propose a modification of the standard branch and bound procedure for combinatorial optimization problems which enables finding best suboptimal solutions. We provide more details by describing an implementation of the algorithm for the travelling salesman problem.

39. *Coplanar graphs*

    Planaran graf sa planarnim komplementom se naziva koplanaran graf. Svi koplanarni grafovi su određeni kombinovanjem matematičkog rezonovanja i kompjuterske pretrage. Pri tome je korišćen programski sistem "Graph". Postoji tačno 2976 koplanarnih grafova.

40. *Some remarks on the problem of multilateral compensation*

    In this note we first give an exact (polynomial time) algorithm for solving the problem of multilateral compensation. For the large scale instances we propose some heuristics for finding suboptimal solutions.

41. *A mathematical approach to solving the problem of multilateral compensation (algorithms and heuristics), (Serbian)*

42. *A study of eigenspaces of graphs*

    We investigate the relationship between the structure of a graph and its eigenspaces. The angles between the eigenspaces and the vectors of a standard basis of R^n play an important role. The key notion is that of a special basis for an eigenspace called a star basis. Star bases enable us to define a canonical basis of R^n associated with a graph, and to formulate an algorithm for graph isomorphism.

43. *Complementary pairs of graphs orientable to line digraphs*

    A graph is orientable to a line digraph (OLD, for short) if its lines can be oriented in such a way that the resulting digraph is the line digraph of some digraph. We find all graphs such that both the graph and its complement are OLD and also characterize these graphs in terms of minimal forbidden subgraphs. As shown, all of these graphs have at most nine points.

44. *Non-complete extended p-sum of graphs, graph angles and star partitions*

    The NEPS (Non-complete Extended p-Sum) of graphs is a graph operation in which the vertex set of the resulting graph is the Cartesian product of the vertex sets of starting graphs. The paper contains a survey on NEPS and some new results concerning graph angles and star partitions of NEPS.

45. *Graph theoretic results obtained by the support of the expert system "GRAPH"*

    An interactive programming package, called GRAPH, an expert system for graph theory, was developed at the University of Belgrade, Faculty of Electrical Engineering during the period 1980-1984. GRAPH was designed to support research in graph theory, among other things, by helping to pose, verify and disprove conjectures. We report here on graph theory results, obtained by several researches in period 1982-1992. which have been obtained with the support of the expert system GRAPH. Most of the results belong to the theory of graph spectra.

46. *On a non-standard network flow problem*

47. *Some remarks on graph equation* $G^2 = \overline{G}$

    We investigate solutions to the graph equation from the title, or more precisely, we investigate those graphs whose square and complement are equal (i.e. isomorphic). Although this equation looks to be simple, it happens to be very hard (when attempting to find a general solution). Here we only offer some interesting observations and/or solutions.

48. *On graphs whose second largest eigenvalue does not exceed* $(\sqrt{5}-1)/2$

    It is well known in the theory of graph spectra that connected graphs except for complete multipartite (including complete) graphs have the second largest eigenvalue greater than 0. Graphs whose second largest eigenvalue does not exceed 1/3 are characterized in Cao and Yuan (1993). In this paper we study the structure of graphs whose second largest eigenvalue does not exceed $(\sqrt{5}-1)/2$.

49. *The nonregular, nonbipartite integral graphs with the maximum degree four*

    An integral graph is a graph whose spectrum is integral. By this paper we start finding all integral graphs with the maximum vertex degree four. Particularly, we find those of them which are nonregular and nonbipartite.

50. *On some algorithmic investigations of star partitions of graphs*

    Star partitions of graphs where introduced in a recent paper by the same authors in order to extend spectral methods in algebraic graph theory. Here it is shown that the corresponding partitioning problem is polynomial. Two algorithms are investigated: the first is based on maximum matching problems for graphs, and the second invokes an algorithm for matroid intersection.

51. *Some notes on graphs whose second largest eigenvalue is less than* $(\sqrt{5}-1)/2$

    It is well known in spectral graph theory that all (connected) graphs except complete graphs and complete multi-partite graphs have second largest eigenvalue greater than 0. Graphs whose second larges eigenvalue does not exceed $1/3$ are characterized in [2]. Some characterizations of graphs whose second largest eigenvalue does not exceed $(\sqrt{5}-1)/2$ are given in [9]. In this paper we prove that graphs whose second largest eigenvalue is less than $(\sqrt{5}-1)/2$ can be characterized by a finite collection of forbidden (induced) subgraphs.

52. *On the index of broken wheels*

    The index of a graph G is the largest eigenvalue of a $(0,1)-$adjacency matrix of G. Let $\mathcal{W}(n,k)$ denote the set of all graphs which can be obtained from an n-cycle by joining and additional ('central') vertex to k of the vertices of the cycle (Such graphs are called broken wheels.) By using a result of Schwenk's to compare the characteristic polynomials of graphs in $\mathcal{W}(n,k)$. we identify the graphs with greatest and least index. We show in fact that the index is greatest when the k 'spokes' are bunched together as closely as possible, and is least when they are spread out as evenly as possible.

53. *Complementary pairs of graphs with second largest eigenvalue not exceeding* $(\sqrt{5}-1)/2$

    We characterize (in terms of minimal forbidden subgraphs) graphs having the following property: both the graph and its complement have the second largest eigenvalue not exceeding $(\sqrt{5}-1)/2$, i.e. the golden section. This characterization also enables us to find explicitely all graphs in question.

54. *The second largest eigenvalue of a graph - A survey*

This is a survey paper on the second largest eigenvalue λ_2 of the adjacency matrix of a graph. Among the topics presented are the graphs with small λ_2, bounds for λ_2, algebraic connectivity, graphs with good expanding properties (such as Ramanujan graphs), rapidly mixing Markov chains etc. Applications to computer science are mentioned. Recent results of the authors are included.

55. *Which bicyclic graphs are reflexive ?*

A bicyclic graph is a connected graph with n vertices and $n+1$ edges. A graph is reflexive if the second largest eigenvalue (of its adjacency matrix) does not exceed 2. In this paper we investigate those bicyclic graphs with (vertex) disjoint cycles which are reflexive.

56. *A note on the second largest eigenvalue of star-like trees*

Star-like trees axe trees homeomorphic to stars. In this paper we identify those star-like trees for which the second largest eigenvalue is extremal - either minimal or maximal - when certain conditions are imposed. We also obtain partial results on the way in which the second largest eigenvalue of a simple class of star-like trees changes under local modifications (graph perturbations). Analogous problems for the largest eigenvalue (known as the index of the graph) have been widely studied in the literature.

57. 4-*regular integral graphs*

Possible spectra of 4-regular integral graphs are determined. Some constructions and a list of 65 known connected 4-regular integral graphs are given.

58. *A database of star complements of graphs*

Let μ be an eigenvalue of the graph G with multiplicity k. A star complement for μ in G is an induced subgraph $H = G - X$ such that $|X| = k$ and μ is not an eigenvalue of $G - X$. The database contains about 1500 triples (G; H; μ) and is available as a supplement to the programming package "Graph". It was produced using (a) "Graph" itself, (b) programs developed independently by M. Lepović, and (c) data from other sources cited in the bibliography. This paper contains a description of the database and a commentary which explains how some interesting graphs can be obtained by extending appropriate star complements.

59. *There are exactly* 150 *connected integral graphs up to* 10 *vertices*

A graph is called integral if its spectrum consists entirely of integers. Using existing graph catalogues we established that there are exactly 150 connected integral graphs up to 10 vertices. Adjacency matrices and/or pictures, and spectra of these graphs together with some comments are given in this paper.

60. *Some characterizations of graphs by star complements*

Let μ be an eigenvalue of the graph G with multiplicity k. A star complement for μ in G is an induced subgraph $H = G - X$ such that $|X| = k$ and μ is not an eigenvalue of $G - X$. Various graphs related to (generalized) line graphs, or their complements are characterized by star complements corresponding to eigenvalues -2 or 1.

61. *Some graphs with extremal Szeged index*

Szeged index of a graph is a graph invariant which "measures" some distance properties of graphs (which are significant in mathematical chemistry). In this paper we identify, among bicyclic and tricyclic graphs, those graphs whose Szeged index is extremal (minimal and maximal).

62. *Some additions to the theory of star partitions*

This paper contains a number of results in the theory of star partitions of graphs. We illustrate a variety of situations which can arise when the Reconstruction Theorem for graphs is used, considering in particular *galaxy* graphs - these are graphs in which every star set is independent. We discuss a recursive ordering of graphs based on the Reconstruction Theorem, and point out the significance of galaxy graphs in this connection.

63. *On generating all integral graphs on* 11 *vertices*

A graph is integral if the spectrum of its adjacency matrix is integral. An evolutionary algorithm for generating integral graphs is described. All connected integral graphs up to 11 vertices are produced. The distribution of some their invariants (maximum and minimum degrees, size, density, the chromatic number, the number of automorphisms, and spectra) are included. New infinite families of bipartite integral graphs are found.

64. *Minimal graphs whose second largest eigenvalue is not less than* $(\sqrt{5}-1)/2$

As is well known in spectral graph theory, all (connected) graphs except complete graphs and complete multi-partite graphs have the second largest eigenvalue greater than 0. Graphs whose second largest eigenvalue does not exceed 1/3 were recently characterized by D. Cao and Y. Hong. In the same paper they posed the problem of characterizing those graphs whose second largest eigenvalue is less than $(\sqrt{5}-1)/2$ (golden ratio). As recently proved by the second author, these graphs can be characterized by a finite collection of forbidden (induced) subgraphs. The aim of the present paper is to describe the above collection within more details: partly by an explicit list while the rest is characterized in an efficient way.

65. *Constructions of the maximal exceptional graphs with largest degree less than* 28

    A graph is said to be exceptional if it is connected, has least eigenvalue greater than or equal to -2, and is not a generalized line graph. Such graphs are known to be representable in the exceptional root system E_8. The 473 maximal exceptional graphs have been found by computer, and the 467 with maximal degree 28 have been characterized. Here we construct the remaining six maximal exceptional graphs by means of representations in E_8.

66. *Variable neighbourhood search for extremal graphs 3. On the largest eigenvalue of color-constrained trees*

    Preprint of the paper [70].

67. *The nonregular, bipartite, integral graphs with maximum degree four - Part I: basic properties*

    A graph is integral if the spectrum (of its adjacency matrix) consists entirely of integers. In this paper, we begin the search of those integral graphs which are nonregular, bipartite and have maximum degree 4. Here, we investigate the structure of these graphs, and provide many properties which facilitate a computer search. Among others, we have shown that any graph in question has not more than 78 vertices.

68. *Some remarks on integral graphs with maximum degree four*

    An integral graph is a graph whose spectrum (of its adjacency matrix) consists entirely of integers. Here we prove some results on bipartite, nonregular integral graphs with maximum degree four. In particular, trees, unicyclic graphs and graphs with some numbers excluded from their spectrum are considered.

69. *Graphs with least eigenvalue -2: The star complement technique*

Let G be a connected graph with least eigenvalue -2 of multiplicity k. A star complement for -2 in G is an induced subgraph $H = G - X$ such that $|X| = k$ and -2 is not an eigenvalue of H. In the case that G is a generalized line graph, a characterization of such subgraphs is used to describe the eigenspace of -2. In some instances, G itself can be characterized by a star complement. If G is not a generalized line graph, G is an exceptional graph, and in this case it is shown how a star complement can be used to construct G without recourse to root systems.

70. *On the largest eigenvalue of color-constrained trees*

In the set of bicolored trees with given numbers of black and of white vertices we describe those for which the largest eigenvalue is extremal (maximal or minimal). The results are first obtained by the automated system AutoGraphiX, developed in GERAD (Montreal), and verified afterwards by theoretical means.

71. *The maximal exceptional graphs with maximal degree less than* 28

A graph is said to be *exceptional* if it is connected, has least eigenvalue greater than or equal to -2, and is not a generalized line graph. Such graphs are known to be representable in the root system E_8. The 473 maximal exceptional graphs were found initially by computer, and the 467 with maximal degree 28 have subsequently been characterized. Here we use constructions in E_8 to prove directly that there are just six maximal exceptional graphs with maximal degree less than 28.

72. *Computer investigations of the maximal exceptional graphs*

A graph is said to be exceptional if it is connected, has least eigenvalue greater or equal to -2, and is not a generalised line graph. Such graphs are known to be representable in the exceptional root system E_8. The 473 maximal exceptional graphs have been found by computer using the star complement technique, and subsequently described using properties od E_8. Here we present some information about these graphs obtained in the computer search: the exceptional star complement, some data on extendability graphs and the corresponding maximal graphs, the maximal exceptional graphs and some of their invariants.

73. *On generating all integral graphs on* 12 *vertices*

A graph is integral if the spectrum of its adjacency matrix is integral. All connected integral graphs on 12 vertices (325) have been generated. The distribution of some of their invariants (maximum and minimum degrees, size, the chromatic number, the number of automorphisms, and spectrum) are included.

74. *The maximal exceptional graphs*

A graph is said to be exceptional if it is connected, has least eigenvalue greater than or equal to -2, and is not a generalized line graph. Such graphs are known to be representable in the exceptional root system E_8. We determine the maximal exceptional graphs by a computer search using the star complement technique, and then show how they can be found by theoretical considerations using a representation of E_8 in R^8. There are exactly 473 maximal exceptional graphs.

75. *A survey on integral graphs*

A graph whose spectrum consists entirely of integers is called an integral graph. We present a survey of results on integral graphs and on the corresponding proof techniques.

76. *Arbitrarily large graphs whose second largest eigenvalue is less than* $(\sqrt{5}-1)/2$

The collection of graphs with the second largest eigenvalue less than the golden section (i.e. $(\sqrt{5}-1)/2$) gives rise to many open questions. The aim of the present paper is to describe (to some extent) those families of graphs from this collection which have an arbitrarily large (i.e. unbounded) number of vertices.

77. *On generating all integral graphs on* 13 *vertices*

A graph is integral if the spectrum of its adjacency matrix is integral. Connected integral graphs on 13 vertices (526) have been generated. The distribution of some of their invariants (maximum and minimum degrees, size, the chromatic number, the number of automorphisms, and spectrum) are included.

78. *A graph and its complement with specified spectral properties*

This article gives a survey of those results in the theory of graph spectra which can be described as characterizations of sets of all graphs G such that both the graph G and its complement have a prescribed property.

79. *On finite and infinite sets of integral graphs*

A graph is integral if the spectrum of its adjacency matrix is integral. An evolutionary algorithm for generating integral graphs of a given order is described. Sets of connected integral graphs up to 12 vertices have been generated combining running this algorithm and testing results with an exact procedure. Sets of cospectral integral graphs and sets of complements of connected integral graphs are given. Some infinite sets of integral graphs are studied.

80. *Two shorter proofs in spectral graph theory*

We give shorter proofs of two inequalities already known in spectral graph theory.

81. *Some new results on graphs with least eigenvalue not less than* -2

We consider some open questions about graphs with least eigenvalue -2. We investigate in particular the star complements for -2 of such graphs.

82. *On graphs whose star complement for* -2 *is a path or a cycle*

It was proved recently by one of the authors that, if H is a path P_t ($t > 2$ with $t \neq 7$ or 8) or an odd cycle C_t ($t > 3$), then there is a unique maximal graph having H as a star complement for -2. The methods employed were analytical in nature, making use of the Reconstruction Theorem for star complements. Here we offer an alternative approach, based on the forbidden subgraph technique. In addition, we resolve the exceptional situations arising when $H = P_7$ or P_8.

83. *Which nonregular bipartite integral graphs with maximum degree four do not have* ± 1 *as eigenvalues?*

A graph is integral if the spectrum (of its adjacency matrix) consists entirely of integers. In this paper we give a partial answer to the question posed in the title.

84. *Connected graphs of fixed order and size with maximal index: Structural considerations*

The largest eigenvalue, or index, of simple graphs is extensively studied in literature. Usually, the authors consider the graphs from some fixed class and identify within it those graphs with maximal (or minimal) index. So far maximal graphs with fixed order, or with fixed size, are identified, but not maximal connected graphs with fixed order and size. In this paper we add some new observations related to the structure of the latter graphs.

85. *Graph theoretical results obtained by the support of the expert system GRAPH - An extended survey*

    An interactive programming package called GRAPH, an expert system for graph theory, was developed at the University of Belgrade, Faculty of Electrical Engineering, during the period 1980-1984. GRAPH was designed to support research in graph theory, among other things, by helping to pose, verify and disprove conjectures. We report here on graph theory results, obtained by several researchers in the period 1982-2001, which have been obtained with the support of the expert system GRAPH. In this way we extend a previous survey [45]. Most of the results belong to the theory of graph spectra.

86. *Graphs with least eigenvalue* -2*; A new proof of* 31 *forbidden subgraphs theorem*

    Generalized line graphs were introduced by Hoffman, Proc. Calgary Internat. Conf. on Combinatorial Structures and their applications, Gordon and Breach, New York (1970); they were characterized in 1980 by a collection of 31 forbidden induced subgraphs, obtained independently by Cvetković et al., Comptes Rendus Math. Rep. Acad. Sci. Canada (1980) and S. B. Rao et al., Proc. Second Symp., Indian Statistical Institute, Calcutta, Lecture Notes in Math., (1981). Here a short new proof of this characterization theorem is given, based on an edge-colouring technique.

87. *Analysis of combinatorial networks with Mathematica*

    This paper focuses on automated computer-aided symbolic analysis of combinational networks. A novel program is presented to carry out the symbolic analysis, and to derive closed-form formulas for the response of combinational networks, for excitations specified by symbols or symbolic expressions. The source code listing of the program is presented. The program operation is fully illustrated by an example. The symbolic analysis addressed in this work can serve as a basis of efficient programs for variety of logic design tasks, including logic simulation, fault simulation, test generation, and symbolic verification. The advantages of the proposed approach are discussed. Scientists, researchers, designers, educators and students dealing with combinational networks can benefit from the symbolic approach considered in this paper.

88. *The index of trees with specified maximum degree*

Let $\mathcal{T}(n, \triangle)$ be the set of all trees on n vertices with a given maximum degree $\triangle$. In this paper we identify in $\mathcal{T}(n, \triangle)$ the tree whose index, i.e. the largest eigenvalue of the adjacency matrix, has the maximum value.

89. *There are* 93 *non-regular, bipartite integral graphs with maximum degree four*

A graph is called integral if the spectrum of its adjacency matrix consists entirely of integers. A class of non-regular bipartite integral graphs with maximum degree four has been characterized by generating all its elements. An exact algorithm for generating these graphs is described; tables containing some invariants and figures showing their structures are given.

90. *Some results on the index of unicyclic graphs*

We identify in some classes of unicyclic graphs (of fixed order and girth) those graphs whose index, i.e. the largest eigenvalue, is maximal. Besides, some (lower and upper) bounds on the indices of the graphs being considered are provided.

91. *Simultaneous editing and multilabelling of graphs in system newGRAPH*

In our research in spectral graph theory we often encounter the need for the simultaneous editing of two or more interdependent graphs (e.g. a graph and its line graph), together with multiple labellings of their vertices and edges. Ocassionally, labellings are of such kind that it could be beneficial to permit the user to modify the labelling and test whether it still satisfies a given property. Here we develop a methodology for treating such situations, which is implemented in system newGRAPH.

92. *More on singular line graphs of trees*

We study those trees whose line graphs are singular. Besides new proofs of some old results, we offer many new results including the computer search which covers the trees with at most twenty vertices.

93. *On graphs with unicyclic star complement for* 1 *as the second largest eigenvalue*

The star complement technique is a spectral tool recently developed for constructing some bigger graphs from their smaller parts, called star complements. Here we first identify among unicyclic graphs those graphs which can be star complements for 1 as the second largest eigenvalue. Using the graphs obtained, we next search for their maximal extensions, either by theoretical means, or by computer aided search.

94. *On generating* 4*-regular integral graphs*

A graph is called integral if all eigenvalues of its adjacency matrix are integers. Exact and randomized algorithms for generating 4-regular graphs and sieving integral graphs are defined. All connected 4-regular integral graphs of order $12 < n < 20$ generated by a computer search are presented and some of their properties are indicated. A new algorithm for constructing 4-regular integral graphs of order $n \geq 20$ based on the experimental results is proposed.

