Petak, 21 mart 2014 godine

Srminar mehanke

Odeljenja za mehaniku i Odeljenja za matematiku

Mathematical Institute SANU Belgrade

Grant OI174001

“Dynamics of hybrid systems with complex structures. Mechanics of Materials”

Report for 2013

Катица Р. (Стевановић) Хедри

Одељење за механику Математичког института САНУ у Београду
и Машински факултет Универзитета у Нишу
Прив. адреса: 18000- Ниш, Србија, ул. Војводе Танкосића 3/22.
e-mail: khedrih@eunet.rs

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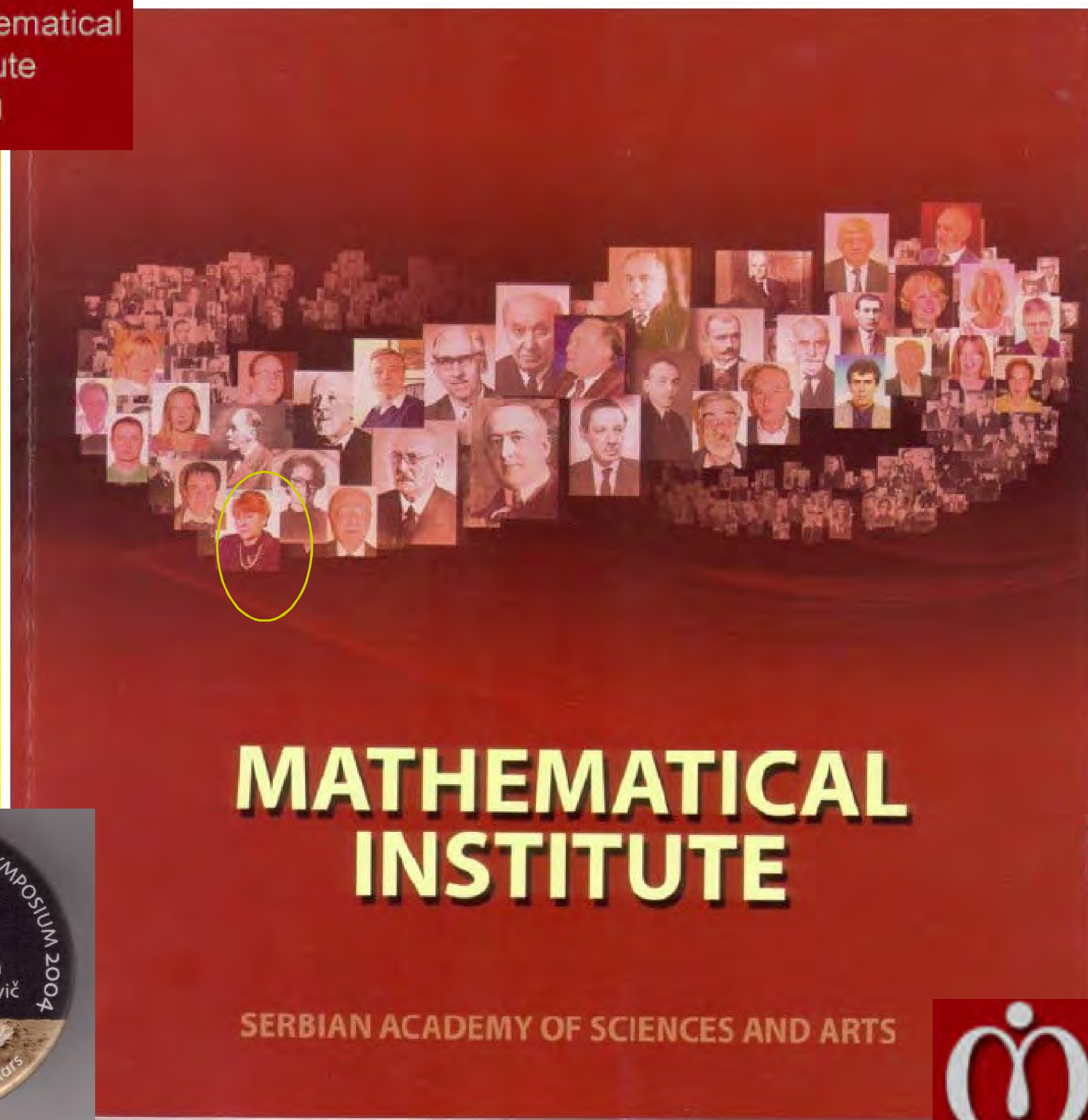

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- Contact Information

Current Projects (2011–2014)
supported by the Ministry of Science, Technology and Development, Republic of Serbia

Fundamental Research | Interdisciplinary | Technological Development | Past Projects

Fundamental Research

PROJECT 174001	**Dynamics of hybrid systems with complex structures. Mechanics of materials (Dinamika hibridnih sistema složenih struktura. Mehanika materijala)** Project leader: ***Professor Dr. Katica R. (Stevanović) Hedrih*** khedrih@eunet.rs Project Description (English | Serbian) List of Researchers Project Activities
PROJECT 174020	**Geometry and Topology of Manifolds, Classical Mechanics and Integrable Dynamical Systems (Geomterija i topologija mnogostrukosti, klasična mehanika i integrabilni dinamički sistemi)** Project leader: ***Dr. Vladimir Dragović*** vladad@mi.sanu.ac.rs Project Description (English | Serbian) List of Researchers

Projekat ON 174005

Viskoelasticnost frakcionog tipa I optimizacija oblika u teorji stapova
Viscoelasticity fractional type and optimization in theory of rods
Project Leader Teodor Atanackovic in 2013
Project Leader in 2014 Dusan Zorica

Petak, 21 mart 2014 godine

Srminar mehanke

Odeljenja za mehaniku i Odeljenja za matematiku

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Катица Р. (Стевановић) Хедрих

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Прив. адреса: 18000- Ниш, Србија, ул. Војводе Танкосића 3/22.
e-mail: khedrih@eunet.rs

Teme su:

1. Analitička mehanika diskretnih sistema frakcionog reda. Dinamika reonomnih i reoloških i sistema sa neholonomnim vezama

2. Nelinearni i retki fenomeni u dinamici hibridnih sistema spregnutih krutih i deformabilnih tela. Prenos energije kroz sistem. Model hibridnog sistema, koji sadrži klatna sa oscilujućim tačkama vešanja duž krivolinijskih putanja (model prema *European patent No. EP1514026*, 28.03.2007), sa ciljem optimizacije kinetičkih parametara stabilnosti i upravljivosti dinamikom istog. Sinhronizacije podsistema hibridnih sistema sa ciljem dobijanja kriterijuma i metodologija za modeliranje prototipa hibridnog sistema. Planiranje teorijskih osnova odgovarajućeg eksperimenta.

3. Modeli bioloških oscilatora i fenomeni dinamike i prenosa signala, informacija i energije kroz njihove kompleksne strukture. Mehanika bio mehaničkih sistema sa spregnutim poljima.

4. Mehanika diskretnih modela kontinuuma - Teorija i primene. Dinamika homogenih struktura spregnutih deformabilnih tela i standardnih elemenata konstitutivnih relacija na bazi linearno elastičnih, nelinearnoelastičnih, visokoelastičnih i/ili naslednih svojstava i/ili svojstava frakcionog reda

5. Fenomeni dinamike sistema sa trenjem, diskontinuitetima svojstava kinetičkih parametara. Vibroudarni sistemi sa trenjem. Fenomeni diskontinuiteta u svojstvima kinetičkih parametara.

6. Dinamika i stabilnost hibridnih sistema u interakciji kruto, čvrsto telo i fluid

7. Dinamika loma i oštećenja materijala po metodi diskretnog kontinuuma. Kinetička stanja dinamike vrha prsline u materijalu sa spregnutim poljima.

8. Upravljanje dinamikama i hibridnim sistemima

Ključne reči:

Hibridni,
nelinearni,
analtička mehanika,
izvod necelog reda,
diskretni kontinuum,
stabilnost,
vibroudar,
upravljanje
Fenomeni nelinearne dinamike
Kvalitativne I matematicke analogije

Овера извештаја	
НИО реализатор	**N**
200029-Математички институт САНУ у Београду	
200104-Математички факултет у Београду	
200105-Машински факултет у Београду	
200109-Машински факултет у Нишу	
200155-Факултет техничких наука, Косовска Митровица	
200131-Технички факултет у Бору	
200107-Факултет инжењерских наука Универзитета у Крагујевцу	
200036-Институт Кирило Савић у Београду	
200223-Иновациони центар Електротехничког факултета д.о.о., Београд	
200252-Државни Универзитет у Новом Пазару	
200133-Технолошки факултет у Лесковцу	
200213-Иновациони центар Машинског факултета у Београду ДОО	

Истраживачи ангажовани у години за коју се подноси извештај

Р.Б.	ЈМБГ	Име	С	Презиме	Титула	Звање	ДАТУМ стицања звања (дд/мм/гггг)	Шифра НИО	Тренутни статус	Тренутни БИМ	БИМ за наредну годину	Остаје на пројекту	Категорија (стање)	Е -
1	2808944735024	Катица	Р	Хедрих-Стевановић	3-Dr	12-Редовни професор	15/11/1975	200029	Да	0	0	Да	А1	kh
2	1408975177656	Јулијана	Д	Симоновић	3-Dr	2-Асистент	07/12/2012	200109	Да	8	8	Да	А1	bj
3	0203950730062	Томислав	Б	Петровић	3-Dr	5-Редовни професор	16/09/1981	200109	Да	8	8	Да	А4	<z
4	0903948910004	Владимир	М	Рачевић	3-Dr	5-Редовни професор	16/06/1985	200155	Да	8	8	Да	А2	an
5	2202949910015	Златибор	С	Васић	3-Dr	5-Редовни професор	22/05/1987	200155	Да	8	8	Да	А7	srd
6	1409968913017	Срђан	В	Јовић	3-Dr	3-Доцент	15/02/2011	200155	Да	8	8	Да	А2	srd
7	1609948715307	Јулка	Д	Кнежевић-Мијановић	3-Dr	5-Редовни професор	17/09/1979	200104	Да	8	8	Да	А4	kn
8	3012950710178	Драгутин	Љ	Дебељковић	3-Dr	5-Редовни професор		200105	Да	8	8	Да	А4	dd
9	0802952710272	Драгомир	Н	Зековић	3-Dr	5-Редовни професор	22/06/1984	200105	Да	8	8	Да	А5	dz
10	1210965740080	Сретен	Б	Стојановић	3-Dr	4-Ванредни професор	16/03/2006	200133	Да	8	8	Да	А3	ssr
11	0108943725026	Вера	Б	Николић-Станојевић	3-Dr	5-Редовни професор	28/03/1988	200252	Да	0	0	Да	А4	ver
12	2110978735026	Анђелка	Н	Хедрих	2-Mr	2-Асистент		200252	Да	0	8	Да	А5	har
13	1006971425050	Ивана	Д	Атанасовска	3-Dr	10-Научни сарадник	10/06/2004	200036	Да	4	4	Да	Т2	ivi
14	2712970725013	Јелена	М	Вељковић-Ђоковић	3-Dr	4-Ванредни професор	21/12/2001	200131	Да	8	8	Да	А4	jel
15	2402956710438	Маринко	Д	Угрчић	3-Dr	5-Редовни професор		200029	Да	6	6	Да	А7	ug

16	0107963785010	Наташа	Р	Тришовић	3-Dr	4-Ванредни професор	06/11/2007	200105	Да	4	4	Да	T1	nt
17	1208948710041	Стеван	М	Максимовић	3-Dr	5-Спољни сарадник	16/07/1999	200029	Да	5	5	Да	A5	<r
18	0310973715274	Катарина	С	Максимовић	3-Dr	23-Спољни сарадник	27/08/2010	200029	Да	3	3	Да	A5	kr
19	1604950725024	Љиљана	Р	Вељовић	3-Dr	3-Доцент	22/07/2011	200107	Да	4	8	Да	A5	ve
20	2206965730057	Милош	М	Јовановић	3-Dr	3-Доцент	27/12/2007	200109	Да	0	0	Да	A6	jn
21	0212971710095	Горан	С	Симеуновић	3-Dr	9-Истраживач сарадник	02/09/2010	200213	Да	12	12	Да	A4	g_
22	2105973742012	Небојша	Ј	Димитријевић	3-Dr	2-Асистент	20/07/2012	200105	Да	8	8	Да	A7	nd
23	3010941710030	Душан	Ј	Микичић	3-Dr	5-Редовни професор		200029	Не	0	0	Не	A4	kh
24	0411921710100	Ђорђе	З	Мушицки	3-Dr	5-Редовни професор		200029	Не	0	0	Не	A4	kh
25	1902940710178	Милутин	М	Марјанов	3-Dr	5-Редовни професор		200029	Не	0	0	Не	A4	m
26	3110986730029	Данило	З	Карличић	2-Mr	9-Истраживач		200029	Да	12	12	Да	A4	da

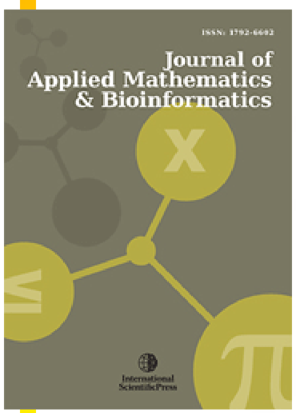
Р.Б.	ЈМБГ	Име	С	Презиме	Титула	Звање	ДАТУМ стицања звања (дд/мм/гггг)	Шифра НИО	Тренутни статус	Тренутни БИМ	БИМ за наредну годину	Остаје на пројекту	Категорија (стање)	Е
						сарадник								
27	2903984730079	Милан	С	Цајић	2-Mr	9-Истраживач сарадник		200029	Да	12	12	Да	A4	ca
28	0305970727227	Тамара	Н	Несторовић-Трајков	3-Dr	20-Странац		200029	Да	0	0	Да	A1	ta
29	0000000GiuReg	Rega		Giuseppe	3-Dr	20-Странац		200029	Да	0	0	Да	A1	G
30	0000000MacJ.	J. A. Tenreiro		Machado	3-Dr	20-Странац		200029	Да	0	0	Да	A1	jt
31	0000000AwrJan	Jan		Awrejcewicz	3-Dr	20-Странац		200029	Да	0	0	Да	A1	av
32	0000000BalJos	Jose Manuel		Balthazar	3-Dr	20-Странац		200029	Да	0	0	Да	A1	jo
33	0000000SinSub	Subhash		Sinha	3-Dr	20-Странац		200029	Да	0	0	Да	A1	<s
34	0000000WarJer	Jerzy		Warminski	3-Dr	20-Странац		200029	Да	0	0	Да	A1	j.v
35	0000000BalDum	Dumitru		Baleanu	3-Dr	20-Странац		200029	Да	0	0	Да	A1	du
36	0000000YabHir	Hiroshi		Yabuno	3-Dr	20-Странац		200029	Да	0	0	Да	A1	ya
37	0000000NayAli	Ali Hasan		Nayfeh	3-Dr	20-Странац		200029	Да	0	0	Да	A1	ar
38	0000000CarMat	Matthew		Cartmell	3-Dr	20-Странац		200029	Да	0	0	Да	A1	<r
39	0000000MikYur	Yuri		Mikhlin	3-Dr	20-Странац		200029	Да	0	0	Да	A1	m
40	1906949720023	Илија	Ж	Николић	3-Dr	5-Редовни професор		200107	Да	4	4	Да	A7	il
41	0309986735063	Марија	Б	Стаменковић	2-Mr	9-Истраживач сарадник		200029	Да	12	12	Да	A4	s_
42	1702986730018	Никола	Д	Нешић	2-Mr	9-Истраживач сарадник		200029	Да	0	0	Да	A4	ni
43	1205987766039	Марија	А	Микић	2-Mr	2-Асистент		200104	Да	8	8	Да	A4	m
44	2605987102388	Љубинко	Б	Кевац	2-Mr	9-Истраживач сарадник		200223	Да	12	12	Да	A4	lj
45	3012987782624	Владимир	Р	Вељић	2-Mr	8-Истраживач приправник		200213	Не	0	0	Не	A4	ve
46	2405986930008	Радослав	Д	Радуловић	2-Mr	2-Асистент		200105	Да	8	8	Да	A4	rr

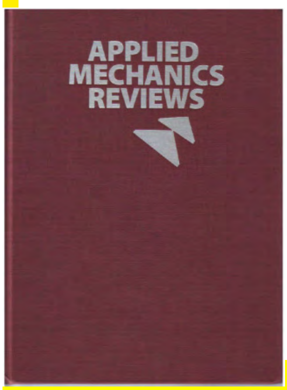
Докторанти ангажовани на пројекту

44 istrazivaca, 12 iz inostranstva, 8 истраживача је има мање од 28 година, svi ostali 25 dr nauka, 6 je sa NULA IM, 19 JE PLACENIH, 5 je doktoriralo od 2011 godine do 2013,

Број референци по категоријама

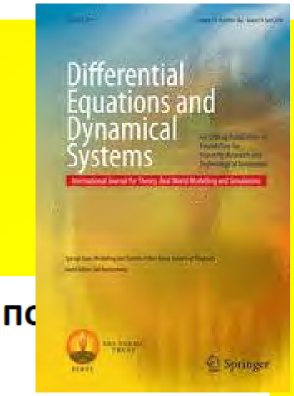
M14	11
M18	1
M21	**4**
M22	**3**
M23	**13**
M24	**13**
M31	2
M32	1
M34	9
M37	1
M32	3
M33	40
M42	5
M51	5
M52	3
M63	3
M66	3

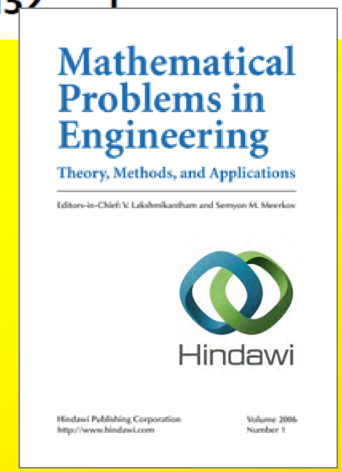


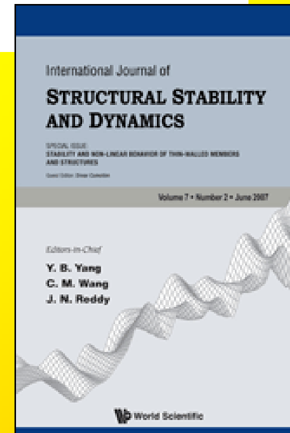


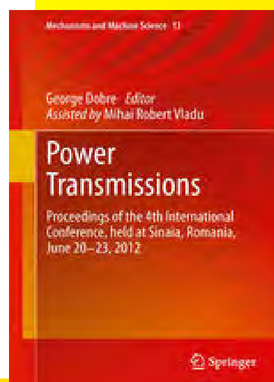
2* Број пленарних предавања и предавања по по

M31	**2**
M32	**1**







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I* Уређивање и садржај специјалног броја часописа

НАУЧНО ДРУШТВО СРБИЈЕ

SCIENTIFIC REVIEW

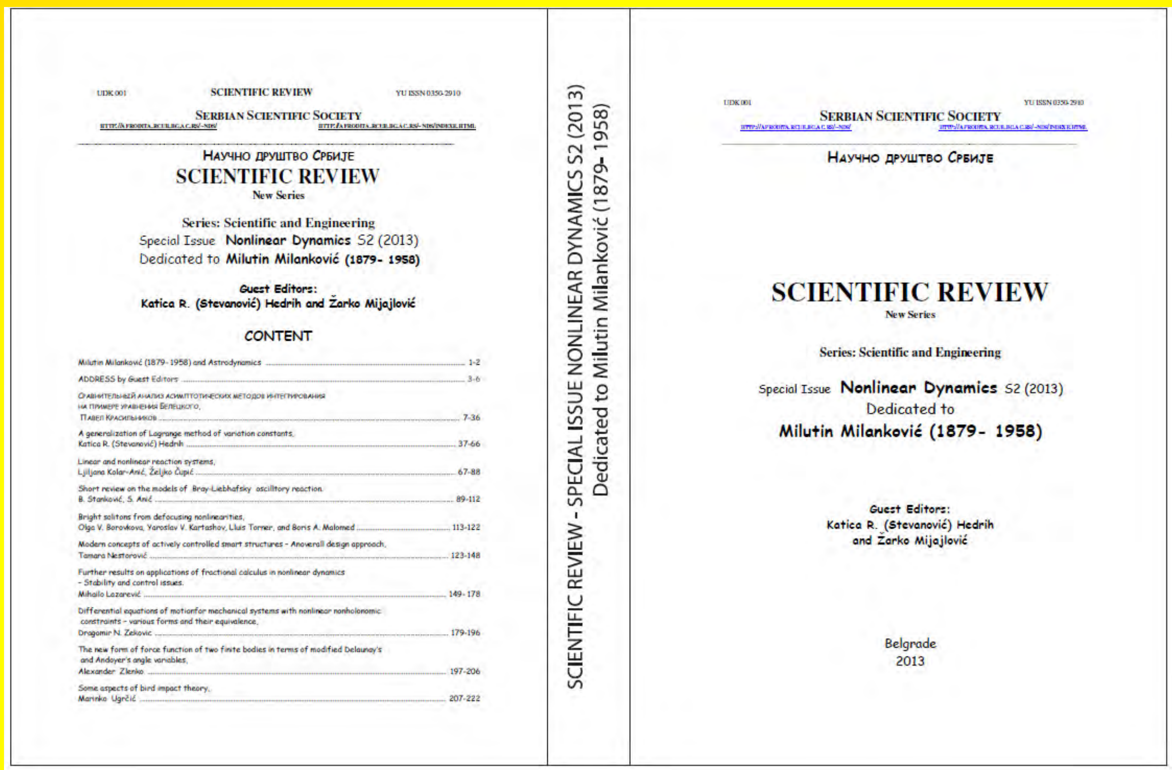
New Series: Series: Scientific and Engineering

Special Issue **Nonlinear Dynamics** S2 (2013)

Dedicated to **Milutin Milanković (1879- 1958)**

Guest Editors: Katica R. (Stevanović) Hedrih and Žarko Mijajlović

Belgrade, 2013.



UDK 001 SCIENTIFIC REVIEW YU ISSN 0350-2910

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SCIENTIFIC REVIEW - SPECIAL ISSUE NONLINEAR DYNAMICS S2 (2013)

Dedicated to Milutin Milanković (1879- 1958)

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Belgrade

2013

THEORETICAL AND APPLIED MECHANICS
TEORIJSKA I PRIMENJENA MEHANIKA

Series: Special Issue - Address to Mechanics, **Vol. 40 (S1), 2012.**

ADDRESS TO MECHANICS:
SCIENCE, TEACHING AND APPLICATIONS
GUEST EDITOR: KATICA R. (STEVANOVIĆ) HEDRIH

SERBIAN SOCIETY OF MECHANICS
SRPSKO DRUŠTVO ZA MEHANIKU

THEORETICAL AND APPLIED MECHANICS
TEORIJSKA I PRIMENJENA MEHANIKA

ISSN 1450-5584

THEORETICAL AND APPLIED MECHANICS

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Series: Special Issue - **Address to Mechanics**, Vol. 40 (S1), 2012.

VOL 1-2 , 2013

Serbian Society of Mechanics

BELGRADE 2012.

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Publication

All underlined issues are free for reading. The others were printed before common practice of www-access.

Volume 40 / 2013
Number 1 / Number 2 / Number 3 / Number 4

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Volume 38 / 2011
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Volume 37 / 2010
Number 1 / Number 2 / Number 3 / Number 4

Volume 36 / 2009
Number 1 / Number 2 / Number 3 / Number 4

Volume 35 / 2008
Number 1 / Number 2 / Number 3 / Number 4

CONGRESS 2013

SERBIAN SOCIETY OF MECHANICS.

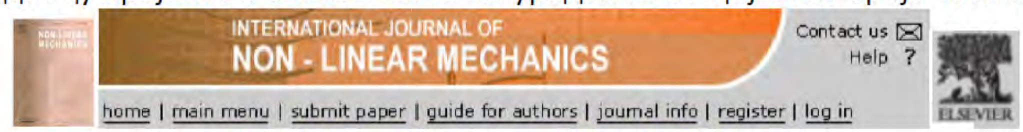
FOURTH SERBIAN (29TH YU) CONGRESS ON THEORETICAL AND APPLIED MECHANICS

4TH -7TH OF JUNE 2013, HOTEL BREZA - VRNJAČKA BANJA, SERBIA

M2: Nonlinear Dynamics – Milutin Milankovic

Corresponding Organizer: Katica Stevanović-Hedrih
Mathematical Institute of the Serbian Academy of Sciences and Arts, Serbia,
Email: khedrih@eunet.rs

II* Позив упућен руководиоцу пројекта **ON174001** за госта уредника специјалног броја часописа



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International Journal of Non-Linear Mechanics (M21)

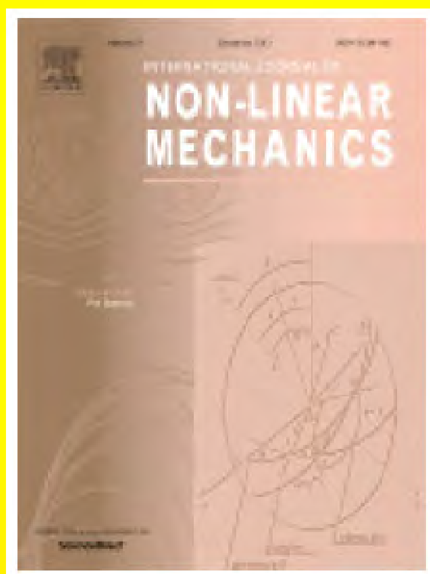
и прихватање садржаја специјалног броја

Special issue: "*Elements of mathematical phenomenology and phenomenological mapping in non-linear dynamics*"

of International Journal Non-Linear Mechanics (IJNM)

по идејама Најзначајнијег дела Михаила Петровића, једног од три студента докторанта импозантног **Poincaré-a (Jules Henri Poincaré** (1854 –1912)).

У току је рецензирање радова подентих за овај број и пријем нових радова.



III* Именовање руководиоца Пројекта **ON174001** у тринаесточлану редакцију интернационалног часописа ''TENSOR'' јапанског друштва **Tensor Society**, који се успешно публикује вћ 74 године и који је јединствен по садржају публикованих радова из области диференијалне геометрије.

IV* Успешност 8 младих истраживача у полагању испита на докторским студијама са просечном оцерном 10 (десет) који су отпочели докторске студије са почетком пројектног циклуса.

V* Пленарна предавања и предавања по позиву руководиоца пројекта у којима је приказо и **увео нову функцију дисипације енергије система фракционих својстава**. Резултати су штампани у изводима, а и предати за штампу у интегралном облику.

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Odeljenja za mehaniku i Odeljenja za matematiku

Mathematical Institute SANU Belgrade

Grant OI174001

“Dynamics of hybrid systems with complex structures. Mechanics of Materials”

Катица Р. (Стевановић) Хедрих

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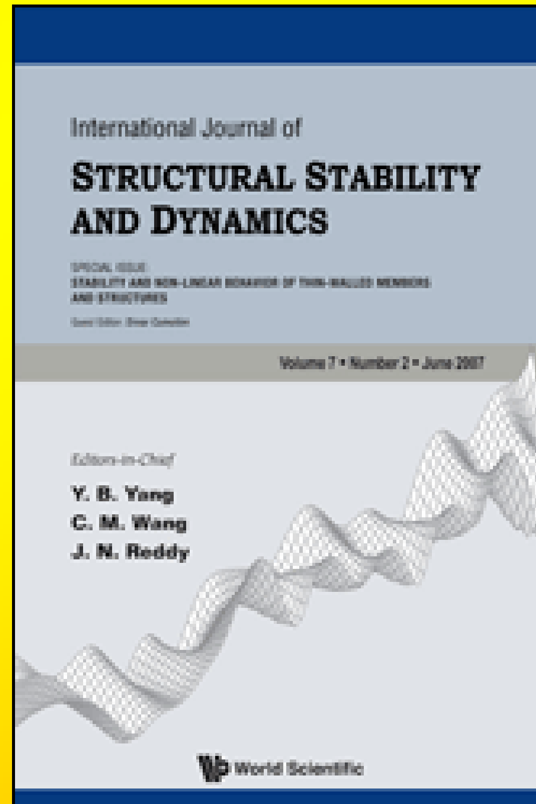
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$\vec{\mathfrak{J}}_{\vec{n}}(O)$

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Hedrih (Stevanoviċ) R. Katica, (2013), Vector method based on mass moment vectors and vector rotators applied to rigid-body multi-coupled rotations around no intersecting axes, International Journal of Structural Stability and Dynamics, Vol. 13, No. 7 (2013) 1340007 (20 pages), #.c World Scientificc Publishing Company, ISSN: 0219-4554, DOI: 10.1142/S0219455413400075
http://dk.doi.org/10.1142/S0219455413400075

Hendrix (Stevanović) K., Veljović Lj., New Vector Description of Kinetic Pressures on Shaft Bearings of a Rigid Body Nonlinear Dynamics with Coupled Rotations around No Intersecting Axes, Acta Polytechnica Hngarica - Journal of Applied Sciences, Special Issue on Applied Mathematics, Guest Editors: Aurél Galántai and Péter T. Nagy, Volume 10, Issue Number 7, 2013, pp. 151-170. ISSN 1785-8860
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Hedrih (Stevanović) K., Milosavljević D., Veljović Lj., Multi-parameter Analysis of a Rigid Body Nonlinear Coupled Rotations around No Intersecting Axes Based on the Vector Method, Adv. Theor. Appl. Mech., Vol. 6, 2013, no. 2, 49 - 70, HIKARI Ltd, www.m-hikari.com, http://dx.doi.org/10.12988/atam.2013.378

http://www.m-hikari.com/atam/atam2013/atam1-4-2013/hedrihATAM1-4-2013.pdf

Katina R. (Stevanović) Hedrih and Marija B. Stamenković, (201`3), Mass moment vector applications to the multi body dynamics with coupled rotations about no intersecting axes, PAMM _ Proc. Appl. Math. Mech. 13, 35 – 36 (2013) / DOI 10.1002/pamm.201310013

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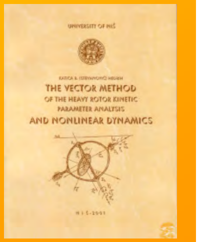


This monograph defines three kinetic vectors fixed to a certain point and axis passing through the given rigid body point. These are:

1 Vector* $\vec{M}_{\vec{n}}^{(O)}$ *of the body mass at the point* O *for the axis oriented by the unit vector* $\vec{n}$*:*

$$\boxed{\vec{M}_{\vec{n}}^{(O)} \overset{def}{=} \iiint\limits_{V} \vec{n}\,dm = \mathrm{M}\,\vec{n}} \qquad dm = \ dV \qquad (1)$$

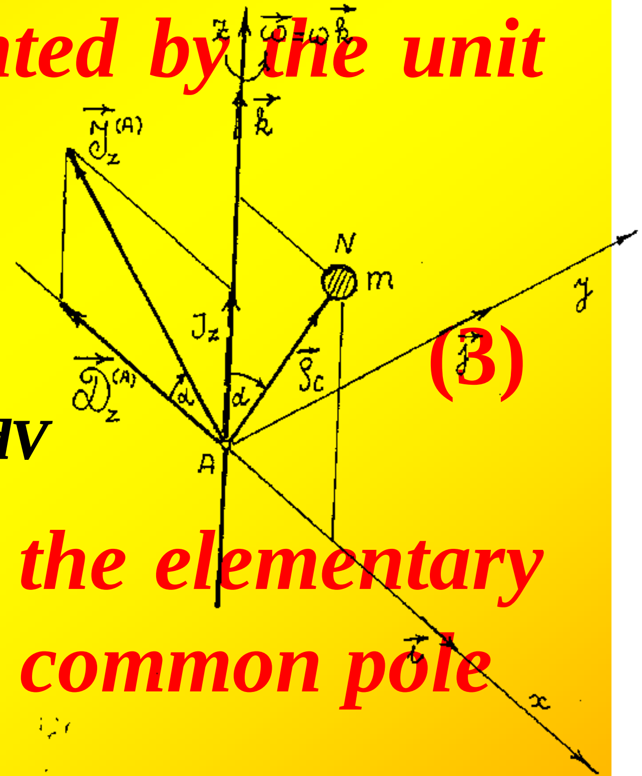
2 Vector* $\vec{\mathfrak{S}}_{\vec{n}}^{(O)}$ *of the body mass static (linear) moment at the point* O *for the axis oriented by the unit vector* $\vec{n}$ *in the form:*

$$\boxed{\vec{\mathfrak{S}}_{\vec{n}}^{(O)} \overset{def}{=} \iiint\limits_{V} [\vec{n}, \vec{\ }]\,dm} \qquad dm = \ dV \qquad (2)$$

3* Vector $\vec{\mathfrak{J}}_{\vec{n}}^{(O)}$ of the body mass inertia moment at the point O for the axis oriented by the unit vector : $\vec{n}$

$$\boxed{\vec{\mathfrak{J}}_{\vec{n}}^{(O)} \overset{def}{=} \iiint\limits_{V} [\vec{\ }, [\vec{n}, \vec{\ }]]dm}$$

$dm = \ dV$



(3)

where $\vec{\ }$ is the position vector of the elementary body mass dm with respect to the common pole O .

The spherical and the devivational parts of the inertia moment vector and of the inertia tensor are analyzed.