95. *On the analytical solution of some families of transcendental equations*

The problem of finding the exact analytical closed-form solution of some families of transcendental equations is studied, in some detail, by the Special Trans Function Theory (STFT). The mathematical genesis of the analytical closed-form solution is presented, and the structure of the theoretical derivation, proofs and numerical results confirm the validity and base principle of the STFT. Undoubtedly, the proposed analytical approach implies the qualitative improvement of the conventional analytical and numerical methods.

96. *Some notes on graphs whose index is close to* 2

We consider two classes of graphs: (i) trees of order n and diameter $d = n - 3$ and (ii) unicyclic graphs of order n and girth $g = n - 2$. Assuming that each graph within these classes has two vertices of degree 3 at distance k, we order by the index (i.e. spectral radius) the graphs from (i) for any fixed k $(1 \leq k \leq d - 2)$, and the graphs from (ii) independently of k.

97. *Path-like graphs ordered by the index*

A path-like graph is a tree on n vertices and diameter $n - 3$. Let $\alpha = \sqrt{2}\sqrt{(\frac{9-\sqrt{33}}{36})^{\frac{1}{3}} + (\frac{9+\sqrt{33}}{36})^{\frac{1}{3}} + 1}$ (note $\alpha \in (2, \sqrt{2}\sqrt{1+\sqrt{2}})$). In this paper we first prove that all path-like graphs have the index in the interval $[2, \sqrt{2}\sqrt{1+\sqrt{2}})$. Next we totally order (with respect to the index) all path-like graphs which have the index in the interval $(\alpha, \sqrt{2}\sqrt{1+\sqrt{2}})$.

98. *The polynomial reconstruction of unicyclic graphs is unique*

We consider the problem of reconstructing the characteristic polynomial of a graph G from its polynomial deck, i.e. the collection $\mathcal{P}(G)$ of characteristic polynomials of its vertex–deleted subgraphs. Here we provide a positive solution for all unicyclic graphs.

99. *Star complements and exceptional graphs*

Let G be a finite graph of order n with an eigenvalue μ of multiplicity k. (Thus the μ-eigenspace of a $(0,1)$-adjacency matrix of G has dimension k.) A star complement for μ in G is an induced subgraph $G - X$ of G such that $|X| = k$ and $G - X$ does not have μ as an eigenvalue. An exceptional graph is a connected graph, other than a generalized line graph, whose eigenvalues lie in $[-2, +\infty)$. We establish some properties of star complements, and of eigenvectors, of exceptional graphs with least eigenvalue -2.

100. *Signless Laplacian of finite graphs*

We survey properties of spectra of signless Laplacians of graphs and discuss possibilities for developing a spectral theory of graphs based on this matrix. For regular graphs the whole existing theory of spectra of the adjacency matrix and of the Laplacian matrix transfers directly to the signless Laplacian, and so we consider arbitrary graphs with special emphasis on the non-regular case. The results which we survey (old and new) are of two types: (a) results obtained by applying to the signless Laplacian the same reasoning as for corresponding results concerning the adjacency matrix, (b) results obtained indirectly via line graphs. Among other things, we present eigenvalue bounds for several graph invariants, an interpretation of the coefficients of the characteristic polynomial, a theorem on powers of the signless Laplacian and some remarks on star complements.

101. *Indices of trees with a prescribed diameter*

The index of a graph is is the largest eigenvalue of its adjacency matrix. Let $T_{n,d}$ be the class of trees with n vertices and diameter d. For all integers n and d with $4 \leq d \leq n-3$ we identify in $T_{n,d}$ the tree with the $K-$th largest index for all k up to $\left[\frac{d}{2}\right] + 1$ if $d \leq n-4$, or for all k up to $\left[\frac{d}{2}\right]$ if $d = n-3$.

102. *On the index of caterpillars*

The index of a graph is the largest eigenvalue of its adjacency matrix. Among the trees with a fixed order and diameter, a graph with the maximal index is a caterpillar. In the set of caterpillars with a fixed order and diameter, or with a fixed degree sequence, we identify those whose index is maximal.

103. *Ordering graphs with index in the interval* $(2, \sqrt{2+\sqrt{5}})$

The index of a graph is the largest eigenvalue (or spectral radius) of its adjacency matrix. We consider the problem of ordering graphs by the index in the class of connected graphs with a fixed order n and index belonging to the interval $(2, \sqrt{2+\sqrt{5}})$. For any fixed n (provided that n is not too small), we order a significant portion of graphs whose indices are close to the end points of the above interval.

104. *Q–integral graphs with edge-degrees at most five*

We consider the problem of determining the Q-integral graphs, i.e. the graphs with integral signless Laplacian spectrum. We find all such graphs with maximum edge–degree 4, and obtain only partial results for the next natural case, with maximum edge–degree 5.

105. *Variable neighborhood search for extremal graphs, 16. Some conjectures related to the largest eigenvalue of a graph*

We consider four conjectures related to the largest eigenvalue of (the adjacency matrix of) a graph (i.e., to the index of the graph). Three of them have been formulated after some experiments with the programming system AutoGraphiX, designed for finding extremal graphs with respect to given properties by the use of variable neighborhood search. The conjectures are related to the maximal value of the irregularity and spectral spread in n-vertex graphs, to a Nordhaus-Gaddum type upper bound for the index, and to the maximal value of the index for graphs with given numbers of vertices and edges. None of the conjectures has been resolved so far. We present partial results and provide some indications that the conjectures are very hard.

106. *Some notes on spectra of cographs*

A cograph is a P_4–free graph. We first give a short proof of the fact that 0 (-1) belongs to the spectrum of a connected cograph (with at least two vertices) if and only if it contains duplicate (resp. codupli-cate) vertices. As a consequence, we next prove that the polynomial reconstruction of graphs whose vertex–deleted subgraphs have the second largest eigenvalue not exceeding $\frac{\sqrt{5}-1}{2}$ is unique.

107. *On the polynomial reconstruction of graphs whose vertex-deleted subgraphs have spectra bounded from below by* -2

We consider the problem of reconstructing the characteristic polynomial of a graph from its polynomial deck, i.e., the collection of characteristic polynomials of its vertex-deleted subgraphs. Here we provide a positive answer to this problem for graphs as in the title, provided they are disconnected. Since the same problem for connected graph was already answered in positive, we have arrived at the positive answer for the entire collection of graphs under considerations.

108. *Trees with minimal index and diameter at most four*

In this paper we consider the trees with fixed order n and diameter $d \leq 4$. Among these trees we identify those trees whose index is minimal.

109. *Graphs for which the least eigenvalue is minimal, I*

Let G be a connected graph whose least eigenvalue $\lambda(G)$ is minimal among the connected graphs of prescribed order and size. We show first that either G is complete or $\lambda(G)$ is a simple eigenvalue. In the latter case, the sign pattern of a corresponding eigenvector determines a partition of the vertex set, and we study the structure of G in terms of this partition. We find that G is either bipartite or the join of two graphs of a simple form.

110. *Eigenvalue bounds for the signless Laplacian*

We extend our previous survey of properties of spectra of signless Laplacians of graphs. Some new bounds for eigenvalues are given, and the main result concerns the graphs whose largest eigenvalue is maximal among the graphs with fixed numbers of vertices and edges. The results are presented in the context of a number of computer-generated conjectures.

111. *Experiences and Achievements*

112. *A sharp lower bound for the least eigenvalue of the signless Laplacian of a non-bipartite graph*

We prove that the minimum value of the least eigenvalue of the signless Laplacian of a connected nonbipartite graph with a prescribed number of vertices is attained solely in the unicyclic graph obtained from a triangle by attaching a path at one of its endvertices.

113. *Graphs for which the least eigenvalue is minimal, II*

We continue our investigation of graphs G for which the least eigenvalue $\lambda(G)$ is minimal among the connected graphs of prescribed order and size.We provide structural details of the bipartite graphs that arise, and study the behaviour of $\lambda(G)$ as the size increases while the order remains constant. The non-bipartite graphs that arise were investigated in a previous paper [F.K. Bell, D. Cvetković, P. Rowlinson, S.K. Simić, Graphs for which the least eigenvalue is minimal, I, Linear Algebra Appl. (2008), doi:10.1016/j.laa.2008.02.032]; here we distinguish the cases of bipartite and non-bipartite graphs in terms of size.

114. *Bidegreed trees with a small index*

Let T_n^Δ be the class of trees on n vertices whose all vertices, other than pendant ones, are of degree Δ (bidegreed trees). In this paper we consider some problems related to the index (largest eigenvalue) of bidegreed trees focusing our attention on those trees with small index. In particular, we identify those trees from T_n^Δ whose index is minimal.

115. *On some forests determined by their Laplacian or signless Laplacian spectrum*

We consider the class of graphs whose each component is either a proper subgraph of some of Smith graphs, or belongs to a precized subset of Smith graphs. We classify the graphs from the considered class into those which are determined, or not determined, by Laplacian, or signless Laplacian spectrum.

116. *Towards a spectral theory of graphs based on the signless Laplacian*

A spectral graph theory is a theory in which graphs are studied by means of eigenvalues of a matrix M which is in a prescribed way defined for any graph. This theory is called M-theory. We outline a spectral theory of graphs based on the signless Laplacians Q and compare it with other spectral theories, in particular with those based on the adjacency matrix A and the Laplacian L. The Q-theory can be composed using various connections to other theories: equivalency with A-theory and L-theory for regular graphs, or with L-theory for bipartite graphs, general analogies with A-theory and analogies with

A-theory via line graphs and subdivision graphs. We present results on graph operations, inequalities for eigenvalues and reconstruction problems.

117. *Schwenk-like formulas for weighted digraphs*

Recently the study of the spectrum of weighted (di)graphs has attracted the interest of many researchers. Here we express the characteristic polynomial of any (square) matrix A in terms of the determinant of the Coates graph of the matrix $B = xI - A$. By doing so we are able to generalize the well-known Schwenk's formulas for simple graphs to weighted digraphs.

118. *Some properties of integral graphs on* 13 *vertices*

A graph is integral if the spectrum of its adjacency matrix is integral. The class of connected integral graphs on 13 vertices have been studied. Almost all of these graphs has been generated using an exact and an evolutionary algorithms. This report is an extension of data collected in the previous report [17] showing 21 new graphs so that the total number of these graphs is now 547. Additionally, for two size intervals (12 to 27 edges and 51 to 78 edges) the exact number of these graphs has been found. For the intermediate size interval (28 to 50 edges) the number of these graphs is not exact and it is possible, that some new graphs can be found in the future research. In this report we update distribution of some invariants, first of all spectrum, size, and degree sequences (and moreover the number of automorphisms and the chromatic number) and add data on some other properties (diameter, graph bandwidth and the order of the largest induced integral subgraph). A partition of this class of graphs according to the number of distinct elements in the degree sequence and two of these subclasses, regular and biregular graphs, are described.

119. *Towards a spectral theory of graphs based on the signless Laplacian, II*

A spectral graph theory is a theory in which graphs are studied by means of eigenvalues of a matrix M which is in a prescribed way defined for any graph. This theory is called M-theory. We outline a spectral theory of graphs based on the signless Laplacians Q and compare it with other spectral theories, in particular to those based on the adjacency matrix A and the Laplacian L. As demonstrated in the first part, the Q-theory can be constructed in part using various connections to other theories: equivalency with A-theory and L-theory for regular graphs, common features with L-theory for bipartite graphs, general analogies with A-theory and analogies with A-theory via line graphs and subdivision graphs. In this part, we introduce notions of enriched and restricted spectral theories and present results on integral graphs, enumeration of spanning trees, characterizations by eigenvalues, cospectral graphs and graph angles.

120. *On the spectral radius of unicyclic graphs with prescribed degree sequence*

We consider the set of unicyclic graphs with prescribed degree sequence. In this set we determine the (unique) graph with the largest spectral radius (or index) with respect to the adjacency matrix. In addition, we give a conjecture about the (unique) graph with the largest index in the set of connected graphs with prescribed degree sequence.

121. *Connected graphs of fixed order and size with maximal index: Some spectral bounds*

The index (or spectral radius) of a simple graph is the largest eigenvalue of its adjacency matrix. For connected graphs of fixed order and size the graphs with maximal index are not yet identified (in the general case). It is known (for a long time) that these graphs are nested split graphs (or threshold graphs). In this paper we use the eigenvector techniques for getting some new (lower and upper) bounds on the index of nested split graphs. Besides we give some computational results in order to compare these bounds.

122. *On the index of necklaces*

We consider the following two classes of simple graphs: open necklaces and closed necklaces, consisting of a finite number of cliques of fixed orders arranged in path-like pattern and cycle-like pattern, respectively. In these two classes we determine those graphs whose index (the largest eigenvalue of the adjacency matrix) is maximal.

123. *On Q-integral $(3, s)$-semiregular bipartite graphs*

A graph is called Q-integral if its signless Laplacian spectrum consists entirely of integers. We establish some general results regarding signless Laplacians of semiregular bipartite graphs. Especially, we consider those semiregular bipartite graphs with integral signless Laplacian spectrum. In some particular cases we determine the possible Q-spectra and consider the corresponding graphs.

124. *Towards a spectral theory of graphs based on the signless Laplacian, III*

This part of our work further extends our project of building a new spectral theory of graphs (based on the signless Laplacian) by some results on graph angles, by several comments and by a short survey of recent results.

125. *Spectral determination of graphs whose components are paths and cycles*

We consider the class of graphs whose each component is either a path or a cycle. We classify the graphs from the considered class into those which are determined, or not determined, by the adjacency spectrum. In addition, we compare this result with the corresponding results for the Laplacian and signless Laplacian spectrum. It turns out that the signless Laplacian spectrum performs the best, confirming some expectations from the literature.

126. *Some notes on threshold graphs*

In this paper we consider threshold graphs (also called nested split graphs) and investigate some invariants of these graphs which can be of interest in bounding the largest eigenvalue of some graph spectra.

127. *Parastrophically uncancellable quasigroup equations*

Krstić initiated the use of cubic graphs in solving quasigroup equations. Based on his work, Krapež and Živković proved that there is a bijective correspondence between classes of parastrophically equivalent parastrophically uncancellable generalized quadratic functional equations on quasigroups and three-connected cubic (multi)graphs. We use the list of such graphs given in the literature to verify existing results on equations with three, four and five variables and to prove new results for equations with six variables. We start with 14 nonisomorphic graphs with ten vertices, choose a set of 14 representative parastrophically nonequivalent equations and give their general solutions. A case of equations with seven and more variables is briefly discussed. The problem of Sokhats'kyi concerning a property which distinguishes visually two parastrophically nonequivalent equations with four variables is solved.

128. *Combinatorial approach for computing the characteristic polynomial of a matrix*

The study of the spectrum of weighted (di)graphs has attracted recently much attention in the literature. Here we use the Coates digraph as a main tool to extend, in a combinatorial way, some well known results from the spectral graph theory on computing the characteristic polynomial of graphs. New results are related to weighted (di)graph, and thus to any square matrix.

129. *On the spectral radius of quasi-k-cyclic graphs*

A connected graph $G = (V_G, E_G)$ is called a quasi-k-cyclic graph, if there exists a vertex $q \in V_G$ such that $G - q$ is a k-cyclic graph (connected with cyclomatic number k). In this paper we identify in the set of quasi-k-cyclic graphs (for $k \leq 3$) those graphs whose spectral radius of the adjacency matrix (and the signless Laplacian if $k \leq 2$) is the largest. In addition, for quasi-unicyclic graphs we identify as well those graphs whose spectral radius of the adjacency matrix is the second largest.

130. *Graphs for small multiprocessor interconnected networks*

Let D be the diameter of a graph G and let λ_1 be the largest eigenvalue of its $(0,1)$-adjacency matrix. We give a proof of the fact that there are exactly 69 non-trivial connected graphs with $(D+1)\lambda_1 \leq 9$. These 69 graphs all have up to 10 vertices and were recently found to be suitable models for small multiprocessor interconnection networks. We also examine the suitability of integral graphs to model multiprocessor interconnection networks, especially with respect to the load balancing problem. In addition, we classify integral graphs with small values of $(D+1)\lambda_1$ in connection with the load balancing problem for multiprocessor systems.

131. *On the spectral radius of cactuses with perfect matchings*

Let $C(2m, k)$ be the set of all cactuses on $2m$ vertices, k cycles, and with perfect matchings. In this paper, we identify in $C(2m, k)$ the unique graph with the largest spectral radius.

132. *Laplacian spectral characterization of disjoint union of paths and cycles*

The Laplacian spectrum of a graph consists of the eigenvalues (together with multiplicities) of the Laplacian matrix. In this article we determine, among the graphs consisting of disjoint unions of paths and cycles, those ones which are determined by the Laplacian spectrum.

For the graphs, which are not determined by the Laplacian spectrum, we give the corresponding cospectral non-isomorphic graphs.

133. *Computing the permanental polynomial of a matrix from a combinatorial viewpoint*

Recently, in the book [A Combinatorial Approach to Matrix Theory and Its Applications, CRC Press (2009)] the authors proposed a combinatorial approach to matrix theory by means of graph theory. In fact, if A is a square matrix over any field, then it is possible to associate to A a weighted digraph G_A, called Coates digraph. Through G_A (hence by graph theory) it is possible to express and prove results given for the matrix theory. In this paper we express the permanental polynomial of any matrix A in terms of permanental polynomials of some digraphs related to G_A.

134. *Graph spectra in Computer Science*

In this paper, we shall give a survey of applications of the theory of graph spectra to Computer Science. Eigenvalues and eigenvectors of several graph matrices appear in numerous papers on various subjects relevant to information and communication technologies. In particular,we survey applications in modeling and searching Internet, in computer vision, data mining, multiprocessor systems, statistical databases, and in several other areas. Some related new mathematical results are included together with several comments on perspectives for future research. In particular, we claim that balanced subdivisions of cubic graphs are good models for virus resistent computer networks and point out some advantages in using integral graphs as multiprocessor interconnection networks.

135. *On bounds for the index of double nested graphs*

The index of a simple graph is the largest eigenvalue of its adjacency matrix. It is well-known that in the set of all connected graphs with fixed order and size the graphs with maximal index are nested split graphs. It was recently observed that double nested graphs assume the same role if we restrict ourselves to bipartite graphs. In this paper we provide some bounds (lower and upper) for the index of double nested graphs. Some computational results are also included.

136. *Graphs whose (signless) Laplacian spectral radius does not exceed the Laplacian Hoffman limit value*

For a graph matrix M, the Hoffman limit value $H(M)$ is the limit (if it exists) of the largest eigenvalue (or, M-index, for short) of $M(H_n)$, where the graph H_n is obtained by attaching a pendant edge to the cycle C_{n-1} of length $n-1$. In spectral graph theory, M is usually either the adjacency matrix A or the Laplacian matrix L or the signless Laplacian matrix Q. The exact values of $H(A)$ and $H(L)$ were first determined by Hoffman and Guo, respectively. Since H_n is bipartite for odd n, we have $H(Q) = H(L)$. All graphs whose A-index is not greater than $H(A)$ were completely described in the literature. In the present paper, we determine all graphs whose Q-index does not exceed $H(Q)$. The results obtained are determinant to describe all graphs whose L-index is not greater then $H(L)$. This is done precisely in Wang et al. [21].

137. *On ordering bicyclic graphs with respect to the Laplacian spectral radius*

A connected graph of order n is bicyclic if it has $n+1$ edges. He et al. [C.X. He, J.Y. Shao, J.L. He, On the Laplacian spectral radii of bicyclic graphs, Discrete Math. 308 (2008) 5981–5995] determined, among the $n-$vertex bicyclic graphs, the first four largest Laplacian spectral radii together with the corresponding graphs (six in total). It turns that all these graphs have the spectral radius greater than $n-1$. In this paper, we first identify the remaining $n-$vertex bicyclic graphs (five in total) whose Laplacian spectral radius is greater than or equal to $n-1$. The complete ordering of all eleven graphs in question was obtained by determining the next four largest Laplacian spectral radii together with the corresponding graphs.

138. *Further results on the least eigenvalue of connected graphs*

In this paper, we identify within connected graphs of order n and size $n+k$ (with $0 \leq k \leq 4$ and $n \geq k+5$) the graphs whose least eigenvalue is minimal. It is also observed that the same graphs have the largest spectral spread if n is large enough.