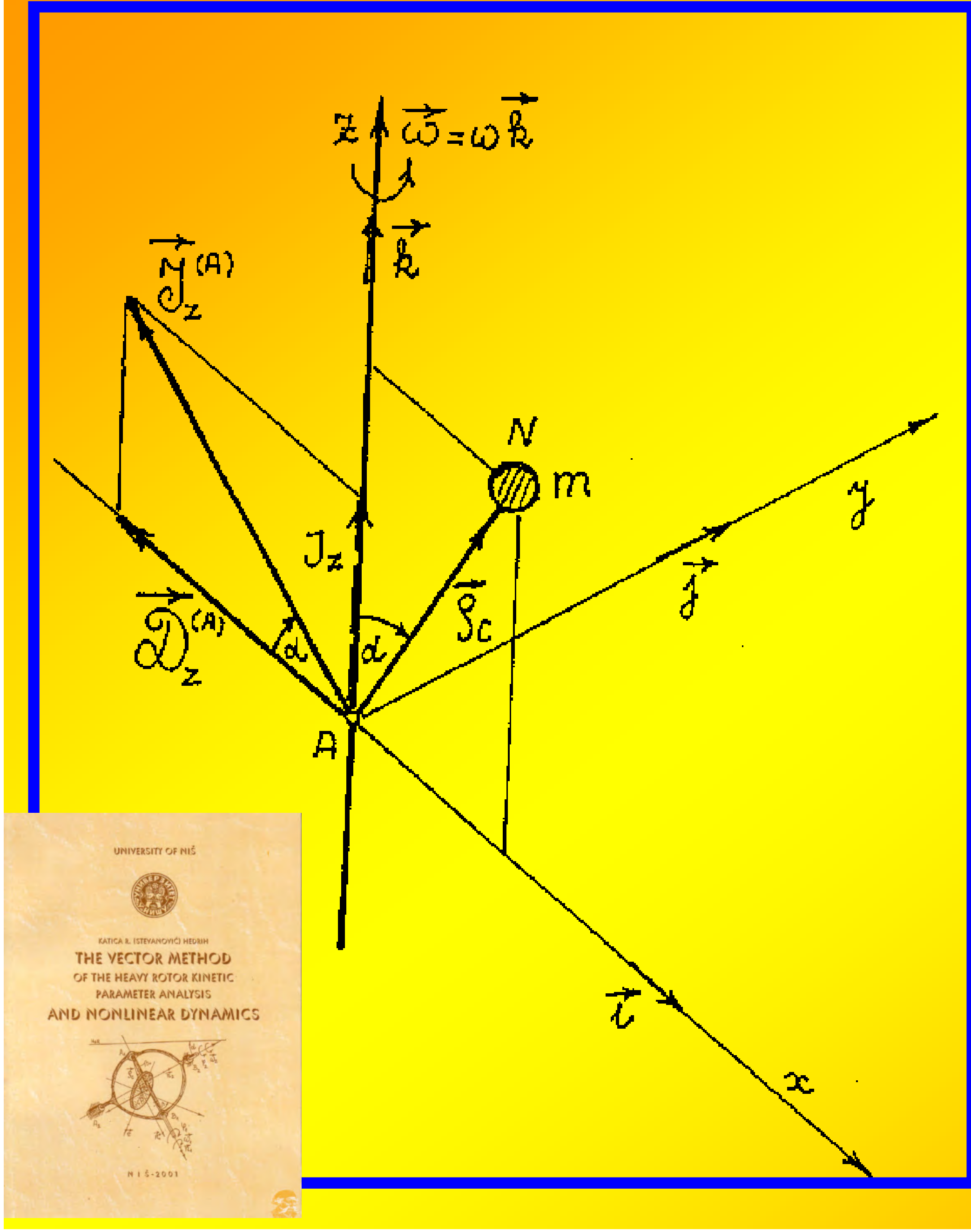


Figure 1. b*

The graphical presentation of the vector of mass particle's mass inertia moment for the reference point and an oriented axis and of the corresponding deviational plane.

The "supports" vectors of the body mass linear moments as well as of the body mass inertia moments for the pole O and axis oriented by unit vector $\vec{n}$ are introduced by definitions and expressions.

The "support" vector $\vec{S}_{\vec{n}}^{(O)}$ of the body mass linear moment and the "support" vector $\vec{\mathfrak{N}}_{\vec{n}}^{(O)}$ of the body mass inertia moment of the body point $N: \overrightarrow{ON} = \vec{\ }$, for the pole in the point O and for the axis oriented by the unit vector $\vec{n}$ are defined by the following expressions:

$$\boxed{\vec{S}_{\vec{n}}^{(O)} \overset{def}{=} \frac{\partial \vec{\mathfrak{S}}_{\vec{n}}^{(O)}}{\partial m} = [\vec{n}, \vec{\ }]}, \quad \boxed{\vec{\mathfrak{N}}_{\vec{n}}^{(O)} \overset{def}{=} \frac{\partial \vec{\mathfrak{J}}_{\vec{n}}^{(O)}}{\partial m} = [\vec{\ }, [\vec{n}, \vec{\ }]]} \quad (4)$$

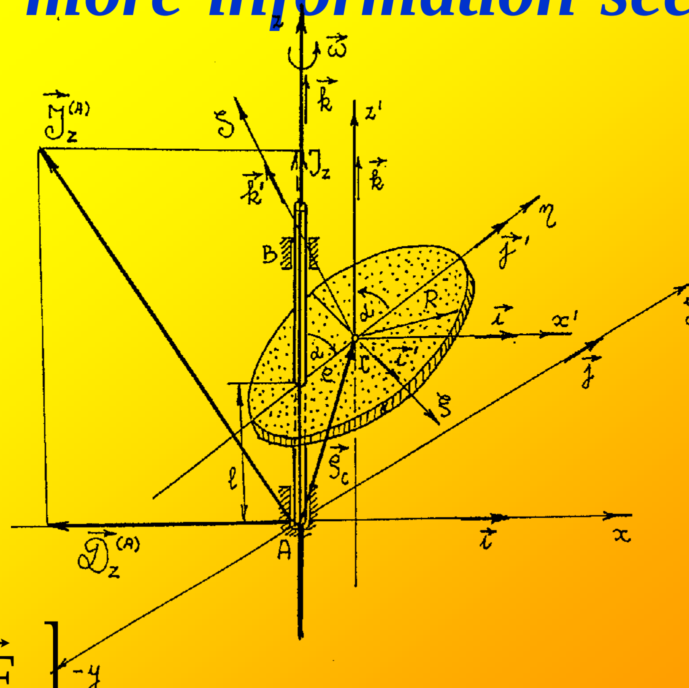
Also, we can conclude that the impact on applications of the use of different possibilities of the phenomenological analogy of different model dynamics and professors, researchers and scientists, with larger area of the own scientific knowledge are also very important for optimization of the teaching processes and the application of Bologna's principle in the original form.

3* Vector $\vec{\Im}_{\vec{n}}^{(O)}$ ***of he body mass inertia moment at the point*** O ***for the axis oriented by the unit vector*** $\vec{n}$ ***:***

$$\vec{\Im}_{\vec{n}}^{(O)} \stackrel{def}{=} \iiint_{V} \vec{\;}, [\vec{n}, \vec{\;}]]\, dn$$

where $\vec{\;}$ ***is the position vector of the elementary body mass with respect to the common pole*** O***. For special cases see Ref. [35].***

We can write two vector equations of dynamic equilibrium for rotation of the body around the stationary axis oriented by the unit vector $\vec{n}$, with bearing A and B , with angular velocity Ω and acceleration $\dot{\Omega}$, and under the action of the active force system $\vec{F}_k$, $k=1,2,\ldots N$ (for more information see Refs. [33] in the following form :

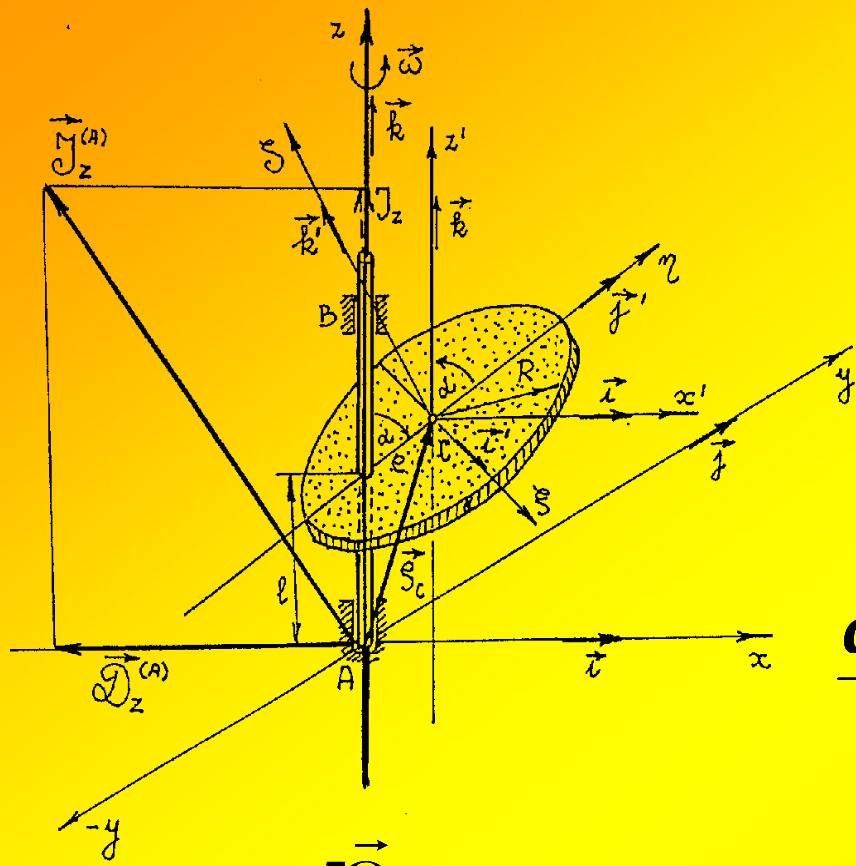


$$\mathfrak{R} = \sqrt{\dot{\Omega}^2 + \Omega^4}$$

$$\frac{d\vec{\mathfrak{K}}}{dt} = \vec{\mathfrak{R}}_1 \left| \vec{\mathfrak{S}}_{\vec{n}}^{(A)} \right| = \sum_{k=1}^{k=N} \vec{F}_k + \vec{F}_A + \vec{F}_B$$

$$\frac{d\vec{\mathfrak{L}}_A}{dt} = \dot{\Omega} \mathrm{J}_{\vec{n}}^{(A)} + \dot{\Omega} \vec{\mathfrak{D}}_{\vec{n}}^{(A)} + \Omega^2 \left[\vec{\Omega}, \vec{\mathfrak{D}}_{\vec{n}}^{(A)} \right] =$$

$$= \dot{\Omega} \mathrm{J}_{\vec{n}}^{(A)} + \left| \vec{\mathfrak{D}}_{\vec{n}}^{(A)} \right| \vec{\mathfrak{R}}_2 = \sum_{k=1}^{k=N} \left[\vec{\rho}_k, \vec{F}_k \right] + \left[\vec{\rho}_B, \vec{F}_B \right]$$



$$\vec{\mathfrak{J}}_{\vec{n}}^{(O)} \overset{def}{=} \iiint\limits_V [\vec{\ }, [\vec{n}, \vec{\ }]] dm$$

$$\vec{\mathfrak{S}}_O^{(\vec{n})} = \iiint\limits_V [\vec{n}, \vec{r}] dm$$

$$\mathfrak{R} = \sqrt{\Omega^2 + \Omega^4}$$

$$\frac{d\vec{\mathfrak{K}}}{dt} = \mathfrak{R}_1 \left|\vec{\mathfrak{S}}_{\vec{n}}^{(A)}\right| = \sum_{k=1}^{k=N} \vec{F}_k + \vec{F}_A + \vec{F}_B$$

$$\frac{d\vec{\mathfrak{L}}_A}{dt} = \dot{\Omega} J_{\vec{n}}^{(A)} + \Omega \vec{\mathfrak{D}}_{\vec{n}}^{(A)} + \Omega \left[\vec{\Omega}, \vec{\mathfrak{D}}_{\vec{n}}^{(A)}\right] =$$

$$= \dot{\Omega} J_{\vec{n}}^{(A)} + \left|\vec{\mathfrak{D}}_{\vec{n}}^{(A)}\right| \vec{\mathfrak{R}}_2 = \sum_{k=1}^{k=N} \left[\vec{\ }_k, \vec{F}_k\right] + \left[\vec{\ }_B, \vec{F}_B\right]$$

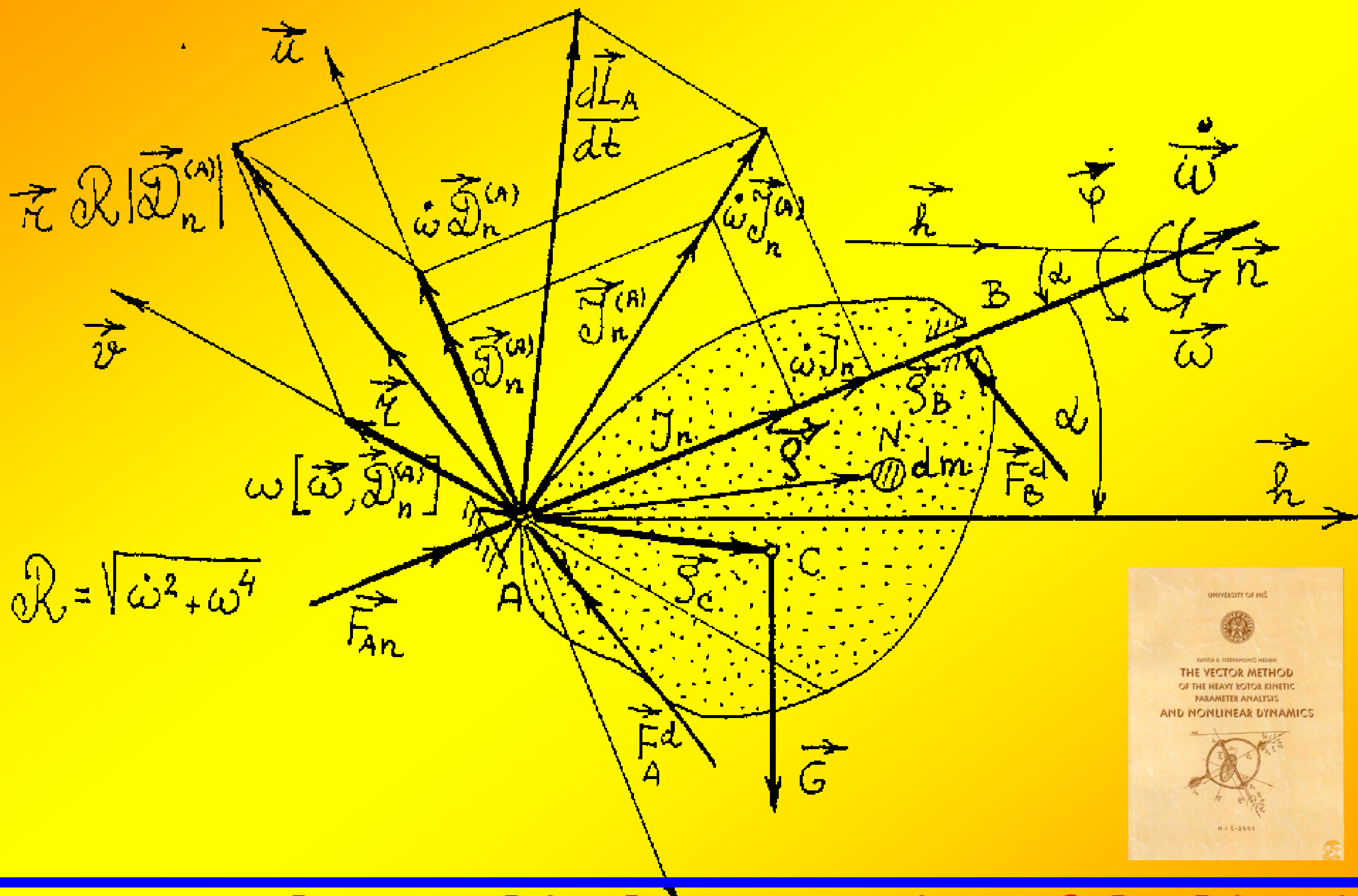


Figure 12. ***The graphical presentation of the kinetic vectors of rotors with inclined rotation axis.***

Following the expressions (67) and (70), as well as the expression (68) and (71), we can write the following two vector equations:

$$\frac{d\vec{\mathfrak{K}}}{dt} = \mathfrak{R}\left|\vec{\mathfrak{S}}_{\vec{n}}^{(A)}\right|\vec{r}_1 = \sum_{k=1}^{k=N}\vec{F}_k + \vec{F}_A + \vec{F}_B + \vec{G} \qquad \textbf{(125)}$$

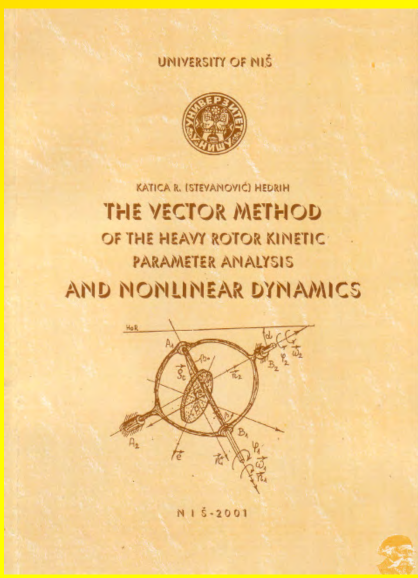
$$\frac{d\vec{\mathfrak{L}}_A}{dt} = \dot{\Omega} J_{\vec{n}}^{(A)} + \left|\vec{\mathfrak{D}}_{\vec{n}}^{(A)}\right|\vec{\mathfrak{R}} = \sum_{k=1}^{k=N}\left[\vec{\ }_k, \vec{F}_k\right] + \left[\vec{\ }_C, \vec{G}\right] + \left[\vec{\ }_B, \vec{F}_B\right] \qquad \textbf{(126)}$$

These two vectorial equations are kinetic equations of dynamic equilibrium of the body rotating around the stationary axis under the action of the active force system $\vec{F}_k$ **.**

If we now multiply scalarly and vectorialy these equations (125) and (126) with the unit vector $\vec{n}$, and, having in mind that the $\vec{\ }_B = {}_B\vec{n}$, we obtain:

1* the rotation equation around the axes oriented by the unit vector $\vec{n}$ in the form:

$$\left(\vec{\mathfrak{J}}_{\vec{n}}^{(A)}, \dot{\vec{\Omega}}\right) = \left(\left[\vec{\ }_C, \vec{G}\right], \vec{n}\right) + \sum_{k=1}^{k=N}\left(\left[\vec{\ }_k, \vec{F}_k\right], \vec{n}\right) \qquad \textbf{(127)}$$



2* the equations for determining the bearings' kinetic pressures, that is, pressures upon the bearings, $\vec{F}_A$ and $\vec{F}_B$, that is, their components in the axis direction $\vec{n}$ and normal to the rotation axis:

$$\vec{F}_{A\vec{n}} = \left(\vec{F}_A, \vec{n}\right)\vec{n} = -\vec{n}\sum_{k=1}^{k=N}\left(\vec{F}_k, \vec{n}\right) - \vec{n}\left(\vec{G}, \vec{n}\right) \qquad \mathbf{(128)}$$

$$\vec{F}_{AT} = -\vec{F}_B + \vec{\mathfrak{R}}_1\left|\vec{\mathfrak{S}}_{\vec{n}}^{(A)}\right| - \left[\vec{n}, \left[\vec{G}, \vec{n}\right]\right] - \sum_{k=1}^{k=N}\left[\vec{n}, \left[\vec{F}_k, \vec{n}\right]\right] \qquad \mathbf{(129)}$$

$$\vec{F}_B = \frac{1}{\quad B}\vec{\mathfrak{R}}\left|\vec{\mathfrak{D}}_{\vec{n}}^{(A)}\right| - \frac{1}{\quad B}\left[\vec{n}, \left[\left[\vec{\ }_C, \vec{G}\right], \vec{n}\right]\right] - \frac{1}{\quad B}\sum_{k=1}^{k=N}\left[\vec{n}, \left[\left[\vec{\ }_k, \vec{F}_k\right], \vec{n}\right]\right] \qquad \mathbf{(130)}$$

where is: $\vec{\mathfrak{R}} = \mathfrak{R}\,\vec{r}$ **,** $\mathfrak{R} = \sqrt{\Omega^2 + \Omega^4}$ **. (131)**

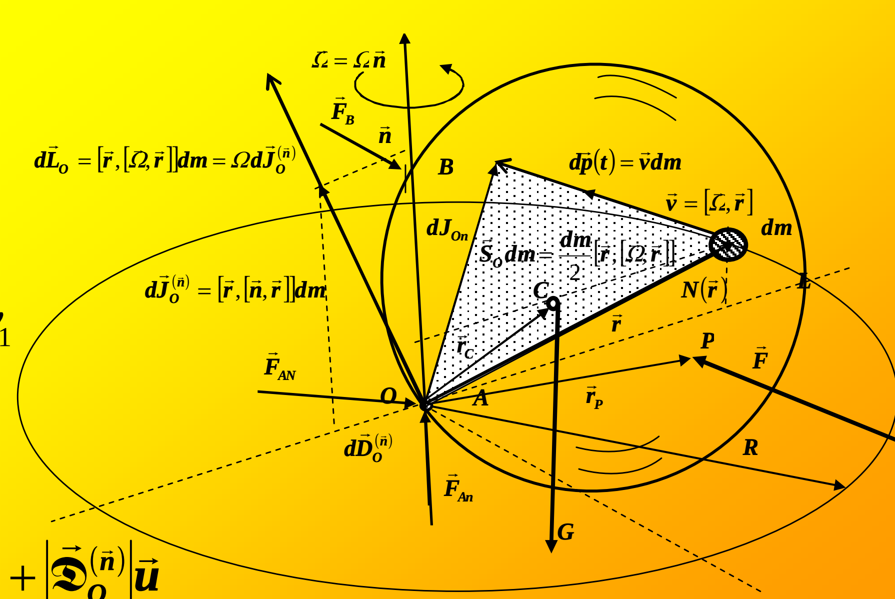
The rotator $\vec{\mathfrak{R}} = \mathfrak{R}\,\vec{r}$ is rotating and increasing by the angular velocity and by the angular acceleration.

Rotation of the elementary material particle around fixed axis – Kinetic presure on bearings

$$\frac{d\vec{p}(t)}{dt} = \dot{\Omega}\vec{S}_O^{(\vec{n})} + \Omega\left[\vec{\Omega},\vec{S}_O^{(\vec{n})}\right] = \dot{\Omega}\vec{S}_O^{(\vec{n})} + \Omega^2\left[\vec{n},\vec{S}_O^{(\vec{n})}\right] = \vec{F}_{AMN} + \vec{F}_{An} + \vec{F}_B + \vec{F} + \vec{G}$$

$$\frac{d\vec{L}_O}{dt} = \dot{\Omega}\vec{J}_O^{(\vec{n})} + \Omega\left[\vec{\Omega},\vec{J}_O^{(\vec{n})}\right] = \dot{\Omega}\vec{J}_O^{(\vec{n})} + \Omega^2\left[\vec{n},\vec{J}_O^{(\vec{n})}\right] = \left[\vec{r}_P,\vec{F}\right] + \left[\vec{r}_C,\vec{G}\right] + \left[\vec{r}_B,\vec{F}_B\right]$$

$$\vec{S}_O^{(\vec{n})} = \iiint\limits_V [\vec{n},\vec{r}]\,dm = [\vec{n},\vec{r}_C]M = \left|\vec{S}_O^{(\vec{n})}\right|\vec{u}_1$$



$$\left[\vec{n},\vec{S}_O^{(\vec{n})}\right] = M\left[\vec{n},[\vec{n},\vec{r}_C]\right] = M\left\langle(\vec{n},\vec{r}_C)\vec{n} - \vec{r}_C\right\rangle = \left|\vec{S}_O^{(\vec{n})}\right|\vec{w}_1$$

$$\left[\vec{n},\vec{J}_O^{(\vec{n})}\right] = \left[\vec{n}, J_{On}\vec{n} + \vec{\mathfrak{D}}_O^{(\vec{n})}\right] = \left[\vec{n},\vec{\mathfrak{D}}_O^{(\vec{n})}\right] = \left|\vec{\mathfrak{D}}_O^{(\vec{n})}\right|\vec{w}$$

$$\vec{J}_O^{(\vec{n})} = \left(\vec{n},\vec{J}_O^{(\vec{n})}\right)\vec{n} + \left[\vec{n},\left[\vec{J}_O^{(\vec{n})},\vec{n}\right]\right] = J_{On}\vec{n} + \vec{\mathfrak{D}}_O^{(\vec{n})} = J_{On}\vec{n} + \left|\vec{\mathfrak{D}}_O^{(\vec{n})}\right|\vec{u}$$

. **For this case the expressions of the linear momentum $\vec{\mathfrak{K}}$ and of the angular momentum $\vec{\mathfrak{L}}_{O_1}$ of the gyro-rotor system are:**

$$\vec{\mathfrak{K}} = \left[\Omega_2, \vec{d}_1\right] \mathrm{M} + \Omega_1 \vec{\mathfrak{S}}^{(O_1)}_{\vec{n}_1} + \Omega_2 \vec{\mathfrak{S}}^{(O_1)}_{\vec{n}_2}$$

$$\vec{\mathfrak{L}}_{O_2} = \Omega_1 \vec{\mathfrak{J}}^{(O_1)}_{\vec{n}_1} + \Omega_2 \vec{\mathfrak{J}}^{(O_1)}_{\vec{n}_2} + \Omega_1 \left[\vec{d}_1, \vec{\mathfrak{S}}^{(O_1)}_{\vec{n}_1}\right] + \Omega_2 \left[\vec{d}_1, \vec{\mathfrak{S}}^{(O_1)}_{\vec{n}_{21}}\right] + \tag{62}$$

$$+ \Omega_2 \vec{\mathfrak{J}}^{(O_2 \to O_1)}_{\vec{n}_2} + \Omega_2 \left[\vec{\ }_C, \vec{\mathfrak{S}}^{(O_2 \to O_1)}_{\vec{n}_1}\right]$$

For special case of the gyro-rotor with many shaft rotor axes with one section O, these expression of the linear momentum and of the angular momentum are very simple:

$$\vec{\mathfrak{K}} = \sum_{i=1}^{i=p} \left|\Omega_i\right| \vec{\mathfrak{S}}^{(O)}_{\vec{n}_i} \qquad \vec{\mathfrak{L}}_O = \sum_{i=1}^{i=p} \left|\Omega_i\right| \vec{\mathfrak{J}}^{(O)}_{\vec{n}_i} \tag{63}$$

$$\frac{d\vec{\mathfrak{L}}_{O_2}}{dt} = \Omega_1 \vec{\mathfrak{J}}^{(O_1)}_{\vec{n}_1} + \Omega_1^2 \left[\vec{n}_1, \vec{\mathfrak{J}}^{(O_1)}_{\vec{n}_1}\right] + \Omega_2 \vec{\mathfrak{J}}^{(O_1)}_{\vec{n}_2} + \Omega_2^2 \left[\vec{n}_2, \vec{\mathfrak{J}}^{(O_1)}_{\vec{n}_2}\right] +$$

$$+ \Omega_1 \Omega_2 \left[\vec{n}_2, \vec{\mathfrak{J}}^{(O_1)}_{\vec{n}_1}\right] + \Omega_1 \Omega_2 \left[\vec{n}_1, \vec{\mathfrak{J}}^{(O_1)}_{\vec{n}_2}\right] + \Omega_1 \Omega_2 \vec{\mathfrak{J}}^{(O_1)}_{[\vec{n}_2, \vec{n}_1]}$$

$$\frac{d\vec{\mathfrak{L}}_{O_2}}{dt} = \Omega_1 \, \mathrm{J}^{(O_1)}_{\vec{n}_1} \, \vec{n}_1 + \Omega_2 \, \mathrm{J}^{(O_1)}_{\vec{n}_2} \, \vec{n}_2 + \vec{\mathfrak{R}}_1 \left| \vec{\mathfrak{D}}^{(O_1)}_{\vec{n}_1} \right| + \vec{\mathfrak{R}}_2 \left| \vec{\mathfrak{D}}^{(O_1)}_{\vec{n}_2} \right| +$$

$$+ \vec{\mathfrak{R}}_{21} \left| \vec{\mathfrak{J}}^{(O_1)}_{\vec{n}_1} \right| + \vec{\mathfrak{R}}_{12} \left| \vec{\mathfrak{J}}^{(O_1)}_{\vec{n}_2} \right| + \vec{\mathfrak{R}}_3 \left| \vec{\mathfrak{J}}^{(O_1)}_{[\vec{n}_2, \vec{n}_1]} \right|$$

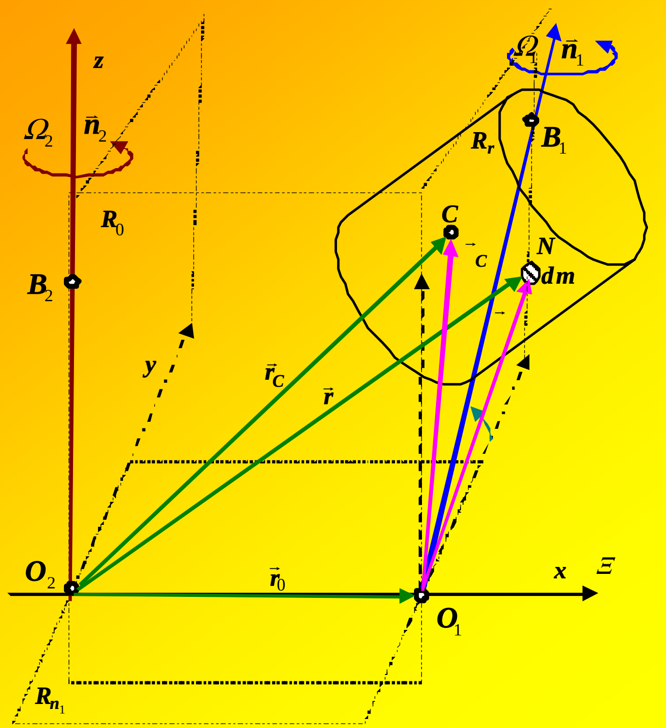
where the kinematic vectors rotator are introduced in following form: $\vec{\mathfrak{R}}_2 = \Omega_2 \vec{u}_2 + \Omega_2^2 \vec{v}_2; \quad \mathfrak{R}_2 = \sqrt{\Omega_2^2 + \Omega_2^4}$

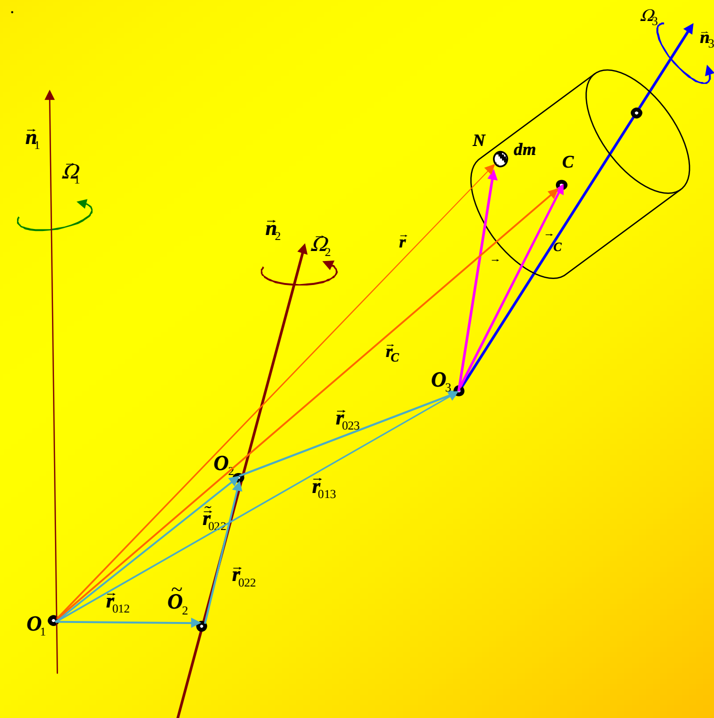
$$\vec{\mathfrak{R}}_3 = \Omega_1 \Omega_2 \vec{u}_3; \quad \mathfrak{R}_3 = \Omega_1 \Omega_2 \qquad \vec{\mathfrak{R}}_1 = \Omega_1 \vec{u}_1 + \Omega_1^2 \vec{v}_1; \quad \mathfrak{R}_1 = \sqrt{\Omega_1^2 + \Omega_1^4}$$

$$\vec{\mathfrak{R}}_{12} = \Omega_1 \Omega_2 [\vec{n}_1, \vec{u}_2]; \quad \mathfrak{R}_{12} = \Omega_1 \Omega_2 \sin \Gamma (=) 0 \qquad \textbf{(65)}$$

$$\vec{\mathfrak{R}}_{21} = \Omega_1 \Omega_2 [\vec{n}_2, \vec{u}_1]; \quad \mathfrak{R}_{21} = \Omega_1 \Omega_2 \sin \vartheta_1$$

Linear momentum of a rigid body coupled multi-rotations around no intersecting axes





$$\vec{\mathfrak{K}} = \vec{\mathfrak{K}}_{\vec{n}_1}^{(O_{1-2})} + \vec{\mathfrak{K}}_{\vec{n}_1}^{(O_2)} + \vec{\mathfrak{K}}_{\vec{n}_2}^{(O_2)} = [\vec{\Omega}_1, \vec{r}_{012}]M + \Omega_1 \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)} + \Omega_2 \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)}$$

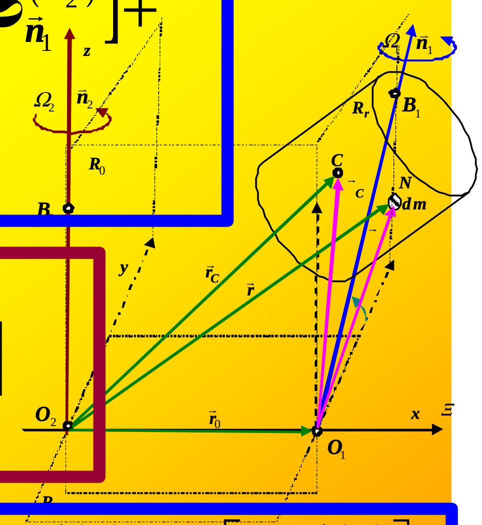
$$\vec{\mathfrak{K}} = \vec{\mathfrak{K}}_{\vec{n}_1}^{(O_{1-3})} + \vec{\mathfrak{K}}_{\vec{n}_2}^{(O_{2-3})} + \vec{\mathfrak{K}}_{\vec{n}_1}^{(O_3)} + \vec{\mathfrak{K}}_{\vec{n}_2}^{(O_3)} + \vec{\mathfrak{K}}_{\vec{n}_3}^{(O_3)} =$$

$$= \Omega_1 [\vec{n}_1, \vec{r}_{012} + \vec{r}_{022} + \vec{r}_{023}]M + \Omega_2 [\vec{n}_2, \vec{r}_{023}]M + \Omega_1 \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)} + \Omega_2 \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)} + \Omega_3 \vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)}$$

$$\vec{\mathfrak{K}} = \vec{\mathfrak{K}}_{\vec n_1}^{(O_{1-2})} + \vec{\mathfrak{K}}_{\vec n_1}^{(O_2)} + \vec{\mathfrak{K}}_{\vec n_2}^{(O_2)} = [\vec\Omega_1, \vec r_{012}]M + \Omega_1 \vec{\mathfrak{S}}_{\vec n_1}^{(O_2)} + \Omega_2 \vec{\mathfrak{S}}_{\vec n_2}^{(O_2)}$$

$$\vec{\mathfrak{L}}_{O_1} = \Omega_1 \vec n_1 r_{012}^2 M + \Omega_1[\vec{\;}_C, [\vec n_1, \vec r_{012}]]M + \Omega_1\left[\vec r_{012}, \vec{\mathfrak S}_{\vec n_1}^{(O_2)}\right] + \Omega_2\left[\vec r_{012}, \vec{\mathfrak S}_{\vec n_2}^{(O_2)}\right] + \Omega_1 \vec{\mathfrak J}_{\vec n_1}^{(O_2)} + \Omega_2 \vec{\mathfrak J}_{\vec n_2}^{(O_2)}$$

$$\frac{d\vec{\mathfrak K}}{dt} = \Omega_1[\vec n_1, \vec r_0]M + \Omega_2^2[\vec n_1,[\vec n_1,\vec r_0]]M + \Omega_1 \vec{\mathfrak S}_{\vec n_1}^{(O_2)} + \Omega_1^2\left[\vec n_1, \vec{\mathfrak S}_{\vec n_1}^{(O_2)}\right] +$$
$$+ \Omega_2\vec{\mathfrak S}_{\vec n_2}^{(O_2)} + \Omega_2^2\left[\vec n_2, \vec{\mathfrak S}_{\vec n_2}^{(O_2)}\right] + 2\Omega_1\Omega_2\left[\vec n_2, \vec{\mathfrak S}_{\vec n_1}^{(O_2)}\right]$$



$$\frac{d\vec{\mathfrak R}}{dt} = \vec{\mathfrak R}_{01}\left|[\vec n_1,\vec r_0]\right|M + \vec{\mathfrak R}_{11}\left|\vec{\mathfrak S}_{\vec n_1}^{(o_2)}\right| + \vec{\mathfrak R}_{022}\left|\vec{\mathfrak S}_{\vec n_2}^{(o_2)}\right| + 2\Omega_1\Omega_2\left[\vec n_2, \vec{\mathfrak S}_{\vec n_1}^{(o_2)}\right]$$

$$\vec{\mathfrak R}_{01} = \dot\Omega_1 \vec u_{01} + \Omega_1^2 \vec v_{01} = \dot\Omega_1\left[\vec n_1, \frac{\vec r_0}{r_0}\right] + \Omega_1^2\left[\vec n_1,\left[\vec n_1,\frac{\vec r_0}{r_0}\right]\right]; \vec{\mathfrak R}_{011} = \dot\Omega_1\vec u_{01} + \Omega_1^2 \vec v_{02} = \dot\Omega_1 \frac{\vec{\mathfrak S}_{\vec n_1}^{(o_2)}}{\mathfrak S_{\vec n_1}^{(o_2)}} + \Omega_1^2\left[\vec n_1, \frac{\vec{\mathfrak S}_{\vec n_1}^{(o_2)}}{\mathfrak S_{\vec n_1}^{(o_2)}}\right]$$

$$\vec{\mathfrak R}_{022} = \dot\Omega_2\vec u_{02} + \Omega_2^2 \vec v_{02} = \dot\Omega_2 \frac{\vec{\mathfrak S}_{\vec n_2}^{(o_2)}}{\mathfrak S_{\vec n_2}^{(o_2)}} + \Omega_2^2\left[\vec n_2, \frac{\vec{\mathfrak S}_{\vec n_2}^{(o_2)}}{\mathfrak S_{\vec n_2}^{(o_2)}}\right]$$

$$\vec{\mathfrak{L}}_{O_1} = \Omega_1 \vec{n}_1 r_{012}^2 M + \Omega_1 \left[\vec{\ }_C, [\vec{n}_1, \vec{r}_{012}]\right] M + \Omega_1 \left[\vec{r}_{012}, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)}\right] + \Omega_2 \left[\vec{r}_{012}, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)}\right] + \Omega_1 \vec{\mathfrak{J}}_{\vec{n}_1}^{(O_2)} + \Omega_2 \vec{\mathfrak{J}}_{\vec{n}_2}^{(O_2)}$$

$$\frac{d\bar{\mathfrak{L}}_{O_1}}{dt} = \bar{\chi}_{12}\left(\bar{r}_0, \bar{\rho}_C, M, \dot{\omega}_1, \dot{\omega}_2, \omega_1, \omega_2, \bar{n}_1, \bar{n}_2\right) + \dot{\omega}_1 \bar{n}_1 r_0^2 M + 2\omega_1\omega_2\left[\bar{n}_1, \bar{\mathfrak{J}}_{\bar{n}_2}^{(O_2)}\right]$$
$$+ \dot{\omega}_1\left(\bar{n}_1, \bar{\mathfrak{J}}_{\bar{n}_1}^{(O_2)}\right)\bar{n}_1 + \dot{\omega}_2\left(\bar{n}_2, \bar{\mathfrak{J}}_{\bar{n}_2}^{(O_2)}\right)\bar{n}_2 + \bar{\mathfrak{R}}_1\left|\bar{\mathfrak{D}}_{\bar{n}_1}^{(O_2)}\right| + \bar{\mathfrak{R}}_2\left|\bar{\mathfrak{D}}_{\bar{n}_2}^{(O_2)}\right|$$

$$\vec{\mathfrak{R}}_1 = \Omega_1 \frac{\vec{\mathfrak{D}}_{\vec{n}_1}^{(O_2)}}{\mathfrak{D}_{\vec{n}_1}^{(O_2)}} + \Omega_1^2\left[\vec{n}_1, \frac{\vec{\mathfrak{D}}_{\vec{n}_1}^{(O_2)}}{\mathfrak{D}_{\vec{n}_1}^{(O_2)}}\right], \quad \vec{\mathfrak{R}}_2 = \Omega_2 \frac{\vec{\mathfrak{D}}_{\vec{n}_2}^{(O_2)}}{\mathfrak{D}_{\vec{n}_2}^{(O_2)}} + \Omega_2^2\left[\vec{n}_2, \frac{\vec{\mathfrak{D}}_{\vec{n}_2}^{(O_2)}}{\mathfrak{D}_{\vec{n}_2}^{(O_2)}}\right]$$

$$\vec{\mathfrak{K}} = \Omega_1\left[\vec{n}_1, \vec{r}_{012} + \vec{r}_{022} + \vec{r}_{023}\right]M + \Omega_2\left[\vec{n}_2, \vec{r}_{023}\right]M + \Omega_1 \vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_1} + \Omega_2 \vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_2} + \Omega_3 \vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_3}$$

$$\frac{d\vec{\mathfrak{K}}}{dt} = \Omega_1\left[\vec{n}_1, \vec{r}_{012} + \vec{r}_{022} + \vec{r}_{023}\right]M + \Omega_1^2\left[\vec{n}_1, \left[\vec{n}_1, \vec{r}_{012} + \vec{r}_{022} + \vec{r}_{023}\right]\right] + \Omega_2\left[\vec{n}_2, \vec{r}_{023}\right]M + \Omega_2^2\left[\vec{n}_2, \left[\vec{n}_2, \vec{r}_{023}\right]\right]M +$$

$$+ 2\Omega_1\Omega_2\left[\vec{n}_1, \left[\vec{n}_2, \vec{r}_{023}\right]\right]M + \Omega_1 \vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_1} + \Omega_1^2\left[\vec{n}_1, \vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_1}\right] + \Omega_2 \vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_2} + \Omega_2^2\left[\vec{n}_2, \vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_2}\right] + \Omega_3 \vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_3} + \Omega_3^2\left[\vec{n}_3, \vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_3}\right]$$

$$+ 2\Omega_1\Omega_2\left[\vec{n}_1, \vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_2}\right] + 2\Omega_1\Omega_3\left[\vec{n}_1, \vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_3}\right] + 2\Omega_2\Omega_3\left[\vec{n}_2, \vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_3}\right]$$

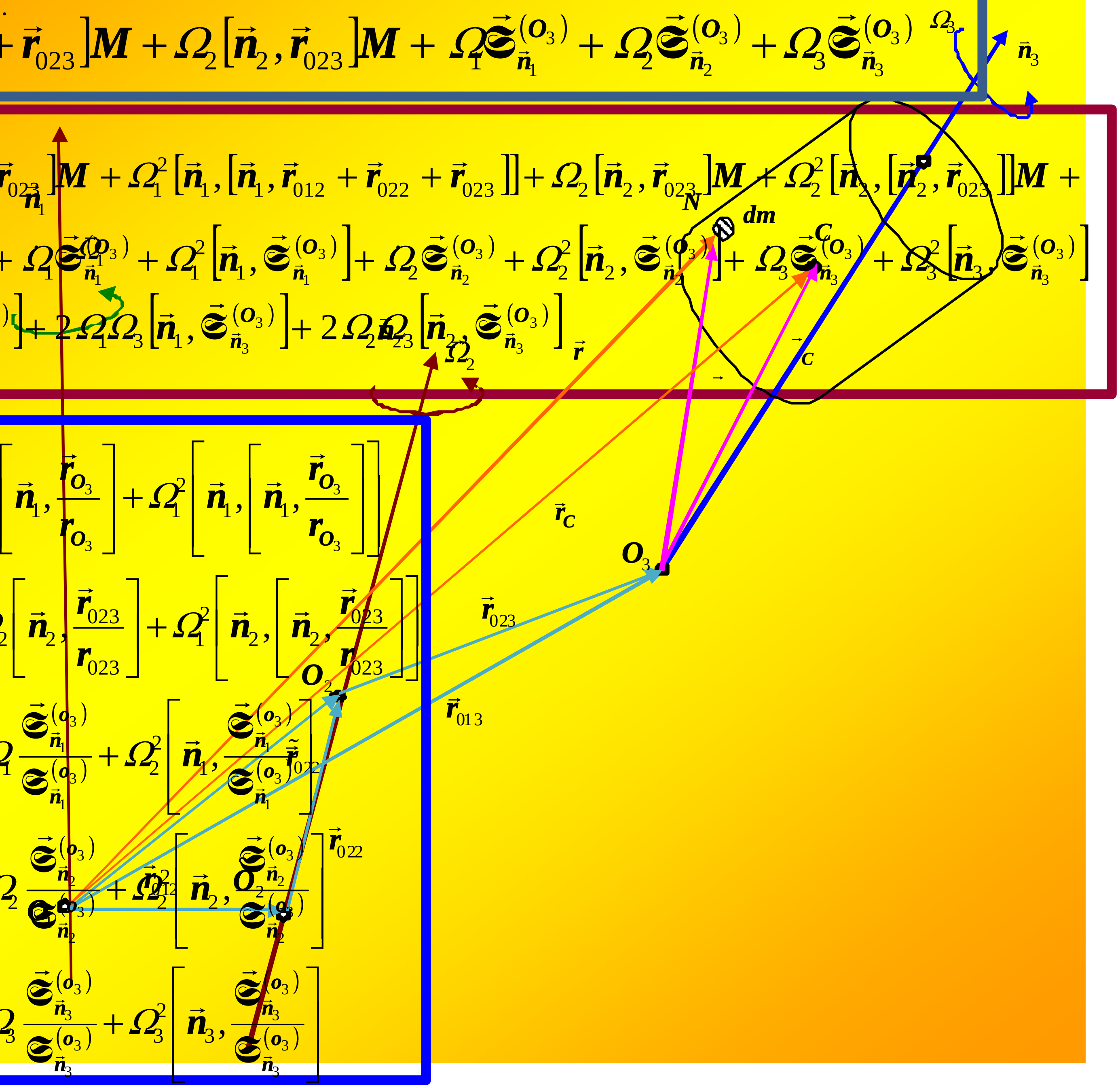
$$\vec{\mathfrak{R}}_{01} = \Omega_1\vec{u}_{01} + \Omega_1^2\vec{v}_{01} = \Omega_1\left[\vec{n}_1, \frac{\vec{r}_{O_3}}{r_{O_3}}\right] + \Omega_1^2\left[\vec{n}_1, \left[\vec{n}_1, \frac{\vec{r}_{O_3}}{r_{O_3}}\right]\right]$$

$$\vec{\mathfrak{R}}_{02} = \Omega_2\vec{u}_{02} + \Omega_2^2\vec{v}_{02} = \Omega_2\left[\vec{n}_2, \frac{\vec{r}_{023}}{r_{023}}\right] + \Omega_1^2\left[\vec{n}_2, \left[\vec{n}_2, \frac{\vec{r}_{023}}{r_{023}}\right]\right]$$

$$\vec{\mathfrak{R}}_{022} = \Omega_1\vec{u}_{02} + \Omega_1^2\vec{v}_{02} = \Omega_1 \frac{\vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_1}}{\mathfrak{S}^{(O_3)}_{\vec{n}_1}} + \Omega_2^2\left[\vec{n}_1, \frac{\vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_1}}{\mathfrak{S}^{(O_3)}_{\vec{n}_1}}\right]$$

$$\vec{\mathfrak{R}}_{022} = \Omega_2\vec{u}_{02} + \Omega_2^2\vec{v}_{02} = \Omega_2 \frac{\vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_2}}{\mathfrak{S}^{(O_3)}_{\vec{n}_2}} + \Omega_2^2\left[\vec{n}_2, \frac{\vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_2}}{\mathfrak{S}^{(O_3)}_{\vec{n}_2}}\right]$$

$$\vec{\mathfrak{R}}_{033} = \Omega_3\vec{u}_{02} + \Omega_3^2\vec{v}_{02} = \Omega_3 \frac{\vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_3}}{\mathfrak{S}^{(O_3)}_{\vec{n}_3}} + \Omega_3^2\left[\vec{n}_3, \frac{\vec{\mathfrak{S}}^{(O_3)}_{\vec{n}_3}}{\mathfrak{S}^{(O_3)}_{\vec{n}_3}}\right]$$



THEOREM 1.

Vector expression of linear momentum derivatives of the rigid N bodies, multi coupled rotations, around no intersecting axis in all cases, placed bodies on the each axis, between other terms, contain sum of products by intensity of rigid N bodies mass linear moment vectors

$$\left|\mathfrak{S}_{\mathfrak{i}\vec{n}_j}^{(o_K)}\right| = \left|\iiint_{V_i}\left[\vec{n}_j, \vec{\rho}_i\right]dm_i\right|, \quad i=1,2,3\ldots N, \quad j=1,2,3\ldots K$$

for the axes oriented by unit vectors of component coupled rotation axes through pole on the rigid N bodies self-rotation axis and vector rotators defined by:

$$\vec{\mathfrak{R}}_{\mathfrak{i}0jj} = \Omega_j \frac{\vec{\mathfrak{S}}_{\mathfrak{i}\vec{n}_j}^{(o_K)}}{\left|\mathfrak{S}_{\mathfrak{i}\vec{n}_j}^{(o_K)}\right|} + \Omega_j^2\left[\vec{n}_j, \frac{\vec{\mathfrak{S}}_{\mathfrak{i}\vec{n}_j}^{(o_K)}}{\left|\mathfrak{S}_{\mathfrak{i}\vec{n}_j}^{(o_K)}\right|}\right] \quad i=1,2,3\ldots N, \quad j=1,2,3\ldots K$$

Where are:

$i=1,2,3\ldots N$ number of bodies,

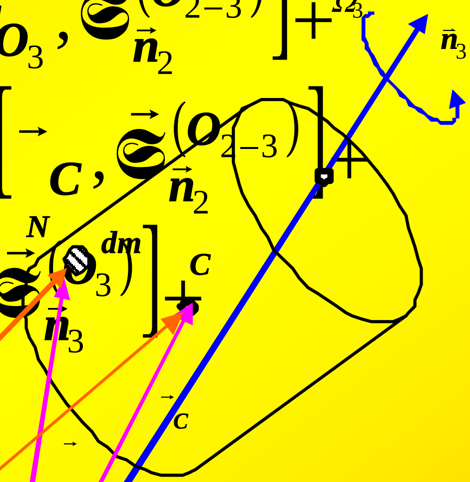
$j=1,2,3\ldots K$ number of axis

$$\vec{\mathfrak{L}}_{O_1} = \Omega_1\left[\vec r_{O_3}, \vec{\mathfrak{S}}^{(O_{1-2-2})}_{\vec n_1}\right] + \Omega_1\left[\vec r_{O_3}, \vec{\mathfrak{S}}^{(O_{2-3})}_{\vec n_1}\right] + \Omega_2\left[\vec r_{O_3}, \vec{\mathfrak{S}}^{(O_{2-3})}_{\vec n_2}\right] +$$

$$+\Omega_1\left[\vec{\ }_C, \vec{\mathfrak{S}}^{(O_{1-2-2})}_{\vec n_1}\right] + \Omega_1\left[\vec{\ }_C, \vec{\mathfrak{S}}^{(O_{2-3})}_{\vec n_1}\right] + \Omega_2\left[\vec{\ }_C, \vec{\mathfrak{S}}^{(O_{2-3})}_{\vec n_2}\right] +$$

$$+\Omega_1\left[\vec r_{O_3}, \vec{\mathfrak{S}}^{(O_3)}_{\vec n_1}\right] + \Omega_2\left[\vec r_{O_3}, \vec{\mathfrak{S}}^{(O_3)}_{\vec n_2}\right] + \Omega_3\left[\vec r_{O_3}, \vec{\mathfrak{S}}^{(O_3)}_{\vec n_3}\right] +$$

$$+\Omega_1 \vec{\mathfrak{J}}^{(O_3)}_{\vec n_1} + \Omega_2 \vec{\mathfrak{J}}^{(O_3)}_{\vec n_2} + \Omega_3 \vec{\mathfrak{J}}^{(O_3)}_{\vec n_3}$$



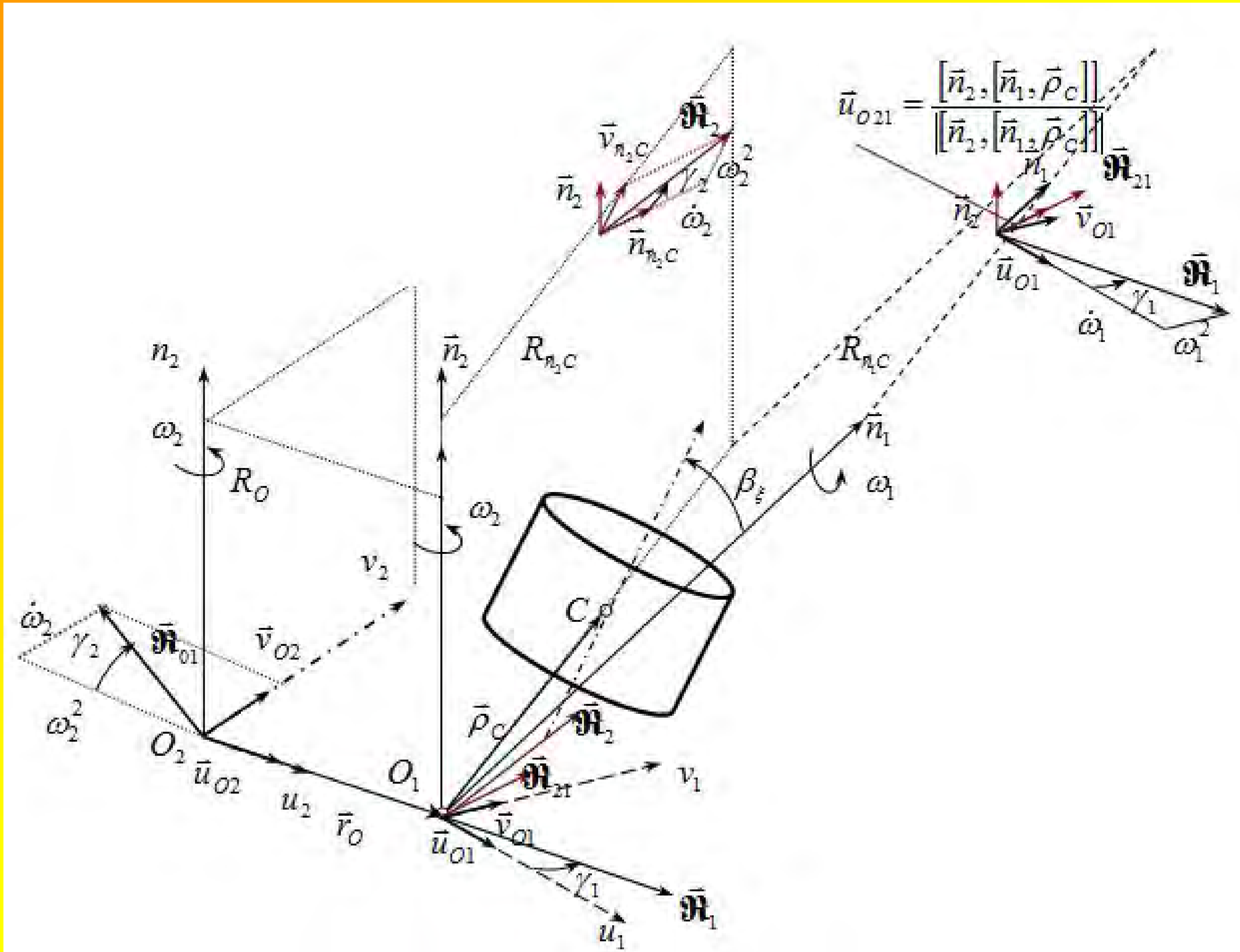
$$\frac{d\vec{\mathfrak{L}}_{O_1}}{dt} = \vec{\ }_{123}\left(\vec r_{O_3}, \vec{\ }_C, \vec{\mathfrak{S}}^{(O_{1-2-2})}_{\vec n_1}, \vec{\mathfrak{S}}^{(O_{2-3})}_{\vec n_1}, \vec{\mathfrak{S}}^{(O_{2-3})}_{\vec n_2}, \vec{\mathfrak{S}}^{(O_3)}_{\vec n_1}, \vec{\mathfrak{S}}^{(O_3)}_{\vec n_2}, \vec{\mathfrak{S}}^{(O_3)}_{\vec n_3}, \dot\Omega_1, \dot\Omega_2, \dot\Omega_3, \Omega_1, \Omega_2, \Omega_3, \vec n_1, \vec n_2, \vec n_3\right) +$$

$$+\Omega_1\left(\vec n_1, \vec{\mathfrak{J}}^{(O_3)}_{\vec n_1}\right)\vec n_1 + \Omega_2\left(\vec n_2, \vec{\mathfrak{J}}^{(O_3)}_{\vec n_2}\right)\vec n_2 + \Omega_3\left(\vec n_3, \vec{\mathfrak{J}}^{(O_3)}_{\vec n_3}\right)\vec n_3 + \vec{\mathfrak{K}}_1\left|\vec{\mathfrak{D}}^{(O_3)}_{\vec n_1}\right| + \vec{\mathfrak{K}}_2\left|\vec{\mathfrak{D}}^{(O_3)}_{\vec n_2}\right| + \vec{\mathfrak{K}}_3\left|\vec{\mathfrak{D}}^{(O_3)}_{\vec n_3}\right| +$$

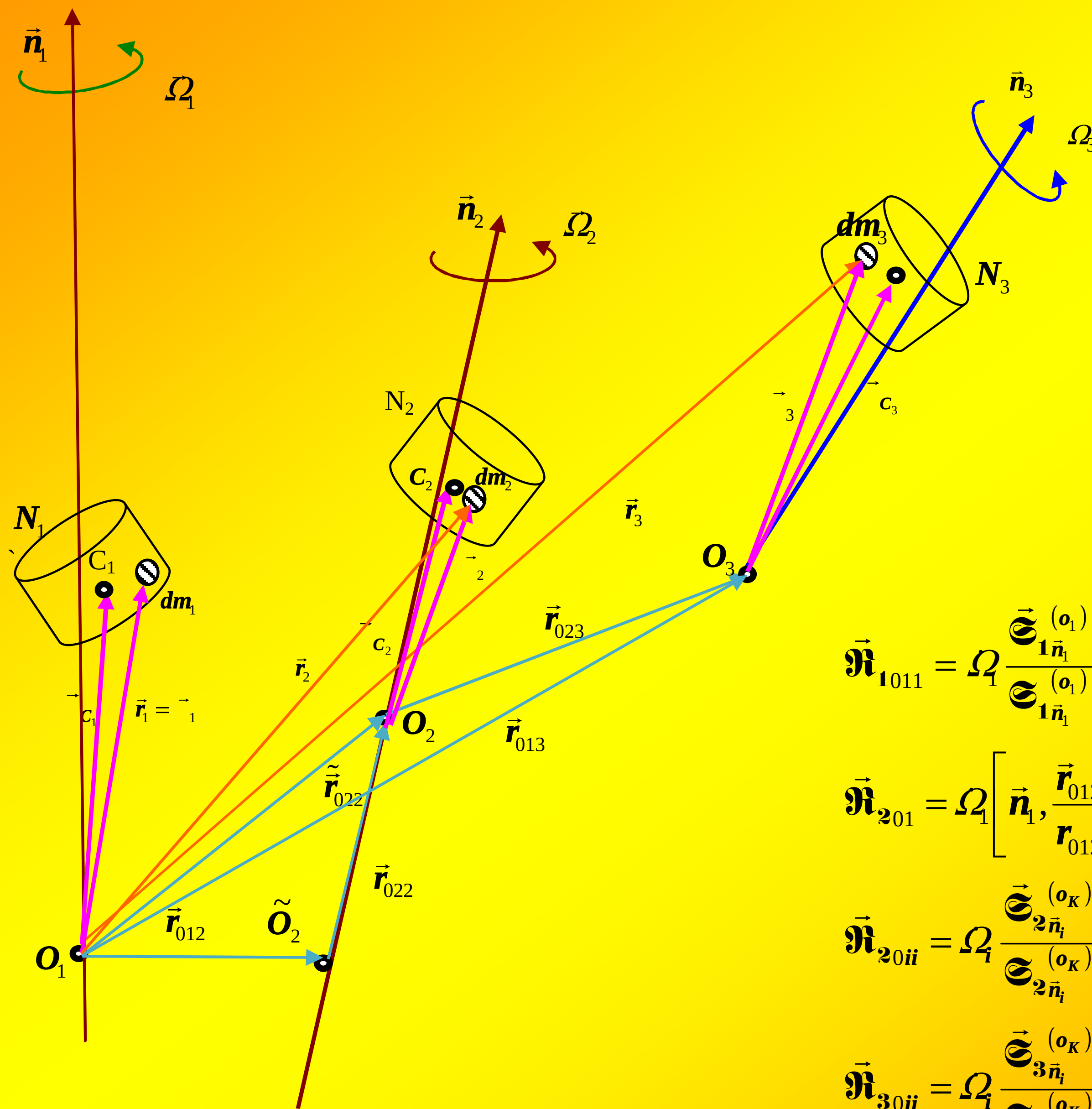
$$+2\Omega_1\Omega_2\left[\vec n_2, \vec{\mathfrak{J}}^{(O_3)}_{\vec n_1}\right] + 2\Omega_2\Omega_3\left[\vec n_3, \vec{\mathfrak{J}}^{(O_3)}_{\vec n_2}\right] + 2\Omega_1\Omega_3\left[\vec n_1, \vec{\mathfrak{J}}^{(O_3)}_{\vec n_3}\right]$$

O_2, $\tilde{\vec r}_{022}$, $\vec r_{013}$, O_3, O_1, $\tilde{\vec r}_{012}$, $\tilde{O}_2$

$$\vec{\mathfrak{R}}_1 = \Omega_1 \frac{\vec{\mathfrak{D}}^{(o_3)}_{\vec n_1}}{\mathfrak{D}^{(o_3)}_{\vec n_1}} + \Omega_1^2\left[\vec n_1, \frac{\vec{\mathfrak{D}}^{(o_3)}_{\vec n_1}}{\mathfrak{D}^{(o_3)}_{\vec n_1}}\right], \quad \vec{\mathfrak{R}}_2 = \Omega_2 \frac{\vec{\mathfrak{D}}^{(o_3)}_{\vec n_2}}{\mathfrak{D}^{(o_3)}_{\vec n_2}} + \Omega_2^2\left[\vec n_2, \frac{\vec{\mathfrak{D}}^{(o_3)}_{\vec n_2}}{\mathfrak{D}^{(o_3)}_{\vec n_2}}\right], \quad \vec{\mathfrak{R}}_3 = \Omega_3 \frac{\vec{\mathfrak{D}}^{(o_3)}_{\vec n_3}}{\mathfrak{D}^{(o_3)}_{\vec n_3}} + \Omega_3^2\left[\vec n_3, \frac{\vec{\mathfrak{D}}^{(o_3)}_{\vec n_3}}{\mathfrak{D}^{(o_3)}_{\vec n_3}}\right]$$



$\vec{u}_{O21} = \dfrac{[\vec{n}_2, [\vec{n}_1, \vec{\rho}_C]]}{|[\vec{n}_2, [\vec{n}_1, \vec{\rho}_C]]|}$
$\vec{v}_{n_2C}$
$\vec{\mathfrak{R}}_2$
$\vec{n}_2$
γ_2
ω_2^2
$\dot{\omega}_2$
$\vec{n}_{n_2C}$
$\vec{n}_1$
$\vec{n}_2$
$\vec{\mathfrak{R}}_{21}$
$\vec{v}_{O1}$
$\vec{u}_{O1}$
$\vec{\mathfrak{R}}_1$
γ_1
$\dot{\omega}_1$
ω_1^2
n_2
$\vec{n}_2$
R_{n_2C}
R_{n_1C}
ω_2
R_O
ω_2
v_2
$\vec{n}_1$
β_ξ
ω_1
C
$\dot{\omega}_2$
γ_2
$\vec{\mathfrak{R}}_{O1}$
$\vec{v}_{O2}$
ω_2^2
$\vec{\rho}_C$
$\vec{\mathfrak{R}}_2$
O_2
$\vec{u}_{O2}$
O_1
$\vec{\mathfrak{R}}_{21}$
v_1
u_2
$\vec{r}_O$
$\vec{v}_{O1}$
$\vec{u}_{O1}$
γ_1
$\vec{\mathfrak{R}}_1$
u_1