139. *Connected graphs of fixed order and size with maximal Q-index: Some spectral bounds*

The Q-index of a simple graph G is the largest eigenvalue of the matrix Q, the signless Laplacian of G. It is well-known that in the set of connected graphs with fixed order and size, the graphs with maximal Q-index are the nested split graphs (also known as threshold graphs).

In this paper we focus our attention on the eigenvector techniques for getting some (lower and upper) bounds on the Q-index of nested split graphs. In addition, we give some computational results in order to compare these bounds.

140. *Some further bounds for the Q-index of nested split graphs*

The Q-index of a simple graph is the largest eigenvalue of its signless Laplacian, or Q-matrix. In our previous paper [the authors, Discrete Appl. Math. 160, No. 4–5, 448–459 (2012; Zbl 1239.05115)] we gave three lower and three upper bounds for the Q-index of nested split graphs, also known as threshold graphs. In this paper, we give another two upper bounds, which are expressed as solutions of cubic equations (in contrast to quadratics from [loc cit.]). Some computational results are also included.

141. *Spectral graph theory in computer science*

In this paper we shall give a survey of applications of the theory of graph spectra to computer science. Eigenvalues and eigenvectors of several graph matrices appear in numerous papers on various subjects relevant to information and communication technologies. In particular, we survey applications in modelling and searching Internet, in computer vision, data mining, multiprocessor systems, statistical databases, and in several other areas.

142. *Graph spectral techniques in computer sciences*

We give a survey of graph spectral techniques used in computer sciences. The survey consists of a description of particular topics from the theory of graph spectra independently of the areas of Computer Science in which they are used. We have described the applications of some important graph eigenvalues (spectral radius, algebraic connectivity, the least eigenvalue etc.), eigenvectors (principal eigenvector, Fiedler eigenvector and other), spectral reconstruction problems, spectra of random graphs, Hoffman polynomial, integral graphs etc. However, for each described spectral technique we indicate the fields in which it is used (e.g. in modelling and searching Internet, in computer vision, pattern recognition, data mining, multiprocessor systems, statistical databases, and in several other areas). We present some novel mathematical results (related to clustering and the Hoffman polynomial) as well.

143. *On eigenspaces of some compound graphs*

In the theory of (simple) graphs the concepts of the line and subdivision graph (as compound graphs) are well-known. It is possible to consider them also in the context of (edge) signed graphs. Some relations between the Laplacian spectrum of signed graphs and adjacency spectra of their associated compound (signed) graphs have been recently established in the literature. In this paper, we study the relations between the corresponding eigenspaces.

144. *Relations between (κ, τ)-regular sets and star complements*

Let G be a finite graph with an eigenvalue μ of multiplicity m. A set X of m vertices in G is called a star set for μ in G if μ is not an eigenvalue of the star complement $G \setminus X$ which is the subgraph of G induced by vertices not in X. A vertex subset of a graph is (κ, τ)-regular if it induces a κ-regular subgraph and every vertex not in the subset has τ neighbors in it. We investigate the graphs having a (κ, τ)-regular set which induces a star complement for some eigenvalue. A survey of known results is provided and new properties for these graphs are deduced. Several particular graphs where these properties stand out are presented as examples.

145. *More on non-regular bipartite graphs with maximum degree four not having ± 1 as eigenvalues*

A graph is integral if the spectrum (of its adjacency matrix) consists entirely of integers. The problem of determining all non-regular bipartite integral graphs with maximum degree four which do not have ± 1 as eigenvalues was posed in K.T. Balińska, S.K. Simić, K.T. Zwierzyński: Which nonregular bipartite integral graphs with maximum degree four do not have ± 1 as eigenvalues? Discrete Math., 286 (2004), 15-25. Here we revisit this problem, and provide its complete solution using mostly the theoretical arguments.

146. *A recursive construction of regular exceptional graphs with least eigenvalue -2*

In spectral graph theory a graph with least eigenvalue -2 is exceptional if it is connected, has least eigenvalue greater than or equal to -2, and it is not a generalized line graph. A (κ, τ)-regular set S of a graph is a vertex subset, inducing a κ-regular subgraph such that every vertex not in S has τ neighbors in S. We present a recursive construction of all regular exceptional graphs as successive extensions by regular sets.

147. *Some new considerations on double nested graphs*

In the set of all connected graphs with fixed order and size, the graphs with maximal index are nested split graphs, also called threshold graphs. It was recently (and independently) observed in Bell et al. (2008) and Bhattacharya et al. (2008) that double nested graphs, also called bipartite chain graphs, play the same role within class of bipartite graphs. In this paper we study some structural and spectral features of double nested graphs. In studying the spectrum of double nested graphs we rather consider some weighted nonnegative matrices (of significantly less order) which preserve all positive eigenvalues of former ones. Moreover, their inverse matrices appear to be tridiagonal. Using this fact we provide several new bounds on the index (largest eigenvalue) of double nested graphs, and also deduce some bounds on eigenvector components for the index. We conclude the paper by examining the questions related to main versus non-main eigenvalues.

148. *On the Laplacian coefficients of signed graphs*

Let $\Gamma = (G, \sigma)$ be a signed graph, where G is its underlying graph and σ its sign function (defined on edges of G). A signed graph Γ', the subgraph of Γ, is its signed TU-subgraph if the signed graph induced by the vertices of Γ' consists of trees and/or unbalanced unicyclic signed graphs. Let $L(\Gamma) = D(G) - A(\Gamma)$ be the Laplacian of Γ. In this paper we express the coefficient of the Laplacian characteristic polynomial of Γ based on the signed TU-subgraphs of Γ, and establish the relation between the Laplacian characteristic polynomial of a signed graph with adjacency characteristic polynomials of its signed line graph and signed subdivision graph. As an application, we identify the signed unicyclic graphs having extremal coefficients of the Laplacian characteristic polynomial.

149. *Graphs with least eigenvalue -2: Ten years on*

The authors' monograph *Spectral Generalizations of Line Graphs* was published in 2004, following the successful use of star complements to complete the classification of graphs with least eigenvalue -2. Guided by citations of the book, we survey progress in this area over the past decade. Some new observations are included.

150. *A note on connected bipartite graphs of fixed order and size with maximal index*

In this paper the unique graph with maximal index (i.e. the largest eigenvalue of the adjacency matrix) is identified among all connected bipartite graphs of order n and size $n+k$, under the assumption that $k \geq 0$ and $n \geq k+5$.

151. *Notes on the second largest eigenvalue of a graph*

For a fixed real number r we give several necessary and/or sufficient conditions for a graph to have the second largest eigenvalue of the adjacency matrix, or signless Laplacian matrix, less then or equal to r.

152. *On the multiplicities of eigenvalues of graphs and their vertex deleted subgraphs: old and new results*

Given a simple graph G, let A_G be its adjacency matrix. A principal submatrix of A_G of order one less than the order of G is the adjacency matrix of its vertex deleted subgraph. It is well-known that the multiplicity of any eigenvalue of A_G and such a principal submatrix can differ by at most one. Therefore, a vertex v of G is a downer vertex (neutral vertex, or Parter vertex) with respect to a fixed eigenvalue μ if the multiplicity of μ in $A_G - v$ goes down by one (resp., remains the same, or goes up by one). In this paper, we consider the problems of characterizing these three types of vertices under various constraints imposed on graphs being considered, on vertices being chosen and on eigenvalues being observed. By assigning weights to edges of graphs, we generalize our results to weighted graphs, or equivalently to symmetric matrices.

153. *Eigenvalue location for chain graphs*

Chain graphs (also called double nested graphs) play an important role in the spectral graph theory since every connected bipartite graph of fixed order and size with maximal largest eigenvalue is a chain graph. In this paper, for a given chain graph G, we present an algorithmic procedure for obtaining a diagonal matrix congruent to , where A is the adjacency matrix of G and x any real number. Using this procedure we show that any chain graph has its least positive eigenvalue greater than , and also prove that this bound is best possible. A similar procedure for threshold graphs (also called nested split graphs) is outlined.

154. *Signed line graphs with least eigenvalue -2: The star complement technique*

We use star complement technique to construct a basis for -2 of signed line graphs using their root signed graphs. In other words, we offer a generalization of the corresponding results known in the literature for (unsigned) graphs in the context of line graphs and generalized line graphs.

155. *On connected graphs with least eigenvalue greater than* -2

Graphs with least eigenvalue greater than or equal to -2 are to a big extent studied by Hoffman and other authors from the early beginning of the spectral graph theory. Most of these results are summarized in the monograph [Cvetković D., Rowlinson P., Simić S., Spectral generalizations of line graphs, on graphs with least eigenvalue -2, Cambridge University Press, 2004], and the survey paper [Cvetković D., Rowlinson P., Simić S., Graphs with least eigenvalue -2: ten years on, Linear Algebra Appl. 484 (2015): 504–539] which is aimed to cover the next 10 years since their monograph appeared. Here, we add some further results. Among others, we identify graphs whose least eigenvalue is greater than -2, but closest to -2 within the graphs of fixed order. Some consequences of these considerations are found in the context of the highest occupied molecular orbital–lowest unoccupied molecular orbital invariants.

156. *On eigenspaces of some compound graphs*

In the theory of (simple) graphs the concepts of the line and subdivision graph (as compound graphs) are well-known. It is possible to consider them also in the context of (edge) signed graphs. Some relations between the Laplacian spectrum of signed graphs and adjacency spectra of their associated compound (signed) graphs have been recently established in the literature. In this paper, we study the relations between the corresponding eigenspaces.

157. *Polynomial reconstruction of signed graphs*

The reconstruction problem of the characteristic polynomial of graphs from their polynomial decks was posed in 1973. So far this problem is not resolved except for some particular cases. Moreover, no counterexample for graphs of order $n > 2$ is known. Here we put forward the analogous problem for signed graphs, and besides some general results, we resolve it within signed trees and unicyclic signed graphs, and also within disconnected signed graphs whose one component is either a signed tree or is unicyclic. A family of counterexamples that was encountered in this paper consists of two signed cycles of the same order, one balanced and the other unbalanced.

158. *Polynomial reconstruction of signed graphs whose least eigenvalue is close to* -2

The polynomial reconstruction problem for simple graphs has been considered in the literature for more than forty years and is not yet resolved except for some special classes of graphs. Recently, the same problem has been put forward for signed graphs. Here we consider the reconstruction of the characteristic polynomial of signed graphs whose vertex-deleted subgraphs have least eigenvalue greater than -2.

159. *Reflexive line graphs of trees*

A graph is reflexive if the second largest eigenvalue of its adjacency matrix is less than or equal to 2. In this paper, we characterize trees whose line graphs are reflexive. It turns out that these trees can be of arbitrary order - they can have either a unique vertex of arbitrary degree or pendant paths of arbitrary lengths, or both. Since the reflexive line graphs are Salem graphs, we also relate some of our results to the Salem (graph) numbers.

160. *On the spectral invariants of symmetric matrices with applications in the spectral graph theory*

We first prove a formula which relates the characteristic polynomial of a matrix (or of a weighted graph), and some invariants obtained from its principal submatrices (resp. vertex deleted subgraphs). Consequently, we express the spectral radius of the observed objects in the form of power series. In particular, as is relevant for the spectral graph theory, we reveal the relationship between spectral radius of a simple graph and its combinatorial structure by counting certain walks in any of its vertex deleted subgraphs. Some computational results are also included in the paper.

161. *Lexicographic polynomials of graphs and their spectra*

For a (simple) graph H and non-negative integers $c_0, c_1, \ldots, c_d$ ($c_d \neq 0$), $p(H) = \sum_{k=0}^{d} c_k \cdot H^k$ is the lexicographic polynomial in H of degree d, where the sum of two graphs is their join and $c_k \cdot H^k$ is the join of c_k copies of H^k. The graph H^k is the kth power of H with respect to the lexicographic product ($H^0 = K_1$). The spectrum (if H is connected and regular) and the Laplacian spectrum (in general case) of $p(H)$ are determined in terms of the spectrum of H and c_k's. Constructions of infinite families of cospectral or integral graphs are announced.

162. *Fast algorithms for computing the characteristic polynomial of some graph classes*

The characteristic polynomial of a graph is the characteristic polynomial of its adjacency matrix. Finding efficient algorithms for computing characteristic polynomial of graphs is an active area of research and for some graph classes, like threshold graphs, there exist very fast algorithms which exploit combinatorial structure of the graphs. In this paper, we put forward some novel ideas based on divisor technique to obtain fast algorithms for computing the characteristic polynomial of threshold and chain graphs.

163. *Tridiagonal matrices and spectral properties of some graph classes*

A graph is called a chain graph if it is bipartite and the neighbourhoods of the vertices in each colour class form a chain with respect to inclusion. In this paper we give an explicit formula for the characteristic polynomial of any chain graph and we show that it can be expressed using the determinant of a particular tridiagonal matrix. Then this fact is applied to show that in a certain interval a chain graph does not have any non-zero eigenvalue. A similar result is provided for threshold graphs.

164. *Reflexive line graphs of trees and Salem numbers*

An elegant full characterization of reflexive line graphs of trees has proved to be quite dificult task. This paper tries to shed some more light on known results about such graphs by providing more numerical details regarding their structural composition. The paper also presents numerous results and ideas on the topic, as well as some observations with respect to the connection with Salem numbers.

165. *On eigenvalue inequalities of a matrix whose graph is bipartite*

We consider the set of real zero diagonal symmetric matrices whose underlying graph, if not told otherwise, is bipartite. Then we establish relations between the eigenvalues of such matrices and those arising from their bipartite complement. Some accounts on interval matrices are provided. We also provide a partial answer to the still open problem posed in (Zhan in SIAM J. Matrix Anal. Appl. 27:851–860, 2006).

166. *Vertex types in threshold and chain graphs*

A graph is called a chain graph if it is bipartite and the neighbourhoods of the vertices in each colour class form a chain with respect to inclusion. A threshold graph can be obtained from a chain graph by making adjacent all pairs of vertices in one colour class. Given a graph G, let λ be an eigenvalue (of the adjacency matrix) of G with multiplicity $k \geq 1$. A vertex v of G is a downer, or neutral, or Parter depending whether the multiplicity of λ in $G - v$ is $k - 1$, or k, or $k + 1$, respectively. We consider vertex types in the above sense in threshold and chain graphs. In particular, we show that chain graphs can have neutral vertices, disproving a conjecture by Alazemi et al.

167. *Vertex types in some lexicographic products of graphs*

Let M be a symmetric matrix, or equivalently, a weighted graph whose edge ij has the weight m_{ij} . The eigenvalues of are the eigenvalues of M. We denote by the principal submatrix of M obtained by deleting from M both the ith row and the ith column. If μ is an eigenvalue of M, and thus of G, of multiplicity k, then vertex i of M is a downer, or a neutral, or a Parter vertex, depending whether the multiplicity of μ in M or, equivalently, in G , is $k - 1, k$, or $k + 1$, respectively. In this paper, for a fixed μ, we consider vertex types according to the above classification in graphs which are generalized lexicographic products of an arbitrary graph over cliques and co-cliques, or connected regular graphs. In addition, we add some comments on constructions of large families of cospectral and integral graphs.

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Kratak prikaz knjiga Slobodana Simića
A Short Rewiew of Books of Slobodan Simić

Spisak knjiga Slobodana Simića sadrži 40 bibliografskih jedinica. Svako ponovljeno izdanje knjiga je posebno navedeno.

Znatan deo spiska zauzima udžbenička literatura.

Pre svega navodimo 11 izdanja udžbenika *Matematika I - Algebra'* (skripta [2], i 10 štampanih izdanja [3], [6], [8], [12], [17], [18], [20], [24], [29], [37]).

Simić je učestvovao u objavljivanju pomoćnih udžbenika (zbirki zadataka):

Zbirka rešenih zadataka iz Matematike I - I deo (7 izdanja [9], [1], [19], [21], [25], [31], [38]);

Zbirka rešenih zadataka iz Matematike I - II deo (6 izdanja [10], [13], [22], [26], [32], [39]).

Ove udžbenike je objavila grupa nastavnika i asistenata Katedre za matematiku Elektrotehničkog fakulteta u Beogradu. Udžbenici se nalaze u upotrebi već nekoliko decenija.

Spomenimo i udžbenik *PASCAL, Standard i PC ekstenzije* [7].

Simić je objavio i nekoliko nestandardnih udžbenika koji sadrže elemente naučnih monografija:

Kombinatorika, klasična i moderna, [1], [5], [30],

Diskretna matematika, Matematika za kompjuterske nauke, [4], [15],

Kombinatorna optimizacija, Matematička teorija i algoritmi, [14],

Odabrana poglavlja iz diskretne matematike, [23], [28], [36],

Diskretna matematika, Osnove kombinatorike i teorije grafova [33].

Posebnu pažnju zaslužuju tri naučne monografije na engleskom jeziku:

Eigenspaces of graphs [16], [34],

Spectral generalizations of line graphs: On graphs with least eigenvalue -2 [27],

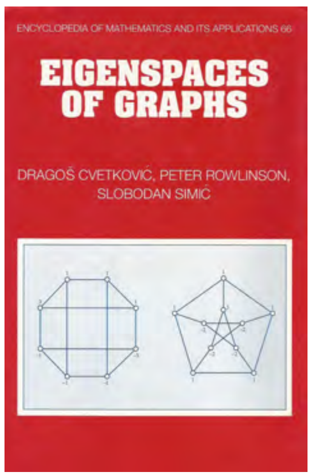
An Introduction to the Theory of Graph Spectra [35].

Ove tri monografije je objavio poznati izdavač naučne literature Cambridge University Press.

Simić je bio jedan od urednika knjige
Grafovi, hemija, računarstvo, optimizacija: Zapisi o radu jednog naučnog projekta [40].

Prefaces for Monographs
Predgovori za monografije

Eigenspaces of graphs



Preface

The foundations of spectral graph theory were laid in the fifties and sixties, as a result of the work of a considerable number of mathematicians. Most of the early results are, like this book, concerned with the relation between spectral and structural properties of a graph. The investigation of such a relationship was proposed explicitly by Sachs [Sac1] and Hoffman [Hof5], although in effect it had already been initiated in an earlier article by Collatz and Sinogowitz [CoSi]. This seminal paper appeared in 1957, but our bibliography contains two references prior to this date: the unpublished thesis of Wei [Wei] from 1952, and a summary (also unpublished) of a 1956 paper by Lihtenbaum [Lih] communicated at the 3rd Congress of Mathematicians of the U.S.S.R.

Another origin of the theory of graph spectra lies beyond mathematics. In quantum chemistry, an approximative treatment of non-saturated hydrocarbons introduced by E. Hückel [Huc] yields a graph-theoretical model of the corresponding molecules in which eigenvalues of graphs represent the energy levels of certain electrons. The connection between Hückel's model of 1931 and the mathematical theory of graph spectra was recognized many years later in [GuPr] and [CvGu1], and thereafter exploited extensively by many authors, both chemists and mathematicians.

In his thesis [Cve7], Cvetković identified 83 papers dealing with eigenvalues of graphs which had appeared before 1970. Ten years later, almost all of the results related to the theory of graph spectra published before 1978 were summarized in the monograph *Spectra of Graphs* by Cvetković, Doob and Sachs, a book which is almost entirely self-contained; only a little familiarity with graph theory and matrix theory is assumed. Its bibliography contains 564 items, most of which were published between 1960 and 1978. It was supplemented in 1988 by *Recent Results in the Theory of Graph Spectra* by Cvetković, Doob, Gutman and Torgašev. This reviews the results in spectral graph theory from the period 1978-1984, and provides over 700 further references from the mathematical and chemical literature. There are additional references from areas such as physics, mechanical engineering, geography and the social sciences. Although many papers contain only minor results, and some present rediscoveries of known results, the large number of references indicates the rapid rate of growth of spectral graph theory. The third edition of *Spectra of Graphs*, published in 1995, contains an appendix which describes recent developments in the subject.

This book deals with eigenspaces of graphs, and although one cannot speak about eigenvectors without mentioning eigenvalues, or vice versa, the emphasis is on those parts of spectral theory where the structure of eigenspaces is a dominant feature, thus complementing the 'eigenvalue part of the theory' described in *Spectra of Graphs*. For the most part, the eigenspaces considered are those of a $(0,1)$-adjacency matrix of a finite undirected graph.

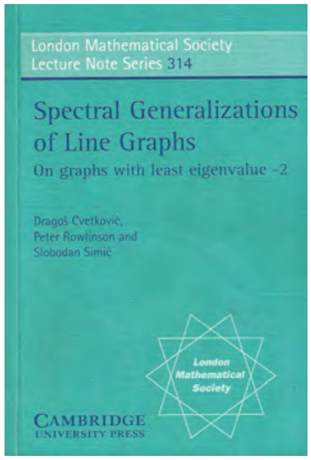
Chapters 1 and 2 review 'old' results on eigenvalues and eigenvectors respectively, while the remaining chapters are devoted to 'new' results and techniques. The eigenspace corresponding to the largest eigenvalue (or *index*) of a connected graph is one-dimensional, and in Chapter 3 a spanning eigenvector is used to identify the graphs with extremal index in various families of graphs. The discussion of graph spectra in the first chapter reveals the limitations of the spectrum as a means of characterizing a graph, and motivates the search for further algebraic invariants such as the graph angles considered in Chapters 4 and 5. Angles also have a role in Chapter 6, where the theory of matrix perturbations is applied to adjacency matrices: one can then describe the behaviour of the index of a graph when it undergoes a local modification such as the addition or deletion of an edge or vertex. Graph angles arise from a geometric approach to eigenspaces that leads in Chapter 7 to the notion of a star partition of vertices, an important concept which enables one to construct 'natural' bases for the eigenspaces of a graph. Implications for the graph isomorphism problem are the subject of current research, and this is described in Chapter 8. Some miscellaneous results are gathered together in Chapter 9, and there are two appendices: one contains some classical results from matrix theory, and the other is a table of graph angles.

The authors are indebted to Mladen Cvetković for assistance with the preparation of a LaTeX version of the first draft of the text. The contents of the second author's article on graph perturbations in *Surveys in Combinatorics 1991* (ed. A.D. Keedwell, Cambridge University Press, 1991) have been included, without significant change, in Chapters 3 and 6. With few other exceptions, the results in Chapters 2 to 9 have not previously appeared in book form.

Finally, the authors gratefully acknowledge individual financial support from the following sources over the past ten years: the British Council, the Carnegie Foundation, the Mathematical Institute of the Serbian Academy of Sciences, the Science and Engineering Research Council, the University of Belgrade and the University of Stirling.

January 1996 D.C., P.R., S.S.

Spectral generalizations of line graphs



Preface

The eigenvalues discussed in this book are those of a $(0, 1)$-adjacency matrix of a finite undirected graph. Line graphs, familiar to graph-theorists for decades, have the property that their least eigenvalue is greater than or equal to -2. This property is shared with generalized line graphs, which can be viewed as line graphs of certain multigraphs. Apart from these classes of examples there are only finitely many further connected graphs with spectrum in the interval $[-2, \infty)$, and these are called exceptional graphs. This book deals with line graphs, generalized line graphs and exceptional graphs, in the context of spectral properties of graphs. Having worked in spectral graph theory for many years, the authors came to see the need for a single source of information on the principal results in this area. Work began early in 2000, and the principal motivation for writing the book at this juncture was the construction of the maximal exceptional graphs in 1999. The working title has become the subtitle on the grounds that 'Graphs with least eigenvalue -2' might appear unreasonably specialized to the casual observer. In fact, the subtitle is not wholly accurate in that it is necessary to treat also the graphs with least eigenvalue greater than -2.

The requirement that the spectrum of a graph lies in $[-2, \infty)$ is a natural one, and in principle not a restriction at all. The reason is to be found in the classical result of H. Whitney, who showed in 1932 that two connected graphs (with more than three vertices) are isomorphic if and only if their line graphs are isomorphic.

The titles of Chapters 2, 3 and 5, namely 'Forbidden subgraphs', 'Root systems' and 'Star complements' reflect three major techniques and three periods in the study of graphs with least eigenvalue -2. Of course, early results were often improved using later techniques, but on considering the interplay between techniques, the authors decided that a presentation broadly in chronological order was the most natural approach.

The forbidden subgraph technique (Chapter 2) was introduced by A. J. Hoffman and others in the 1960s. It is based on the fact that the property of having least eigenvalue greater than or equal to -2 is a hereditary property, that is, a property which the graph shares with all its induced subgraphs. For any hereditary property $\mathcal{P}$ we can consider graphs without property $\mathcal{P}$ which are minimal with respect to the induced subgraph relation: such graphs are the minimal forbidden subgraphs for graphs with property $\mathcal{P}$. For graphs with least eigenvalue greater than or equal to -2, the collection of minimal forbidden subgraphs is finite.

The subject of Chapter 3 is the root system technique introduced by P. J. Cameron, J. M. Goethals, J. J. Seidel and E. E. Shult [CaGSS] in 1976. Root systems were already known in the theory of Lie algebras and in other parts of mathematics, and it turned out that graphs with least eigenvalue -2 can be elegantly described by means of root systems. The description relies on the use of Gram matrices of certain sets of vectors to represent the graphs in question. Generalized line graphs (including line graphs) can be represented in the root system D_n for some n while the existence of the exceptional root system E_8 in 8-dimensional Euclidean space (containing extremely densely packed sets of vectors at 60 and 90 degrees) accounts for the existence of graphs with least eigenvalue -2 which are not generalized line graphs. Chapter 4 uses the tools introduced in Chapter 3 to investigate regular graphs; many spectral characterization theorems for regular line graphs are presented, among them some results from Chapter 2 in an improved form with shorter proofs.

The star complement technique was introduced into the study of graphs with least eigenvalue -2 by the authors of this book in 1998 [CvRS3]. One of the main results presented in Chapter 5 is a characterization of exceptional graphs by exceptional star complements, and this enables all of the maximal exceptional graphs to be constructed (Chapter 6).

Preliminary results in spectral graph theory are given in Chapter 1, while Chapter 7 contains miscellaneous results that do not fit readily into the earlier chapters. It is relatively straightforward to describe a means of constructing exceptional graphs, but the results of the construction make for a fairly elaborate picture. Accordingly the technical descriptions of the

187 regular exceptional graphs and the 473 maximal exceptional graphs are consigned to the Appendix. The authors are grateful to M. Lepović (University of Kragujevac, Serbia & Montenegro) for his assistance in completing the tables in the Appendix, which throughout the book are referred to as Tables A1 to A7. Table A2 contains a description of the 573 exceptional graphs with least eigenvalue greater than -2.

The book brings together many independent discoveries and overlapping results, and provides over 250 references to the literature. The vast majority of the material has not previously appeared in book form. The classification by P. J. Cameron *et al* [CaGSS] using root systems has been summarized in various forms in the monographs [BrCN], [CaLi] and [GoRo]. In this book an outline appears in Section 3.5, following the presentation of a lesser known approach due to M. Doob and D. Cvetković [CvDo]. Further, we acknowledge a debt to [BrCN], Chapter 3 as the source of our proof of Theorem 4.1.5, and as a guide to results on lattices.

Inevitably it has been necessary to limit the scope of the book. A more ambitious work on graphs with least eigenvalue -2 could elaborate not only on the connections with Lie Algebras and lattices but also on the relation to distance-regular graphs, association schemes, block designs, signed graphs, Coxeter systems, Weyl groups and many other combinatorial or algebraic objects. We have merely drawn attention to such connections by short comments and relevant references at the appropriate places. Many of these links to other mathematical areas are described in the book [BrCN] and the expository paper [CaST].

The authors are grateful for financial support from the United Kingdom Engineering & Physical Sciences Research Council (EPSRC), the Serbian Academy of Science & Arts, the Serbian Ministry for Science, Technology and Development, the Universities of Belgrade and Stirling, and the University of Montenegro (S.S. in the period 2000-2002).

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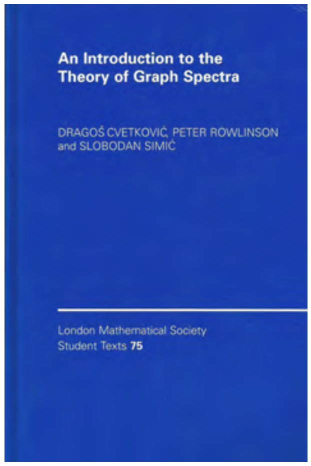
August 2003

An Introduction to the Theory of Graph Spectra



Preface

This book has been written primarily as an introductory text for graduate students interested in algebraic graph theory and related areas. It is also intended to be of use to mathematicians working in graph theory and combinatorics, to chemists who are interested in quantum chemistry, and in part to physicists, computer scientists and electrical engineers using the theory of graph spectra in their work. The book is almost entirely self-contained; only a little familiarity with graph theory and linear algebra is assumed.

In addition to more recent developments, the book includes an up-to-date treatment of most of the topics covered in *Spectra of Graphs* by D. Cvetković, M. Doob and H. Sachs, where spectral graph theory was characterized as follows:

> The theory of graph spectra can, in a way, be considered as an attempt to utilize linear algebra including, in particular, the well-developed theory of matrices, for the purposes of graph theory and its applications. However, that does not mean that the theory of graph spectra can be reduced to the theory of matrices; on the contrary, it has its own characteristic features and specific ways of reasoning fully justifying it to be treated as a theory in its own right.

Spectra of Graphs has been out of print for some years; it first appeared in 1980, with a second edition in 1982 and a Russian edition in 1984. The third English edition appeared in 1995, with new material presented in two

Appendices and an additional Bibliography of over 300 items. The original edition summarized almost all results related to the theory of graph spectra published before 1978, with a bibliography of 564 items. A review of results in spectral graph theory which appeared mostly between 1978 and 1984 can be found in *Recent Results in the Theory of Graph Spectra* by D. Cvetković, M. Doob, I. Gutman and A. Torgašev. This second monograph, published in 1988, contains over 700 further references, reflecting the rapid growth of interest in graph spectra. Today we are witnessing an explosion of the literature on the topic: there exist several thousand papers in mathematics, chemistry, physics, computer science and other scientific areas which develop or use some parts of the theory of graph spectra. Consequently a truly comprehensive text with a complete bibliography is no longer practicable, and we have concentrated on what we see as the central concepts and the most useful techniques.

The monograph *Spectra of Graphs* has been used for many years both as an introductory text book and as a reference book. Since it is no longer available, we decided to write a new book which would nowadays be more suitable for both purposes. In this sense, the book is a replacement for *Spectra of Graphs* ; but it is not a substitute because *Spectra of Graphs* will continue to serve as a reference for more advanced topics not covered here. The content has been influenced by our previous books from the same publisher, namely *Eigenspaces of Graphs* and *Spectral Generalizations of Line Graphs: on Graphs with Least Eigenvalue* -2. Nevertheless, very few sections of the present text are taken from these more specialized sources. For further reading we recommend not only the books mentioned above but also [BrCN], [Big], [Chu] and [GoRo].

The spectra considered here are those of the adjacency matrix, the Laplacian, the normalized Laplacian, the signless Laplacian and the Seidel matrix of a finite simple graph. In Chapters 2-6, the emphasis is on the adjacency matrix. In Chapter 1, we introduce the various matrices associated with a graph, together with the notation and terminology used throughout the book. We include proofs of the necessary results in matrix theory usually omitted from a first course on linear algebra, but we assume familiarity with the fundamental concepts of graph theory, and with basic results such as the orthogonal diagonalizability of symmetric matrices with real entries. Chapter 2 is concerned with the effects of constructing new graphs from old, and graph angles are used in place of walk generating functions to provide streamlined proofs of some classical results. Chapter 3 deals with the relations between the spectrum and structure of a graph, while Chapter 4 discusses the extent to which the spectrum can characterize a graph. Chapter 5 explores the relation between structure and just one eigenvalue, a relation made precise by the relatively recent notion of a star complement. Chapter 6 is concerned with spectral techniques used to prove graph-theoretical results which themselves make no reference to eigenvalues. Chapter 7 is devoted to the Laplacian, the normalized Laplacian and the signless Laplacian; here the emphasis is on the Laplacian because the normalized Laplacian is the subject of the monograph *Spectral Graph Theory* by F. R. K. Chung, while the theory of the signless Laplacian is still in its infancy. In Chapter 8 we discuss sundry topics which did not fit readily into earlier sections of the book, and in Chapter 9 we provide a small selection of applications, mostly outwith mathematics.

The tables in the Appendix provide lists of the various spectra, characteristic polynomials and angles of all connected graphs with up to 5 vertices, together with relevant data for connected graphs with 6 vertices, trees with up to 9 vertices, and cubic graphs with up to 12 vertices. We are indebted to M. Lepović for creating the graph catalogues for Tables A1, A3, A4 and A5, and for computing the data. We are grateful to D. Stevanović for the graph diagrams which appear with these tables: they were produced using *Graphviz* (open source graph visualization software developed byÊÊ AT&T, `http://www.graphviz.org/`Ê), in particular, the programs 'circo' (Tables A1,A3,A5) and 'neato' (Table A4). Table A2 is taken from *Eigenspaces of Graphs*.

Chapters 2,4 and 9 were drafted by D. Cvetković, Chapters 1,5 and 6 by P. Rowlinson, and Chapters 3,7,8 by S. Simić. However, each of the authors added contributions to all of the chapters, which were then re-written in an effort to refine the text and unify the material. Hence all three authors are collectively responsible for the book. We have endeavoured to find a style which is concise enough to enable the extensive material to be treated in a book of limited size, yet intuitive enough to make the book readily accessible to the intended readership. The choice of consistent notation was a challenge because of conflicts in the 'standard' notation for several of the topics covered; accordingly we hope that readers will understand if their preferred notation has not been used. The proofs of some straightforward results in the text are relegated to the exercises. These appear at the end of the relevant chapter, along with notes which serve as a guide to a bibliography of over 500 selected items.

Autumn 2008

D. Cvetković
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Graph Equations in the Work of Slobodan Simić
Grafovske jednačine u delu Slobodana Simića

Dragoš Cvetković

The term “graph equation” was mentioned informally for the first time in [2]. It was noticed that using graph spectra one can find regular solutions of “graph equation” $T(G) = L(H)$, where G and H are graphs, $T(G)$ is the total graph of G and $L(H)$ is the line graph of H.

About that time Slobodan Simić and I were collecting results which can be classified as graph equations. We defined “graph equation” simply as an equation in which unknowns are graphs.

Graph equations are formally defined in [S7] in an obvious and natural way. Graph inequalities are also introduced.

In 1975 we published the joint paper [3] where we solved graph equations $L(G) = T(H)$ and $L(G) = \overline{T(H)}$. This was for the first time that a paper of Simić has been published in an internation journal (nowadays at SCI-list). The paper was published in this journal on recommendation of F. Harary who was in 1974 in Belgrade and liked very much our result.

Simić has also solved [S5] graph equation $L^n(G) = \overline{G}$.

We started building a theory of graph equations which is described in papers [4] and [5]. Graph equations are naturally classified by the number of unknowns.

The problem of characterizing some classes of graphs can be formulated in terms of graph equations. For example one could consider under which conditions the graph equation $L(G) = H$, where H is a given graph, has solutions in G. The well-known characterization of line graphs by nine forbidden induced subgraph, due to L.W. Beineke [1], applies to this graph equation.

The equality in graph equations denotes graph isomorphism. However, one can consider generalized graph equations with other graph equivalence relation as equality. For example, Simić and I found graphs which are switching equivalent to their line graphs [S10] and Slobodan found graphs which are switching equivalent to their complementary line graphs [S15], [S19].

An interesting case happened in 1977. I proposed to the Japanese mathematician J. Akiyama that S. Simić, J. Akiyama and K. Kaneko publish a joint paper [S9] since they had all come independently to the same results. All three colleagues agreed. This case was later described by F. Harary in his article on independent discoveries in graph theory [6]:

"Here are two more draw stories both involving the Japanese graph theorist, Jin Akiyama. By way of background, I met in person both Dragoš Cvetković and his doctoral student S. Simić on arrival at Belgrade airport in 1974. At about the same time, Akiyama and his student K. Kaneko were also deriving graph equations for line graphs and n-th power graphs. Friendly correspondence between Tokyo and Belgrade led to a triply joint paper which is to appear in 1980[1]."

Slobodan Simić considered several other graph theory problems in terms of graph equation [S11], [S12], [S31], [S35].

"Graph equations" was just the title of Simić's master thesis [7] and these equations were also one of the main subjects in his doctoral thesis [8]. Early work of Slobodan has developed much in terms of graph equations. Since graph spectra are useful in solving graph equation, Slobodan gradually moved in direction of spectral graph theory.

Graph equations attracted several researchers abroad, both under our influence and independently [5]. Slobodan and I were thinking at a moment of publishing a book, jointly with some Japanese mathematicians, but that was not realized.

By nineteen eighties we both felt that the area of graph equations was exhausted and that important new results was hard to obtain. Nevertheless Simić published results on graph equations even nineteen nineties (see, for example, [S47]).

[1] In fact, the paper had already appeared in 1978.

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Integral Graphs
Integralni grafovi

Dragan Stevanović

Integral graphs are graphs for which all eigenvalues of the associated graph matrix are integers. Historically, the adjacency matrix represents the matrix associated to the graph in this definition, although nowadays there are quite a few results on graphs that are integral with respect to Laplacian, signless Laplacian and distance matrices as well.

The quest for characterizing integral graphs was initiated by Harary and Schwenk at a conference in 1973, fully described in their seminal paper [1], where they gave several examples of integral graphs and indicated several graph operations that can be used to construct new integral graphs from the existing ones. They had also properly indicated that the general problem of characterizing integral graphs appears intractable, so that the forthcoming research was mostly focused to characterizing integral graphs within particular classes of graphs.

When Slobodan joined the great search for integral graphs some ten years later, this research topic was still very young, as only a handful of research results were published at that time. In June 1974, a year after integral graphs were introduced, Frank Harary visited Dragoš Cvetković in Belgrade, for which occasion Ivan Gutman also arrived from Zagreb to Belgrade [2]. Later that year the second paper [3] on integral graphs appeared, in which Cvetković, Gutman and Trinajstić determined nonregular integral graphs with the maximum vertex degree three. Bussemaker and Cvetković [4] in 1976 and, independently, Schwenk [5] in 1978 further characterized cubic integral graphs. In 1979, Watanabe constructed a family of integral trees of diameter four in [6], and together with Schwenk [7] in the same year characterized integral starlike trees and integral double starlike trees in which vertices of degree at least three are adjacent. Esser and Harary [8] in 1980 showed that a few operations on digraphs enable one to represent the spectra of the resulting digraphs in terms of the starting spectra, and used them to construct infinite families of integral digraphs. Roitman [9] showcased an infinite family of integral complete tripartite graphs in 1984.

Nonregular graphs with maximum degree four

Slobodan's first result on integral graphs was a characterization of nonregular nonbipartite connected integral graphs with maximum vertex degree four, obtained jointly with Zoran Radosavljević [S22]. Let $\lambda_1 > \lambda_2 \geq \cdots \geq \lambda_n$ be the eigenvalues of the adjacency matrix of such graph. Since the graph is not regular, $\lambda_1 < 4$, and the nontrivial case is $\lambda_1 = 3$ (as $\lambda_1 \leq 2$ would lead to already characterized integral Smith graphs). Further, the graph is not bipartite, so $\lambda_n > -\lambda_1$ and the nontrivial case is $\lambda_n = -2$ (as $\lambda_n = -1$ implies that the graph is complete and hence regular). This implies that such integral graphs are either generalized line graphs or exceptional graphs that stem from the root system E_8. As such graphs have no more than six distinct eigenvalues, their diameter is at most five, which together with the maximum vertex degree four, implies that they belong to a finite set of feasible candidate graphs. Further discussion with the help of the expert system GRAPH enabled Slobodan and Zoran to obtain the following characterization [S22].