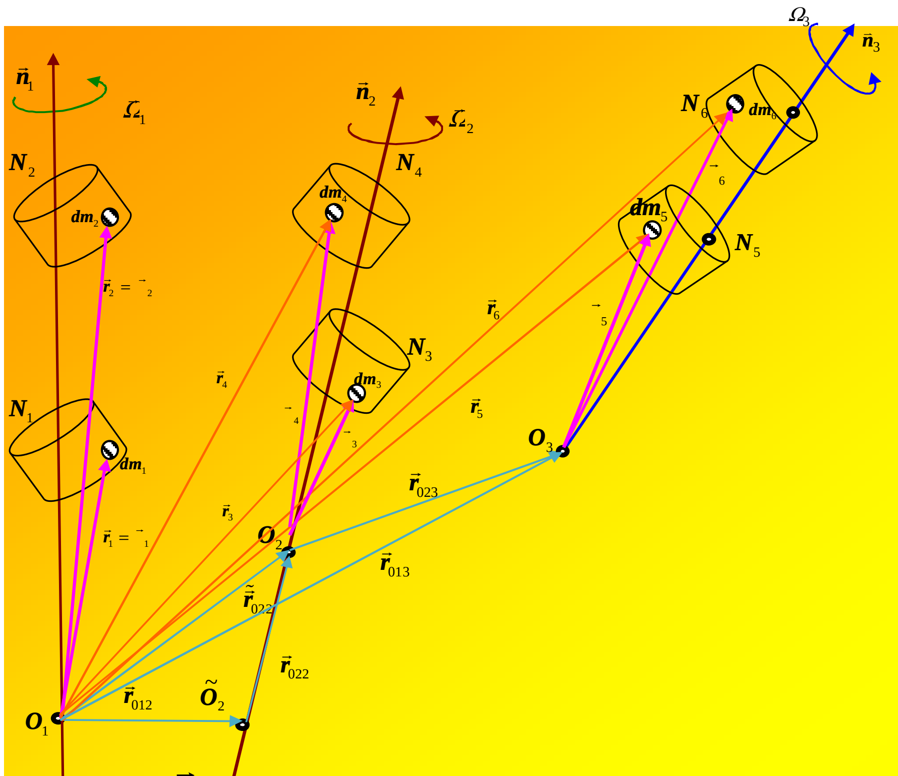


$$\vec{\mathfrak{R}}_{1011} = \Omega_1 \frac{\vec{\mathfrak{S}}^{(o_1)}_{1\vec{n}_1}}{\mathfrak{S}^{(o_1)}_{1\vec{n}_1}} + \Omega_1^2 \left[\vec{n}_1, \frac{\vec{\mathfrak{S}}^{(o_1)}_{1\vec{n}_1}}{\mathfrak{S}^{(o_1)}_{1\vec{n}_1}}\right]$$

$$\vec{\mathfrak{R}}_{201} = \Omega_1 \left[\vec{n}_1, \frac{\vec{r}_{012}}{r_{012}}\right] + \Omega_1^2 \left[\vec{n}_1, \left[\vec{n}_1, \frac{\vec{r}_{012}}{r_{012}}\right]\right]$$

$$\vec{\mathfrak{R}}_{20ii} = \Omega_i \frac{\vec{\mathfrak{S}}^{(o_K)}_{2\vec{n}_i}}{\mathfrak{S}^{(o_K)}_{2\vec{n}_i}} + \Omega_i^2 \left[\vec{n}_i, \frac{\vec{\mathfrak{S}}^{(o_K)}_{2\vec{n}_i}}{\mathfrak{S}^{(o_K)}_{2\vec{n}_i}}\right] \quad i = 1,2 \quad K = 2$$

$$\vec{\mathfrak{R}}_{30ii} = \Omega_i \frac{\vec{\mathfrak{S}}^{(o_K)}_{3\vec{n}_i}}{\mathfrak{S}^{(o_K)}_{3\vec{n}_i}} + \Omega_i^2 \left[\vec{n}_i, \frac{\vec{\mathfrak{S}}^{(o_K)}_{3\vec{n}_i}}{\mathfrak{S}^{(o_K)}_{3\vec{n}_i}}\right] \quad i = 1,2,3 \quad K = 3$$



$$\vec{\mathfrak{R}}_{1011} = \Omega_1 \frac{\vec{\mathfrak{S}}^{(o_1)}_{1\vec n_1}}{\mathfrak{S}^{(o_1)}_{1\vec n_1}} + \Omega_1^2\left[\vec n_1, \frac{\vec{\mathfrak{S}}^{(o_1)}_{1\vec n_1}}{\mathfrak{S}^{(o_1)}_{1\vec n_1}}\right], \quad \vec{\mathfrak{R}}_{2011} = \Omega_1 \frac{\vec{\mathfrak{S}}^{(o_1)}_{2\vec n_1}}{\mathfrak{S}^{(o_1)}_{2\vec n_1}} + \Omega_1^2\left[\vec n_1, \frac{\vec{\mathfrak{S}}^{(o_1)}_{2\vec n_1}}{\mathfrak{S}^{(o_1)}_{2\vec n_1}}\right]$$

$$\vec{\mathfrak{R}}_{301} = \Omega_1\left[\vec n_1, \frac{\vec r_{012}}{r_{012}}\right] + \Omega_1^2\left[\vec n_1, \left[\vec n_1, \frac{\vec r_{012}}{r_{012}}\right]\right]$$

$$\vec{\mathfrak{R}}_{30ii} = \Omega_i \frac{\vec{\mathfrak{S}}^{(o_K)}_{3\vec n_i}}{\mathfrak{S}^{(o_K)}_{3\vec n_i}} + \Omega_i^2\left[\vec n_i, \frac{\vec{\mathfrak{S}}^{(o_K)}_{3\vec n_i}}{\mathfrak{S}^{(o_K)}_{3\vec n_i}}\right] \quad i = 1,2 \quad K = 2$$

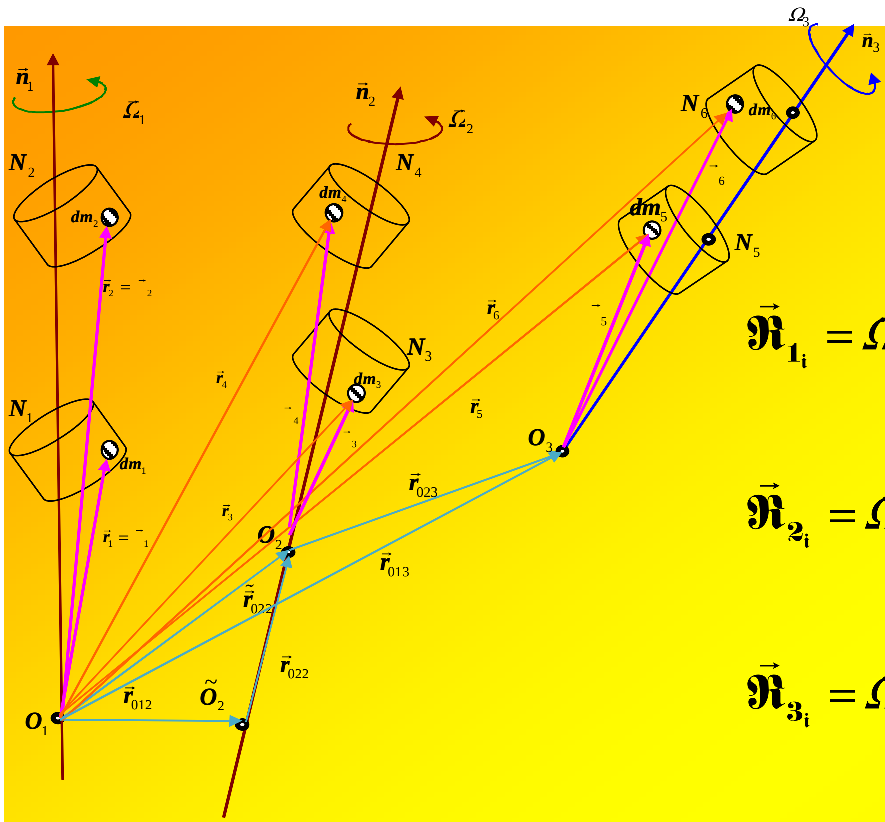
$$\vec{\mathfrak{R}}_{40ii} = \Omega_i \frac{\vec{\mathfrak{S}}^{(o_K)}_{4\vec n_i}}{\mathfrak{S}^{(o_K)}_{4\vec n_i}} + \Omega_i^2\left[\vec n_i, \frac{\vec{\mathfrak{S}}^{(o_K)}_{4\vec n_i}}{\mathfrak{S}^{(o_K)}_{4\vec n_i}}\right] \quad i = 1,2 \quad K = 2$$

$$\vec{\mathfrak{R}}_{50ii} = \Omega_i \frac{\vec{\mathfrak{S}}^{(o_K)}_{5\vec n_i}}{\mathfrak{S}^{(o_K)}_{5\vec n_i}} + \Omega_i^2\left[\vec n_i, \frac{\vec{\mathfrak{S}}^{(o_K)}_{5\vec n_i}}{\mathfrak{S}^{(o_K)}_{5\vec n_i}}\right] \quad i = 1,2,3 \quad K = 3$$

$$\vec{\mathfrak{R}}_{60ii} = \Omega_i \frac{\vec{\mathfrak{S}}^{(o_K)}_{6\vec n_i}}{\mathfrak{S}^{(o_K)}_{6\vec n_i}} + \Omega_i^2\left[\vec n_i, \frac{\vec{\mathfrak{S}}^{(o_K)}_{6\vec n_i}}{\mathfrak{S}^{(o_K)}_{6\vec n_i}}\right] \quad i = 1,2,3 \quad K = 3$$

$$\begin{aligned}
\sum_{i=1}^{6} \frac{d\vec{\mathfrak{K}}_i}{dt} = {} & \vec{\mathfrak{R}}_{1011}\left|\mathfrak{S}^{(o_1)}_{1\vec n_1}\right| + \vec{\mathfrak{R}}_{2011}\left|\mathfrak{S}^{(o_1)}_{2\vec n_1}\right| + \vec{\mathfrak{R}}_{301}\left|[\vec n_1, \vec r_{012}]\right| M_3 + \vec{\mathfrak{R}}_{3011}\left|\mathfrak{S}^{(o_2)}_{3\vec n_1}\right| + \vec{\mathfrak{R}}_{3022}\left|\mathfrak{S}^{(o_2)}_{3\vec n_2}\right| + 2\Omega_1\Omega_2\left[\vec n_2, \vec{\mathfrak{S}}^{(O_2)}_{3\vec n_1}\right] \\
& + \vec{\mathfrak{R}}_{401}\left|[\vec n_1, \vec r_{012}]\right| M_4 + \vec{\mathfrak{R}}_{4011}\left|\mathfrak{S}^{(o_2)}_{4\vec n_1}\right| + \vec{\mathfrak{R}}_{4022}\left|\mathfrak{S}^{(o_2)}_{4\vec n_2}\right| + 2\Omega_1\Omega_2\left[\vec n_2, \vec{\mathfrak{S}}^{(O_2)}_{4\vec n_1}\right] + \\
& + \vec{\mathfrak{R}}_{501}\left|[\vec n_1, \vec r_{O_3}]\right| M_5 + \vec{\mathfrak{R}}_{502}\left|[\vec n_2, \vec r_{023}]\right| M_5 + 2\Omega_1\Omega_2\left[\vec n_1, [\vec n_2, \vec r_{023}]\right] M_5 + \vec{\mathfrak{R}}_{5011}\left|\mathfrak{S}^{(o_3)}_{5\vec n_1}\right| + \\
& + \vec{\mathfrak{R}}_{5022}\left|\mathfrak{S}^{(o_3)}_{5\vec n_2}\right| + \vec{\mathfrak{R}}_{5033}\left|\mathfrak{S}^{(o_3)}_{5\vec n_3}\right| + 2\Omega_1\Omega_2\left[\vec n_1, \vec{\mathfrak{S}}^{(O_3)}_{5\vec n_2}\right] + 2\Omega_1\Omega_3\left[\vec n_1, \vec{\mathfrak{S}}^{(O_3)}_{5\vec n_3}\right] + 2\Omega_2\Omega_3\left[\vec n_2, \vec{\mathfrak{S}}^{(O_3)}_{5\vec n_3}\right] + \\
& + \vec{\mathfrak{R}}_{601}\left|[\vec n_1, \vec r_{O_3}]\right| M_6 + \vec{\mathfrak{R}}_{602}\left|[\vec n_2, \vec r_{023}]\right| M_6 + 2\Omega_1\Omega_2\left[\vec n_1, [\vec n_2, \vec r_{023}]\right] M_6 + \vec{\mathfrak{R}}_{6011}\left|\mathfrak{S}^{(o_3)}_{6\vec n_1}\right| + \\
& + \vec{\mathfrak{R}}_{6022}\left|\mathfrak{S}^{(o_3)}_{6\vec n_2}\right| + \vec{\mathfrak{R}}_{6033}\left|\mathfrak{S}^{(o_3)}_{6\vec n_3}\right| + 2\Omega_1\Omega_2\left[\vec n_1, \vec{\mathfrak{S}}^{(O_3)}_{6\vec n_2}\right] + 2\Omega_1\Omega_3\left[\vec n_1, \vec{\mathfrak{S}}^{(O_3)}_{6\vec n_3}\right] + 2\Omega_2\Omega_3\left[\vec n_2, \vec{\mathfrak{S}}^{(O_3)}_{6\vec n_3}\right]
\end{aligned}$$

$$
\begin{aligned}
&\sum_{i=1}^{6}\frac{d\vec{\mathfrak{L}}_{iO_1}}{dt}=\Omega_1\left(\vec{n}_1,\vec{\mathfrak{J}}_{1\vec{n}_1}^{(O_1)}\right)\vec{n}_1+\vec{\mathfrak{K}}_{1_1}\left|\vec{\mathfrak{D}}_{1\vec{n}_1}^{(O_1)}\right|+\Omega_1\left(\vec{n}_1,\vec{\mathfrak{J}}_{2\vec{n}_1}^{(O_1)}\right)\vec{n}_1+\vec{\mathfrak{K}}_{2_1}\left|\vec{\mathfrak{D}}_{2\vec{n}_1}^{(O_1)}\right|+\\
&+\vec{\ }_{3_{12}}\left(\vec{r}_{012},\vec{\ }_{C_3},M_3,\vec{\mathfrak{S}}_{3\vec{n}_1}^{(O_2)},\vec{\mathfrak{S}}_{3\vec{n}_2}^{(O_2)},\dot{\Omega}_1,\dot{\Omega}_2,\Omega_1,\Omega_2,\vec{n}_1,\vec{n}_2,\right)+2\Omega_1\Omega_2\left[\vec{n}_1,\vec{\mathfrak{J}}_{3\vec{n}_2}^{(O_2)}\right]+\\
&+\vec{\mathfrak{K}}_{3_1}\left|\vec{\mathfrak{D}}_{3\vec{n}_1}^{(O_2)}\right|+\vec{\mathfrak{K}}_{3_2}\left|\vec{\mathfrak{D}}_{3\vec{n}_2}^{(O_2)}\right|+\Omega_1\left(\vec{n}_1,\vec{\mathfrak{J}}_{3\vec{n}_1}^{(O_2)}\right)\vec{n}_1+\Omega_2\left(\vec{n}_2,\vec{\mathfrak{J}}_{3\vec{n}_2}^{(O_2)}\right)\vec{n}_2+\\
&+\vec{\ }_{4_{12}}\left(\vec{r}_{012},\vec{\ }_{C_4},M_4,\vec{\mathfrak{S}}_{4\vec{n}_1}^{(O_2)},\vec{\mathfrak{S}}_{4\vec{n}_2}^{(O_2)},\dot{\Omega}_1,\dot{\Omega}_2,\Omega_1,\Omega_2,\vec{n}_1,\vec{n}_2,\right)+2\Omega_1\Omega_2\left[\vec{n}_1,\vec{\mathfrak{J}}_{4\vec{n}_2}^{(O_2)}\right]+\\
&+\vec{\mathfrak{K}}_{4_1}\left|\vec{\mathfrak{D}}_{4\vec{n}_1}^{(O_2)}\right|+\vec{\mathfrak{K}}_{4_2}\left|\vec{\mathfrak{D}}_{4\vec{n}_2}^{(O_2)}\right|+\Omega_1\left(\vec{n}_1,\vec{\mathfrak{J}}_{4\vec{n}_1}^{(O_2)}\right)\vec{n}_1+\Omega_2\left(\vec{n}_2,\vec{\mathfrak{J}}_{4\vec{n}_2}^{(O_2)}\right)\vec{n}_2+\\
&+\vec{\ }_{5_{123}}\left(\vec{r}_{5O_3},\vec{\ }_{C_5},\vec{\mathfrak{S}}_{5\vec{n}_1}^{(O_{1-2-2})},\vec{\mathfrak{S}}_{5\vec{n}_1}^{(O_{2-3})},\vec{\mathfrak{S}}_{5\vec{n}_2}^{(O_{2-3})},\vec{\mathfrak{S}}_{5\vec{n}_1}^{(O_3)},\vec{\mathfrak{S}}_{5\vec{n}_2}^{(O_3)},\vec{\mathfrak{S}}_{5\vec{n}_3}^{(O_3)},\dot{\Omega}_1,\dot{\Omega}_2,\dot{\Omega}_3,\Omega_1,\Omega_2,\Omega_3,\vec{n}_1,\vec{n}_2,\vec{n}_3\right)+\\
&+\vec{\mathfrak{K}}_{5_1}\left|\vec{\mathfrak{D}}_{5\vec{n}_1}^{(O_3)}\right|+\vec{\mathfrak{K}}_{5_2}\left|\vec{\mathfrak{D}}_{5\vec{n}_2}^{(O_3)}\right|+\vec{\mathfrak{K}}_{5_3}\left|\vec{\mathfrak{D}}_{5\vec{n}_3}^{(O_3)}\right|+\Omega_1\left(\vec{n}_1,\vec{\mathfrak{J}}_{5\vec{n}_1}^{(O_3)}\right)\vec{n}_1+\Omega_2\left(\vec{n}_2,\vec{\mathfrak{J}}_{5\vec{n}_2}^{(O_3)}\right)\vec{n}_2+\Omega_3\left(\vec{n}_3,\vec{\mathfrak{J}}_{5\vec{n}_3}^{(O_3)}\right)\vec{n}_3+\\
&+2\Omega_1\Omega_2\left[\vec{n}_2,\vec{\mathfrak{J}}_{5\vec{n}_1}^{(O_3)}\right]+2\Omega_2\Omega_3\left[\vec{n}_3,\vec{\mathfrak{J}}_{5\vec{n}_2}^{(O_3)}\right]+2\Omega_1\Omega_3\left[\vec{n}_1,\vec{\mathfrak{J}}_{5\vec{n}_3}^{(O_3)}\right]+\\
&+\vec{\ }_{6_{123}}\left(\vec{r}_{6O_3},\vec{\ }_{C_6},\vec{\mathfrak{S}}_{6\vec{n}_1}^{(O_{1-2-2})},\vec{\mathfrak{S}}_{6\vec{n}_1}^{(O_{2-3})},\vec{\mathfrak{S}}_{6\vec{n}_2}^{(O_{2-3})},\vec{\mathfrak{S}}_{6\vec{n}_1}^{(O_3)},\vec{\mathfrak{S}}_{6\vec{n}_2}^{(O_3)},\vec{\mathfrak{S}}_{6\vec{n}_3}^{(O_3)},\Omega_1,\Omega_2,\Omega_3,\vec{n}_1,\vec{n}_2,\vec{n}_3\right)+\\
&+\vec{\mathfrak{K}}_{6_1}\left|\vec{\mathfrak{D}}_{6\vec{n}_1}^{(O_3)}\right|+\vec{\mathfrak{K}}_{6_2}\left|\vec{\mathfrak{D}}_{6\vec{n}_2}^{(O_3)}\right|+\vec{\mathfrak{K}}_{6_2}\left|\vec{\mathfrak{D}}_{6\vec{n}_3}^{(O_3)}\right|+\Omega_1\left(\vec{n}_1,\vec{\mathfrak{J}}_{6\vec{n}_1}^{(O_3)}\right)\vec{n}_1+\Omega_2\left(\vec{n}_2,\vec{\mathfrak{J}}_{6\vec{n}_2}^{(O_3)}\right)\vec{n}_2+\Omega_3\left(\vec{n}_3,\vec{\mathfrak{J}}_{6\vec{n}_3}^{(O_3)}\right)\vec{n}_3\\
&+2\Omega_1\Omega_2\left[\vec{n}_2,\vec{\mathfrak{J}}_{6\vec{n}_1}^{(O_3)}\right]+2\Omega_2\Omega_3\left[\vec{n}_3,\vec{\mathfrak{J}}_{6\vec{n}_2}^{(O_3)}\right]+2\Omega_1\Omega_3\left[\vec{n}_1,\vec{\mathfrak{J}}_{6\vec{n}_3}^{(O_3)}\right]
\end{aligned}
$$



$$\vec{\mathfrak{R}}_{1_i} = \Omega_i \frac{\vec{\mathfrak{D}}^{(o_K)}_{1\vec{n}_i}}{\mathfrak{D}^{(o_K)}_{1\vec{n}_i}} + \Omega_i^2\left[\vec{n}_i, \frac{\vec{\mathfrak{D}}^{(o_K)}_{1\vec{n}_i}}{\mathfrak{D}^{(o_K)}_{1\vec{n}_i}}\right] \quad i=1 \quad K=1$$

$$\vec{\mathfrak{R}}_{2_i} = \Omega_i \frac{\vec{\mathfrak{D}}^{(o_K)}_{2\vec{n}_i}}{\mathfrak{D}^{(o_K)}_{2\vec{n}_i}} + \Omega_i^2\left[\vec{n}_i, \frac{\vec{\mathfrak{D}}^{(o_K)}_{2\vec{n}_i}}{\mathfrak{D}^{(o_K)}_{2\vec{n}_i}}\right] \quad i=1 \quad K=1$$

$$\vec{\mathfrak{R}}_{3_i} = \Omega_i \frac{\vec{\mathfrak{D}}^{(o_K)}_{3\vec{n}_i}}{\mathfrak{D}^{(o_K)}_{3\vec{n}_i}} + \Omega_i^2\left[\vec{n}_i, \frac{\vec{\mathfrak{D}}^{(o_K)}_{3\vec{n}_i}}{\mathfrak{D}^{(o_K)}_{3\vec{n}_i}}\right] \quad i=1,2 \quad K=2$$

$$\vec{\mathfrak{R}}_{4_i} = \Omega_i \frac{\vec{\mathfrak{D}}^{(o_K)}_{4\vec{n}_i}}{\mathfrak{D}^{(o_K)}_{4\vec{n}_i}} + \Omega_i^2\left[\vec{n}_i, \frac{\vec{\mathfrak{D}}^{(o_K)}_{4\vec{n}_i}}{\mathfrak{D}^{(o_K)}_{4\vec{n}_i}}\right] \quad i=1,2 \quad K=2$$

$$\vec{\mathfrak{R}}_{5_i} = \Omega_i \frac{\vec{\mathfrak{D}}^{(o_K)}_{5\vec{n}_i}}{\mathfrak{D}^{(o_K)}_{5\vec{n}_i}} + \Omega_i^2\left[\vec{n}_i, \frac{\vec{\mathfrak{D}}^{(o_K)}_{5\vec{n}_i}}{\mathfrak{D}^{(o_K)}_{5\vec{n}_i}}\right] \quad i=1,2,3 \quad K=3$$

$$\vec{\mathfrak{R}}_{6_i} = \Omega_i \frac{\vec{\mathfrak{D}}^{(o_K)}_{6\vec{n}_i}}{\mathfrak{D}^{(o_K)}_{6\vec{n}_i}} + \Omega_i^2\left[\vec{n}_i, \frac{\vec{\mathfrak{D}}^{(o_K)}_{6\vec{n}_i}}{\mathfrak{D}^{(o_K)}_{6\vec{n}_i}}\right] \quad i=1,2,3 \quad K=3$$

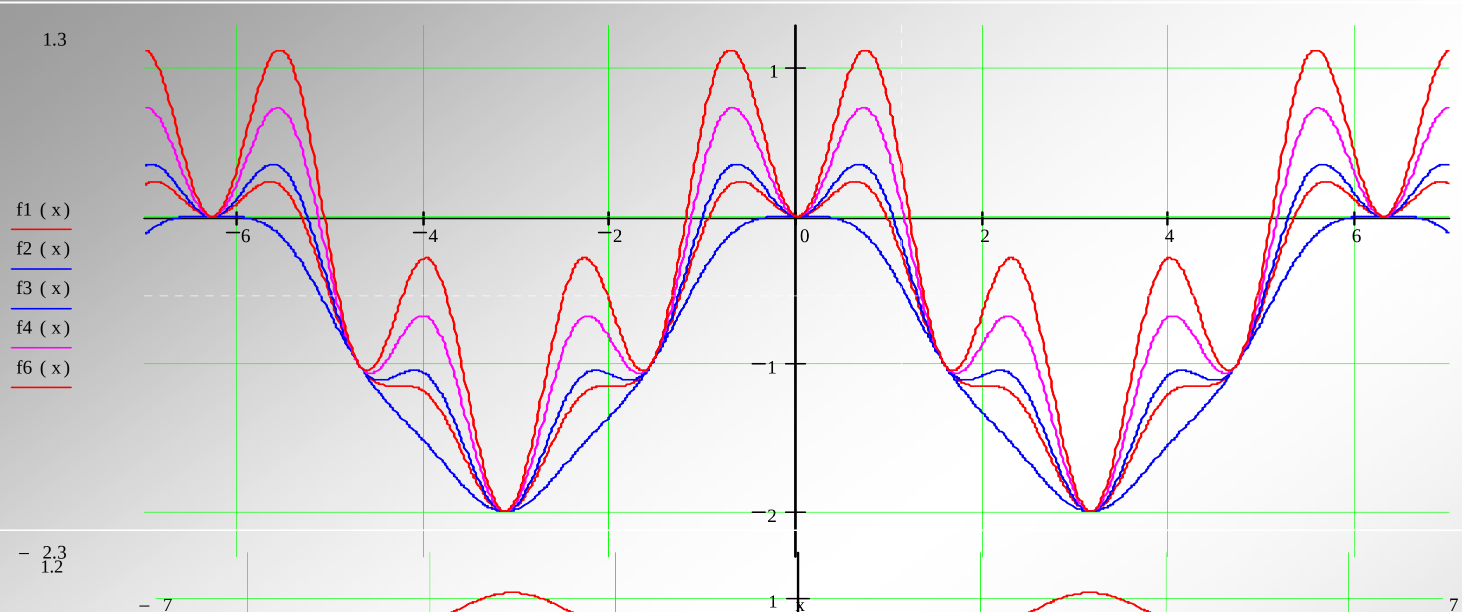
THEOREM 2.

Vector expression of angular momentum derivatives of the rigid N bodies, multi coupled rotations, around no intersecting axis in all cases, placed bodies on the each axis, between other terms, contain sum of products by intensity of rigid N bodies mass deviation moment vectors

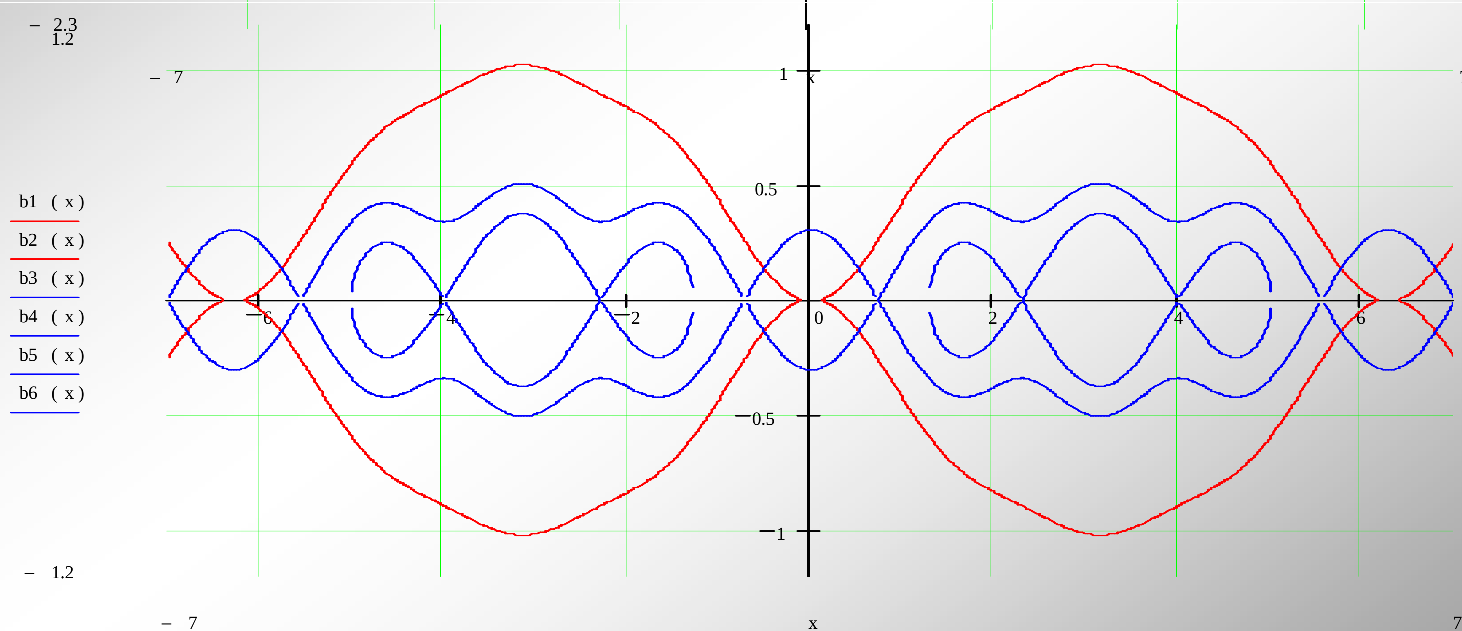
$$\left|\mathfrak{D}_{\mathrm{i}\vec{n}_j}^{(o_K)}\right| = \left\|\left[\vec{n}_j, \left[\iiint\limits_{V_i}\left[\vec{n}_j, \vec{\;}_i\right] dm_i, \vec{n}_j\right]\right]\right\|, \quad i = 1,2,3\ldots N, \quad j = 1,2,3\ldots K$$

for the axes oriented by unit vectors of component coupled rotation axes through pole on the rigid N bodies self-rotation axis and vector rotators defined by:

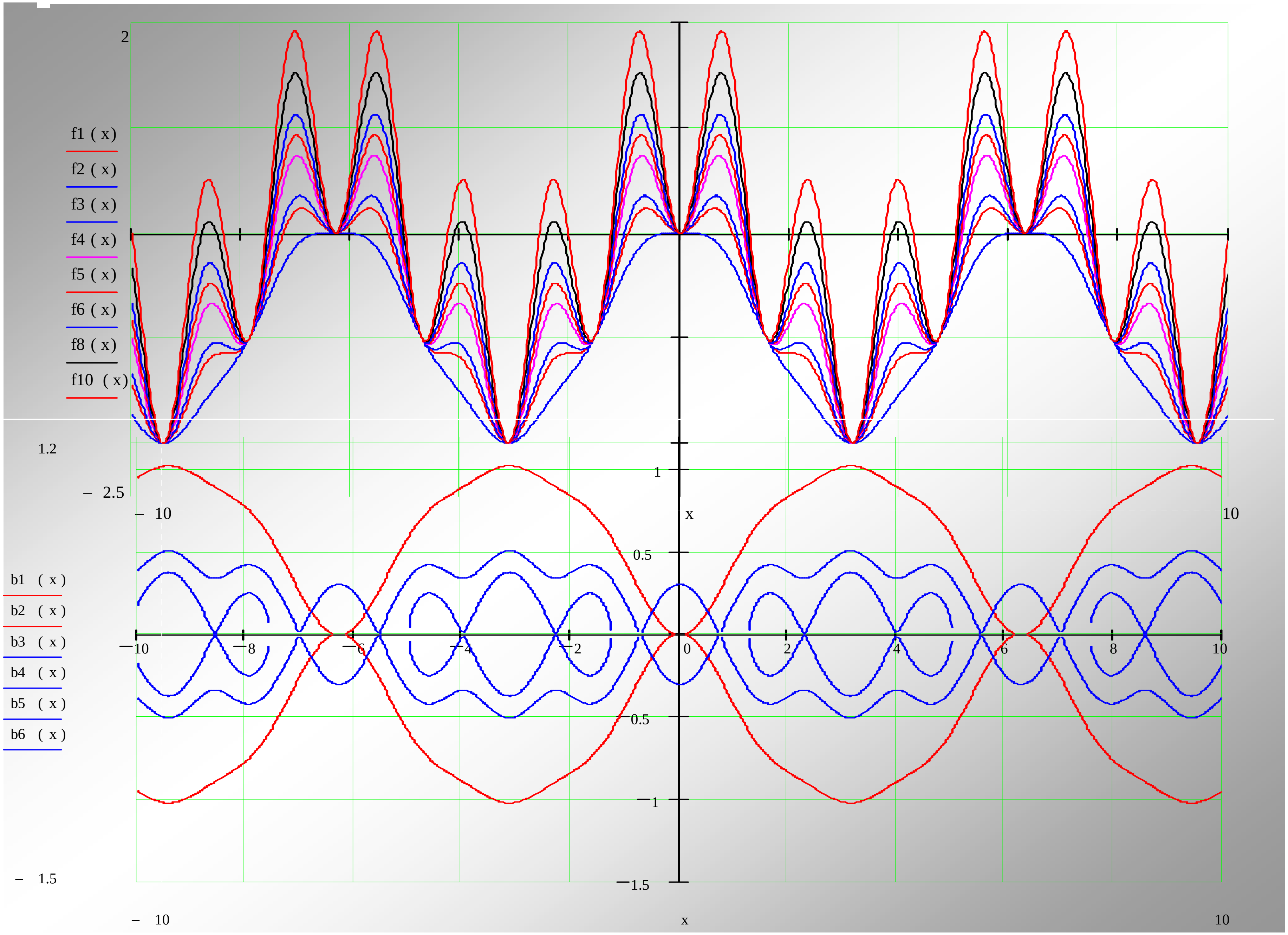
$$\vec{\mathfrak{R}}_{\mathrm{i}_\mathrm{j}} = \Omega_j \frac{\vec{\mathfrak{D}}_{\mathrm{i}\vec{n}_j}^{(o_K)}}{\left|\mathfrak{D}_{\mathrm{i}\vec{n}_j}^{(o_K)}\right|} + \Omega_j^2 \left[\vec{n}_j, \frac{\vec{\mathfrak{D}}_{\mathrm{i}\vec{n}_j}^{(o_K)}}{\left|\mathfrak{D}_{\mathrm{i}\vec{n}_j}^{(o_K)}\right|}\right] \quad i = 1,2,3\ldots N, \quad j = 1,2,3\ldots K$$