Theorem 1 *There are exactly* 13 *nonregular nonbipartite connected integral graphs with maximum vertex degree four, of which five are generalized line graphs (shown in Fig. 1), while eight are exceptional graphs (shown in Fig. 2).*

Slobodan attended the fifth workshop "Graphs '96" organized by AGH University in Kraków in November 1996, where he met Krystyna Balińska and Krzysztof Zwierzyński from Poznań who will become his main collaborators in further work on integral graphs. Together they started to study nonregular bipartite graphs with maximum degree four. Denote the set of these graphs by $\mathcal{S}$. The graphs from $\mathcal{S}$ satisfy $\lambda_n = -3$, so that one can no longer rely on generalized line graphs and exceptional graphs for their characterization. In paper [S67], published in 2001, a number of structural properties of graphs from $\mathcal{S}$ were described, related to subgraphs formed by vertices at distance at least three from a fixed vertex, that eventually implied that any graph in $\mathcal{S}$ has at most 78 vertices. All graphs in $\mathcal{S}$ with up to 16 vertices were also enumerated computationally. Further properties

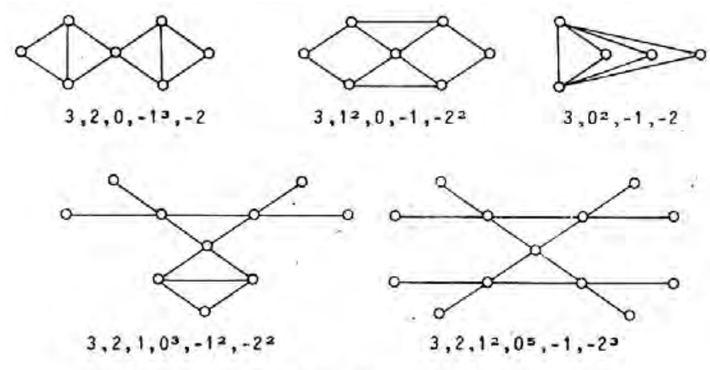

Fig. 1 Nonregular nonbipartite connected integral graphs with maximum vertex degree four, that are generalized line graphs (reprinted from [S22]).

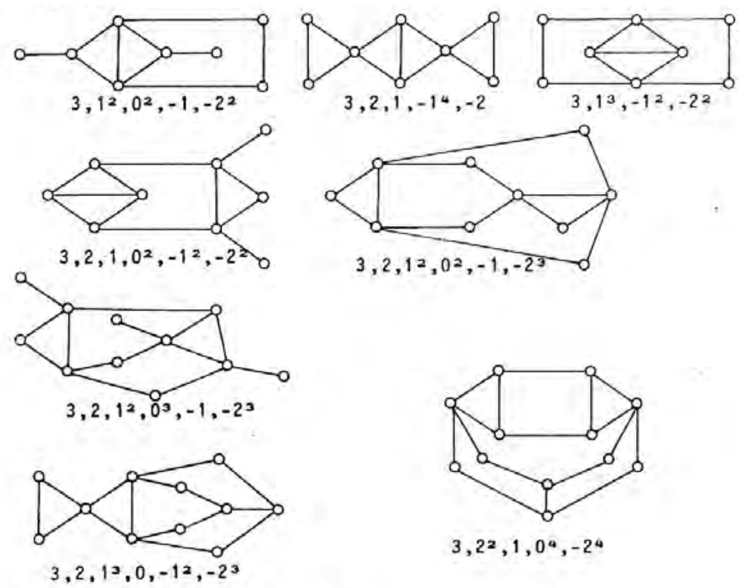

Fig. 2 Nonregular nonbipartite connected integral graphs with maximum vertex degree four, that are exceptional graphs (reprinted from [S22]).

of graphs in $\mathcal{S}$ were obtained in [S68], published in the same year, where it was shown that the star $K_{1,4}$ is the only tree in $\mathcal{S}$, that $\mathcal{S}$ contains no unicyclic graphs and that there are only graphs in $\mathcal{S}$ that do not have eigenvalue ± 2. shown in Fig. 3. Properties of graphs from $\mathcal{S}$ that do not have

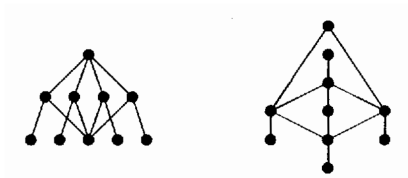

Fig. 3 The only two graphs in $\mathcal{S}$ that do not have ± 2 as an eigenvalue (reprinted from [S68]).

eigenvalue ± 1 were studied in [S83], where 83 feasible degree sequences of such graphs were determined, implying that any such graph also has at most 29 vertices. The task of characterizing graphs in $\mathcal{S}$ was by designing and running an algorithm for constructing all nonregular bipartite graphs with the maximum degree four. The algorithm was designed by Slobodan and Mirko Lepović, and implemented and run by Mirko. The algorithm completed computationally in [S89], starts from a single root vertex r of degree four and proceeds by adding vertices in layers according to their distance from r. Layers are processed sequentially, so that all vertices of one layer are added before starting to build the next layer. Vertices are added to a layer in all possible ways, such that the degree of any vertex is at most four, that $\lambda_1(G') \leq 3$ and $\lambda_2(G') \leq 2$, where G' is the graph induced by currently constructed layers, and that G' is not isomorphic to a previously considered subgraph. This exact algorithm found that $\mathcal{S}$ contains a total of 93 connected nonregular bipartite graphs with maximum degree four, which are listed in [S89] together with their spectra and other invariants. Slobodan returned once more to graphs from $\mathcal{S}$ that do not contain eigenvalue ± 1 in [S145], in order to produce a theoretical proof that, in addition to the star $K_{1,4}$, $\mathcal{S}$ contains only three more such graphs, depicted in Fig. 4. The primary goal of this theoretical study was to show that most of the degree sequences found in [S83] are not feasible. The easier part of the task was the observation that the number q of subgraphs isomorphic to the cycle C_4 satisfies

$$q = 5a + \frac{1}{4}(90 - 16v_4 - 9v_3 - 4v_2)$$

where a is the multiplicity of eigenvalue 2 and v_i is the number of vertices of degree i for $i \in \{2, 3, 4\}$, and that $v_3 \equiv 2 \pmod 4$. These observations were sufficient to show that 39 degree sequences from [S83] are infeasible. The lengthier part of the proof relies on further observation that for graphs

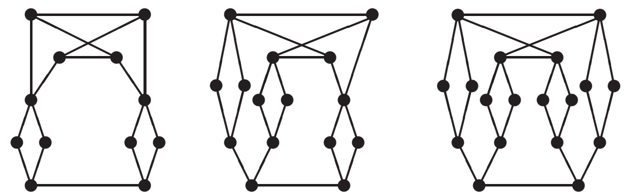

Fig. 4 The three graphs in $\mathcal{S}$, apart from the star $K_{1,4}$, that do not have ± 1 as an eigenvalue (reprinted from [S145]).

in $\mathcal{S}$ that do not have eigenvalue ± 1 holds

$$p + 2(e + h) = 114 + 4a + v_3, \tag{1}$$

where p, e and h are, respectively, the number of subgraphs isomorphic to the path P_4, the cycle C_4 with a pendant edge and the cycle C_6. The proof is then divided into a large number of cases in which parts of the hypothetical graphs with remaining infeasible degree sequences are constructed theoretically until a contradiction to either (1) or the Interlacing theorem is encountered.

Enumeration of integral graphs

Slobodan was also interested in computational enumeration of small integral graphs, the topic to which a series of papers [S59, S63, S73, S118] is dedicated. The first paper [S59], in which integral graphs with up to ten vertices were enumerated, was a result of two independent computer searches, one performed by Mirko Lepović and another by Krystyna Balińska. In the remaining three papers, that deal with the numbers of integral graphs with 11, 12 and 13 vertices, respectively, enumerations are performed by Krzysztof Zwierzyński, at the Poznán Supercomputing and Networking Center. Interestingly, they first devised an evolutionary algorithm to find integral graphs as optimal values of several fitness functions that measure distances of graph eigenvalues to the corresponding nearest integers, probably in order to quickly find as many integral graphs as possible, but later performed exhaustive searches in order to confirm the complete counts for 11 and 12 vertices and the partial count for 13 vertices. The numbers of identified connected integral graphs in these papers are given in Table 1, while their drawings and other invariants are given in the respective papers.

n	1	2	3	4	5	6	7	8	9	10	11	12	13
Int. graphs	1	1	1	2	3	6	7	22	24	83	113	325	$\geq$547

Table 1 The numbers of connected integral graphs with given number of vertices [S59, S63, S73, S118].

Signless Laplacian integral graphs

Slobodan also studied graphs with integral signless Laplacian spectrum (the so-called Q-integral graphs) together with his former PhD student Zoran Stanić. For a given edge e in a graph, its edge-degree is equal to the number of edges incident to e in the graph. Thanks to the fact that a graph is Q-integral if and only if its line graph is integral, Slobodan and Zoran [S104] used the existing knowledge on integral graphs with maximum vertex degree four to show the following preliminary result.

Theorem 2 *There are exactly 26 connected Q-integral graphs with maximum edge-degree at most four.*

Their further focus was then put on edge-regular graphs with edge-degree five, which are necessarily (r, s)-semiregular bipartite graphs with $r+s = 7$. They were able to show that the only connected Q-integral (r, s)-semiregular bipartite graph are $K_{1,6}$ for $(r, s) = (1, 6)$ and $K_{2,5}$ and the subdivision graph of K_6 for $(r, s) = (2, 5)$. For $(r, s) = (3, 4)$ they determined the list of 16 feasible spectra for such Q-integral graphs, and managed to show that three of these spectra yield a unique graph, while further four of these spectra are infeasible.

In [S123] they further studied Q-spectral properties of (r, s)-semiregular bipartite graphs, and obtained further feasible spectra of Q-integral graphs when $r = 3$ and $s > 5$. In particular, if such graph does not have Q-eigenvalue 1 then its spectrum is either one of three particular spectra or belongs to one of two infinite families of spectra, and Slobodan and Zoran managed to describe corresponding graphs for both infinite families and one of the particular spectra. On the other hand, if such graph does not have induced quadrangles or hexagons, then it must have the spectrum $[9, 8^{63}, 7^{90}, 6^{14}, 3^{182}, 2^{90}, 1^{63}, 0]$, while existence of a graph with this spectrum was left open.

Slobodan's highly cited papers [S119, S124] on spectral theory of graphs based on the signless Laplacian, written together with Dragoš Cvetković, also contain reviews of results of other researchers on Q-integral graphs.

Other topics on integral graphs

In addition to topics elaborated in previous sections, Slobodan wrote three more papers on integral graphs.

4-regular integral graphs were one of the topics of my PhD thesis research, so that Dragoš Cvetković, Slobodan and I first prepared a preliminary paper [S57] in 1998 in which we obtained the feasible spectra of such graphs, brought an upper bound on the number of vertices and compiled the list of known 4-regular integral graphs.

Another opportunity to collaborate with Slobodan appeared a few years later, when together with Krystyna Balińska, Dragoš Cvetković and Zoran Radosavljević, we prepared an extensive survey of results on integral graphs [S75] until 2002. The new version of such survey is now long overdue thanks to many new developments on the topic of integral graphs.

The last Slobodan's paper on integral graphs that we cover here is the paper [S130], written together with Dragoš Cvetković, Tatjana Davidović and Aleksandar Ilić, in which they argued that integral graphs may be suitable candidates for multiprocessor interconnection networks, as they have not only integer eigenvalues, but also the corresponding eigenvectors can be chosen to have integer components only, so that dynamic load balancing algorithms could be, in principle, performed in integer arithmetic. These arguments were later analyzed in more details in [10].

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The Largest Eigenvalue of a Graph
Najveća sopstvena vrednost grafa

Milica Anđelić

An overview

There is an extensive amount of literature connecting the largest eigenvalue (also called the index) of a graph and the graph structure. Along his remarkable career, Slobodan K. Simić was passionated in studying the index of different types of graphs. About 30 of his publications were devoted to this subject.

His interest in this topic had started in 1986, when he published together with V. Lj. Kocić a paper on the largest eigenvalue of some homeomorphic graphs [S23]. In this paper, they considered the graphs obtained from the cycle C_n on $n \geq 6$ vertices by adding an edge between two vertices at distance k ($k = 2, 3, \ldots, \lfloor \frac{n}{2} \rfloor$). They proved that the index of these graphs is monotone with respect to k. Moreover, they noticed that the monotonicity holds even for some larger classes of graphs, like parallel paths, graphs homeomorphic to a multigraph consisting of k parallel edges, and cycles with vertex in common, i.e., graphs that are homeomorphic to a graph consisting of k loops sharing a common vertex.

In [S27] the ordering of unicycle graphs with respect to the largest eigenvalue, so called λ_1-ordering, was provided. The paper also includes some relations between the graph structure and its largest eigenvalue. Applying these results to unicyclic graphs of fixed order, some facts about the λ_1-ordering involving the girth were obtained.

Paper [S32] deals with bicyclic graphs. Among all connected graphs with two independent cycles those whose largest eigenvalue is minimal were identified.

Jointly with F.K. Bell, in [S52] Slobodan considered the index of 'broken wheels'. These are connected graphs which can be obtained from an n-cycle by joining an additional 'central' vertex to k of the vertices of the cycle. They used the famous result of Schwenk to compare the characteristic polynomials of graphs in $\mathcal{W}(n,k)$, broken wheels on $n+1$ vertices and k 'spokes'. They identified the broken wheel graphs with greatest and least index in $\mathcal{W}(n,k)$, showing that the index is the greatest when the k 'spokes' are bunched together as closely as possible, and is the least when they are spread out as evenly as possible.

The largest eigenvalue of color-constrained trees was the topic of [S66], where Slobodan, together with G. Caporossi, D. Cvetković, and P. Hansen considered the set of bicolored trees with given numbers of black and of white vertices. They determined those for which the largest eigenvalue is extremal (maximal or minimal). The results were motivated by the automated system AutoGraphiX, developed in GERAD (Montreal), and verified afterwards by theoretical means.

In 2003 Slobodan started his fruitful and long lasting cooperation with the researchers from the University of Messina, Italy. As a result of this collaboration the PhD thesis of Francesco Belardo was defended in 2007, under a joint supervision of Slobodan and Enzo M. Li Marzi. Their joint work dealt mainly with the largest eigenvalue of different types of graphs. The first publication [S84] goes back to 2004, where some accounts on the structure of graphs of fixed order and size with maximal index were given. They considered how the relocation of some edges, based on the entry of corresponding vertices in the Perron vector, influenced the largest eigenvalue of a graph. By this follows that the graphs with maximal index are $2K_2, C_4$ and P_4 free, i.e., they do not contain any of these graphs as an induced subgraph.

For the set of trees $\mathcal{T}(n,\Delta)$ on n vertices with a given maximum degree Δ, in [S88] the trees with the maximum index were determined.

In [S90] F. Belardo, E. M. Li Marzi, and Slobodan identified in some classes of unicyclic graphs of fixed order and girth those graphs whose index is maximal. Besides, they provided some (lower and upper) bounds on the indices of the graphs being considered.

The paper [S96] dealt with two classes of graphs, both having the index close to 2:

(i) trees of order n and diameter $d = n-3$;

(ii) unicyclic graphs of order n and girth $g = n-2$.

Assuming that each graph within these classes has two vertices of degree 3 at distance k, the authors ordered by the index the graphs from (i) for any fixed k $(1 \leq k \leq d-2)$, and the graphs from (ii) independently of k.

In [S97] together with his Italian colleagues Slobodan considered path-like graphs, i.e., trees on n vertices and diameter $n-3$. They proved that all path-like graphs have the index in the interval $(2, \sqrt{2}\sqrt{1+\sqrt{2}})$. They also completely ordered all path-like graphs which have the index in the interval $(\alpha, \sqrt{2}\sqrt{1+\sqrt{2}})$, where $\alpha = \sqrt{2}\sqrt{(\frac{9-\sqrt{33}}{36})^{1/3} + (\frac{9+\sqrt{33}}{36})^{1/3} + 1}$.

The index of the class of trees $\mathcal{T}_{n,d}$ with n vertices and diameter d was the subject of [S101], realized as a joint work of Slobodan and B. Zhou. For all integers n and d with $4 \leq d \leq n-3$ all trees in $\mathcal{T}_{n,d}$ with k-th largest index for all k up to $\lfloor d/2 \rfloor + 1$ if $d \leq n-4$, or for all k up to $\lfloor d/2 \rfloor$ if $d = n-3$ were identified.

Among the trees with a fixed order and diameter, a graph with the maximal index is a caterpillar. In the set of caterpillars with a fixed order and diameter, or with a fixed degree sequence, in [S102] those whose index is maximal were characterized. For this publication Slobodan and his coauthors received the award from the journal Discrete Mathematics for the most cited paper in $2005-2010$ period.

The problem of ordering graphs by the index in the class of connected graphs with a fixed order n and index belonging to the interval $(2, \sqrt{2+\sqrt{5}})$ is the topic of [S103]. For any fixed n (provided that n is not too small), the authors order a significant portion of graphs whose indices are close to the end points of the above interval.

Let $\mathcal{T}^n_\Delta$ be the class of trees on n vertices whose all vertices, other than pendant ones, are of degree Δ (bidegreed trees). In [S114] some problems related to the index of bidegreed trees focusing on those trees with small index were under the consideration. In particular, those trees from $\mathcal{T}^n_\Delta$ with minimal index were identified.

In [S120] unicyclic graphs with prescribed degree sequence were considered. In this set the (unique) graph with the largest spectral radius with respect to the adjacency matrix was determined. In addition, a conjecture about the (unique) graph with the largest index in the set of connected graphs with prescribed degree sequence was included.

In [S121] Slobodan, D. Tošić, F. Belardo and E. Li Marzi applied so called "the eigenvector techniques" for getting some new lower and upper bounds on the index of nested split graphs. The technique is based on estimation of the entries of the principal eigenvector, followed by the application of the Rayleigh principle. Computational results are provided in order to get better insight to the quality of the obtained bounds. Similar results, are obtained in [S139] and [S140] for the signless Laplacian index of nested split graphs.

Open and closed necklaces, are simple graphs consisting of a finite number of cliques of fixed orders arranged in path-like pattern and cycle-like pattern, respectively. In these two classes in [S122] those graphs whose index is maximal were determined.

A connected graph (V_G, E_G) is called a quasi-k-cyclic graph, if there exists a vertex q such that $G - q$ is a k-cyclic graph (connected with cyclomatic number k). In [S129], jointly with X. Geng and S. Li, in the set of quasi-k-cyclic graphs (for $k \leq 3$) those graphs whose spectral radius of the adjacency matrix (and the signless Laplacian if $k \leq 2$) is the largest are determined. In addition, for quasi-unicyclic graphs those graphs whose spectral radius of the adjacency matrix is the second largest are identified, as well .

For the set $C(2m, k)$ of all cactuses on $2m$ vertices, k cycles, and with perfect matchings in collaboration with Z. Huang and H. Deng, Slobodan identified the unique graph with the largest spectral radius.

In [S135] the focus was put on double nested graphs. These graphs assume the same role as threshold graphs if we restrict ourselves to bipartite graphs, i.e., in the class of connected bipartite graphs of fixed order and size those with maximal index are double nested graphs. The paper provides some bounds, both lower and upper, for the index of double nested graphs. All the results are accompanied with computational experiments. These results were part of my PhD thesis defended in 2011 at Department of Mathematics, University of Aveiro, under the joint supervision of Slobodan and Prof. Domingos Cardoso. Some new aspects and results on this topic were published in [S147]. In studying the spectrum of double nested graphs we considered some weighted nonnegative matrices of significantly less order than the order of a graph which preserved all positive eigenvalues. Moreover, their inverse matrices appeared to be tridiagonal. This work was realized as a part of bilateral project "Applications of graph spectra in computer science" funded by Portuguese Foundation for Science and Technology and Serbian Ministry of Science and Technology during the two year period $2013 - 2014$.

In [S150], the unique double nested graph with maximal index was identified among all connected bipartite graphs of order n and size $n + k$, under the assumption that $k \geq 0$ and $n \geq k + 5$. On this problem Slobodan collaborated with M. Petrović, from the University of Kragujevac.

Slobodan was eager to provide an answer to a long lasting open problem that dealt with the structure of threshold graphs of fixed order and size having the maximal index. In [S160], some accounts toward the general solution of this problem were given. A formula which related the characteristic polynomial of a matrix (or of a weighted graph), and some invariants obtained from its principal submatrices (resp. vertex deleted subgraphs) was provided. Consequently, the spectral radius of the observed objects was expressed in the form of power series. In particular, the relationship between spectral radius of a simple graph and its combinatorial structure by counting certain walks in any of its vertex deleted subgraphs was revealed.

In 2006, Slobodan focused his research interest on signless Laplacian spectrum of graphs, mainly on the signless Laplacian index of graphs. In collaboration with D. Cvetković in [S116], [S119] and [S124], some structural considerations of graphs with different constraints on the largest signless Laplacian index were given. It turned out that if the order and size are fixed the graph in question is a nested split graph.

For a graph matrix M, the Hoffman limit value $H(M)$ is the limit, if it exists, of the largest eigenvalue of $M(H_n)$, where the graph H_n is obtained by attaching a pendant edge to the cycle C_{n-1} of length $n-1$. The exact values of $H(A)$ and $H(L)$ were first determined by Hoffman and Guo, respectively. Since H_n is bipartite for odd n, we have $H(Q) = H(L)$. All graphs whose A-index is not greater than $H(A)$ were completely described in the literature. In [S136], all graphs whose Q-index does not exceed $H(Q)$ are determined. The results obtained are determinant to describe all graphs whose L-index is not greater then $H(L)$.

A connected graph of order n is bicyclic if it has $n+1$ edges. In [C.X. He, J.Y. Shao, J.L. He, On the Laplacian spectral radii of bicyclic graphs, Discrete Math. 308 (2008) 5981–5995] the authors determined, among the n-vertex bicyclic graphs, the first four largest Laplacian spectral radii together with the corresponding graphs (six in total). It turns that all these graphs have the spectral radius greater than $n-1$. In [S137], the remaining n-vertex bicyclic graphs (five in total) whose Laplacian spectral radius is greater than or equal to $n-1$ are determined. The complete ordering of all eleven graphs in question was obtained by determining the next four largest Laplacian spectral radii together with the corresponding graphs.

In [S166] an interesting interlacing relations between adjacency eigenvalues of bipartite graphs and those arising from their bipartite complement were established. In particular it was shown that $\lambda_2(G) \leq \lambda_1(G^b)$, where G^b stands for the bipartite complement of a bipartite graph G.

Basic tools

It is natural to expect that the largest eigenvalue changes if G is perturbed. In his papers Slobodan considered different types of graph perturbations and their effects on the largest eigenvalue. Here we present some, most common in his papers.

Lemma 1 *If G' is a graph obtained from a connected graph G obtained by adding an edge, then*

$$\lambda_1(G') > \lambda_1(G).$$

The following two results are the part of standard folklore of graph perturbations. The first one is about the perturbation known as the em simultaneous rotations; the second is about the *local switching*. It is worth mentioning that the local switchings preserve the degree sequences.

Proposition 1 *Let r, s bw two vertices of a connected graph G of order n and let $N(r) \setminus N(s) = \{v_1, \ldots, v_t\}$ be the neighbours of r not adjacent to s. Let G' be a graph obtained from G by relocating the edge rv_i to the position of non-edge sv_i for $i = 1, \ldots, t$. Let $\mathbf{x} = (x_1, \ldots, x_n)$ be a Perron vector of G. If $x_s \geq x_r$, then $\lambda_1(G') > \lambda_1(G)$.*

Proposition 2 *Let G' be a graph obtained from a connected graph G by the local switching, that consists of the deletion of edges $e = st$ and $f = uv$, followed by the addition of edges $e' = sv$ and $f' = tu$. Let $\mathbf{x} = (x_1, \ldots, x_n)$ be a Perron vector of G. If $(x_s - x_u)(x_v - x_t) \geq 0$, then $\lambda_1(G') \geq \lambda_1(G)$. The equality holds if and only if $x_s = x_u$ and $x_v = x_t$.*

In some of the papers the results of Hoffman and Smith were employed. An *internal path* $v_1 \ldots v_{r-1}v_r$ in a graph is a path joining vertices v_1 and v_r which are both of degree equal to 2. By C_n and W_n we denote the cycle and the double snake (the tree on n vertices having two vertices of degree 3 which are at distance $n-5$).

Lemma 2 *Let G' be a graph obtained from a $G \neq C_n, W_n$ by inserting in an edge e a vertex of degree 2. Then we have:*

- *If e does not lie on an internal path, then $\lambda_1(G') > \lambda_1(G)$.*
- *If e lies on an internal path, then $\lambda_1(G') < \lambda_1(G)$.*

If $G = C_n(W_n)$ and $G' = C_{n+1}(W_{n+1})$, then $\lambda_1(G') = \lambda_1(G) = 2$.

Lemma 3 *Let $G(l, m)$ be a graph obtained from a non-trivial graph G by adding at some fixed vertex r two hanging paths whose lengths are l and m ($l \leq m \leq 1$). Then*

$$\lambda_1(G(l, m)) > \lambda_1(G(l+1, m-1)).$$

Useful tools in studying the largest the eigenvalue are divisors. A partition $\{V_1, \ldots, V_k\}$ of the vertex set of a graphs G is an *equitable partition*, if each vertex from some cell, say V_i, has the same number d_{ij} of neighbors in any cell V_j (including $j = i$). Let D be the multidigraph having as its vertex set the cell of the equitable partition, and d_{ij} parallel arcs from the ith cell V_i to the jth cell V_j, where $1 \leq i, j \leq k$. If is, D is called a *divisor* of G. Its adjacency matrix is called the *divisor matrix*. A remarkable property of the characteristic polynomials of the latter two adjacency matrices is that the characteristic polynomial of a divisor divides the characteristic polynomial of the graph. Moreover the index of the graph is included in the spectrum of the divisor. As a consequence it follows.

Lemma 4 *Let G be a graph and D any divisor of G. Then $\lambda_1(G) = \lambda_1(D)$.*

Slobodan's talent and experience led to many novel ideas. Besides being specialist in Graph Theory he also possessed an excellent general mathematical knowledge. Therefore, in many of his proofs one can find a mixture of analysis, combinatorics, algebra, logic... His approach in studying the index of graphs was a milestone, and the path he built many authors followed afterwards.

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Connected Graphs of Fixed Order and Size with Minimum Index
Povezani grafovi sa zadatim brojem čvorova i grana sa minimalnim indeksom

Francesco Belardo

One of the most studied invariants in Spectral Graph Theory is the *largest eigenvalue* of the adjacency matrix of graphs. The largest eigenvalue is studied also under the name of *spectral radius*, since by the Perron-Frobenius theorem we get that the largest eigenvalue is the spectral radius, or *index*, as it was named in [1]. We shall use the term index, as preferred by Slobodan. The "success" behind the index can be motivated by several reasons. One of them is that the index is the easiest eigenvalue to study: it maximizes the Rayleigh quotient, it is simple (when the graph is connected), and it gets an eigenvector with positive components. However, most papers on the index are concerned with the maximization of the index within a class of graphs, because the special features of the index favor augmentation rather than decrement. We refer the reader to Chapter 6 of the monograph Eigenspaces of Graphs [2] to see a collection of perturbations on the graph structure and their effect on the variation of the index, and to [7] for a recent monograph devoted to the index.

A significant part of Slobodan's research was dedicated to the index, and naturally many of his results are related to the identification of graphs, belonging to some given class, whose index is maximal. Here, we want to discuss a smaller portion of Slobodan's research on the index which contains the possibly most interesting results in this topic: connected graphs of fixed order and size minimizing the index.

The celebrated paper [1] by L. Collatz and U. Sinogowitz, considered as the first paper on graph spectra written by mathematicians, contains one of the most known results in Spectral Graph Theory. This result offers the general solution for connected graphs of given order.

Theorem 1 *Let G be a connected graph of order n, and $\rho(G)$ its index. Then*

$$2\cos\frac{\pi}{n+1} \leq \rho(G) \leq n-1,$$

where the left (right) equality holds iff G is the path P_n (resp., the clique K_n).

The problem becomes more interesting when we fix, in addition to the order n, the number of edges m, and since the graphs are connected, we can use the cyclomatic number $c = m - n + 1$ as a parameter. Therefore let us denote by $\mathcal{G}_{n,c}$ the class of connected graphs on n vertices and cyclomatic number c, and let $\hat{G}$ be a graph with minimal index in $\mathcal{G}_{n,c}$.

From Theorem 1, we can obviously claim that for $c = 0$, the tree $\hat{G}$ minimizing the index in $\mathcal{G}_{n,0}$ is the path P_n. If we consider unicyclic graphs, so graphs in $\mathcal{G}_{n,1}$, the answer easily comes again, since the cycle C_n has index 2, and any graph containing a cycle gets a larger index. So $\hat{G} = C_n$ has the minimal index amongst the unicyclic graphs.

However, when we speak of c-cyclic graphs, where $c > 1$, the problem gets quite challenging. Slobodan, in [5], gave a solution for the bicyclic case $c = 2$ and attacked the general problem. Let us consider in more details his results. We first need the *Hoffman-Smith lemma*, which gives an important tool to decrease the index and an important restriction on the structure of index minimizers in $\mathcal{G}_{n,c}$, as well. A path $v_0 v_1 \cdots v_k$ (with possibly $v_0 = v_k$) is an *internal path* of some graph if $\deg(v_0), \deg(v_k) \geq 3$, while $\deg(v_i) = 2$ for each $i = 1, \ldots, k-1$ (see Fig. 1).

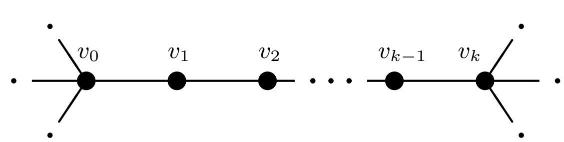


Fig. 1 An internal path with distinct endvertices.

Lemma 2 (Hoffman-Smith lemma) *Let G' be a graph obtained from a connected graph G by inserting a vertex of degree 2 in an edge e. Then we have:*

(i) if $G \neq W_n$ and e lies on an internal path then $\rho(G') < \rho(G)$;

(ii) if $G \neq C_n$ and e does not lie on an internal path then $\rho(G') > \rho(G)$.

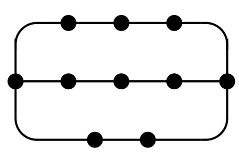
If G is the double-snake W_n or the cycle C_n then $\rho(G') = \rho(G) = 2$.

The following corollary is a straightforward consequence of Lemma 2.

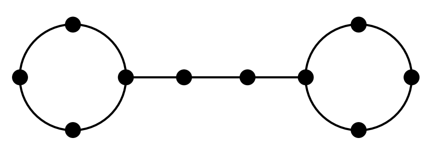
Corollary 3 *If $c \geq 1$ and $\hat{G} \in \mathcal{G}_{n,c}$ is a graph with minimal index, then $\delta(G) \geq 2$.*

Since vertices of degree 1 are forbidden, then the candidate to be a minimizer consists of vertices of degree 3 or greater joined by internal paths. That is all one can deduce from the powerful Hoffman-Smith lemma. Therefore, a minimizer consists of internal paths, whose lengths are unknown. Slobodan attacked the case $c = 2$ and, with the cooperation of V.Lj. Kocić, in [6] they proved, using calculus, that in a θ-graph (a graph consisting of three paths whose endvertices are pairwise identified, cf. Fig. 2) making the internal paths of almost the same length leads to the minimal index. A few years later in [5], Slobodan completed the research as he identified the minimizers of $\mathcal{G}_{n,2}$.

Theorem 4 *Let $\theta(a, b, c)$ be a theta-graph and $D(a, b, c)$ a dumbbell-graph, where a, b and c denote the lengths of the internal paths (b is the length of the joining path for the dumbbell). If $c = 2$ and $\hat{G} \in \mathcal{G}_{n,c}$ is a graph with minimal index, then $\hat{G} = \theta(k, k, n+1-2k)$ or $\hat{G} = D(k, n+1-2k, k)$, where $k = \lceil n/3 \rceil$.*



$\theta(4, 4, 3)$



$D(4, 3, 4)$

Fig. 2 Minimizers in $\mathcal{G}_{10,2}$.

In 1993, Y. Hong in [4] considered this problem and he posed the famous question:

Conjecture 5 (Hong's Conjecture) *Let G be a simple connected non-regular graph with n vertices and e edges. We denote by $\delta(G)$ and $\Delta(G)$ the minimum and the maximum degree, respectively, of G. If G has the smallest possible spectral radius, is it true that $\Delta(G) - \delta(G) \leq 1$?*

Informally, Hong's conjecture says that minimizers are either regular or almost regular graphs, and the conjecture is obviously coherent with the so far known results. For example, when n divides $2m$, then the conjecture is true, as there are regular graphs on n vertices, m edges that are ρ-regular, where $\rho = 2m/n$. After the bicyclic case, the next step is to consider minimizers in $\mathcal{G}_{n,3}$. In 2009, 20 years after the publication of [5], Slobodan proposed me to attack this problem, as he got some minimizers by brute force search from some colleague (whose name I do not recall). In fact, we had the minimizers for several values of the order up to 18. By experiments with the computer system newGraph, we managed to conjecture the minimizers for the 3-cyclic case. The description of the minimizers was very meaningful to Slobodan, as he recognized a common feature from his old investigation on the bicyclic graphs:

Conjecture 6 *If $c \geq 1$ and $\hat{G} \in \mathcal{G}_{n,c}$ is a graph with minimal index, then the lengths of internal paths pairwise differ at most by* 1.

Hence, all minimizers consists of internal paths and their lengths are almost equal. To prove the above conjecture we tried the approach used in [6], without success. Slobodan then tried a slightly different approach, that is to simultaneously consider the eigenvalue equations for the index (denoted by ρ) at the extremal vertices of the internal paths. We will outline it in the sequel.

Firstly, let G be a graph consisting of internal paths joined at their endvertices. Take any internal path labelled as in Fig. 1, and let $a = x_0, x_1, \ldots, x_{k-1}, x_k = b$ be the components of the Perron vector (the eigenvector related to the index ρ) corresponding to the vertices of P_{k+1}. For each internal vertex v_i $(i = 1, \ldots, k-1)$ we have the following ρ-eigenvalue equation:

$$\rho x_i = x_{i+1} + x_{i-1}. \tag{1}$$

Equation (1) can be seen as a linear second–order homogeneous difference equation with constant coefficients.

$$x_{i+1} - \rho\, x_i + x_{i-1} = 0.$$

Since $\rho > 2$, there exists $t > 0$ such that $\rho = 2\cosh(t)$. Solving the system of equations $x_{i+1} - 2\cosh(t)x_i + x_{i-1} = 0$, for $i = 1, \ldots, k-1$, which stems from the eigenvalue equations (restricted to P_{k+1}) we get

$$x_i = a\frac{\sinh(k-i)t}{\sinh(kt)} + b\frac{\sinh(it)}{\sinh(kt)}.$$

We next evaluate the Perron vector at vertices of degree ≥ 3. By x_j^1 we denote the Perron component (computed from above) of a vertex adjacent to the endvertices.

$$\rho a = \sum_{j=1}^{d(v_a)} x_j^1$$

and by substituting the values of each x_j^1:

$$2\cosh(t)a = \sum_{j=1}^{d(v_a)} \left[a\frac{\sinh(l_j-1)t}{\sinh(l_jt)} + b_j\frac{\sinh(t)}{\sinh(l_jt)}\right]$$

$$\sum_{j=1}^{d(v_a)} \tanh\left(\frac{l_j}{2}t\right) = (d(v_a)-2)\coth(t) + \sum_{j=1}^{d(v_a)} \frac{b_j/a - 1}{\sinh(l_jt)}.$$

We sum up all equations taken over all v_{a_i}s in G:

$$2\sum_{i=1}^{q} \tanh\left(\frac{l_i}{2}t\right) = 2(q-p)\coth(t) + \sum_{i=1}^{q} \frac{a_i/b_i + b_i/a_i - 2}{\sinh(l_it)},$$

where q is the number of internal paths, and p is the number of vertices of degree ≥ 3.

Let $w_i = \frac{1}{2}(a_i/b_i + b_i/a_i) - 1$ be the weight of the i-th internal path $P^{(i)}$. Since $q - p = c - 1$, we get

$$\sum_{i=1}^{q} \tanh\left(\frac{l_i}{2}t\right) = (c-1)\coth(t) + \sum_{i=1}^{q} \frac{w_i}{\sinh(l_it)}.$$

For simplicity, we define

$$\begin{aligned}\sum_{i=1}^{q} \tanh\left(\frac{l_i}{2}t\right) &= \Phi(l_1, l_2, \ldots, l_q),\\ \sum_{i=1}^{q} \frac{w_i}{\sinh(l_it)} &= \phi(l_1, l_2, \ldots, l_q; w_1, w_2, \ldots, w_q).\end{aligned}$$

Then we have:

$$\Phi = (c-1)\coth(t) + \phi. \tag{2}$$

Equation (2) represents the generic equation of $\rho = 2\cosh t$ for a c-cyclic graph consisting of internal paths of length l_is. If we want the minimizer $\hat{G}$, then ρ as a solution of (2) must be the smallest possible. The latter can be obtained by maximizing Φ and by minimizing ϕ. Evidently, if all l_is are equal then Φ gets the largest possible, while $\phi = 0$. Furthermore, it is not difficult to prove that the endvertices of the internal paths should have degree equal to 3 (as an induced $K_{1,4}$ would push the index above $4/\sqrt{3}$). This last fact means that for n large enough, the minimizers are obtained from cubic graphs by subdividing their edges. As an example of basic configuration, the tricyclic cubic minimal configurations are depicted in Fig. 3. A c-cyclic cubic configuration is given in Fig. 4.

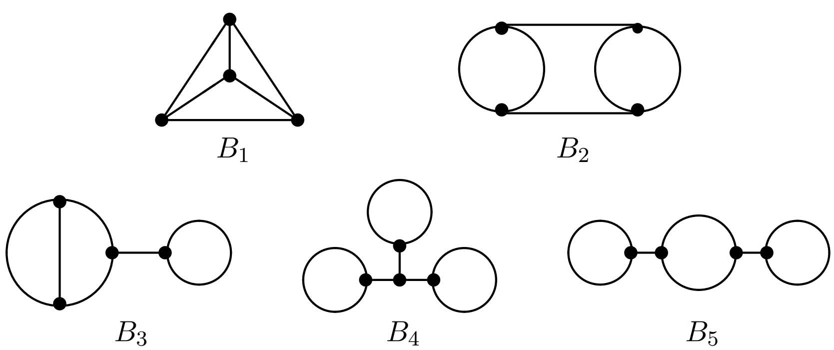


Fig. 3 Cubic tricyclic minimal configurations.

Here is the (yet unpublished) result we got on the minimizers of $\mathcal{G}_{n,c}$:

Theorem 7 *Let $\hat{G}$ be a graph with minimal index in $\mathcal{G}_c(n)$, where $n = (3k+2)(c-1)$ for some $k \geq 0$. Then $\hat{G} = B^{(k)}$, where B is some cubic multi-graph on $2(c-1)$ vertices and $B^{(k)}$ is obtained by inserting k vertices in each internal edge of B.*

Additionally, in view of Hoffman-Smith lemma, we have the following corollary which validates Hong's conjecture for c-cyclic graphs of order large enough to build a c-cyclic cubic basic configuration.

Corollary 8 *Let $\hat{G}$ be a graph with minimal index in $\mathcal{G}_{n,c}$, if $n \geq 2(c-1)$ then $\Delta(\hat{G}) = 3$ and $\delta(\hat{G}) = 2$.*

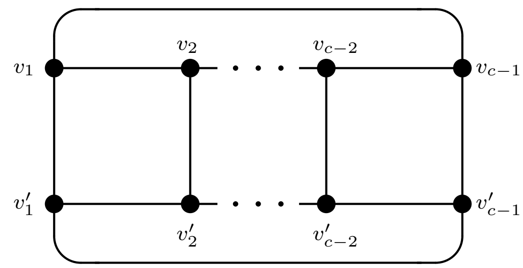


Fig. 4 A c-cyclic cubic basic configuration.

From Theorem 7 we can deduce the minimizers $\hat{G} \in \mathcal{G}_{n,3}$ for $n = 6k + 4$ and $k \geq 1$. In Fig. 5 we have such an example for $k = 2$.