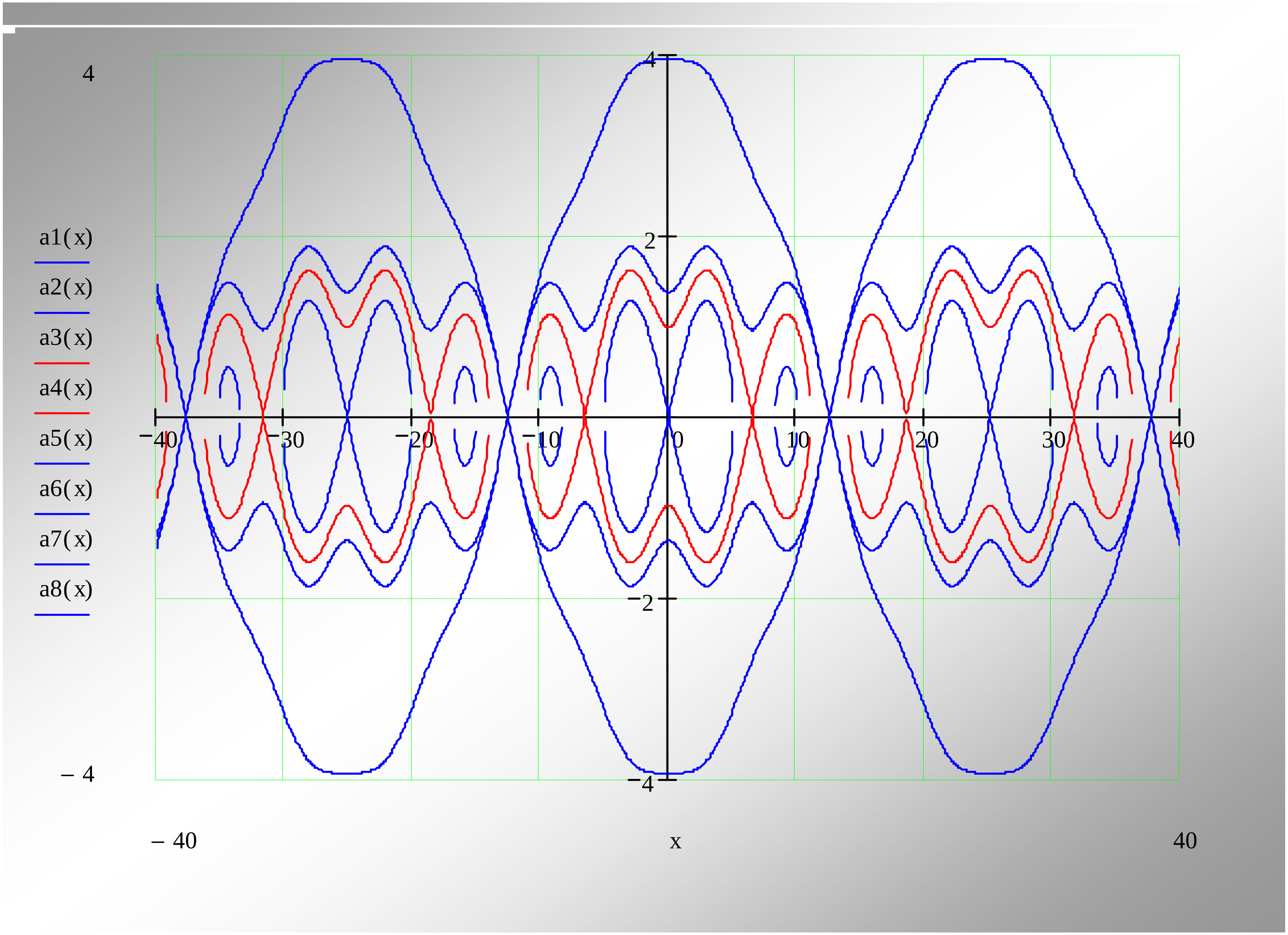


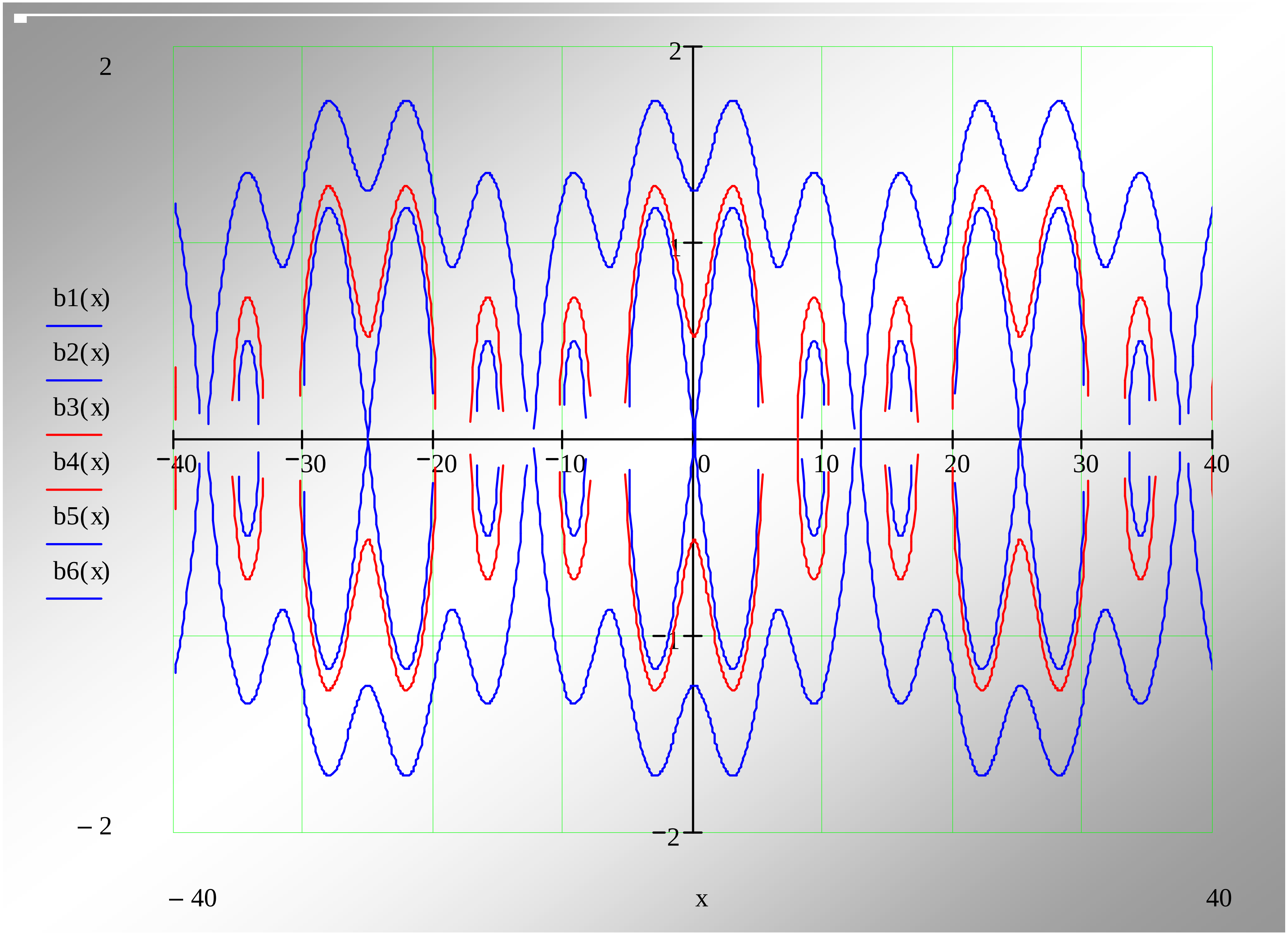
1.3
f1 (x)
f2 (x)
f3 (x)
f4 (x)
f6 (x)
−2.3
−7
x
7

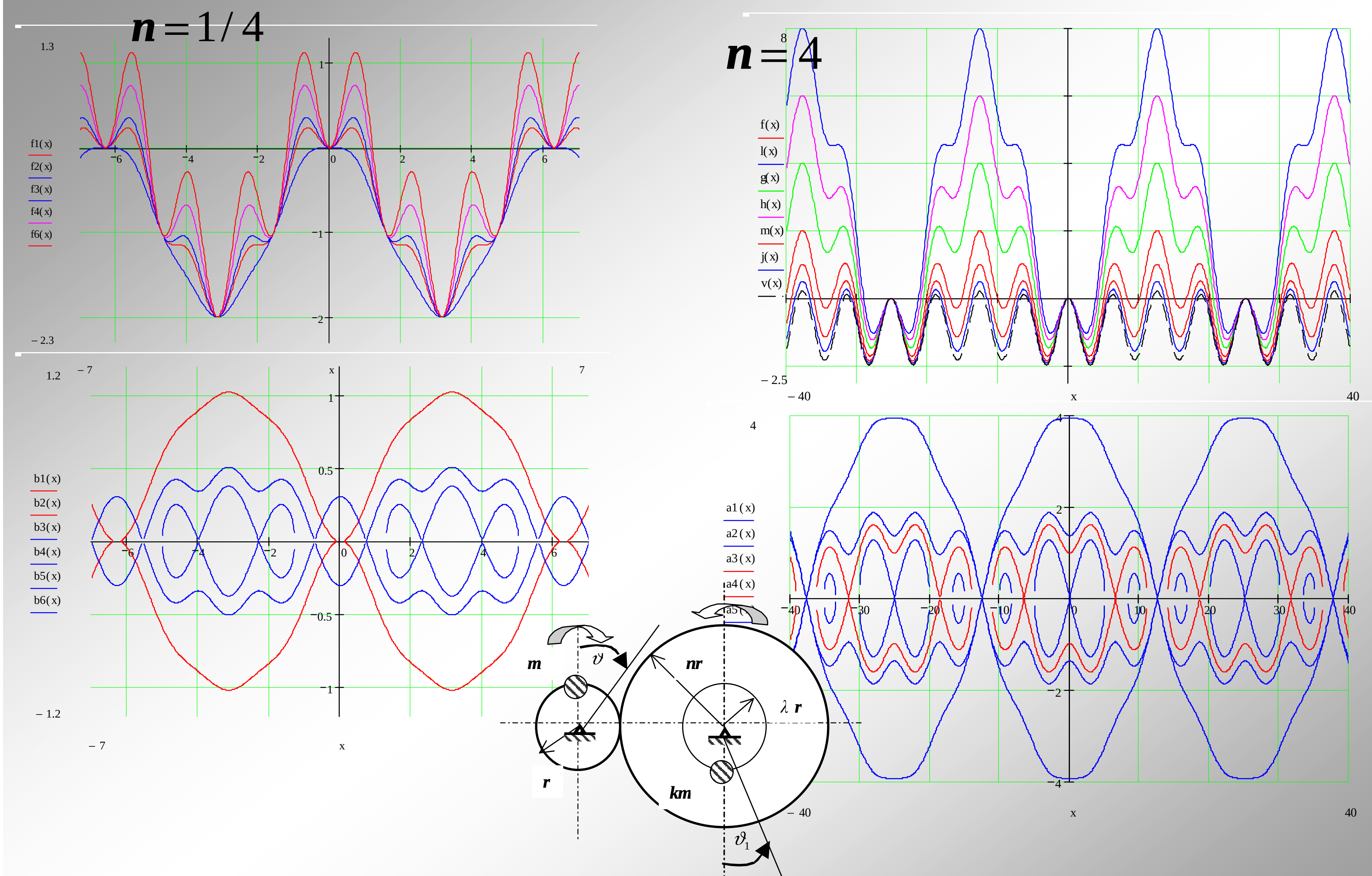


1.2
b1 (x)
b2 (x)
b3 (x)
b4 (x)
b5 (x)
b6 (x)
−1.2
−7
x
7


f1 (x)
f2 (x)
f3 (x)
f4 (x)
f5 (x)
f6 (x)
f8 (x)
f10 (x)
2
− 2.5
− 10
x
10
b1 (x)
b2 (x)
b3 (x)
b4 (x)
b5 (x)
b6 (x)
1.2
− 1.5
1
0.5
−0.5
−1
−1.5
−10
−8
−6
−4
−2
0
2
4
6
8
10
− 10
x
10


4
a1(x)
a2(x)
a3(x)
a4(x)
a5(x)
a6(x)
a7(x)
a8(x)
− 4
− 40
x
40
4
2
−2
−4
−40
−30
−20
−10
0
10
20
30
40


2
−2
b1(x)
b2(x)
b3(x)
b4(x)
b5(x)
b6(x)
−40
−30
−20
−10
0
10
20
30
40
1
−1
x
− 40
40



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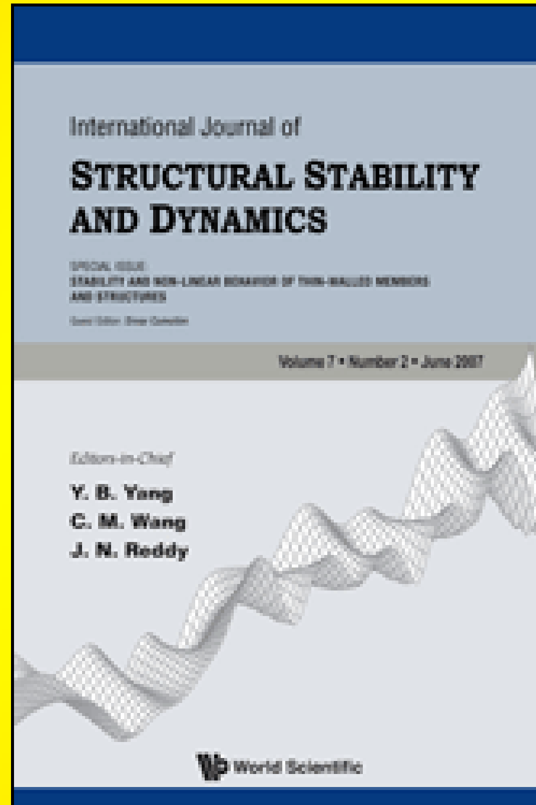

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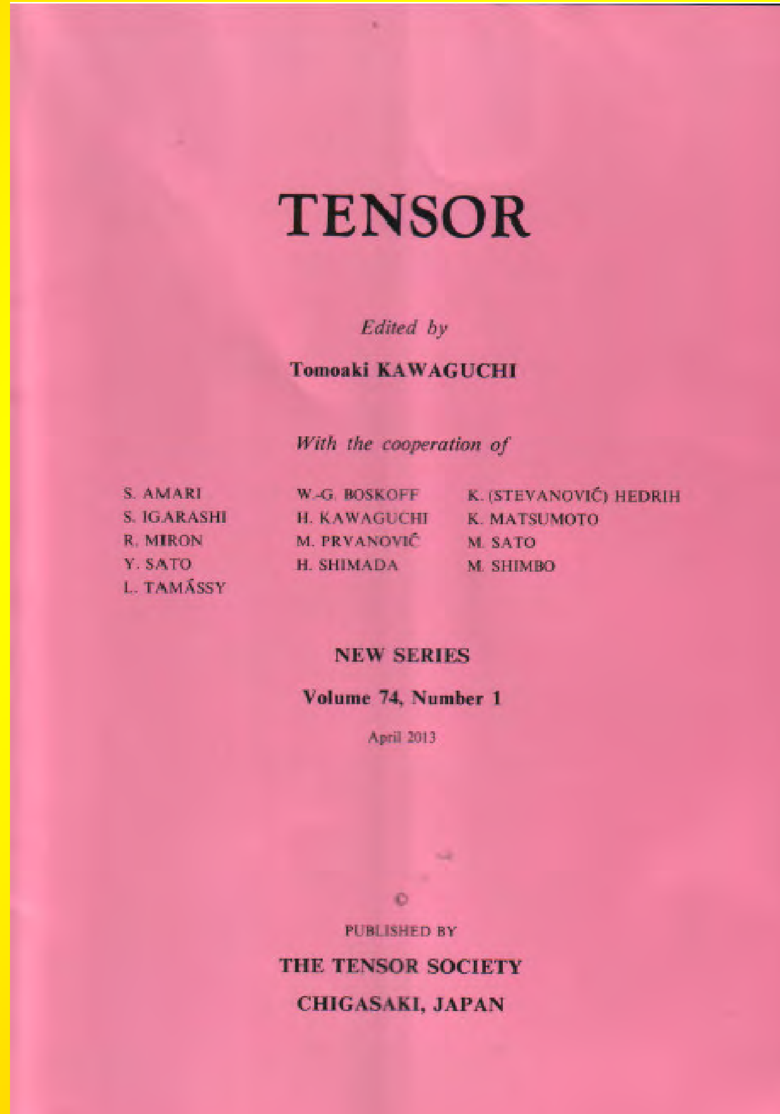
In Romaina Academy of Sciences (Depatmeny in Iashi)


Some new Mechanics based on Lagrangian and
Hamiltonian Geometries*
Radu Miron
1 Introduction
ACADEMIA ROMANA


Generalized function of fractional order
dissipation of system energy
ACADEMIA ROMÂNA



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Elements of mathematical phenomenology in dynamics of multi-body system with fractional order discrete continuum layers

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http://afrodita.rcub.bg.ac.rs/~nds/ ,http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

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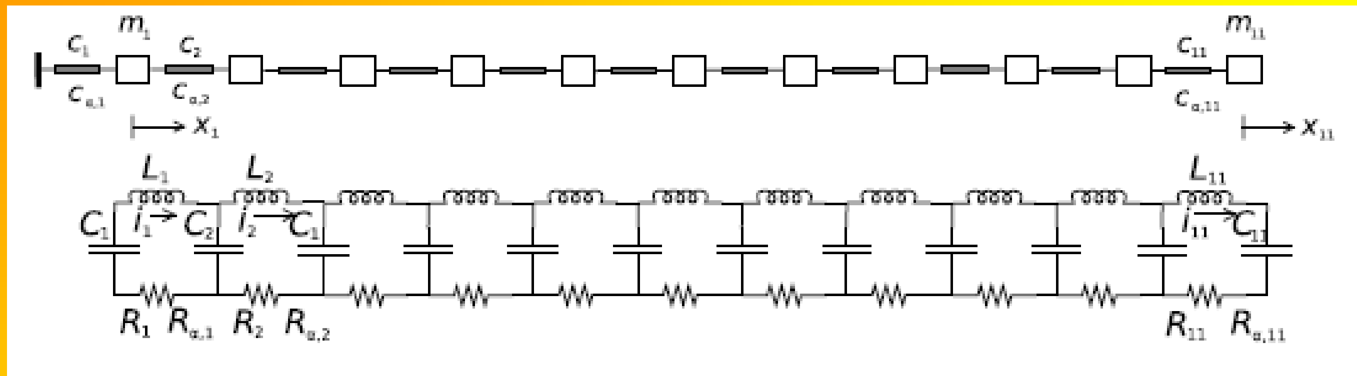
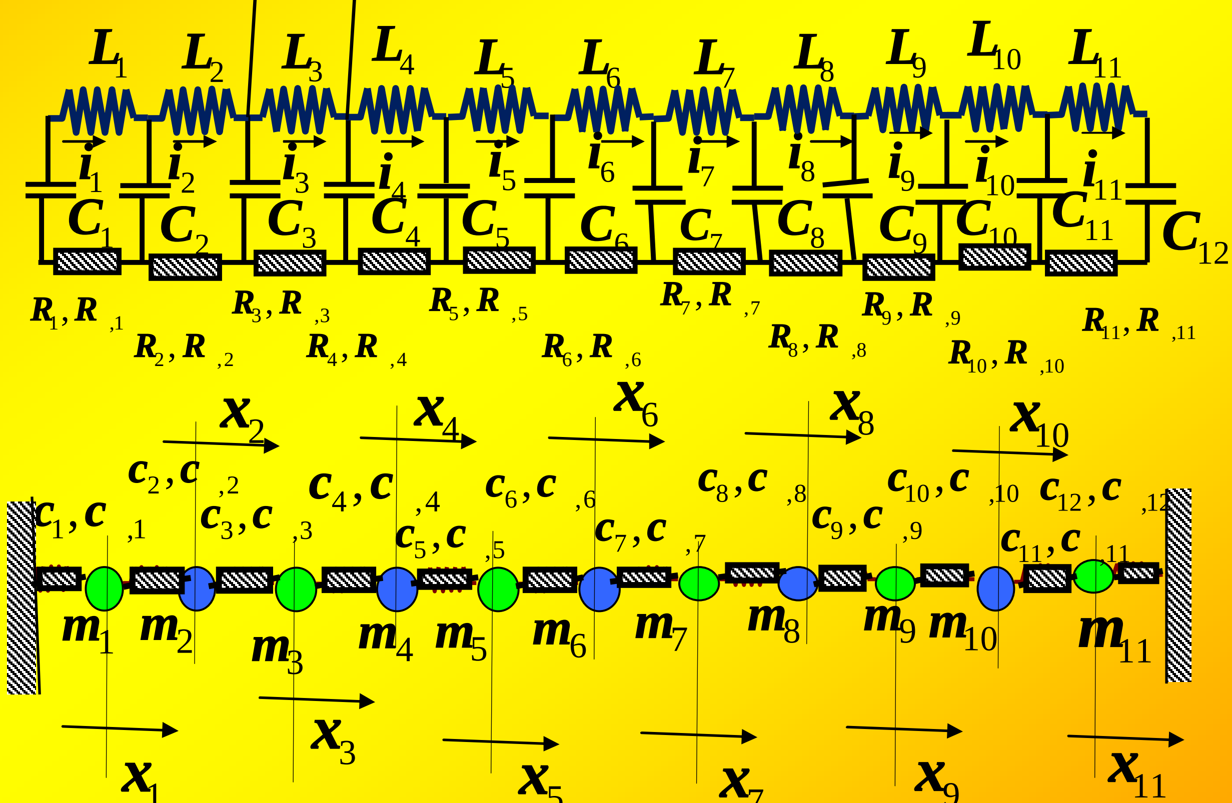
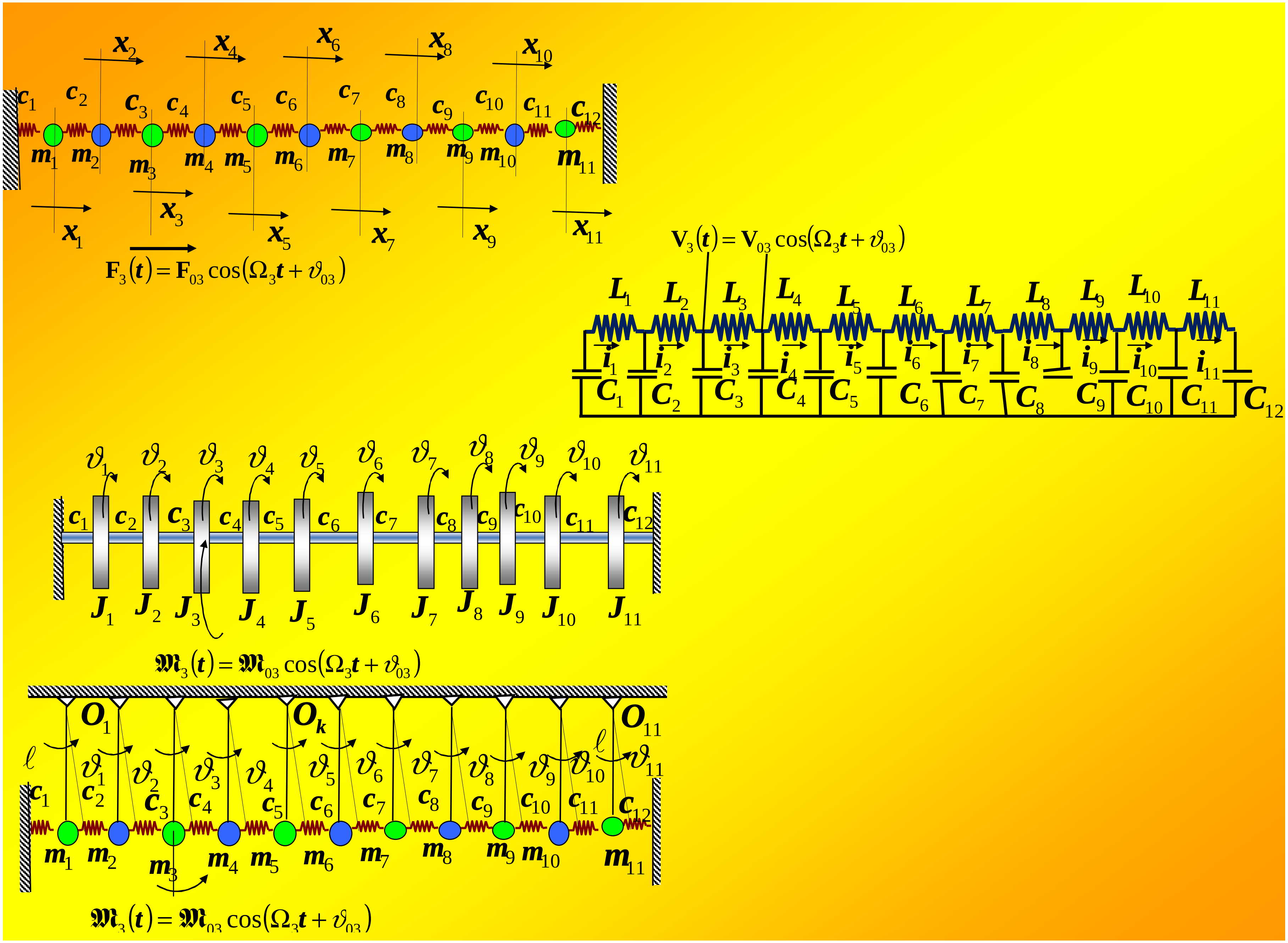
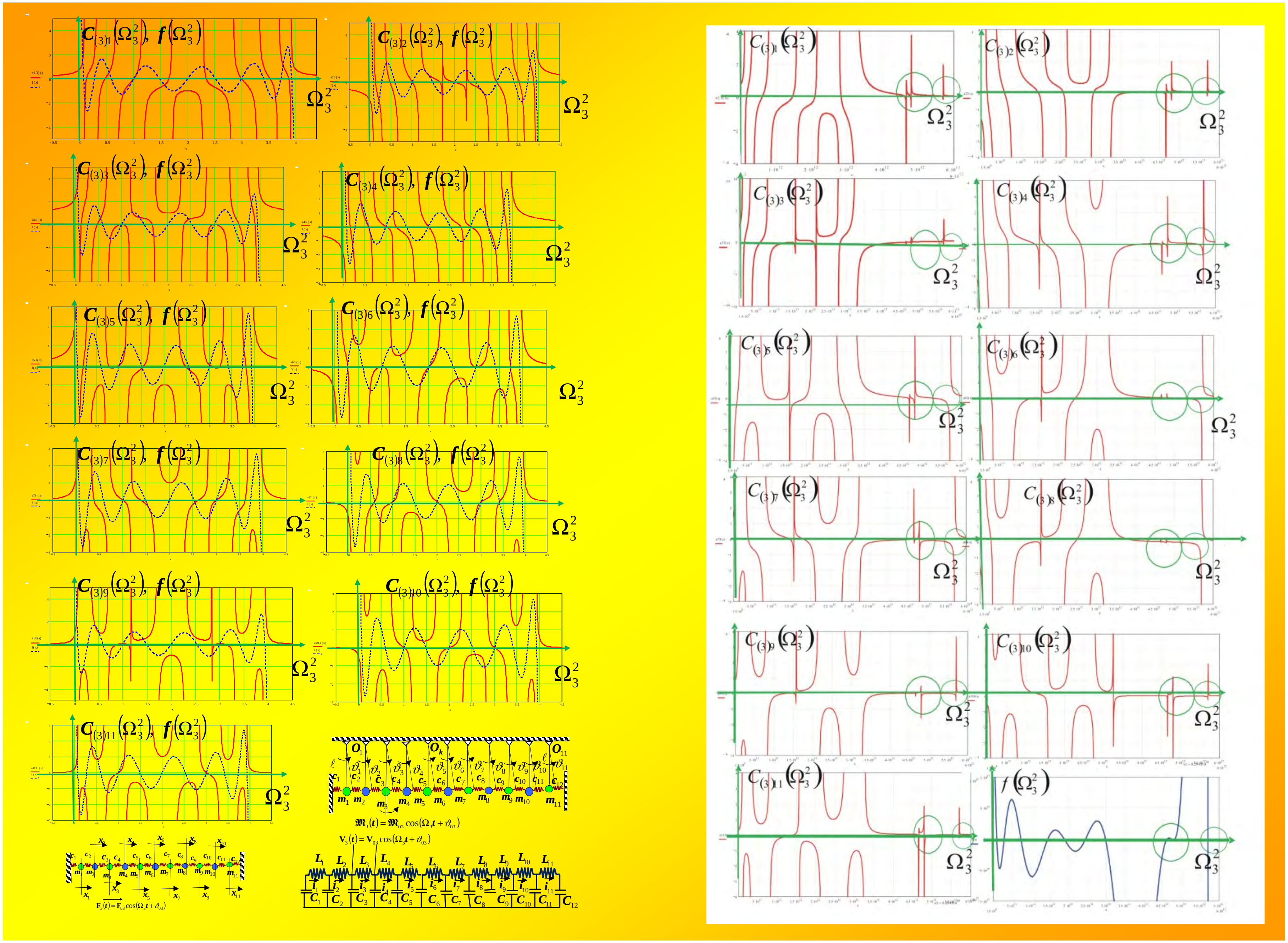


$$V_3(t) = V_{03}\cos(\Omega_3 t + \vartheta_{03})$$

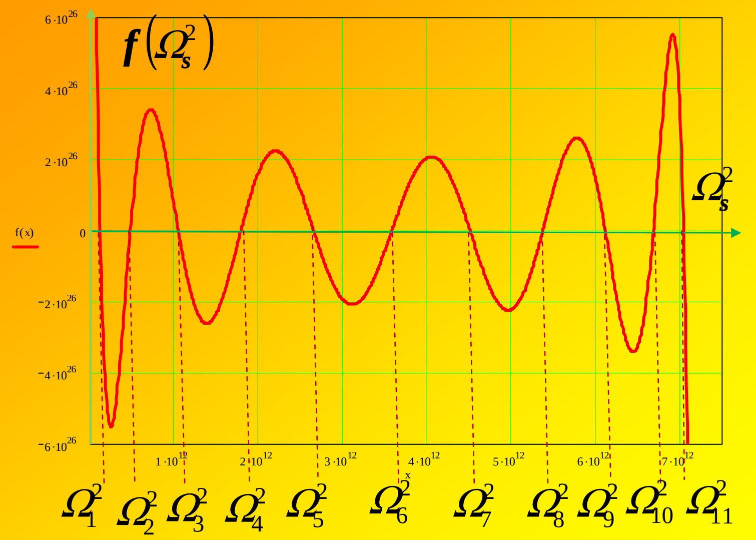


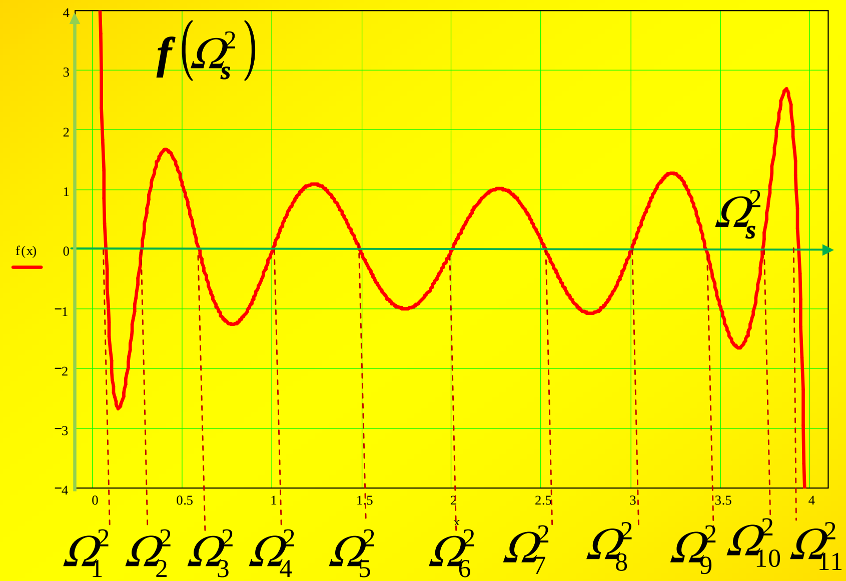


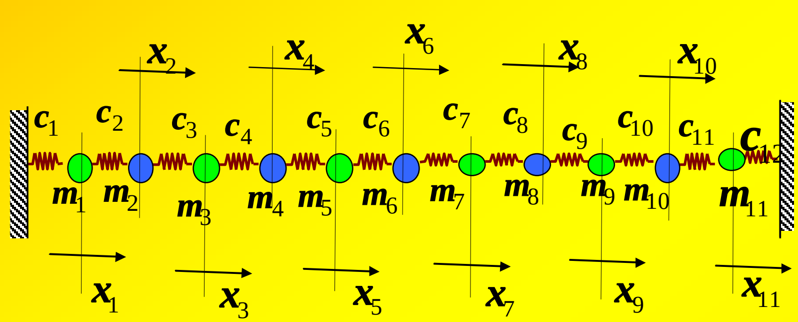
F3(t) = F03 cos(Ω3t + ϑ03)
V3(t) = V03 cos(Ω3t + ϑ03)
𝔐3(t) = 𝔐03 cos(Ω3t + ϑ03)
𝔐3(t) = 𝔐03 cos(Ω3t + ϑ03)