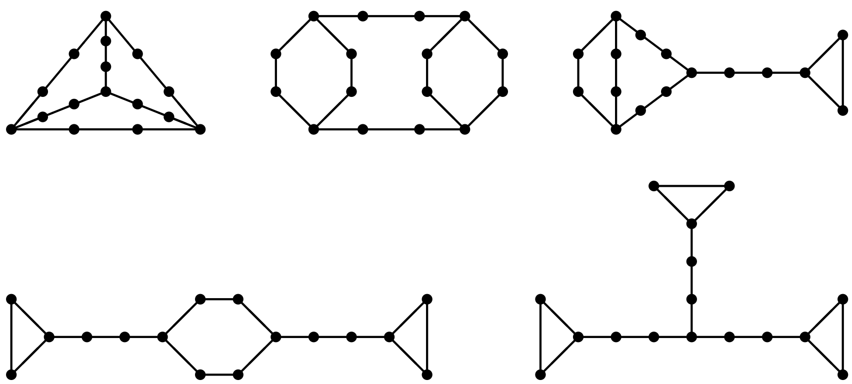


Fig. 5 Minimizers in $\mathcal{G}_{16,2}$.

If Conjecture 6 holds, we can deduce minimizers of $\mathcal{G}_{n,c}$ for some further values of n, in addition to those described in Theorem 7. In particular, if we know that the internal paths have almost equal lengths and as long as the endvertices of the internal paths get the same weight, we have $\phi = 0$ in (2). The latter condition is realized in each of subcases from the theorem below.

Conjecture 9 *Let $\hat{G}$ be a graph with minimal index in $\mathcal{G}_{n,c}$, where $n = (3k + t)(c - 1)$ for some $k \in \mathbb{N}$ and $t = 2, 3, 4$. Further assume that in $\hat{G}$ all internal paths are of almost equal lengths. If B is some cubic multi-graph on $2(c - 1)$ vertices, then*

- *if* $t = 2$*, then* $\hat{G} = B^{(k)}$*, where* $B^{(k)}$ *is obtained from* B *by inserting* k *vertices in each edge of* B*;*
- *if* $t = 3$*, then* $\hat{G}$ *is obtained from* $B^{(k)}$ *by inserting one more vertex in each path originating from a perfect matching of* B*;*
- *if* $t = 4$*, then* $\hat{G}$ *is obtained from* $B^{(k+1)}$ *by removing a vertex in each path originating from a perfect matching of* B*.*

Slobodan was somewhat unsatisfied with the above results, as he wanted to complete the investigations on the tricyclic case. Hence, we kept these computations unpublished, trying to complete the tricyclic case. For several years Slobodan used to attack this problem during his summer holidays, but with no luck. He used to call this problem as "the nightmare".

A couple of years ago, Sebi Cioabă contacted me as he got with one collaborator some progress on the tricyclic case, but unfortunately their results have overlapped with ours. However, we all decided to join forces. After the premature death of Slobodan, 30 years after the publication of [5], we have decided to publish these results in a forthcoming paper, which will result in a posthumous publication of Slobodan.

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Graphs with Bounded Second Largest Eigenvalue
Grafovi sa ograničenom drugom najvećom sopstvenom vrednošću

Zoran Stanić

Graphs whose second largest eigenvalue (of the adjacency matrix) is comparatively small have received a great deal of attention in literature. Such graphs have a specified structure; for example, in some particular cases, they have a low connectivity. There is also a number of applications in chemistry and physics. The results obtained before 2001 have been given an adequate attention in the survey paper [S54] and the book [3]. In particular, all results concerning graphs whose second largest eigenvalue does not exceed $\frac{1}{3}$, $\sqrt{2}-1$ or $\frac{\sqrt{5}-1}{2}$ and some results concerning graphs with $\lambda_2 \leq 1$ can be found in at least one of these references. The results obtained after 2001, which treat bounds equal to 1, $\sqrt{2}$, $\sqrt{3}$ or 2, are surveyed in [4].

For any real number k, the property $\lambda_2 \leq k$ is a hereditary property which means that if, for any graph G, $\lambda_2(G) \leq k$ then, for any induced subgraph H of G, $\lambda_2(H) \leq k$. If it occurs that $\lambda_2(G) \leq k$, but at the same time no supergraph of G satisfies the same inequality, then G is called the *maximal graph* for $\lambda_2 \leq k$. Similarly, if $\lambda_2(G) > k$ and at the same time no induced subgraph of G satisfies the same inequality, then G is called the *minimal forbidden graph* for $\lambda_2 \leq k$.

In the majority of his work on this topic, Slobodan devoted his attention to graphs whose second largest eigenvalue does not exceed $\frac{\sqrt{5}-1}{2}$; so-called the *golden section bound*. The notation $\sigma = \frac{\sqrt{5}-1}{2}$ was introduced in [S48].

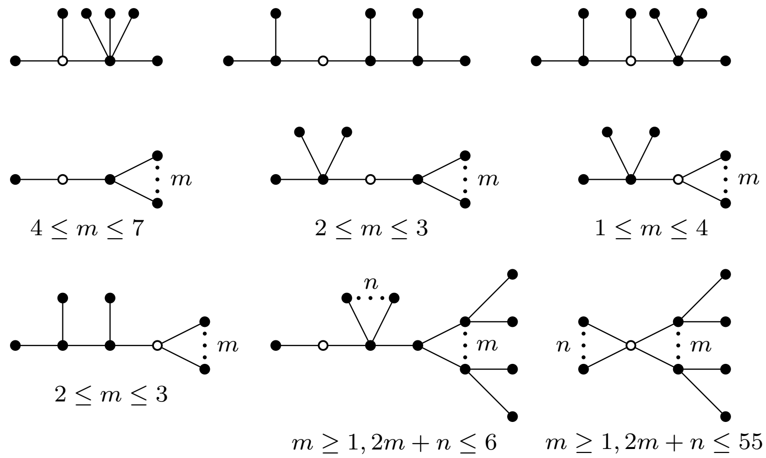

Fig. 1 Rooted trees associated with graphs satisfying $\lambda_2 < \sigma$.

It is worth mentioning that, at present moment, these graphs are not fully determined!

The paper [S48], written jointly with Cvetković, was the initial one. It contains some structural properties of such graphs. For example, since the second largest eigenvalue of the path P_4 is equal to σ, it follows that P_5 is a forbidden induced subgraph for $\lambda_2 \leq \sigma$.

In my opinion, the most significant results concerning the golden section bound are reported in [S51, S64]; some of them can also be found in [S54].

In order to consider the graphs with $\lambda_2 < \sigma$, Slobodan introduced the family of graphs denoted by Φ, which is the smallest family of graphs that contains K_1 and is closed under adding isolated vertices and taking joins of graphs. He also proved that every graph with $\lambda_2 < \sigma$ belongs to Φ (but not vice versa).

To any graph G from Φ, Slobodan associated a weighted rooted tree T_G in the following way: if $H = (H_1 \nabla H_2 \nabla \cdots \nabla H_m) \cup nK_1$ is any subrepresentation of G then a subtree T_H with a root v (of weight n) represents H whereas for each i $(1 \leq i \leq m)$ there is a vertex v_i (a direct successor of v in T_H) representing a root of H_i. It turns out that the set of graphs with $\lambda_2 < \sigma$ falls into a finite number of structured types of graph in Φ. These types are illustrated in Fig. 1 by the corresponding representing trees with emphasized roots.

It has also been proved in [S51] that the set of minimal forbidden subgraphs for $\lambda_2 < \sigma$ is finite. They all belong to Φ except for P_4 and $2K_2$.

Concerning the graphs with $\lambda_2 \leq \sigma$, we quote the following two results

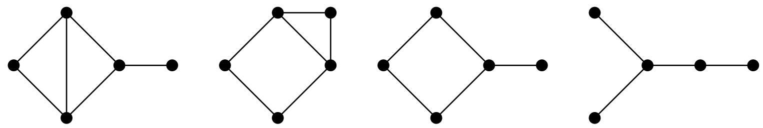

Fig. 2 Rooted trees associated with graphs satisfying $\lambda_2 < \sigma$.

of [S64].

Theorem 1 *A graph with $\lambda_2 \leq \sigma$ has at most one non-trivial component G for which one of the following holds:*

(i) G is a complete multipartite graph,

(ii) G is an induced subgraph of C_5,

(iii) G contains a triangle.

It has also been proved that the set of minimal forbidden subgraphs for $\lambda_2 \leq \sigma$ is finite. The next theorem provides more details.

Theorem 2 *If G is a minimal forbidden subgraph for $\lambda_2 \leq \sigma$ then either*

(i) G is $2K_2$ or one of the graphs depicted in Fig. 2 or

(ii) G belongs to Φ.

The remaining Slobodan's papers concerning the golden section bound are [S53, S76]. In the former one, Slobodan characterized (again, in terms of minimal forbidden subgraphs) graphs having the following property: both the graph and its complement have the second largest eigenvalue not exceeding σ. Using this characterization, he also explicitly determined such graphs. Although they appear sporadically in his later works (for example, in [S106]), in the latter reference, Slobodan concluded his work on graphs with $\lambda_2 \leq \sigma$. There, he described (to some extent) the infinite families of these which have an arbitrarily large number of vertices.

The next interesting topic are graphs with $\lambda_2 \leq 2$. These graphs are also known as the *reflexive graphs* due to their significance in the theory of reflexive groups. Slobodan dealt with these graph in only one of his papers [S55] (joint work with Radosavljević), but there the authors established possibly the most important result concerning tree-like reflexive graphs (also known as cacti). This result, which is known as the RS-theorem (where the acronym refers to the authors), reads as follows.

Theorem 3 *Let G be a graph with a cut vertex u. Then*

(i) if at least two components of $G-u$ are the supergraphs of Smith graphs, and if at least one of them is a proper supergraph, then $\lambda_2(G) > 2$;

(ii) if at least two components of $G-u$ are Smith graphs, while the rest are subgraphs of Smith graphs, then $\lambda_2(G) = 2$;

(iii) if at most one component of $G-u$ is a Smith graph, while the rest are proper subgraphs of Smith graphs, then $\lambda_2(G) < 2$.

(We believe that the reader is familiar with Smith graphs.) In relation to the previous theorem, if after removing a vertex u we get one proper supergraph, while the rest are proper subgraphs of Smith graphs, the theorem does not answer the question whether the graph is reflexive or not. This fact initialized an intensive theoretical research performed by Radosavljević and his research group. We mention that two Ph.D. theses are based on these results. A survey can be found in [2]. The reminder of [S55] is devoted to determination of bicyclic graphs with $\lambda_2 \leq 2$.

In [S56], together with Bell, Slobodan considered the second largest eigenvalue of graphs that are homeomorphic to stars, so called star-like trees. They identified those star-like trees for which the second largest eigenvalue is extremal (either minimal or maximal), when certain conditions are imposed. They also provided a valuable consideration on the way in which the second largest eigenvalue of such trees changes under local graph perturbations.

In [S151], together with Anđelić, da Fonseca and Živković, Slobodan gave certain necessary conditions for a graph to have the second largest eigenvalue of the adjacency matrix or the signless Laplacian matrix less than or equal to an arbitrary real constant.

The remaining Slobodan's research on graphs with bounded second largest eigenvalue is based on so-called *star complement technique* – a spectral tool developed for constructing some bigger graphs from their smaller parts, called star complements. In [S93], together with me, Slobodan used this technique to determine all the unicyclic graphs which can be star complements for 1 as the second largest eigenvalue. Using the graphs obtained, we next determined all graphs with which contain such star complements and satisfy $\lambda_2 = 1$. Similar technique is frequently used in search on so-called exceptional graphs for -2 as the least eigenvalue. In particular, the regular ones are determined, and since their complements satisfy $\lambda_2 = 1$, these results can also be considered in the framework of graphs with bounded second largest eigenvalue. Slobodan's contribution can be found in [S146, S149], but also in one of his well-known monographs [1].

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Graphs with Least Eigenvalue at Least -2
Grafovi sa najmanjom sopstvenom vrednošću većom ili jednakom -2

Dragoš Cvetković

It was very well known for a long time that the least eigenvalue of a *line graph* is greater than or equal to -2. However, this spectral property line graphs share with *generalized line graphs* and with a finite number of connected graphs called *exceptional graphs*. These facts explain the title of the scientific monograph [2] that Slobodan Simić published jointly with the other two authors.

The monograph is dedicated to authors' late parents, in particular to Olga Simić (1916-2002) and Kosta Simić (1907-1998).

Simić's results on the subject are well described in the monograph. 48 papers of Slobodan has been included into the bibliography of this monograph. Slobodan published several additional papers on the subject after 2004.

Monograph chapters: 1. Introduction, 2. Forbidden subgraphs, 3. Root systems, 4. Regular graphs, 5. Star complements, 6. The maximal exceptional graphs, 7. Miscellaneous results.

The notion of a line graph is a very natural concept in graph theory. It attracted Simić already in the first of his papers (see, for example, [S3], [S5], [S6] and [S9]). In these early papers line graphs appeared in the context of the so called graph equations.

First substantial paper on graphs with least eigenvalue at least -2 was the paper [S16] on generalized line graphs. It appeared in Journal of Graph Theory. The coauthors were Canadian colleague Michael Doob and myself. Results of the paper are previously published in [S14].

One of the main results in these papers is a characterization of generalized line graphs by a collection of 31 forbidden induced subgraphs. This is an analogy with the well-known characterization of line graphs by nine forbidden induced subgraph, due to L.W. Beineke [1].

Many years later a new proof of 31 forbidden subgraphs theorem [S86] has been published following an idea of Slobodan Simić.

The paper [S21] on generalized line graphs with only two authors (Radosavljević and Simić) was already in print, when a paper by M. Syslo and J. Topp with the same result was submitted to a domestic journal. At my suggestion, all interested parties agreed that the names of our Polish colleagues should be added to the list of authors. Such an unbelievable case of independent discoveries had already occurred with S. Simić (see the paper on graph equations in this book).

Main reason for the decision to publish the monograph [2] was the publication of papers [S69] and [S74] in good journals.

Paper [S69] outlined a new theory of graphs with least eigenvalue at least -2 based on star complements (see Chapter 5 of the monograph).

The maximal exceptional graphs were determined in [S74] (see Chapter 6 of the monograph). A special hard case was considered in [S71]. Detailed results on computer investigations of maximal exceptional graphs are collected in the report [S72].

Ten years later the authors of the monograph [2] published expository paper [3] describing how the monograph was used by researchers.

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Spectral Reconstructions
Spektralne rekonstrukcije

Zoran Stanić

A *spectral reconstruction* is a wide notion and, it the most general form, it can be interpreted as the problem in which we deal with a spectrum of (some matrix associated with) a graph or spectra of some related graphs, and try to deduce some information about the graph itself. For example, it is known that, in general, a graph cannot be reconstructed from its spectrum – in other words, there are graphs that share the same spectrum. On the contrary, some particular graphs are reconstructible from their spectrum. This, for example holds for paths and cycles in the case of the spectrum of their adjacency matrix. It is known for a long time that some structural properties of a graph are reconstructible from its spectrum; not to be listed here.

The question of spectral reconstruction in some form appears, at least sporadically, in a large number of Slobodan's papers. Some of them are [S42, S44, S49, S62, S65, S78, S96]. In what follows, I give a review on the three major topics considered in his work:

(i) reconstruction of the characteristic polynomial of a graph,

(ii) characterizations of graphs on the basis of their star complements and

(iii) spectral determination of graphs.

(i) The characteristic polynomial reconstruction conjecture is a spectral counterpart to the classical Ulam's conjecture (stating that the graph is reconstructible from the collection of its vertex-deleted subgraphs). This problem was posed by Cvetković in 1973 and reads: Is it true that, for $n \geq 2$ (n being the number of vertices), the characteristic polynomial $\Phi(G)$

of a graph G is determined uniquely by its polynomial deck, i.e., by the collection of the characteristic polynomials of its vertex-deleted subgraphs? Both conjectures are still open, and no counterexamples are known.

It is known that $\Phi(G)$ is determined up to the constant term. There is also a long list of invariants or graph properties that can be deduced from its polynomial deck [S98]. A number of results on this topic are obtained by Cvetković, Gutman, Lepović, Hagos, Sciriha and some other authors. The corresponding references are cited in [S98]. For example, the conjecture is affirmatively resolved in case of trees.

In his first paper on this topic [S36], Slobodan proved that the polynomial reconstruction is unique for connected graphs such that spectra of their vertex-deleted subgraphs are bounded by -2.

When I met Slobodan in 2004, he proposed this topic to me and suggested to try to resolve the conjecture in case of unicyclic graph (which was the next natural step after the trees). Later on, we published the five joint papers on this topic [S98, S106, S107, S157, S158], he became my mentor, and a part of the obtained results was included in my Ph.D. thesis. So, in [S98] we proved the uniqueness of the polynomial reconstruction for unicyclic graphs. In [S106], together with Bıyıkoğlu, we proved the uniqueness for graphs whose vertex-deleted subgraphs have the second largest eigenvalue not exceeding $\frac{\sqrt{5}-1}{2}$. I mention in passing that this paper contains another significant result – a short proof of the fact that 0 (resp. -1) belongs to the spectrum of a connected cograph (with at least two vertices) if and only if it contains duplicate (resp. co-duplicate) vertices. This problem was posed by Royle [3], and the corresponding result is Slobodan's personal contribution to the joint paper. Using some ideas from [S36], we extended the result reported therein by including the possibility that the graph under consideration is disconnected. In the latter two joint papers, we transferred the conjecture to so-called signed graphs and we extended the results of [S98, S107] with some restrictions and some additional results.

(ii) Let μ be an eigenvalue with multiplicity k of a graph G. A *star complement* for μ in G is an induced subgraph $H = G - X$, such that $|X| = k$ and μ is not an eigenvalue of H. It is known that the least eigenvalue of a connected graph G is greater than or equal to -2 if and only if G is a generalized line graph or so-called exceptional graph for this property. In an intensive research, which was mostly joint with Cvetković and Rowlinson, Slobodan considered determination of exceptional graphs, their characterization by means of minimal forbidden subgraphs, but also a question on whether a graph whose least eigenvalue is bounded by -2 is a line graph, a generalized (but not line) graph or an exceptional graph. This research was crowned by the monograph [1]. Particular results can be found in [S60, S65, S69, S71, S72, S74, S81, S86, S155].

(iii) A graph is said to be determined by the spectrum of some associated matrix if there is no other graph sharing the same spectrum. A natural question that arises immediately asks for graphs which are determined by the spectrum (or, in the context of spectral reconstruction, it asks for graphs which are reconstructible from thier spectrum). Independently of the associated matrix, for almost all graphs the answer to this question is still unknown. The question itself goes back for about 60 years, and originates from chemistry. In [2] van Dam and Haemers reposted the same question, and after this paper a number of results appeared in various literature. This topic also appears in many Slobodan's publications, and here I restrict myself on presentation of the results reported in [S115, S125 S132].

In [S115] Slobodan and me considered graphs that are related to Smith graphs. Precisely, we considered the class of graphs whose each component is either a proper subgraph of some Smith graphs or belongs to a fixed subset of Smith graphs. Then, we classified the graphs from the considered class into those which are determined, or not determined, by the Laplacian or the signless Laplacian spectrum. In the next paper, together with Cvetković, we considered the class of graphs each of whose components is either a path or a cycle and classified the graphs into those which are determined and those which are not determined by the spectrum of the adjacency matrix. We also compared the obtained results with the corresponding results for the Laplacian and the signless Laplacian spectrum. In this way we confirmed some expectations that cospectrality as a phenomenon appears most rarely in the case of the signless Laplacian spectrum. The same class of graphs, but in the context of Laplacian spectrum, was considered in the third paper (together with Wang, Huang, Belardo and Li Marzi).

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Signed Graphs and Their Eigenvalues
Označeni grafovi i njihove sopstvene vrednosti

Francesco Belardo

A *signed graph* Γ is a pair (G, σ), where $G = (V, E)$ is a graph, called the *underlying graph*, and $\sigma\colon E \longrightarrow \{-1, +1\}$ is the *sign function*. The edge set of a signed graph is composed of subsets of positive and negative edges. Signed graphs first appeared in the context of social (signed) networks describing the relation of being friend (edge positively valuated) or enemy (negative edge) between two actors. Nowadays signed graphs appear in the literature of several disciplines as, for example, quantum computing.