$C_{(3)1}(\Omega_3^2)$, $f(\Omega_3^2)$
$C_{(3)2}(\Omega_3^2)$, $f(\Omega_3^2)$
$C_{(3)3}(\Omega_3^2)$, $f(\Omega_3^2)$
$C_{(3)4}(\Omega_3^2)$, $f(\Omega_3^2)$
$C_{(3)5}(\Omega_3^2)$, $f(\Omega_3^2)$
$C_{(3)6}(\Omega_3^2)$, $f(\Omega_3^2)$
$C_{(3)7}(\Omega_3^2)$, $f(\Omega_3^2)$
$C_{(3)8}(\Omega_3^2)$, $f(\Omega_3^2)$
$C_{(3)9}(\Omega_3^2)$, $f(\Omega_3^2)$
$C_{(3)10}(\Omega_3^2)$, $f(\Omega_3^2)$
$C_{(3)11}(\Omega_3^2)$, $f(\Omega_3^2)$
Ω_3^2
$\mathbf{F}_3(t) = \mathbf{F}_{03}\cos(\Omega_3 t + \vartheta_{03})$
$\mathfrak{M}_3(t) = \mathfrak{M}_{03}\cos(\Omega_3 t + \vartheta_{03})$
$V_3(t) = V_{03}\cos(\Omega_3 t + \vartheta_{03})$
$C_{(3)1}(\Omega_3^2)$
$C_{(3)2}(\Omega_3^2)$
$C_{(3)3}(\Omega_3^2)$
$C_{(3)4}(\Omega_3^2)$
$C_{(3)5}(\Omega_3^2)$
$C_{(3)6}(\Omega_3^2)$
$C_{(3)7}(\Omega_3^2)$
$C_{(3)8}(\Omega_3^2)$
$C_{(3)9}(\Omega_3^2)$
$C_{(3)10}(\Omega_3^2)$
$C_{(3)11}(\Omega_3^2)$
$f(\Omega_3^2)$


f(Ω_s^2)
f(x)
6·10^26
4·10^26
2·10^26
0
−2·10^26
−4·10^26
−6·10^26
1·10^12
2·10^12
3·10^12
4·10^12
5·10^12
6·10^12
7·10^12
Ω_s^2
Ω_1^2 Ω_2^2 Ω_3^2 Ω_4^2 Ω_5^2 Ω_6^2 Ω_7^2 Ω_8^2 Ω_9^2 Ω_10^2 Ω_11^2


f(Ω_s^2)
f(x)
4
3
2
1
0
−1
−2
−3
−4
0
0.5
1
1.5
2
2.5
3
3.5
4
Ω_s^2
Ω_1^2 Ω_2^2 Ω_3^2 Ω_4^2 Ω_5^2 Ω_6^2 Ω_7^2 Ω_8^2 Ω_9^2 Ω_10^2 Ω_11^2


x_2 x_4 x_6 x_8 x_10
c_1 c_2 c_3 c_4 c_5 c_6 c_7 c_8 c_9 c_10 c_11 c_12
m_1 m_2 m_3 m_4 m_5 m_6 m_7 m_8 m_9 m_10 m_11
x_1 x_3 x_5 x_7 x_9 x_11

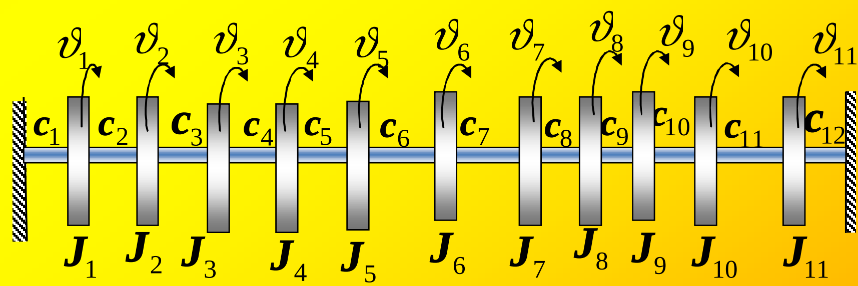

ϑ_1 ϑ_2 ϑ_3 ϑ_4 ϑ_5 ϑ_6 ϑ_7 ϑ_8 ϑ_9 ϑ_10 ϑ_11
c_1 c_2 c_3 c_4 c_5 c_6 c_7 c_8 c_9 c_10 c_11 c_12
J_1 J_2 J_3 J_4 J_5 J_6 J_7 J_8 J_9 J_10 J_11

Table 1: Mathematical analogy between kinetic and material parameters of linear mechanical system and electrical linear system with one dof

	Linear differential equations	Generalized system coordinate	Inertial coefficient	Elastic rigidity coefficient	Dissipation coefficient	External input	Eigen circular frequency Eigen resonant frequency
1	$m\ddot{x} + b\dot{x} + cx = F_0 \cos \Omega t$	x, displacement	m, mass	c, rigidity	b, damping force coefficient	$F_0 \cos \Omega t$, force	$\Omega_{res} = \omega = \sqrt{\frac{c}{m}}$
2	$L\ddot{q} + R\dot{q} + \frac{1}{C} q = V_0 \cos \Omega t$	q, charge	L, inductance	$\frac{1}{C}$, capacitance	R, resistance	$V_0 \cos \Omega t$, voltage	$\Omega_{res} = \omega = \sqrt{\frac{1}{CL}}$

Table 2: Mathematical analogy between kinetic and material parameters of FO inertia less mechanical visco-elastic element and FO inertia less electrical resistance element, $0 \le \alpha \le 1$

	Constitutive relation of the FO element	Generalized system coordinate	Inertial coefficient	Elastic rigidity coefficient	FO coefficients	FO differential equations
1	$P(t) = -\{c_0 x(t) + c_\alpha D_t^\alpha [x(t)]\}$	x, displacement	m, mass	c_0, rigidity	$c_{(\alpha)}$, damping FO force coefficient	$m\ddot{x} + c_{(\alpha)} D_t^\alpha [x] + cx = 0$
2	$V(t) = -\left\{\frac{1}{C_0} q(t) + R_\alpha D_t^\alpha [q(t)]\right\}$	q, charge	L, inductance	C_0, capacitance	$R_{(\alpha)}$, fractional resistance	$L\ddot{q} + R_{(\alpha)} D_t^\alpha [q] + \frac{1}{C_0} q = 0$

Table 3: Mathematical and qualitative analogies between energies and measurement of FO energy dissipations (degradations) and eigen characteristic numbers of a FO mechanical oscillator and an electrical FO oscillator with one dof, and phenomenological mapping to corresponding kinetic parameters of eigen FO modes in FO mechanical chain and electrical FO chain system $0 < \alpha \le 1$

	Kinetic energy	Potential energy	Generalized function of fractional order energy dissipation	Characteristic numbers for eigen FO normal modes
1	$\mathbf{E}_k = \frac{1}{2} m [\dot{x}(t)]^2$, mass	$\mathbf{E}_{p,\alpha} = \frac{1}{2} c_0 [x(t)]^2$, elastic element	$\Phi_\alpha = \frac{1}{2} c_\alpha \langle D_t^\alpha [x(t)] \rangle^2$	$\omega_S^2 = \frac{c}{m}$, $\omega_{(\alpha)}^2 = \kappa_\alpha \frac{c}{m}$
2	$\mathbf{E}_k = \frac{1}{2} L [\dot{q}(t)]^2$, inductance	$\mathbf{E}_{p,\alpha} = \frac{1}{2} \frac{1}{C_0} [q(t)]^2$, capacitor	$\Phi_\alpha = \frac{1}{2} R_\alpha \langle D_t^\alpha [q(t)] \rangle^2$	$\omega^2 = \frac{1}{LC_0}$, $\omega_{(\alpha)}^2 = \kappa_\alpha \frac{1}{LC_0}$

Table 4: Mathematical and qualitative analogies between matrix FODE of FO dynamics on a mechanical chain system with finite number of dof and an electrical chain system with corresponding finite number of loops and their eigen FO modes and eigen characteristic numbers and corresponding constitutive relations of inertia less standard FO mechanical viscoeelastic element and FO electrical resistor-capacitive element included in the corresponding analogous systems: $0 < \alpha \leq 1$, $s = 1, \cdots, n$. Phenomenological mapping between eigen FO modes of mechanical and electrical chains

	Constitutive relation of the FO element	Matrix FODE	Independent eigen fractional order normal oscillators	Characteristic numbers for eigen FO normal modes:
1	$P(t) = -\left[c_0 x(t) + c_\alpha D_t^\alpha[x(t)]\right]$	$\mathbf{A}\{\ddot{x}\} + \mathbf{C}_\alpha\{D_t^\alpha\{x\}\} + \mathbf{C}\{x\} = \{0\}$	$\ddot{\xi}_s + \omega_{(\alpha)s}^2 D_t^\alpha[\xi_s] + \omega_s^2 \xi_s = 0$	$\omega_s^2 = 2\frac{c}{m}\left(1 - \cos\frac{(2s-1)\pi}{2n+1}\right)$, $\omega_{(\alpha)s}^2 = 2\kappa_\alpha \frac{c}{m}\left(1 - \cos\frac{(2s-1)\pi}{2n+1}\right)$
2	$V(t) = -\left\{\frac{1}{C_0} q(t) + R_\alpha D_t^\alpha[q(t)]\right\}$	$\mathbf{L}\{\ddot{q}\} + \mathbf{R}_\alpha\{D_t^\alpha\{q\}\} + \mathbf{C}^*\{q\} = \{0\}$	$\ddot{\xi}_s + \omega_{(\alpha)s}^2 D_t^\alpha[\xi_s] + \omega_s^2 \xi_s = 0$	$\omega_s^2 = 2\frac{1}{LC_0}\left(1 - \cos\frac{(2s-1)\pi}{2n+1}\right)$, $\omega_{(\alpha)s}^2 = 2\kappa_\alpha \frac{1}{LC_0}\left(1 - \cos\frac{(2s-1)\pi}{2n+1}\right)$

Table 5: Mathematical and qualitative analogies between energies and measurement of FO energy dissipations (degradations) and eigen characteristic numbers of a FO mechanical chain system with finite number of dof and a electrical FO system with corresponding finite number loops, and phenomenological mapping to corresponding kinetic parameters of eigen FO modes in FO mechanical chain and electrical FO chain system $0 < \alpha \leq 1$, η_s, $s = 1, \cdots, n$

	Kinetic energy	Potential energy	Generalized function of fractional order energy dissipation	Characteristic numbers for eigen FO normal modes
1	$2\mathbf{E}_k = (\dot{x})\mathbf{A}\{\dot{x}\}$ $2\mathbf{E}_k = \sum_{s=1}^{s=n} \dot{\eta}_s^2$	$2\mathbf{E}_p = (x)C\{x\}$ $2\mathbf{E}_p = \sum_{s=1}^{s=n} \omega_s^2 \eta_s^2$	$2\mathbf{P}_{\alpha\neq 0} = (D_t^\alpha\{x\})C_\alpha\{D_t^\alpha\{x\}\}$ $2\mathbf{P}_\alpha = \sum_{s=1}^{s=n} \omega_{(\alpha),s}^2 (D_t^\alpha[\eta_s])^2$	$\omega_s^2 = 2\frac{c}{m}\left(1 - \cos\frac{(2s-1)\pi}{2n+1}\right)$ $\omega_{(\alpha)s}^2 = 2\kappa_\alpha \frac{c}{m}\left(1 - \cos\frac{(2s-1)\pi}{2n+1}\right)$
2	$2\mathbf{E}_k = (\dot{q})\mathbf{L}\{\dot{q}\}$ $2\mathbf{E}_k = \sum_{s=1}^{s=n} \dot{\eta}_s^2$	$2\mathbf{E}_p = (q)\mathbf{C}^*\{q\}$ $2\mathbf{E}_p = \sum_{s=1}^{s=n} \omega_s^2 \eta_s^2$	$2P_{\alpha\neq 0} = (D_t^\alpha\{q\})\mathbf{R}_\alpha\{D_t^\alpha\{q\}\}$ $2\mathbf{P}_\alpha = \sum_{s=1}^{s=n} \omega_{(\alpha),s}^2 (D_t^\alpha[\eta_s])^2$	$\omega_s^2 = 2\frac{1}{LC_0}\left(1 - \cos\frac{(2s-1)\pi}{2n+1}\right)$ $\omega_{(\alpha)s}^2 = 2\kappa_\alpha \frac{1}{LC_0}\left(1 - \cos\frac{(2s-1)\pi}{2n+1}\right)$

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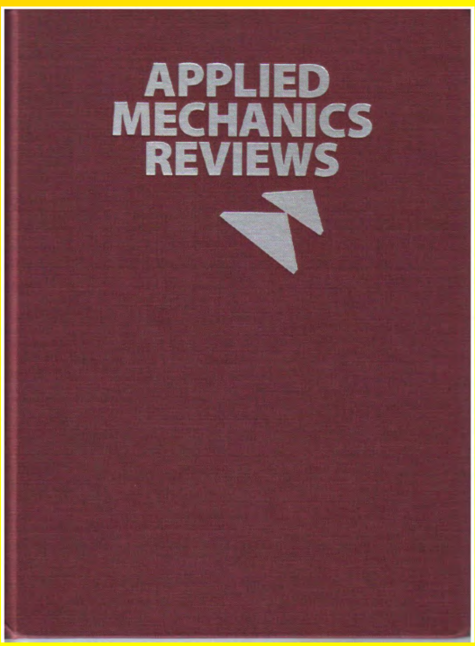
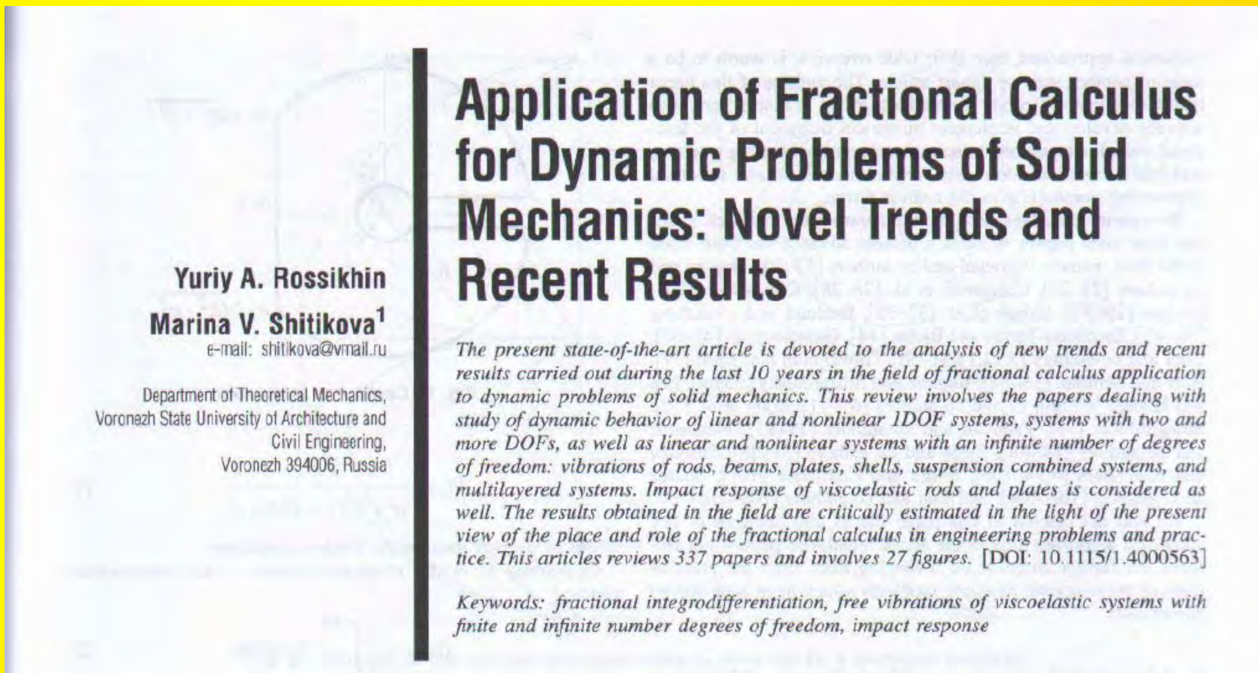
2012, Part 1, 29-83, DOI: 10.1007/978-1-4614-0454-5_2

Energy and Nonlinear Dynamics of Hybrid Systems

Katica R. (Stevanović) Hedrih

Katica R. (Stevanović) Hedrih, (2012), Energy and Nonlinear Dynamics of Hybrid Systems, Chapter in Book: Dynamical Systems and Methods, 2012, Part 1, 29-83, DOI: 10.1007/978-1-4614-0454-5_2





Application of Fractional Calculus for Dynamic Problems of Solid Mechanics: Novel Trends and Recent Results

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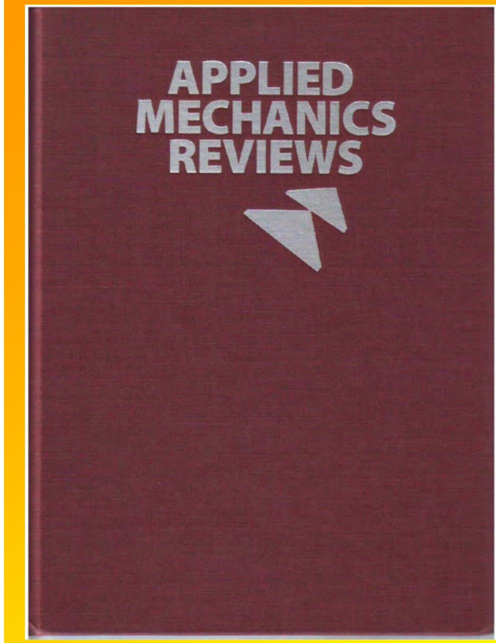
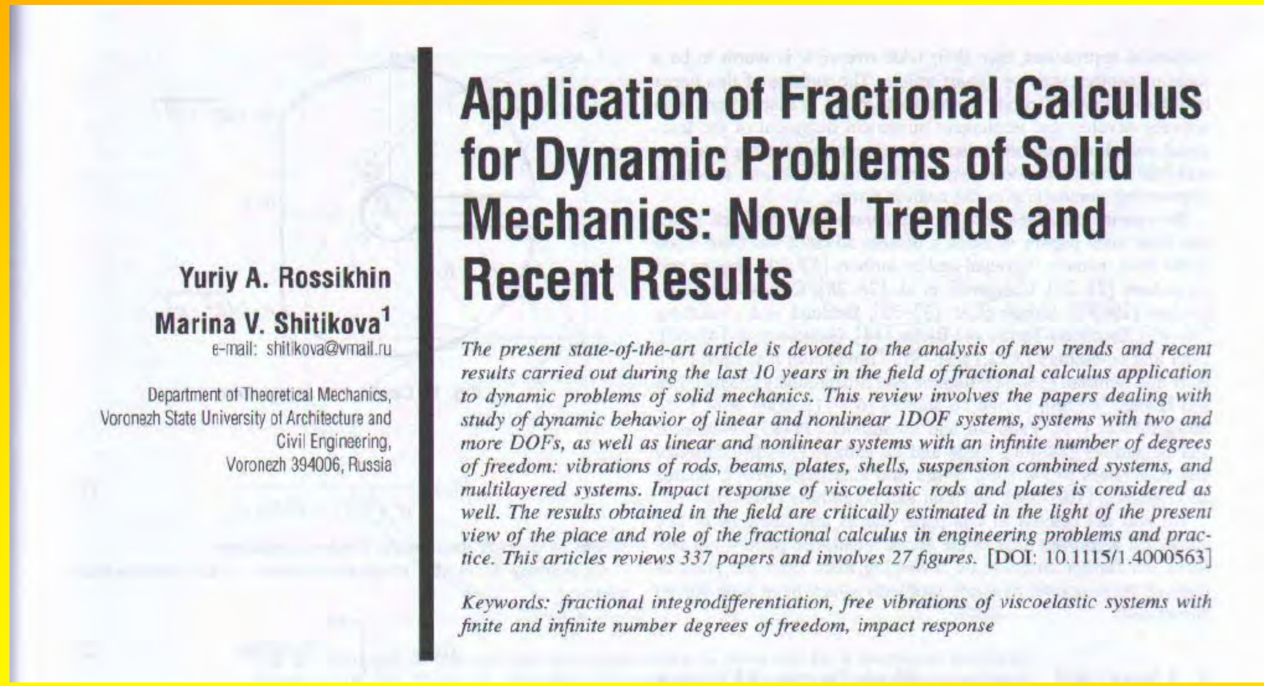
Voronezh 394006, Russia

The present state-of-the-art article is devoted to the analysis of new trends and recent results carried out during the last 10 years in the field of fractional calculus application to dynamic problems of solid mechanics. This review involves the papers dealing with study of dynamic behavior of linear and nonlinear 1DOF systems, systems with two and more DOFs, as well as linear and nonlinear systems with an infinite number of degrees of freedom: vibrations of rods, beams, plates, shells, suspension combined systems, and multilayered systems. Impact response of viscoelastic rods and plates is considered as well. The results obtained in the field are critically estimated in the light of the present view of the place and role of the fractional calculus in engineering problems and practice. This articles reviews 337 papers and involves 27 figures. [DOI: 10.1115/1.4000563]

Keywords: fractional integrodifferentiation, free vibrations of viscoelastic systems with finite and infinite number degrees of freedom, impact response





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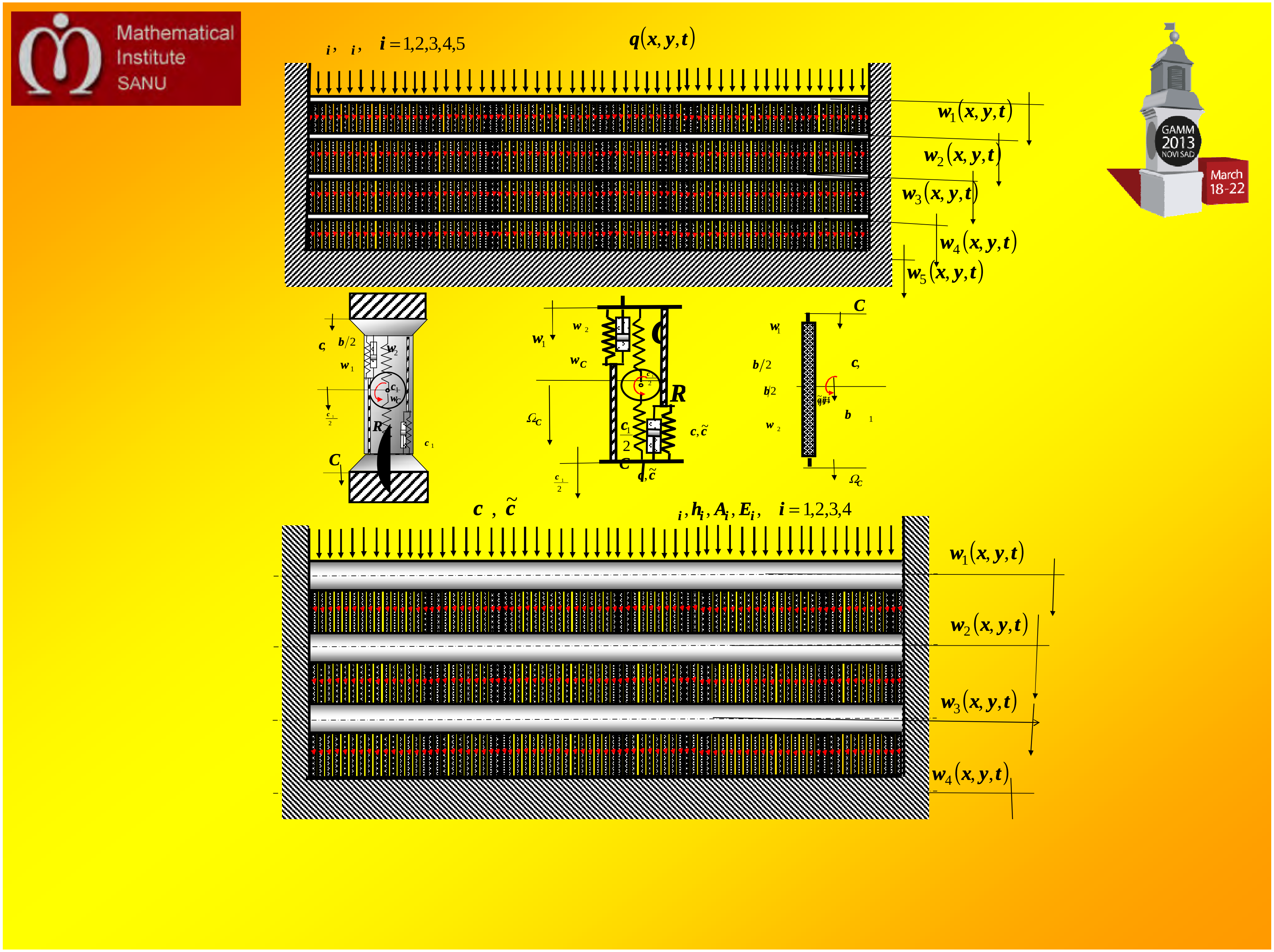

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$i = 1,2,3,4,5$
$q(x,y,t)$
$w_1(x,y,t)$
$w_2(x,y,t)$
$w_3(x,y,t)$
$w_4(x,y,t)$
$w_5(x,y,t)$
R
C
$c,\tilde{c}$
$i = 1,2,3,4$
$w_1(x,y,t)$
$w_2(x,y,t)$
$w_3(x,y,t)$
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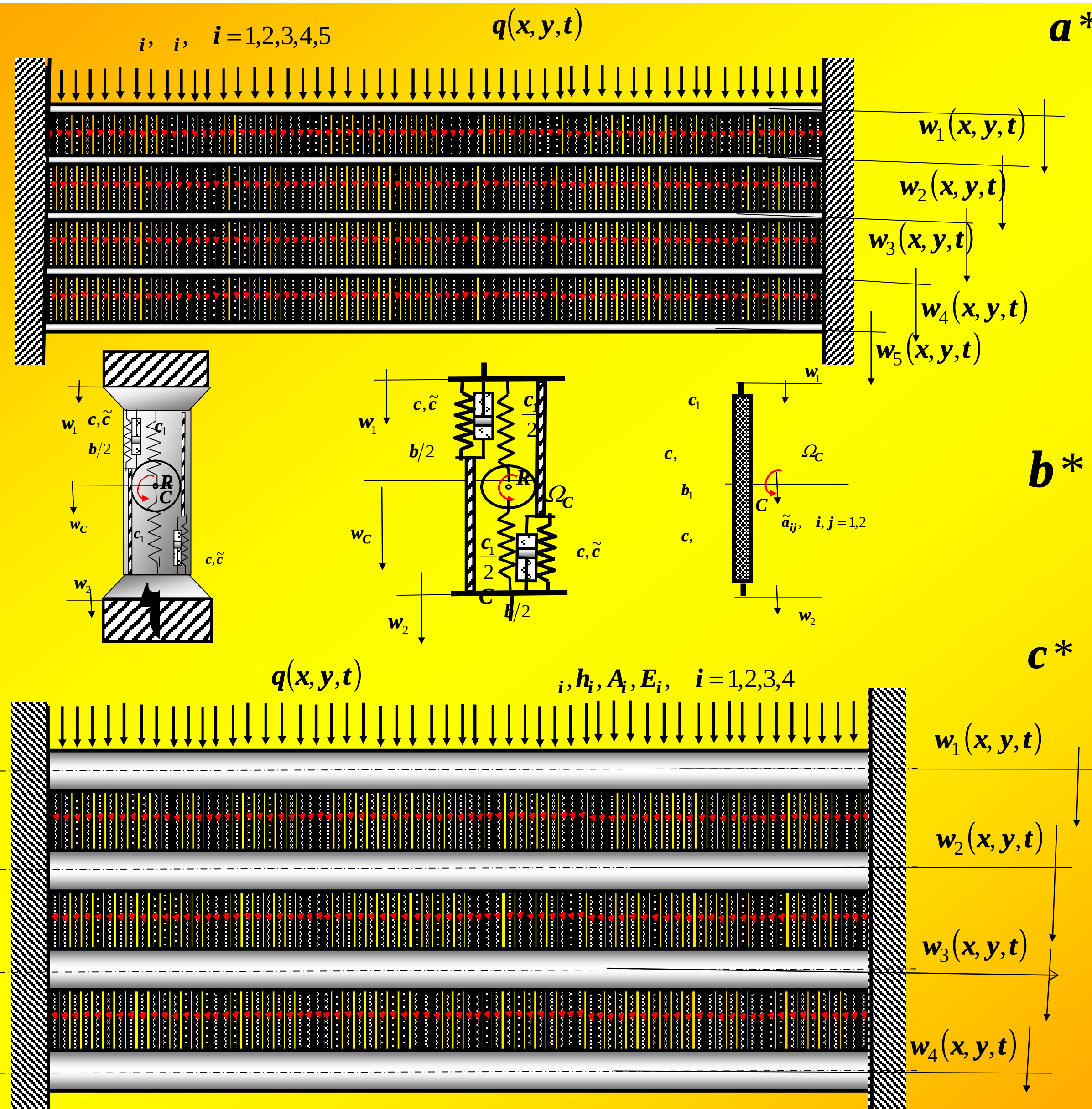

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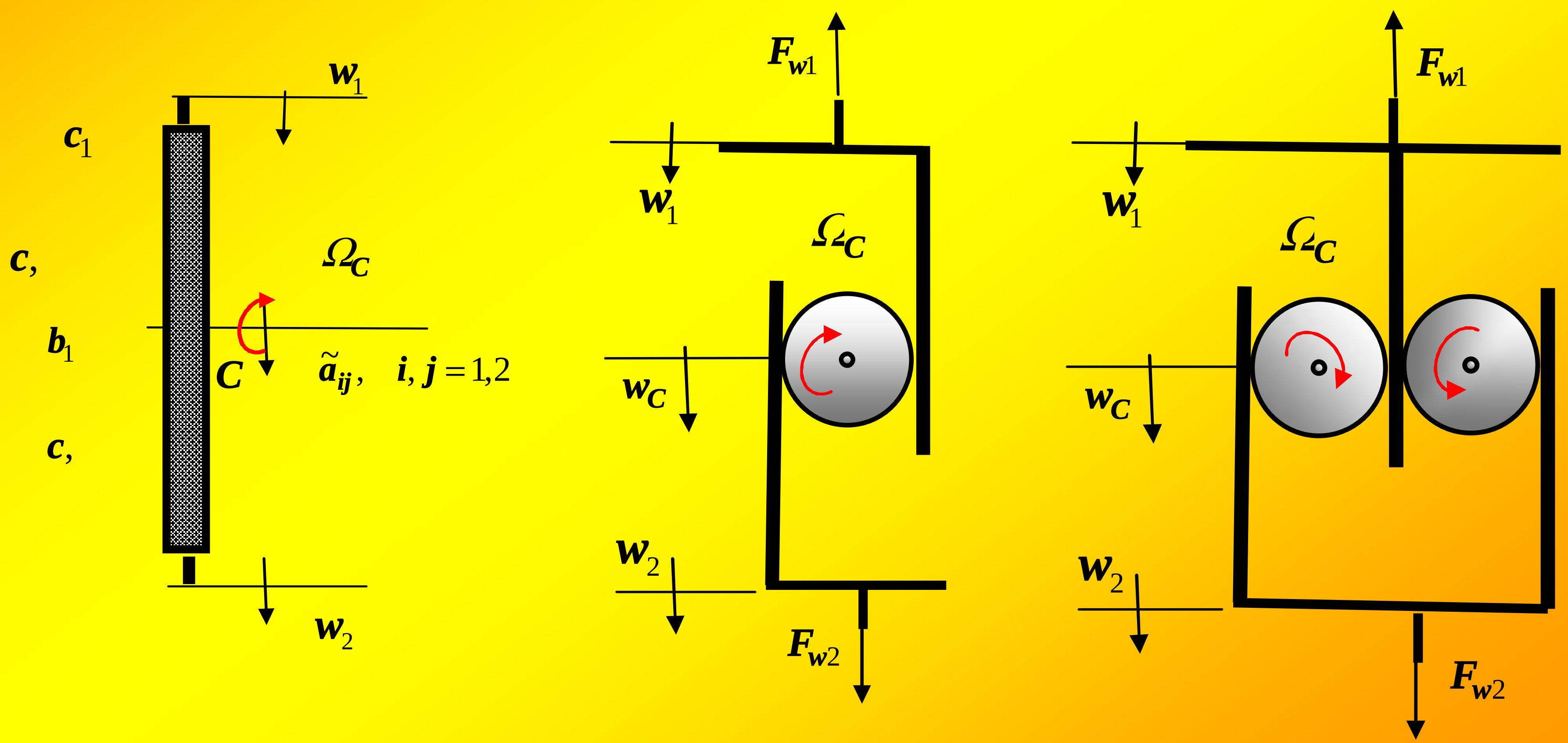
Standard element with translator and rotator inertia properties

We introduced standard element with **translator and rotator inertia properties**, taking into account mass and mass inertia moments and realized by a rolling disk or sphere