Similarly to unsigned graphs, matrices can be used to study signed graphs, and we get a Spectral Theory of Signed Graphs which nicely encapsulates the usual spectral graph theory. For instance, if a signed graph has all positive edges, then its Laplacian matrix is the usual Laplacian of the underlying graph, while if all edges are negative, then its Laplacian is the signless Laplacian of the underlying graph. I have written some papers on the Laplacian and signless Laplacian of graphs, and several of them with Slobodan. When I first met the signed graphs in 2013, I realized that we could use our experience to develop a spectral theory for signed graphs that could generalize that of simple unsigned graphs. After my first paper on the least Laplacian eigenvalue of signed graphs [1], I asked Slobodan's support to further develop the theory. After some discussions, he recognized the potential of such spectral theory based on signed graphs, and he became excited in studying this new topic.

The first paper was devoted to the coefficients of the Laplacian polynomial: we gave a combinatorial interpretation in terms of signed TU-subgraphs, which generalizes the TU-subgraphs from the signless Laplacian theory. For the proof, we decided to reuse the "unsigned" proof (used in his famous paper [S100] but we needed to develop the concept of signed line graph. On the other hand, such concept was already developed by T. Zaslavsky in several papers around 30 years earlier, but we were not satisfied with his signing of the line graph of the underlying graph (see, for example [2]). In fact, we arrived to a signature which is the opposite signature (all edges get reversed signs). Our definition of signed line graph was more natural to us, as it is coherent to the (signless) Laplacian theory of unsigned graphs, and to the concept of generalized line graph used to describe the graphs with least eigenvalue not less than -2, as well. This led to a series of passionate emails between Slobodan and Tom where both discussed the concept of signed line graph and their point of view on this respect.

The crucial role is taken by the incidence matrix. With signed graphs, instead of one arrow we give two arrows assigned to edges. This yields to *bi-directed graphs*. More precisely, if $\Gamma = (G, \sigma)$ is a signed graph, we define a bi-orientation η on G, as G_η, where

$$\eta : V(G) \times E(G) \to \{-1, +1, 0\}$$

satisfies the following three conditions:

(*i*) $\eta(u, vw) = 0$ whenever $u \neq v, w$;

(*ii*) $\eta(v, vw) = +1$ (or -1) if an arrow at v is going into (resp. out of) v;

(*iii*) $\eta(v, vw)\eta(w, vw) = -\sigma(vw)$.

In a few words, the positive and negative edges are represented by bi-directed edges according to the Figure 1.

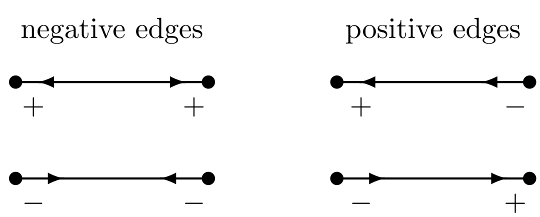

Fig. 1 Bidirected edges

Once the incidence matrix $B = B_\eta$ is given, we can define two new signed graphs, as in Figure 2.

Theorem 1 describes the relation among the polynomials of the signed graphs depicted in Figure 2. Observe that both (signatures of) the signed

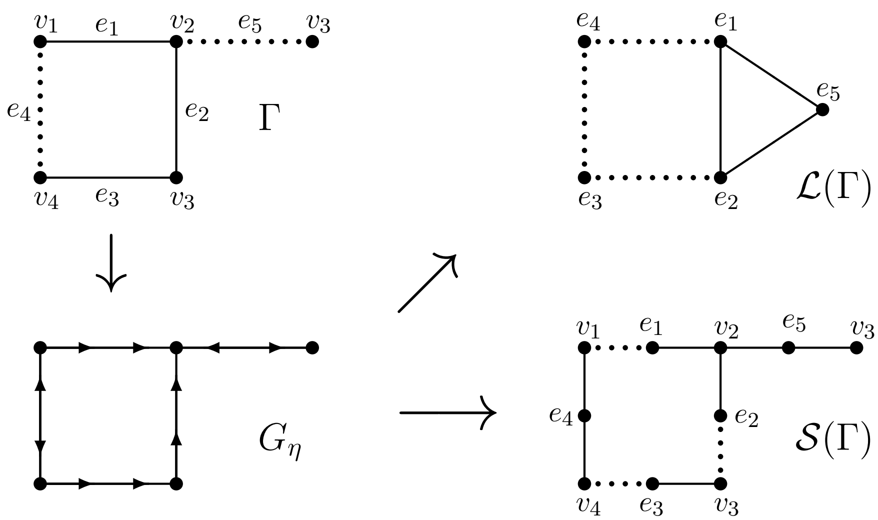

Fig. 2 A signed graph and the corresponding signed subdivision and line graph.

line graph and the signed subdivision graph depend on the bi-orientation η given on the underlying graph G of Γ. However a different η' gives an equivalent (and cospectral) signed compound graph. Furthermore, as far as we know, no definition has been given, prior to ours, for the signed subdivision graph. The signed line graph $\dot{\mathcal{L}}(\Gamma)$ defined by T. Zaslavsky has opposite signature (and then spectrum). but it has the advantage of the following equality: $\dot{\mathcal{L}}(G, -) = -\mathcal{L}(G)$, which easily permits iterations of the line graph operator.

Theorem 1 *Let Γ be a signed graph of order n and size m, and let $\phi(\Gamma)$ and $\psi(\Gamma)$ be its adjacency and Laplacian characteristic polynomials, respectively. Then it holds*

1^o $\phi(\mathcal{L}(\Gamma), x) = (x+2)^{m-n}\psi(\Gamma, x+2)$,

2^o $\phi(\mathcal{S}(\Gamma), x) = x^{m-n}\psi(\Gamma, x^2)$.

In 2015, Slobodan and I were visiting University of Primorska (Koper, Slovenia) at the same time. During the visit period we worked on signed graphs. We discussed several new ideas and most of them were concretized in several papers.

In the first paper, we considered together with I. Sciriha, the eigenspaces of the signed line graphs and signed subdivisions graphs in terms of the eigenspace of the root signed graph. This paper is a generalization to signed graph of an older one, written by Slobodan and I. Sciriha. One of the results therein reads:

Theorem 2 *Let $B = B_\eta$ be the bi-directed incidence matrix associated to G_η corresponding to a connected signed graph Γ. Then we have:*

1^o $\{\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_s}\}$ *is a L-eigenbasis of* Γ *for* $\mu \neq 0$ *if and only if* $\{B^\top \mathbf{x_1}, B^\top \mathbf{x_2}, \ldots, B^\top \mathbf{x_s}\}$ *is an A-eigenbasis of* $\mathcal{L}(\Gamma)$ *for* $\mu - 2$*;*

2^o $\{\mathbf{y_1}, \mathbf{y_2}, \ldots, \mathbf{y_t}\}$ *is an A-eigenbasis of* $\mathcal{L}(\Gamma)$ *for* $\lambda \neq -2$ *if and only if* $\{B\mathbf{y_1}, B\mathbf{y_2}, \ldots, B\mathbf{y_t}\}$ *is a L-eigenbasis of* Γ_η *for* $\lambda + 2$.

Moreover, in the same paper, we gave a similar result for the eigenspaces of the signed subdivision graph. Also, we considered the eigenspaces of -2 in signed line graphs, and of 0 in the signed subdivision graphs. The eigenspace of -2 in signed line graphs is described by means of star complements, but for its proof we had another paper [S154] prepared almost at the same time.

In a subsequent paper [S155], together with T. Pisanski, we used the eigenvalues of signed line graph to identify the (unsigned) connected graph whose least adjacency eigenvalue is closest (but different) to -2. In fact, we make use of the "signed" spectral theory to derive a result in the usual spectral theory. Let $T(a, b, c)$ be the T-shaped tree of order $a + b + c + 1$ consisting of 3 paths of length $a + 1$, $b + 1$ and $c + 1$ sharing exactly one endvertex. We got the following result

Theorem 3 *Let G be a connected graph on n vertices whose least eigenvalue (for the adjacency spectrum) is minimal but strictly greater than -2. Then the following holds:*

(i) *if* $n \neq 6, 7, 8$ *then* $G = T(1, 1, n - 3)$*;*

(ii) *if* $n = 6, 7, 8$ *then* $G = T(1, 2, n - 4)$.

Additionally, we were also able to give an application to HOMO-LUMO. In fact, the HOMO-LUMO is a spectral invariant considered in Chemical Graph Theory corresponding to the gap between the least positive eigenvalue and the largest nonnegative eigenvalue. Using the eigenvalues of the signed subdivision graph, the least eigenvalue becomes the smallest positive eigenvalue in the subdivision graph, so that we could compute the HOMO-LUMO for some graphs.

The last paper I had with Slobodan on signed graphs was [S154], where, together with E.M. Li Marzi, we considered the powerful star complement technique in terms of the spectra of signed graphs. In this paper we described the eigenbasis for -2 in signed line graphs. Such eigenspace can be directly obtained from a connected spanning signed subgraph Φ whose signed line graph does not have -2 as an eigenvalue (the so-called *signed foundation*). A foundation Φ is either a spanning tree whenever Γ is balanced, or it is a unbalanced unicyclic graph. From one-edge extensions of the foundation Φ, namely $\Phi + e$, we obtain three kinds of spanning subgraphs of Γ: those containing either a balanced cycle, or the *double-unbalanced infinite graph*, or the *double-unbalanced dumbbell* (i.e. graphs obtained from two unbalanced cycles joined by a path, possibly of length zero – see Figure 3). By properly weighting the edges of $\Phi + e$ (that is a subgraph of Γ) we get a (-2)-eigenvector for $\mathcal{L}(\Gamma)$. The edges corresponding to nonzero entries of the (-2)-eigenvector are called *heavy edges*, while the others are the *light edges*. Let Θ be the subgraph of $\Phi + e$ consisting of all heavy edges of $\Phi + e$. Since Θ consists of heavy edges, then Θ is said to be the *core subgraph* of $\Phi + e$. For each $e \in E(\Gamma \setminus \Phi)$ we get a different $\Phi + e$ (with a corresponding core subgraph Θ) from which we build a (-2)-eigenvector, which will be linearly independent from those similarly obtained. We have that Θ is either a balanced cycle, or a double-unbalanced infinite graph, or a double-unbalanced dumbbell (see again Figure 3).

The following theorems are proved in [S154].

Theorem 4 *Let Θ be a balanced cycle and $\Theta_{\mathcal{L}}$ be its signed line graph. Then, under the above notation, the vector $\mathbf{a} = (a_0, a_1, \dots, a_{q-1})^\top$, where*

$$a_i = (-1)^i \Big[\prod_{s=1}^{i} \nu(s) \Big] a_0 \qquad (i = 0, 1, \dots, q-1) \quad \text{and}$$

$$\nu(s) \;=\; \sigma_L(e_{s-1}e_s) \;=\; \eta(s, e_{s-1})\eta(s, e_s),$$

is an eigenvector of $\Theta_{\mathcal{L}}$ for -2. Moreover, it can be extended to a (-2)-eigenvector of $\mathcal{L}(\Gamma)$ by putting zeros at all other entries.

Theorem 5 *Let Θ be a double-unbalanced infinite graph and $\Theta_{\mathcal{L}}$ be its signed line graph. Then, under the above notation, the vector $\mathbf{a}' \dot{+} \mathbf{a}''$, where*

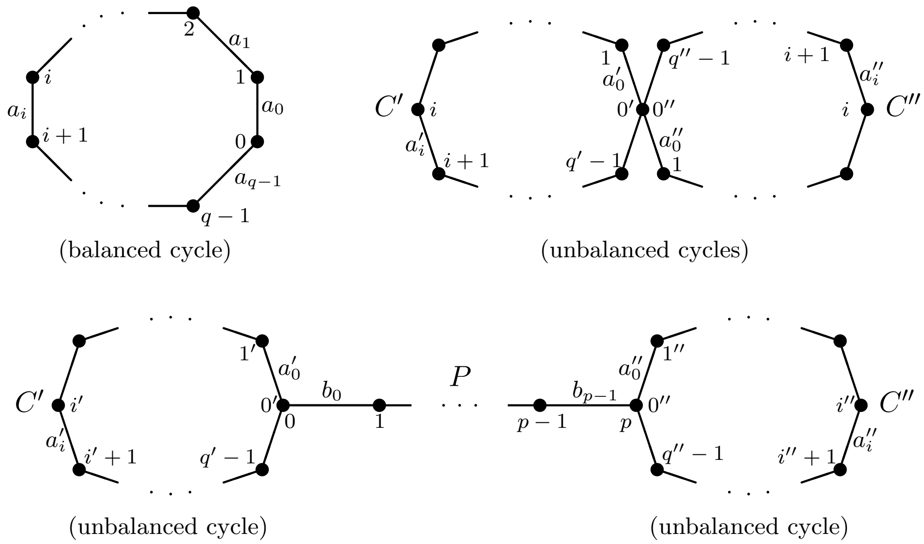

Fig. 3 Three types of core subgraphs.

$\mathbf{a}' = (a_0', a_1', \ldots, a_{q'-1}')^\top$, $\mathbf{a}'' = (a_0'', a_1'', \ldots, a_{q''-1}'')^\top$, *and*

$$a_i' = (-1)^i \Big[\prod_{s=1}^{i} \nu(s)\Big] a_0' \quad (i = 0, 1, \ldots, q'-1);$$
$$a_i'' = (-1)^i \Big[\prod_{s=1}^{i} \nu(s)\Big] a_0'' \quad (i = 0, 1, \ldots, q''-1);$$

is an eigenvector of $\Theta_{\mathcal{L}}$ *for* -2 *provided* $a_0' \neq 0$ *is arbitrary and* $a_0'' = -\widehat{\nu}(0', 0'')a_0'$, *where* $\widehat{\nu}(0', 0'') = \eta(0', e_0')\eta(0'', e_0'')$. *Moreover, it can be extended to a* (-2)*-eigenvector of* $\mathcal{L}(\Gamma)$ *by putting zeros at all other entries.*

Theorem 6 *Let* Θ *be a double-unbalanced dumbbell and* $\Theta_{\mathcal{L}}$ *be its signed line graph. Then, under the above notation, the vector* $\mathbf{a}' \dot{+} \mathbf{b} \dot{+} \mathbf{a}''$, *where* $\mathbf{a}' = (a_0', a_1', \ldots, a_{q'-1}')^\top$, $\mathbf{b} = (b_0, b_1, \ldots, b_{p-1})^\top$, $\mathbf{a}'' = (a_0'', a_1'', \ldots, a_{q''-1}'')^\top$, *and*

$$\begin{aligned} a_i' &= (-1)^i\Big[\prod_{s=1}^{i}\nu(s)\Big]a_0' \quad (i=0,1,\ldots,q'-1),\\ b_i &= (-1)^i\Big[\prod_{s=1}^{i}\nu(s)\Big]b_0 \quad (i=0,1,\ldots,p-1),\\ a_i'' &= (-1)^i\Big[\prod_{s=1}^{i}\nu(s)\Big]a_0'' \quad (i=0,1,\ldots,q''-1), \end{aligned}$$

is an eigenvector of $\Theta_{\mathcal{L}}$ *for* -2 *provided* $b_0 \neq 0$ *is arbitrary,* $a_0' = -\frac{1}{2}\widehat{\nu}(0,0')b_0$ *and* $a_0'' = -\frac{1}{2}\widehat{\nu}(p,0'')c_0$*, where* $\widehat{\nu}(0,0') = \eta(0,e_0')\eta(0,f_0)$ *and* $\widehat{\nu}(p,0'') = \eta(p,f_{p-1})\eta(0'',e_0'')$ *. Moreover, it can be extended to a* (-2)*-eigenvector of* $\mathcal{L}(\Gamma)$ *by putting zeros at all other entries.*

Slobodan, with Z. Stanić, had some further papers on signed graphs, in which they studied the *Polynomial Reconstruction Problem* (PRP, for short). The PRP is a classical problem in Spectral Graph Theory, as it is a spectral variant of the Ulam conjecture. The PRP asks whether for a graph G the polynomials of its vertex-deleted subgraphs (the *polynomial deck*) determines the characteristic polynomial of G. The PRP gets a negative answer if there exists two non cospectral graphs sharing the same polynomial deck. So far, for (unsigned) graphs there are no such examples known for $n > 2$. Of course, the same question can be considered in terms of signed graphs.

Slobodan and Z. Stanić wrote two papers on this topic. In their first paper [S157], they defined the problem and gave some restrictions to the existence of a counterexample. They further considered disconnected signed graphs and unicyclic graphs (trees degenerate in the usual theory, as they are balanced), and they proved, among others, the following theorem.

Theorem 7 *If* (U,σ) *is a non-regular signed unicyclic graph, then the polynomial reconstruction is unique. If* (G,σ) *is a disconnected signed graphs that have a unicyclic component, then the polynomial reconstruction of is unique, as well.*

Notably, they found that if we consider regular unicyclic graphs, namely cycles, then we get a counterexample pair. Let C_n^+ (resp. C_n^-) denote a balanced (resp. unbalanced) cycle, and $\phi(\Gamma)$ be the adjacency characteristic polynomial of Γ. Then, it is possible to verify that

$$\phi(C_n^+) = \phi(C_n^-) - 4.$$

Clearly, the signed cycles share the same polynomial deck (consisting of n paths on $n-1$ vertices) but their polynomials are not the same. It remains as an open problem to identify a counterexample in which the two signed graphs do not share the underlying graph. Additionally, in a sequel paper [S158], they considered the polynomial reconstruction problem for signed graphs whose least eigenvalue exceeds -2.

By this I have concluded the description of the main results obtained by Slobodan in the Spectral Theory of Signed Graphs. We had in mind several other projects and, in 2016, I was waiting for Slobodan to visit me in Naples to work on further papers in this topic. Unfortunately, Slobodan never came to Naples, as he started to face his problems which, sadly, led to a premature conclusion of his remarkable career. Without these unlucky events, this list would have been much longer.

References

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Special Issue of DMGT in Memory of Slobodan K. Simić
Specijalna sveska časopisa DMGT u spomen na Slobodana K. Simića

Milica Anđelić
Francesco Belardo
Zoran Stanić

In 2018, as Slobodan's former Ph.D. students, we decided to prepare a special issue of some international mathematical journal to be dedicated to his 70th birthday, and to his remarkable scientific career. Slobodan was a member of the Editorial Board of the Polish journal named *Discussiones Mathematicae Graph Theory* (in short, DMGT), whose Editor-in-Chief is Mieczysław Borowiecki, a longtime friend of Slobodan. DMGT has a long tradition and publishes high-quality refereed original papers, furthemore, it is indexed in all relevant international reviewing databases. So this journal appeared to be a natural choice. Eventually, we made an agreement with DMGT for such a special issue to include scientific contributions dedicated to Slobodan on invitation basis.

According to the journal policy, we were allowed to have around 20 papers which will be published in the first issue of 2020. The unfortunate and premature Slobodan's death lead us to change the scope of the issue to the career and memory of Professor Slobodan K. Simić. Among the received contributions, one is a biographical note co-authored by his mentor Dragoš Cvetković and Peter Rowlinson. The others are written by around 50 well-known mathematicians working in spectral graph theory; many of them were Slobodan's former co-authors, collaborators and close friends.

We are indebted to the Editorial Board of DMGT, and in particular to Professor Mieczysław Borowiecki, for allowing us to organize the issue devoted to Slobodan. We are very grateful to all invited authors, who immediately accepted our call and submitted their high-quality manuscripts concerning contemporary topics in (spectral) graph theory.

We hope that this special issue will leave a significant trace in mathematical society, and keep memory of our honorable mentor Slobodan K. Simić.

Neke fotografije
Some Photos

S. Simić i D. Cvetković na konferenciji u Mađarskoj 1973. godine

(Capo Mulini) Acireale, Conference "Combinatorics 2004"

Relaksacija na Zlatiboru, 2005.

Britanski matematičar Peter Rowlinson sa Simićem i suprugom, Škotska 2005

Profesor Francis Bell i Slobodan Simić sa suprugama, Škotska 2005

Sa sinom Stefanom, Sicilija 2006

Italijanski matematičar E.M. Li Marzi i Simić sa suprugama,
Taormina, Sicilija, 2006.

Lj. Branković, S. Simić, D. Cardoso i H. Sachs na konferenciji SGA2006
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Veče u Skadarliji, konferencija MAGT 2006

Bagnara Calabra, u pozadini Sicilija, 2010

Italijanski matematičar Francesco Belardo i Simić, Montalbano Elicona, u pozadini Castle Federico II, 2010

S. Simić i D. Cvetković u svom radnom kabinetu u
Matematičkom institutu SANU 2014. godine