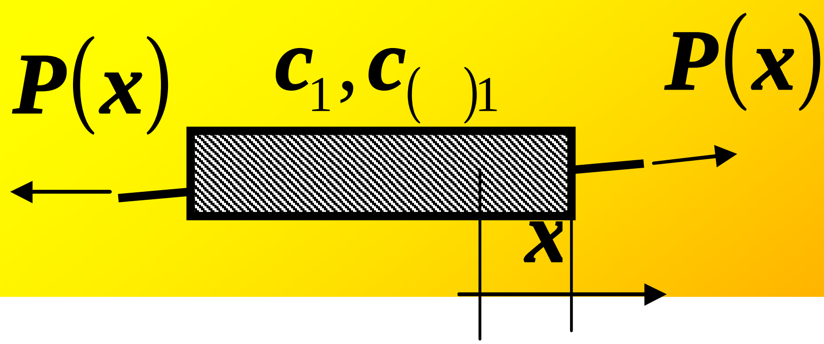


Standard light fractional order element

$$P(t) = -\left\{c_0 x(t) + c\, \mathfrak{D}_t \left[x(t)\right]\right\} \qquad 0 < \quad \leq 1$$

$$\mathfrak{D}_t \left[x(t)\right] = \frac{d \quad x(t)}{dt} = x^{(\ )}(t) = \frac{1}{\Gamma(1- \ )} \frac{d}{dt} \int_0^t \frac{x(\ )}{(t- \ )} d$$

$$0 < \quad \leq 1$$



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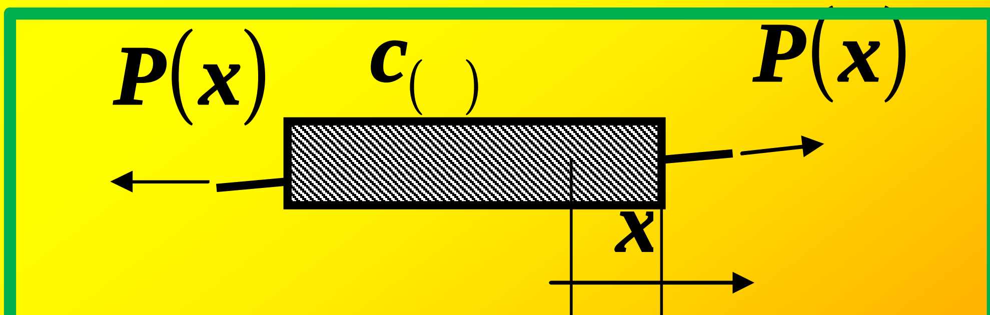
THE 13th INTERNATIONAL CONFERENCE OF TENSOR SOCIETY ON DIFFERENTIAL GEOMETRY AND ITS APPLICATIONS, AND INFORMATICS BESIDES.
The 86th Anniversary of Radu MIRON'S Birth.
Faculty of Math. Al. I. Cuza Univ. Iași, Romania — テンソル学会 (TENSOR SOCIETY) — Inst. of Math. "Octav Mayer" Romanian Academy

Generalized function of fractional order dissipation of system energy

$$2\mathrm{P}_{\ \neq 0} = \left(\mathfrak{D}_t \{x\}\right) C \left\{\mathfrak{D}_t \{x\}\right\}, \qquad \textit{for} \qquad \neq 0$$

$$\mathfrak{D}_t \left[x(t)\right] = \frac{d\ \ x(t)}{dt} = x^{(\ )}(t) = \frac{1}{\Gamma(1-\ \ )}\frac{d}{dt}\int_0^t \frac{x(\ )}{(t-\ )} d$$

$$0 < \quad \le 1$$

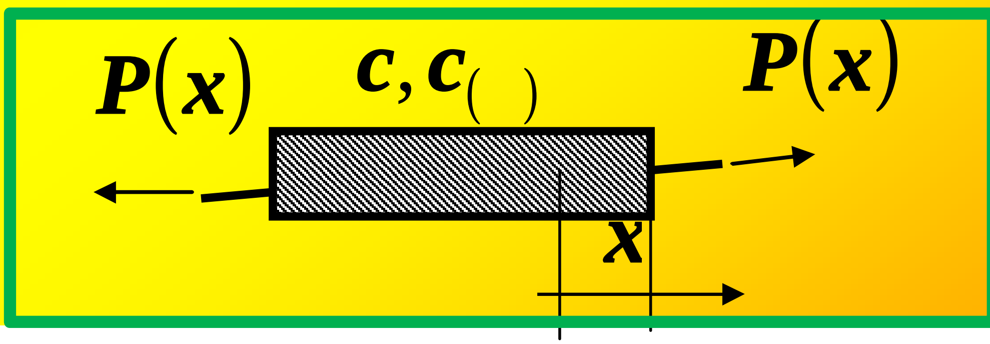


Generalized forces $\{\mathrm{F}_{\neq 0}\}$ of system no ideal visoelastic creep fractional order disipaon of system energy for $0 < \quad \leq 1 \qquad \neq 0$

for generalied coordinates $\{x\}$

$$\{\mathrm{F}_{\neq 0}\} = -\frac{\partial \mathrm{P}}{\partial\left(\mathfrak{D}_{t}\{x\}\right)} = -C\left\{\mathfrak{D}_{t}\{x\}\right\}, \quad \textit{for} \quad \neq 0$$

$$0 < \quad \leq 1$$



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Generalized forces $\{\mathbf{F}_{\neq 0}\}$ of system no ideal visoelastic creep fractional order disipaon of system energy for $0 < \quad \leq 1 \qquad \neq 0$

for generalied coordinates $\{x\}$

$$\boxed{\{\mathbf{F}_{\neq 0}\} = -\frac{\partial \mathbf{P}}{\partial\left(\mathfrak{D}_t \{x\}\right)} = -C \{\mathfrak{D}_t \{x\}\}, \quad \textit{for} \quad \neq 0 \quad 0 < \quad \leq 1}$$

Generalized disipatve forces $\{\mathbf{F}_{=1}\}$ of system dissipative (no conservative) properties for $=1$

for generalied coordinates $\{x\}$

$$\boxed{\{\mathbf{F}_{=1}\} = -\frac{\partial \Phi}{\partial \{\dot{x}\}} = -\frac{\partial \mathbf{P}_{=1}}{\partial \{\dot{x}\}} = -C_{=1}\{\dot{x}\}, \quad \textit{for} \quad =1}$$

Generalized function of fractional order dissipation of system energy

$$\{x\} \qquad x_k \qquad k = 1,2,3,\ldots,n$$

$$\boxed{\mathbf{A} = \left(a_{kj}\right)_{\rightarrow\; j=1,2,3,\ldots,n}^{\downarrow\; k=1,2,3,\ldots,n}}$$

Matrix of coefficients of system mass inertia properties

$$\boxed{\mathbf{C} = \left(c_{kj}\right)_{\rightarrow\; j=1,2,3,\ldots,n}^{\downarrow\; k=1,2,3,\ldots,n}}$$

Matrix of coefficients of system rigidity properties

$$\boxed{\mathbf{C} = \left(c_{\partial,kj}\right)_{\rightarrow\; j=1,2,3,\ldots,n}^{\downarrow\; k=1,2,3,\ldots,n}}$$

Matrix of coefficients of system viscoelastic creep fractional order properties

$$\boxed{2\mathrm{E}_k = (\dot{x})\mathrm{A}\{\dot{x}\}}$$

$$\boxed{2\mathrm{E}_p = (x)C\{x\}}$$

$$\boxed{2\mathrm{P}_{\neq 0} = \left(\mathfrak{D}_{\underset{\downarrow}{t}}\{x\}\right)C\left\{\mathfrak{D}_{\underset{\downarrow}{t}}\{x\}\right\}, \quad \textit{for} \quad \neq 0 \quad 0 < \quad \leq 1}$$

$$\boxed{2\Phi = 2\mathrm{P}_{=1} = (\dot{x})C_{=1}\{\dot{x}\}, \quad \textit{for} \quad = 1}$$

$$\boxed{2\mathrm{E}_{p,} = 2\mathrm{P}_{=0} = (x)C_{=0}\{x\}, \quad \textit{for} \quad = 0}$$

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Generalized forces of system inertia $\{\mathbf{F}_j\}$ for generalized coordinates $\{x\}$

$$\boxed{\{\mathbf{F}_j\} = -\left(\frac{d}{dt}\frac{\partial \mathbf{E}_k}{\partial\{\dot{x}\}} - \frac{\partial \mathbf{E}_k}{\partial\{x\}}\right) = -\mathbf{A}\{\ddot{x}\}}$$

Generalized forces $\{\mathbf{F}_c\}$ of system ideal elastic (conservative) properties for generalized coordinates $\{x\}$

$$\boxed{\{\mathbf{F}_c\} = -\frac{\partial \mathbf{E}_p}{\partial\{x\}} = -C\{x\}}$$

:

Generalized forces $\{F_{\neq 0}\}$ **of system no ideal visoelastic creep fractional order disipaon of system energy for** $0 < \quad \leq 1 \qquad \neq 0$

for generalied coordinates $\{x\}$

$$\{F_{\neq 0}\} = -\frac{\partial P}{\partial\left(\mathfrak{D}_t \{x\}\right)} = -C \{\mathfrak{D}_t \{x\}\}, \quad \textit{for} \quad \neq 0 \quad 0 < \quad \leq 1$$

Generalized disipatve forces $\{F_{=1}\}$ **of system dissipative (no conservative) properties for** $=1$

for generalied coordinates $\{x\}$

$$\{F_{=1}\} = -\frac{\partial \Phi}{\partial\{\dot{x}\}} = -\frac{\partial P_{=1}}{\partial\{\dot{x}\}} = -C_{=1}\{\dot{x}\}, \quad \textit{for} \quad =1$$

Generalized forces $\{\mathbf{F}_{=0}\}$ of system with additional elastic (conservative) properties for $=0$ for generalized coordinates $\{x\}$

$$\{\mathbf{F}_{=0}\} = -\frac{\partial \mathbf{E}_{p,\alpha=0}}{\partial\{x\}} = -\frac{\partial \mathbf{P}_{=0}}{\partial\{x\}} = -\boldsymbol{C}_{=0}\{x\}, \quad \textit{for} \qquad = 0$$

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Matrix fractional order differential equation of discrete fractional order system free vibrations

$$\left\{\mathbf{F}_{j}\right\}+\left\{\mathbf{F}_{c}\right\}+\left\{\mathbf{F}\quad\right\}=\left\{0\right\}$$

$$\frac{d}{dt}\frac{\partial \mathbf{E}_{k}}{\partial\{\dot{x}\}}-\frac{\partial \mathbf{E}_{k}}{\partial\{x\}}+\frac{\partial \mathbf{E}_{p}}{\partial\{x\}}+\frac{\partial \mathbf{P}}{\partial\left(\mathfrak{D}_{t}\ \{x\}\right)}+\frac{\partial \Phi}{\partial\{\dot{x}\}}=0$$

$$\mathbf{A}\{\ddot{x}\}+\boldsymbol{C}\ \left\{\mathfrak{D}_{t}\ \{x\}\right\}+\boldsymbol{C}\{x\}=\{0\}$$

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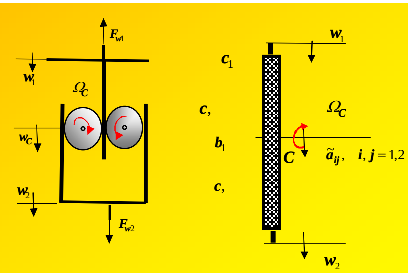


Generalized forces for generalized coordinates $w_k(x,y,t)$ and $w_{k+1}(x,y,t)$

$$Q_{w_1}^{elem-sloja}=\left[F_{j1}+F_{e1}+F_{w1}\right]_{w_1 elem-sloja}=-\left\langle \frac{d}{dt}\frac{\partial \mathrm{E}_k^{\,elem-sloja}}{\partial\left(\frac{\partial w_1}{\partial t}\right)}-\frac{\partial \mathrm{E}_k^{\,elem-sloja}}{\partial w_1}\right\rangle-\frac{\partial \mathrm{E}_p^{\,elem-sloja}}{\partial w_1}-\frac{\partial \Phi^{elem-sloja}}{\partial\left(\frac{\partial w_1(x,y,t)}{\partial t}\right)}=Q_{w_1}^{elem-placa}$$

$$Q_{w_2}^{elem-sloja}=\left[F_{j2}+F_{e2}+F_{w2}\right]_{w_2 elem-sloa}=-\left\langle \frac{d}{dt}\frac{\partial \mathrm{E}_k^{\,elem-sloja}}{\partial\left(\frac{\partial w_2}{\partial t}\right)}-\frac{\partial \mathrm{E}_k^{\,elem-sloja}}{\partial w_1}\right\rangle-\frac{\partial \mathrm{E}_p^{\,elem-sloja}}{\partial w_2}-\frac{\partial \Phi^{elem-sloja}}{\partial\left(\frac{\partial w_2(x,y,t)}{\partial t}\right)}=Q_{w_2}^{elem-placa}$$

$$Q_{w_k}^{elem}=-\frac{1}{4}m\left[\left(\frac{\partial^2 w_{k+1}}{\partial t^2}+\frac{\partial^2 w_k}{\partial^2 t}\right)-\frac{i_C^2}{R^2}\left(\frac{\partial^2 w_{k+1}}{\partial t^2}-\frac{\partial^2 w_k}{\partial^2 t}\right)\right]-$$
$$-\left\{c_{0k,k+1}\left[w_{k+1}(x,y,t)-w_k(x,y,t)\right]+c_{0N,k,k+1}\left[w_{k+1}(x,y,t)-w_k(x,y,t)\right]^3\right\}-b_{k,k+1}\left[\frac{\partial w_{k+1}(x,y,t)}{\partial t}-\frac{\partial w_k(x,y,t)}{\partial t}\right]$$

$$Q_{w_{k+1}}^{elem}=-\frac{1}{4}m\left[\left(\frac{\partial^2 w_{k+1}}{\partial t^2}+\frac{\partial^2 w_k}{\partial^2 t}\right)+\frac{i_C^2}{R^2}\left(\frac{\partial^2 w_{k+1}}{\partial t^2}-\frac{\partial^2 w_k}{\partial^2 t}\right)\right]+$$
$$+\left\{c_{0,k,k+1}\left[w_{k+1}(x,y,t)-w_k(x,y,t)\right]+c_{0N,k,k+1}\left[w_{k+1}(x,y,t)-w_k(x,y,t)\right]^3\right\}+b_{k,k+1}\left[\frac{\partial w_{k+1}(x,y,t)}{\partial t}-\frac{\partial w_k(x,y,t)}{\partial t}\right]$$

Governing partial fractional order differential equations of a hybrid multi deformable beam system transversal oscillations on a discrete continuum layer with visco-elastic and translator and rotator inertia properties

$$
{}_{1}\boldsymbol{A}_{1}\frac{\partial^{2}w_{1}(x,t)}{\partial t^{2}}=-\mathfrak{B}_{1}\frac{\partial^{4}w_{1}(x,t)}{\partial x^{4}}-\frac{1}{4}m_{1}\left[\left(\frac{\partial^{2}w_{2}(x,t)}{\partial t^{2}}+\frac{\partial^{2}w_{1}(x,t)}{\partial^{2}t}\right)-\left(\frac{\partial^{2}w_{2}(x,t)}{\partial t^{2}}-\frac{\partial^{2}w_{1}(x,t)}{\partial^{2}t}\right)\right]+
$$

$$
+c_{0(1.2)}\left[w_{2}(x,t)-w_{1}(x,t)\right]+c_{\ (1,2)}\mathfrak{D}_{t}\left[w_{2}(x,t)-w_{1}(x,t)\right]+b\left[\frac{\partial w_{2}(x,t)}{\partial t}-\frac{\partial w_{1}(x,t)}{\partial t}\right]+q_{1}(x,t)
$$

$$
{}_{2}\boldsymbol{A}_{2}\frac{\partial^{2}w_{2}(x,t)}{\partial t^{2}}=-\mathfrak{B}_{2}\frac{\partial^{4}w_{2}(x,t)}{\partial x^{4}}-\frac{1}{4}m_{1}\left[\left(\frac{\partial^{2}w_{2}(x,t)}{\partial t^{2}}+\frac{\partial^{2}w_{1}(x,t)}{\partial^{2}t}\right)+\left(\frac{\partial^{2}w_{2}(x,t)}{\partial t^{2}}-\frac{\partial^{2}w_{1}(x,t)}{\partial^{2}t}\right)\right]-c_{0(1.2)}\left[w_{2}(x,t)-w_{1}(x,t)\right]-
$$

$$
-c_{\ (1,2)}\mathfrak{D}_{t}\left[w_{2}(x,t)-w_{1}(x,t)\right]-b\left[\frac{\partial w_{2}(x,t)}{\partial t}-\frac{\partial w_{1}(x,t)}{\partial t}\right]-\frac{1}{4}m_{2}\left[\left(\frac{\partial^{2}w_{3}(x,t)}{\partial t^{2}}+\frac{\partial^{2}w_{2}(x,t)}{\partial^{2}t}\right)-\left(\frac{\partial^{2}w_{3}(x,t)}{\partial t^{2}}-\frac{\partial^{2}w_{2}(x,t)}{\partial^{2}t}\right)\right]-
$$

$$
+c_{0(2.3)}\left[w_{3}(x,t)-w_{2}(x,t)\right]+c_{\ (2,3)}\mathfrak{D}_{t}\left[w_{3}(x,t)-w_{2}(x,t)\right]-b\left[\frac{\partial w_{2}(x,t)}{\partial t}-\frac{\partial w_{3}(x,t)}{\partial t}\right]-q_{2}(x,t)=0
$$

$$
{}_{3}\boldsymbol{A}_{3}\frac{\partial^{2}w_{3}((x,t)}{\partial t^{2}}=-\mathfrak{B}_{3}\frac{\partial^{4}w_{3}(x,t)}{\partial x^{4}}-\frac{1}{4}m_{2}\left[\left(\frac{\partial^{2}w_{3}(x,t)}{\partial t^{2}}+\frac{\partial^{2}w_{2}(x,t)}{\partial^{2}t}\right)+\left(\frac{\partial^{2}w_{3}(x,t)}{\partial t^{2}}-\frac{\partial^{2}w_{2}(x,t)}{\partial^{2}t}\right)\right]-
$$

$$
-c_{0(2.3)}\left[w_{3}(x,t)-w_{2}(x,t)\right]-c_{\ (2,3)}\mathfrak{D}_{t}\left[w_{3}(x,t)-w_{2}(x,t)\right]-
$$

$$
-b\left[\frac{\partial w_{3}(x,t)}{\partial t}-\frac{\partial w_{2}(x,t)}{\partial t}\right]-\frac{1}{4}m_{0}\left[\left(\frac{\partial^{2}w_{4}(x,t)}{\partial t^{2}}+\frac{\partial^{2}w_{3}(x,t)}{\partial^{2}t}\right)-\left(\frac{\partial^{2}w_{4}(x,t)}{\partial t^{2}}-\frac{\partial^{2}w_{3}(x,t)}{\partial^{2}t}\right)\right]-
$$

$$
+c_{0(1.2)}\left[w_{4}(x,t)-w_{3}(x,t)\right]+c_{\ (3,4)}\mathfrak{D}_{t}\left[w_{4}(x,t)-w_{3}(x,t)\right]+c_{0N(3,4)}\left[w_{4}(x,t)-w_{3}(x,t)\right]^{3}-
$$

$$
-b\left[\frac{\partial w_{3}(x,t)}{\partial t}-\frac{\partial w_{4}(x,t)}{\partial t}\right]-q_{3}(x,t)
$$

Governing partial fractional order differential equations of a hybrid multi deformable plate system transversal oscillations on a discrete continuum layer with viscoelastic and translator and rotator inertia properties

$$
{}_1h_1\frac{\partial^2 w_1(x,y,t)}{\partial t^2}=-\mathfrak{D}_1\Delta\Delta w_1(x,y,t)-\frac{1}{4}m_1\left[\left(\frac{\partial^2 w_2}{\partial t^2}+\frac{\partial^2 w_1}{\partial^2 t}\right)-\left(\frac{\partial^2 w_2}{\partial t^2}-\frac{\partial^2 w_1}{\partial^2 t}\right)\right]+
$$

$$
+c_{0(1.2)}\left[w_2(x,y,t)-w_1(x,y,t)\right]+c_{(1,2)}\mathfrak{D}_t\left[w_2(x,y,t)-w_1(x,y,t)\right]+
$$

$$
+b\left[\frac{\partial w_2(x,y,t)}{\partial t}-\frac{\partial w_1(x,y,t)}{\partial t}\right]+q_1(x,y,t)
$$

$$
{}_2A_2\frac{\partial^2 w_2(x,y,t)}{\partial t^2}=-\mathfrak{D}_2\Delta\Delta w_2(x,y,t)-\frac{1}{4}m_1\left[\left(\frac{\partial^2 w_2(x,y,t)}{\partial t^2}+\frac{\partial^2 w_1(x,y,t)}{\partial^2 t}\right)+\left(\frac{\partial^2 w_2(x,y,t)}{\partial t^2}-\frac{\partial^2 w_1(x,y,t)}{\partial^2 t}\right)\right]-
$$

$$
-b\left[\frac{\partial w_2(x,y,t)}{\partial t}-\frac{\partial w_1(x,y,t)}{\partial t}\right]-\frac{1}{4}m_2\left[\left(\frac{\partial^2 w_3(x,y,t)}{\partial t^2}+\frac{\partial^2 w_2(x,y,t)}{\partial^2 t}\right)-\left(\frac{\partial^2 w_3(x,y,t)}{\partial t^2}-\frac{\partial^2 w_2(x,y,t)}{\partial^2 t}\right)\right]-
$$

$$
-c_{0(1.2)}\left[w_2(x,y,t)-w_1(x,y,t)\right]-c_{(1,2)}\mathfrak{D}_t\left[w_2(x,y,t)-w_1(x,y,t)\right]+
$$

$$
+c_{0(2.3)}\left[w_3(x,t)-w_2(x,t)\right]+c_{(2,3)}\mathfrak{D}_t\left[w_3(x,t)-w_2(x,t)\right]-
$$

$$
-b\left[\frac{\partial w_2(x,y,t)}{\partial t}-\frac{\partial w_3(x,y,t)}{\partial t}\right]-q_2(x,y,t)
$$

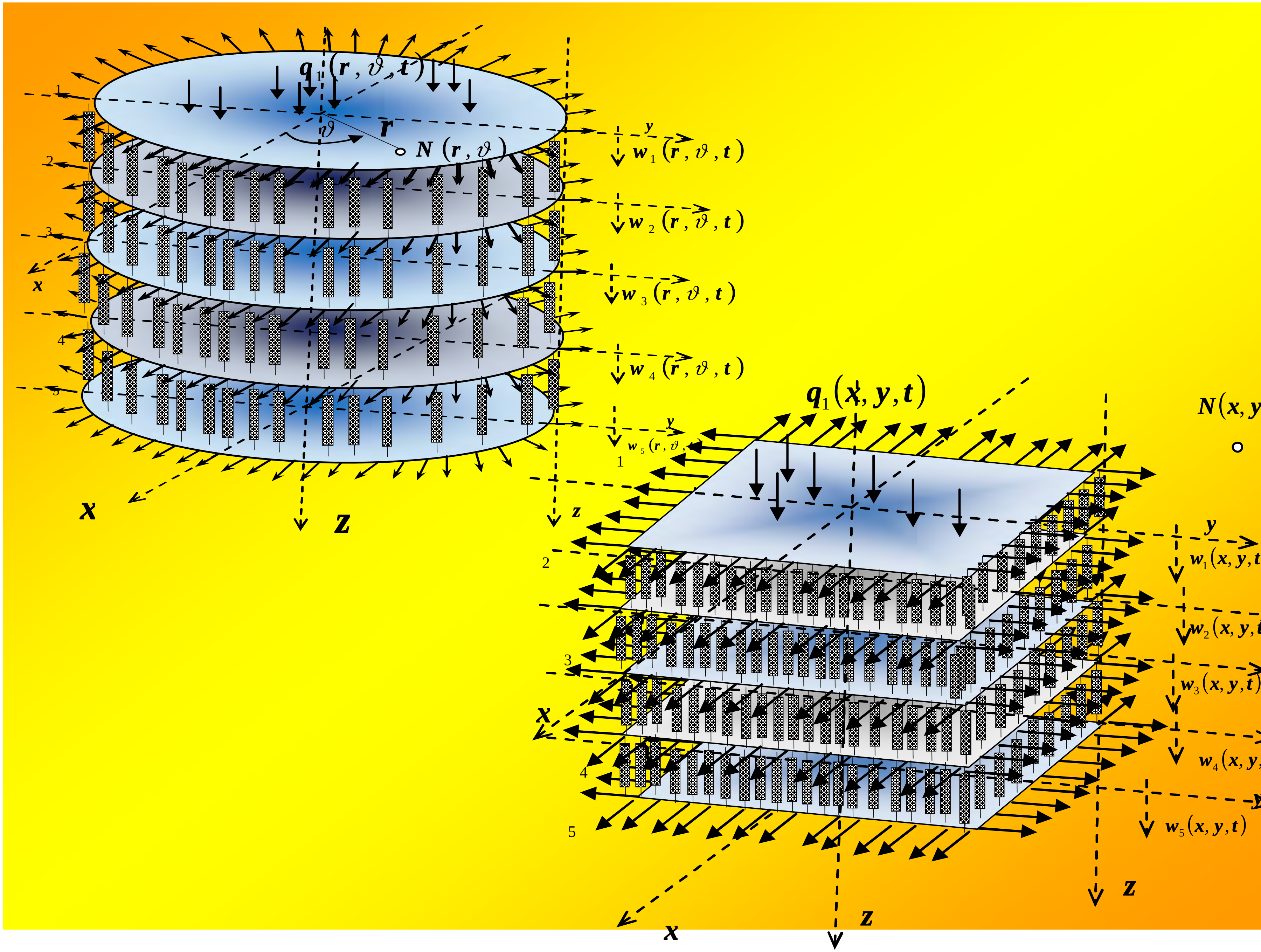
$$
{}_3A_3\frac{\partial^2 w_3(x,y,t)}{\partial t^2}=-\mathfrak{D}_3\Delta\Delta w_3(x,y,t)-\frac{1}{4}m_2\left[\left(\frac{\partial^2 w_3(x,y,t)}{\partial t^2}+\frac{\partial^2 w_2(x,y,t)}{\partial^2 t}\right)+\left(\frac{\partial^2 w_3(x,y,t)}{\partial t^2}-\frac{\partial^2 w_2(x,y,t)}{\partial^2 t}\right)\right]-
$$

$$
--b\left[\frac{\partial w_3(x,y,t)}{\partial t}-\frac{\partial w_2(x,y,t)}{\partial t}\right]-\frac{1}{4}m_0\left[\left(\frac{\partial^2 w_0(x,y,t)}{\partial t^2}+\frac{\partial^2 w_3(x,y,t)}{\partial^2 t}\right)-\left(\frac{\partial^2 w_0(x,y,t)}{\partial t^2}-\frac{\partial^2 w_3(x,y,t)}{\partial^2 t}\right)\right]-
$$

$$
-c_{0(2.3)}\left[w_3(x,y,t)-w_2(x,y,t)\right]-c_{(2,3)}\mathfrak{D}_t\left[w_3(x,y,t)-w_2(x,y,t)\right]+
$$

$$
+c_{0(1.2)}\left[w_4(x,y,t)-w_3(x,y,t)\right]+c_{(3,4)}\mathfrak{D}_t\left[w_4(x,y,t)-w_3(x,y,t)\right]+c_{0N(3,4)}\left[w_4(x,y,t)-w_3(x,y,t)\right]^3-
$$

$$
-b\left[\frac{\partial w_3(x,y,t)}{\partial t}-\frac{\partial w_0(x,y,t)}{\partial t}\right]-q_3(x,y,t)
$$



q1(r, ϑ, t)
r
ϑ
N(r, ϑ)
1
2
3
4
5
x
y
w1(r, ϑ, t)
w2(r, ϑ, t)
w3(r, ϑ, t)
w4(r, ϑ, t)
x
z
z
q1(x, y, t)
N(x, y)
y
w1(x, y, t)
w2(x, y, t)
w3(x, y, t)
w4(x, y, t)
w5(x, y, t)
x
x
z
z

Governing partial fractional order differential equations of a hybrid multi deformable membrane system transversal oscillations on a discrete continuum layer with visco-elastic and translator and rotator inertia properties

$$
{}_1\frac{\partial^2 \boldsymbol{w}_1(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})}{\partial \boldsymbol{t}^2} = {}_1\boldsymbol{c}_1^2 \Delta \boldsymbol{w}_1(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) - \frac{1}{4}\boldsymbol{m}_1\left[\left(\frac{\partial^2 \boldsymbol{w}_2}{\partial \boldsymbol{t}^2} + \frac{\partial^2 \boldsymbol{w}_1}{\partial^2 \boldsymbol{t}}\right) - \left(\frac{\partial^2 \boldsymbol{w}_2}{\partial \boldsymbol{t}^2} - \frac{\partial^2 \boldsymbol{w}_1}{\partial^2 \boldsymbol{t}}\right)\right] +
$$

$$
+\boldsymbol{c}_{0(1.2)}\left[\boldsymbol{w}_2(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) - \boldsymbol{w}_1(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})\right] + \boldsymbol{c}\ {}_{(1,2)}\mathfrak{D}_{\boldsymbol{t}}\left[\boldsymbol{w}_2(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) - \boldsymbol{w}_1(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})\right] +
$$

$$
+\boldsymbol{b}\left[\frac{\partial \boldsymbol{w}_2(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})}{\partial \boldsymbol{t}} - \frac{\partial \boldsymbol{w}_1(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})}{\partial \boldsymbol{t}}\right] + \boldsymbol{q}_1(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})
$$

$$
{}_2\frac{\partial^2 \boldsymbol{w}_2(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})}{\partial \boldsymbol{t}^2} = {}_2\boldsymbol{c}_2^2 \Delta \boldsymbol{w}_2(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) - \frac{1}{4}\boldsymbol{m}_1\left[\left(\frac{\partial^2 \boldsymbol{w}_2}{\partial \boldsymbol{t}^2} + \frac{\partial^2 \boldsymbol{w}_1}{\partial^2 \boldsymbol{t}}\right) + \left(\frac{\partial^2 \boldsymbol{w}_2}{\partial \boldsymbol{t}^2} - \frac{\partial^2 \boldsymbol{w}_1}{\partial^2 \boldsymbol{t}}\right)\right] -
$$

$$
-\boldsymbol{b}\left[\frac{\partial \boldsymbol{w}_2(\boldsymbol{x},\boldsymbol{t})}{\partial \boldsymbol{t}} - \frac{\partial \boldsymbol{w}_1(\boldsymbol{x},\boldsymbol{t})}{\partial \boldsymbol{t}}\right] - \frac{1}{4}\boldsymbol{m}_2\left[\left(\frac{\partial^2 \boldsymbol{w}_3}{\partial \boldsymbol{t}^2} + \frac{\partial^2 \boldsymbol{w}_2}{\partial^2 \boldsymbol{t}}\right) - \left(\frac{\partial^2 \boldsymbol{w}_3}{\partial \boldsymbol{t}^2} - \frac{\partial^2 \boldsymbol{w}_2}{\partial^2 \boldsymbol{t}}\right)\right] -
$$

$$
-\boldsymbol{c}_{0(1.2)}\left[\boldsymbol{w}_2(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) - \boldsymbol{w}_1(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})\right] - \boldsymbol{c}\ {}_{(1,2)}\mathfrak{D}_{\boldsymbol{t}}\left[\boldsymbol{w}_2(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) - \boldsymbol{w}_1(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})\right] +
$$

$$
+\boldsymbol{c}_{0(2.3)}\left[\boldsymbol{w}_3(\boldsymbol{x},\boldsymbol{t}) - \boldsymbol{w}_2(\boldsymbol{x},\boldsymbol{t})\right] + \boldsymbol{c}\ {}_{(2,3)}\mathfrak{D}_{\boldsymbol{t}}\left[\boldsymbol{w}_3(\boldsymbol{x},\boldsymbol{t}) - \boldsymbol{w}_2(\boldsymbol{x},\boldsymbol{t})\right] - \boldsymbol{b}\left[\frac{\partial \boldsymbol{w}_2(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})}{\partial \boldsymbol{t}} - \frac{\partial \boldsymbol{w}_3(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})}{\partial \boldsymbol{t}}\right] - \boldsymbol{q}_2(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})
$$

$$
{}_3\frac{\partial^2 \boldsymbol{w}_3(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})}{\partial \boldsymbol{t}^2} = {}_2\boldsymbol{c}_3^2 \Delta \boldsymbol{w}_3(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) - \frac{1}{4}\boldsymbol{m}_2\left[\left(\frac{\partial^2 \boldsymbol{w}_3}{\partial \boldsymbol{t}^2} + \frac{\partial^2 \boldsymbol{w}_2}{\partial^2 \boldsymbol{t}}\right) + \left(\frac{\partial^2 \boldsymbol{w}_3}{\partial \boldsymbol{t}^2} - \frac{\partial^2 \boldsymbol{w}_2}{\partial^2 \boldsymbol{t}}\right)\right] -
$$

$$
-\boldsymbol{b}\left[\frac{\partial \boldsymbol{w}_3(\boldsymbol{x},\boldsymbol{t})}{\partial \boldsymbol{t}} - \frac{\partial \boldsymbol{w}_2(\boldsymbol{x},\boldsymbol{t})}{\partial \boldsymbol{t}}\right] - \frac{1}{4}\boldsymbol{m}_0\left[\left(\frac{\partial^2 \boldsymbol{w}_0}{\partial \boldsymbol{t}^2} + \frac{\partial^2 \boldsymbol{w}_3}{\partial^2 \boldsymbol{t}}\right) - \left(\frac{\partial^2 \boldsymbol{w}_0}{\partial \boldsymbol{t}^2} - \frac{\partial^2 \boldsymbol{w}_3}{\partial^2 \boldsymbol{t}}\right)\right] - -\boldsymbol{b}\left[\frac{\partial \boldsymbol{w}_3(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})}{\partial \boldsymbol{t}} - \frac{\partial \boldsymbol{w}_0(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})}{\partial \boldsymbol{t}}\right] - \boldsymbol{q}_3(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) -
$$

$$
-\boldsymbol{c}_{0(2.3)}\left[\boldsymbol{w}_3(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) - \boldsymbol{w}_2(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})\right] - \boldsymbol{c}\ {}_{(2,3)}\mathfrak{D}_{\boldsymbol{t}}\left[\boldsymbol{w}_3(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) - \boldsymbol{w}_2(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})\right] +
$$

$$
+\boldsymbol{c}_{0(1.2)}\left[\boldsymbol{w}_4(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) - \boldsymbol{w}_3(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})\right] + \boldsymbol{c}\ {}_{(3,4)}\mathfrak{D}_{\boldsymbol{t}}\left[\boldsymbol{w}_4(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) - \boldsymbol{w}_3(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})\right] + \boldsymbol{c}_{0\boldsymbol{N}(3,4)}\left[\boldsymbol{w}_4(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) - \boldsymbol{w}_3(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})\right]^3 -
$$

Solution of the governing partial fractional order differential equations of a hybrid multi deformable membrane system transversal oscillations on a discrete continuum layer with visco-elastic and translator and rotator inertia properties

$$\left[1+\frac{1}{4}\;{}_{1,2}\left(1+\;\right)\right]\ddot{T}_{1(nm)}(t)+c_1^2k_{nm}^2T_{1(nm)}(t)+a_{0(1,2)}^2T_{1(nm)}(t)+2\varDelta_{1,2}\dot{T}_{1(nm)}(t)-$$
$$+a^2{}_{(1,2)}\mathfrak{D}_t\left[T_{1(nm)}(t)\right]-a_{0(1,2)}^2T_{2(nm)}(t)-a^2{}_{(1,2)}\mathfrak{D}_t\left[T_{2(nm)}(t)\right]-$$
$$-2\varDelta_{1,2}\dot{T}_{2(nm)}(t)-\frac{1}{4}\;{}_{1,2}\left(1-\;\right)\ddot{T}_{2(nm)}(t)=h_{01,nm}\sin\left(\Omega_{1,nm}t+\vartheta_{1,nm}\right)$$

$$\left[1+\frac{1}{4}\tilde{\;}_{1,2}\left(1+\;\right)+\frac{1}{4}\;{}_{2,3}\left(1+\;\right)\right]\ddot{T}_{2(nm)}(t)+c_2^2k_{nm}^2T_{2(nm)}(t+\left[\tilde{a}_{0(1,2)}^2+a_{0(2,3)}^2\right]T_{2(nm)}(t)+2\left[\tilde{\varDelta}_{1,2}+2\varDelta_{2,3}\right]\dot{T}_{2(nm)}(t)+$$
$$+\left[\tilde{a}^2{}_{(1,2)}+a^2{}_{(2,3)}\right]\mathfrak{D}_t\left[T_{2(nm)}(t)\right]-\tilde{a}_{0(1,2)}^2T_{1(nm)}(t)-\tilde{a}^2{}_{(1,2)}\mathfrak{D}_t\left[T_{1(nm)}(t)\right]-a_{0(2,3)}^2T_{3(nm)}(t)-a^2{}_{(2,3)}\mathfrak{D}_t\left[T_{3(nm)}(t)\right]+$$
$$+\frac{1}{4}\tilde{\;}_{1,2}\left(1-\;\right)\ddot{T}_{1(nm)}(t)-2\tilde{\varDelta}_{1,2}\dot{T}_{1(nm)}(t)+\frac{1}{4}\;{}_{2,3}\left(1-\;\right)\ddot{T}_{3(nm)}(t)-2\varDelta_{2,3}\dot{T}_{3(nm)}(t)=$$
$$=h_{02,nm}\sin\left(\Omega_{2nm}t+\vartheta_{2,nm}\right)$$

$$\left[1+\frac{1}{4}\tilde{\;}_{2,3}\left(1+\;\right)+\frac{1}{4}\;{}_{3,4}\left(1+\;\right)\right]\ddot{T}_{3(nm)}(t)+c_3^2k_{nm}^2T_{3(nm)}(t)+\left[\tilde{a}_{0(2,3)}^2+a_{0(3,4)}^2\right]T_{3(nm)}(t)+$$
$$+2\left[\tilde{\varDelta}_{2,3}+2\varDelta_{3,4}\right]\dot{T}_{3(nm)}(t)+\frac{1}{4}\tilde{\;}_{2,3}\left(1-\;\right)\ddot{T}_{2(nm)}(t)-\tilde{a}_{0(2,3)}^2T_{2(nm)}(t)-2\tilde{\varDelta}_{2,3}\dot{T}_{2(nm)}(t)+$$
$$+\left[\tilde{a}^2{}_{(2,3)}+a^2{}_{(3,4)}\right]\mathfrak{D}_t\left[T_{3(nm)}(t)\right]-\tilde{a}^2{}_{(2,3)}\mathfrak{D}_t\left[T_{2(nm)}(t)\right]\approx$$
$$\approx-a_{NL,3,4}^2\tilde{g}_{nm}\left[T_{3(nm)}(t)\right]^3+\frac{1}{4}\;{}_{3,4}\left(1-\;\right)\Omega_{0nm}^2w_{0nm}\sin\left(\Omega_{0nm}t+\vartheta_{0nm}\right)+a_{0(3,4)}^2w_{0nm}\sin\left(\Omega_{0nm}t+\vartheta_{0nm}\right)+$$
$$+a^2{}_{(3,4)}w_{0nm}\mathfrak{D}_t\left[\sin\left(\Omega_{0nm}t+\vartheta_{0nm}\right)\right]+2\varDelta_{3,4}\Omega_{0nm}w_{0nm}\cos\left(\Omega_{0nm}t+\vartheta_{0nm}\right)+h_{03,nm}\sin\left(\Omega_{3nm}t+\vartheta_{3,nm}\right)$$

$$T_{1(nm)}(t)=\sum_{s=1}^{s=3} A_{(nm)1}^{(s)} \cos\left(\tilde{\Omega}_{(nm)s} t + \quad_{(nm)s}\right)=\sum_{s=1}^{s=3} \mathbf{K}_{(nm)31}^{(s)} \left(\tilde{\Omega}_{nm(s)}^{2}\right) C_{(nm)s} \cos\left(\tilde{\Omega}_{(nm)s} t + \quad_{(nm)s}\right)$$

$$T_{2(nm)}(t)=\sum_{s=1}^{s=3} A_{(nm)2}^{(s)} \cos\left(\tilde{\Omega}_{(nm)s} t + \quad_{(nm)s}\right)=\sum_{s=1}^{s=3} \mathbf{K}_{(nm)32}^{(s)} \left(\tilde{\Omega}_{nm(s)}^{2}\right) C_{(nm)s} \cos\left(\tilde{\Omega}_{(nm)s} t + \quad_{(nm)s}\right)$$

$$T_{3(nm)}(t)=\sum_{s=1}^{s=3} A_{(nm)3}^{(s)} \cos\left(\tilde{\Omega}_{(nm)s} t + \quad_{(nm)s}\right)=\sum_{s=1}^{s=3} \mathbf{K}_{(nm)33}^{(s)} \left(\tilde{\Omega}_{nm(s)}^{2}\right) C_{(nm)s} \cos\left(\tilde{\Omega}_{(nm)s} t + \quad_{(nm)s}\right)$$

$$T_{1(nm)}(t)=\sum_{s=1}^{s=3} \mathbf{K}_{(nm)31}^{(s)} \left(\tilde{\Omega}_{nm(s)}^{2}\right) \Xi_{(nm)s}(t)$$

$$T_{2(nm)}(t)=\sum_{s=1}^{s=3} \mathbf{K}_{(nm)32}^{(s)} \left(\tilde{\Omega}_{nm(s)}^{2}\right) \Xi_{(nm)s}(t)$$

$$T_{3(nm)}(t)=\sum_{s=1}^{s=3} \mathbf{K}_{(nm)33}^{(s)} \left(\tilde{\Omega}_{nm(s)}^{2}\right) \Xi_{(nm)s}(t)$$

$$T_{k(nm)}(t)=\sum_{s=1}^{s=3} \mathbf{K}_{(nm)3k}^{(s)} C_{(nm)s} \cos\left(\tilde{\Omega}_{(nm)s} t + \quad_{(nm)s}\right)=\sum_{s=1}^{s=3} \mathbf{K}_{(nm)3k}^{(s)} \Xi_{(nm)s}(t)$$

$$\left\langle \ddot{\boldsymbol{\varXi}}_{(\boldsymbol{nm})1}(\boldsymbol{t})+\tilde{\boldsymbol{\varOmega}}^{2}_{\boldsymbol{nm}(1)}\boldsymbol{\varXi}_{(\boldsymbol{nm})1}(\boldsymbol{t})+\tilde{\boldsymbol{\varOmega}}^{2}_{(\boldsymbol{nm})(1)}\mathfrak{D}_{\boldsymbol{t}}\left[\boldsymbol{\varXi}_{(\boldsymbol{nm})1}(\boldsymbol{t})\right]\right\rangle=\frac{\begin{vmatrix}\boldsymbol{h}_{01,\boldsymbol{nm}}\sin\left(\Omega_{1,\boldsymbol{nm}}\boldsymbol{t}+\vartheta_{1,\boldsymbol{nm}}\right) & \mathbf{K}^{(2)}_{(\boldsymbol{nm})31} & \mathbf{K}^{(3)}_{(\boldsymbol{nm})31}\\ \boldsymbol{h}_{02,\boldsymbol{nm}}\sin\left(\Omega_{2,\boldsymbol{nm}}\boldsymbol{t}+\vartheta_{2,\boldsymbol{nm}}\right) & \mathbf{K}^{(2)}_{(\boldsymbol{nm})32} & \mathbf{K}^{(3)}_{(\boldsymbol{nm})32}\\ \boldsymbol{h}_{03,\boldsymbol{nm}}\sin\left(\Omega_{3,\boldsymbol{nm}}\boldsymbol{t}+\vartheta_{3,\boldsymbol{nm}}\right) & \mathbf{K}^{(2)}_{(\boldsymbol{nm})33} & \mathbf{K}^{(3)}_{(\boldsymbol{nm})33}\end{vmatrix}}{\begin{vmatrix}\mathbf{K}^{(1)}_{(\boldsymbol{nm})31} & \mathbf{K}^{(2)}_{(\boldsymbol{nm})31} & \mathbf{K}^{(3)}_{(\boldsymbol{nm})31}\\ \mathbf{K}^{(1)}_{(\boldsymbol{nm})32} & \mathbf{K}^{(2)}_{(\boldsymbol{nm})32} & \mathbf{K}^{(3)}_{(\boldsymbol{nm})32}\\ \mathbf{K}^{(1)}_{(\boldsymbol{nm})33} & \mathbf{K}^{(2)}_{(\boldsymbol{nm})33} & \mathbf{K}^{(3)}_{(\boldsymbol{nm})33}\end{vmatrix}}$$

$$\left\langle \ddot{\boldsymbol{\varXi}}_{(\boldsymbol{nm})2}(\boldsymbol{t})+\tilde{\boldsymbol{\varOmega}}^{2}_{\boldsymbol{nm}(2)}\boldsymbol{\varXi}_{(\boldsymbol{nm})\boldsymbol{s}}2(\boldsymbol{t})+\tilde{\boldsymbol{\varOmega}}^{2}_{(\boldsymbol{nm})(2)}\mathfrak{D}_{\boldsymbol{t}}\left[\boldsymbol{\varXi}_{(\boldsymbol{nm})2}(\boldsymbol{t})\right]\right\rangle=\frac{\begin{vmatrix}\mathbf{K}^{(1)}_{(\boldsymbol{nm})31} & \boldsymbol{h}_{01,\boldsymbol{nm}}\sin\left(\Omega_{1,\boldsymbol{nm}}\boldsymbol{t}+\vartheta_{1,\boldsymbol{nm}}\right) & \mathbf{K}^{(3)}_{(\boldsymbol{nm})31}\\ \mathbf{K}^{(1)}_{(\boldsymbol{nm})32} & \boldsymbol{h}_{02,\boldsymbol{nm}}\sin\left(\Omega_{3,\boldsymbol{nm}}\boldsymbol{t}+\vartheta_{2,\boldsymbol{nm}}\right) & \mathbf{K}^{(3)}_{(\boldsymbol{nm})32}\\ \mathbf{K}^{(1)}_{(\boldsymbol{nm})33} & \boldsymbol{h}_{03,\boldsymbol{nm}}\sin\left(\Omega_{3,\boldsymbol{nm}}\boldsymbol{t}+\vartheta_{3,\boldsymbol{nm}}\right) & \mathbf{K}^{(3)}_{(\boldsymbol{nm})33}\end{vmatrix}}{\begin{vmatrix}\mathbf{K}^{(1)}_{(\boldsymbol{nm})31} & \mathbf{K}^{(2)}_{(\boldsymbol{nm})31} & \mathbf{K}^{(3)}_{(\boldsymbol{nm})31}\\ \mathbf{K}^{(1)}_{(\boldsymbol{nm})32} & \mathbf{K}^{(2)}_{(\boldsymbol{nm})32} & \mathbf{K}^{(3)}_{(\boldsymbol{nm})32}\\ \mathbf{K}^{(1)}_{(\boldsymbol{nm})33} & \mathbf{K}^{(2)}_{(\boldsymbol{nm})33} & \mathbf{K}^{(3)}_{(\boldsymbol{nm})33}\end{vmatrix}}$$

$$\left\langle \ddot{\boldsymbol{\varXi}}_{(\boldsymbol{nm})3}(\boldsymbol{t})+\tilde{\boldsymbol{\varOmega}}^{2}_{\boldsymbol{nm}(3)}\boldsymbol{\varXi}_{(\boldsymbol{nm})3}(\boldsymbol{t})+\tilde{\boldsymbol{\varOmega}}^{2}_{(\boldsymbol{nm})(3)}\mathfrak{D}_{\boldsymbol{t}}\left[\boldsymbol{\varXi}_{(\boldsymbol{nm})3}(\boldsymbol{t})\right]\right\rangle=\frac{\begin{vmatrix}\mathbf{K}^{(1)}_{(\boldsymbol{nm})31} & \mathbf{K}^{(2)}_{(\boldsymbol{nm})31} & \boldsymbol{h}_{01,\boldsymbol{nm}}\sin\left(\Omega_{1,\boldsymbol{nm}}\boldsymbol{t}+\vartheta_{1,\boldsymbol{nm}}\right)\\ \mathbf{K}^{(1)}_{(\boldsymbol{nm})32} & \mathbf{K}^{(2)}_{(\boldsymbol{nm})32} & \boldsymbol{h}_{02,\boldsymbol{nm}}\sin\left(\Omega_{2,\boldsymbol{nm}}\boldsymbol{t}+\vartheta_{2,\boldsymbol{nm}}\right)\\ \mathbf{K}^{(1)}_{(\boldsymbol{nm})33} & \mathbf{K}^{(2)}_{(\boldsymbol{nm})33} & \boldsymbol{h}_{03,\boldsymbol{nm}}\sin\left(\Omega_{3,\boldsymbol{nm}}\boldsymbol{t}+\vartheta_{3,\boldsymbol{nm}}\right)\end{vmatrix}}{\begin{vmatrix}\mathbf{K}^{(1)}_{(\boldsymbol{nm})31} & \mathbf{K}^{(2)}_{(\boldsymbol{nm})31} & \mathbf{K}^{(3)}_{(\boldsymbol{nm})31}\\ \mathbf{K}^{(1)}_{(\boldsymbol{nm})32} & \mathbf{K}^{(2)}_{(\boldsymbol{nm})32} & \mathbf{K}^{(3)}_{(\boldsymbol{nm})32}\\ \mathbf{K}^{(1)}_{(\boldsymbol{nm})33} & \mathbf{K}^{(2)}_{(\boldsymbol{nm})33} & \mathbf{K}^{(3)}_{(\boldsymbol{nm})33}\end{vmatrix}}$$

Theorem 1:

Generalized forces $Q_{w_k}^{elem-sloja}$ and $Q_{w_{k+1}}^{elem-sloja}$ of interaction between two deformable bodies coupled by standard discrete continuum layer with known kinetic $\mathrm{E}_k^{elem-sloja}$ and potential $\mathrm{E}_p^{elem-sloja}$ energies and known function of energy dissipation $\Phi^{elem-sloja}$ for generalized coordinates $w_k(x,y,t)$ and $w_{k+1}(x,y,t)$ displacement of deformable bodies at the point of contacts with discrete continuum layer are in the following forms:

$$Q_{w_k}^{elem-sloja}=-\left\langle \frac{d}{dt}\frac{\partial \mathrm{E}_k^{elem-sloja}}{\partial\left(\frac{\partial w_k(x,y,t)}{\partial t}\right)}-\frac{\partial \mathrm{E}_k^{elem-sloja}}{\partial w_k(x,y,t)}\right\rangle-\frac{\partial \mathrm{E}_p^{elem-sloja}}{\partial w_k(x,y,t)}-\frac{\partial \Phi^{elem-sloja}}{\partial\left(\frac{\partial w_k(x,y,t)}{\partial t}\right)}=Q_{w_k(x,y,t)}^{elem-ploca}$$

$$Q_{w_{k+1}}^{elem-sloja}=-\left\langle \frac{d}{dt}\frac{\partial \mathrm{E}_k^{elem-sloja}}{\partial\left(\frac{\partial w_{k+1}(x,y,t)}{\partial t}\right)}-\frac{\partial \mathrm{E}_k^{elem-sloja}}{\partial w_{k+1}(x,y,t)}\right\rangle-\frac{\partial \mathrm{E}_p^{elem-sloja}}{\partial w_{k+1}(x,y,t)}-\frac{\partial \Phi^{elem-sloja}}{\partial\left(\frac{\partial w_{k+1}(x,y,t)}{\partial t}\right)}=Q_{w_{k+1}(x,y,t)}^{elem-ploca}$$

expressed by energies and energy dissipation which posses discrete continuum layer.

Theorem 2.

Dynamics of hybrid system which contain deformable bodies (beams, plates or membranes) coupled by discrete continuum fractional order layers with equal boundary conditions and with displacements $w_k(x,y,t)$ and $w_{k+1}(x,y,t)$ is described by corresponding system of coupled partial fractional order differential equations. Systems of coupled partial fractional order differential equations for the cases of the hybrid systems containing coupled beams, or coupled plates or coupled membranes by discrete continuum fractional order layers are in mathematical analogy.

Theorem 3.

Dynamics of hybrid system which contain deformable bodies (beams, plates or membranes) coupled by discrete continuum fractional order layers with equal boundary conditions and with displacements $w_k(x,y,t)$ and $w_{k+1}(x,y,t)$ and described by corresponding system of coupled partial fractional order differential equations, in each eigen amplitude mode $W_{nm}(x,y)$ from set of infinite number is described by corresponding like -frequency eigen time functions N

$$T_{k(nm)}(t)=\sum_{s=1}^{s=N}\mathbf{K}_{(nm)Nk}^{(s)}\Xi_{(nm)s}(t)$$

where $\Xi_{(nm)s}(t)$, $s=1,2,3\ldots,N$ are normal main coordinates of corresponding subsystem in eigen amplitude mode $W_{nm}(x,y)$

These normal coordinates are analogous to the corresponding normal coordinates of corresponding linear system dynamics in same eigen amplitude mode. .

Theorem 4.

Dynamics of hybrid system which contain deformable bodies (beams, plates or membranes) coupled by discrete continuum fractional order layers with equal boundary conditions and with displacements $w_k(x,y,t)$ and $w_{k+1}(x,y,t)$ and described by corresponding system of coupled partial fractional order differential equations, in each eigen amplitude mode $W_{nm}(x,y)$ from set of infinite number is described by corresponding like -frequency eigen time functions N

$$T_{k(nm)}(t) = \sum_{s=1}^{s=N} \mathbf{K}_{(nm)Nk}^{(s)} \Xi_{(nm)s}(t)$$

where $\Xi_{(nm)s}(t)$, $s = 1,2,3...,N$ are normal main coordinates of corresponding subsystem in eigen amplitude mode $W_{nm}(x,y)$

These normal coordinates are analogous to the corresponding normal coordinates of corresponding linear system dynamics in same eigen amplitude mode. .

These normal fractional order time modes $\Xi_{(nm)s}(t)$ $s = 1,2,3\ldots, N$ are described by system of independent ordinary fractional order differential equations in the forms:

$$\ddot{\Xi}_{(nm)s}(t) + \tilde{\omega}^2_{nm(s)}\,\Xi_{(nm)s}(t) + \tilde{\omega}^2_{(nm)(s)}\mathfrak{D}_t\left[\Xi_{(nm)s}(t)\right] = 0$$

with two sets of characteristic numbers: $\tilde{\omega}^2_{nm(s)}$ and $\tilde{\omega}^2_{(nm)(s)}$

First set $\tilde{\omega}^2_{nm(s)}$ of characteristic numbers are square of eigen circular frequencies, same as for corresponding linear system, and seconds set $\tilde{\omega}^2_{(nm)(s)}$ of characteristic numbers correspond to fractional properties of eigen like one frequency fractional order mode.

Theorem 5.

In dynamics of hybrid system which contain deformable bodies (beams, plates or membranes) coupled by discrete continuum fractional order layers with equal boundary conditions and with displacements $w_k(x, y, t)$ and $w_{k+1}(x, y, t)$ and described by corresponding system of coupled partial fractional order differential equations, in each eigen amplitude mode $W_{nm}(x, y)$ from set of infinite number, described by corresponding like -frequency eigen time functions N

$$T_{k(nm)}(t) = \sum_{s=1}^{s=N} \mathrm{K}_{(nm)Nk}^{(s)} \Xi_{(nm)s}(t) \qquad W_{nm}(x, y)$$

where $\Xi_{(nm)s}(t)$, $s = 1,2,3..., N$ displacements of partial, independent fractional order oscillators described by system of independent ordinary fractional order differential equations in the forms:

$$\ddot{\Xi}_{(nm)s}(t)+\tilde{\Omega}^{2}_{nm(s)}\,\Xi_{(nm)s}(t)+\tilde{\tilde{\Omega}}^{2}_{(nm)(s)}\mathfrak{D}_{t}\left[\Xi_{(nm)s}(t)\right]=0$$

with two sets of characteristic numbers: $\tilde{\Omega}^{2}_{nm(s)}$ and . $\tilde{\tilde{\Omega}}^{2}_{(nm)(s)}$

and two complement modes: firs $\left[\Xi_{(nm)s}(t)\right]_{Like\ \ \cos\left(\tilde{\Omega}_{(nm)(s)}t+\ \ _{(nm)(s)}\right)}$

fractional order like cos mode and second $\left[\Xi_{(nm)s}(t)\right]_{Like\ \ \sin\left(\tilde{\Omega}_{(nm)(s)}t+\ \ _{(nm)(s)}\right)}$
fractional order like sin mode expressed by series along time in the following analytical forms:

$$\left[\Xi_{(nm)s}(t)\right]_{Like\ \ \cos\left(\tilde{\Omega}_{(nm)(s)}t+\ \ _{(nm)(s)}\right)}=\sum_{k=0}^{\infty}(-1)^{k}\tilde{\Omega}^{2k}_{(nm)(s)}t^{2k}\sum_{j=0}^{k}\binom{k}{j}\frac{(\mp 1)^{j}\tilde{\tilde{\Omega}}^{2j}_{(nm)(s)}t^{-\ \ j}}{\tilde{\Omega}^{2j}_{(nm)(s)}\Gamma(2k+1-\ \ j)}$$

$$\left[\Xi_{(nm)s}(t)\right]_{Like\ \ \sin\left(\tilde{\Omega}_{(nm)(s)}t+\ \ _{(nm)(s)}\right)}=\sum_{k=0}^{\infty}(-1)^{k}\tilde{\Omega}^{2k}_{(nm)(s)}t^{2k+1}\sum_{j=0}^{k}\binom{k}{j}\frac{(\mp 1)^{j}\tilde{\tilde{\Omega}}^{2j}_{(nm)(s)}t^{-\ \ j}}{\tilde{\Omega}^{2j}_{(nm)(s)}\Gamma(2k+2-\ \ j)}$$

$$\Xi_{(nm)s}(t)=\Xi_{(nm)s}(0)\sum_{k=0}^{\infty}(-1)^{k}\tilde{\Omega}^{2k}_{(nm)(s)}t^{2k}\sum_{j=0}^{k}\binom{k}{j}\frac{(\mp 1)^{j}\tilde{\tilde{\Omega}}^{2j}_{(nm)(s)}t^{-\ \ j}}{\tilde{\Omega}^{2j}_{(nm)(s)}\Gamma(2k+1-\ \ j)}+$$

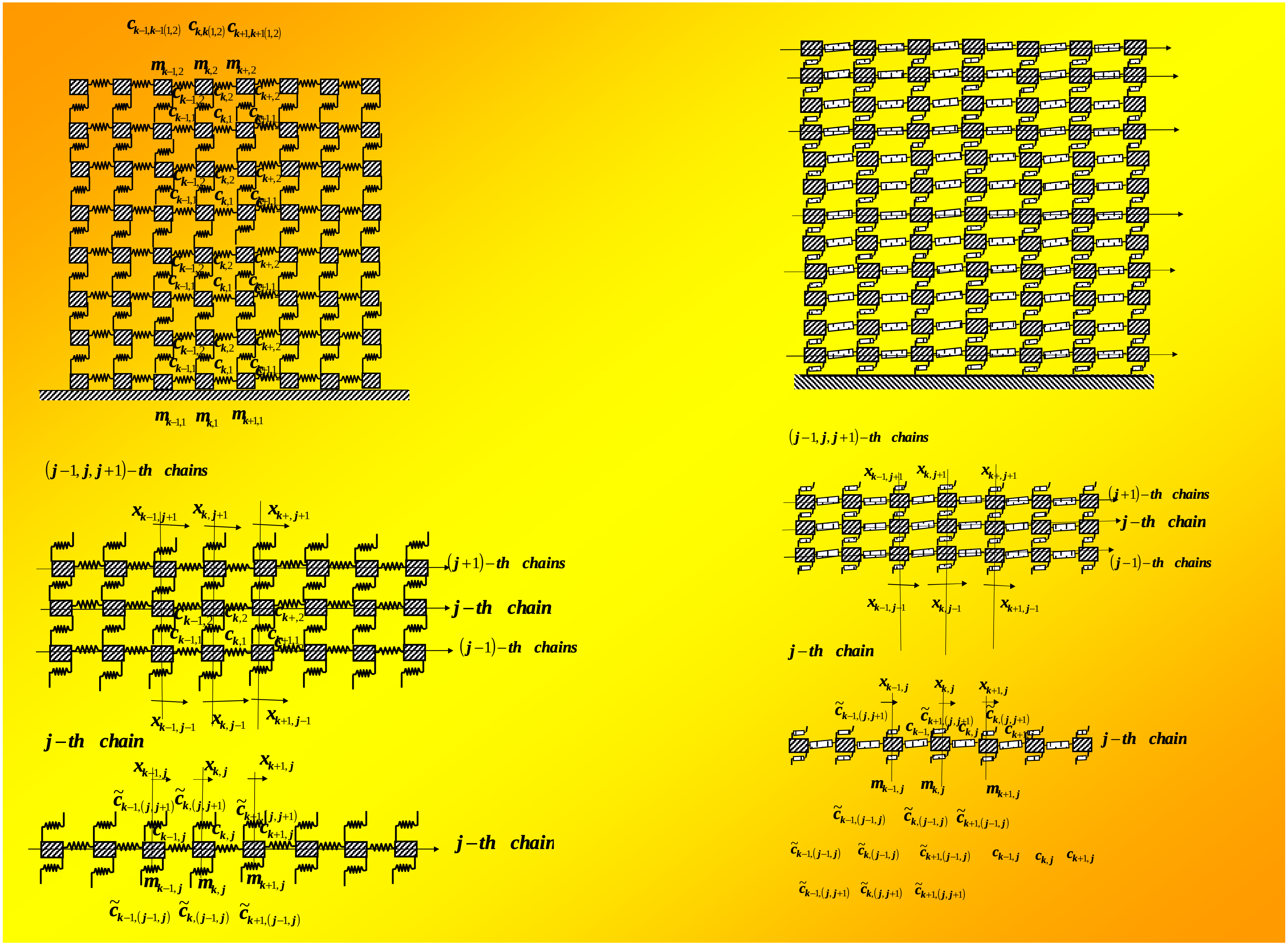
$$+\dot{\Xi}_{(nm)s}(0)\sum_{k=0}^{\infty}(-1)^{k}\tilde{\Omega}^{2k}_{(nm)(s)}t^{2k+1}\sum_{j=0}^{k}\binom{k}{j}\frac{(\mp 1)^{j}\tilde{\tilde{\Omega}}^{2j}_{(nm)(s)}t^{-\ \ j}}{\tilde{\Omega}^{2j}_{(nm)(s)}\Gamma(2k+2-\ \ j)}$$

Theorem 6:

Conriseder system of fractional oreder differential equation is linear and main coordinates of corresponding system of linear differential equations are analogous to normal coordinates of ftactional order differential equations.

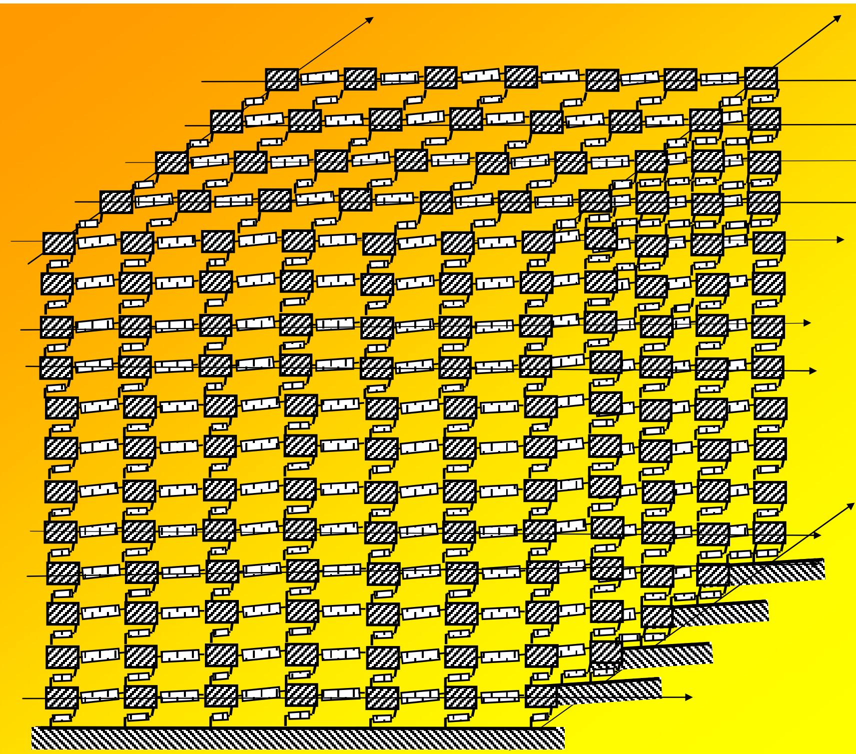
Theorem 7:

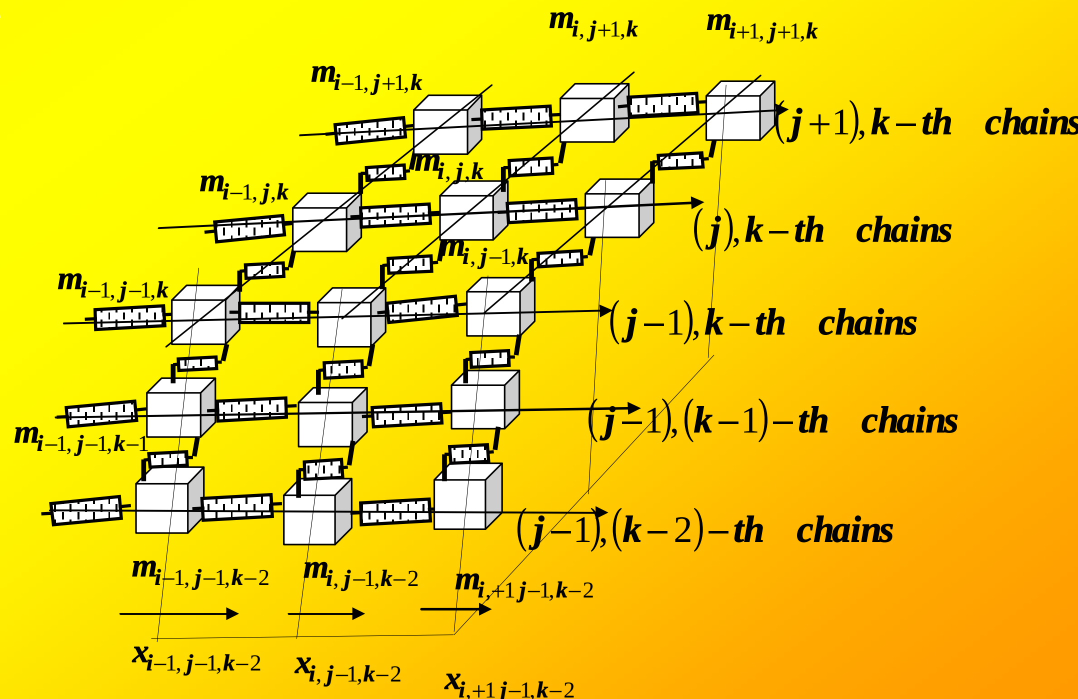
Considered system of fractional order differential equations described by like N frequensy fractional order modes with corresponding eigen circular frequencies and corresponding characteristic numbers desribing fractional order properties of fractional order like one frequency vibrations.

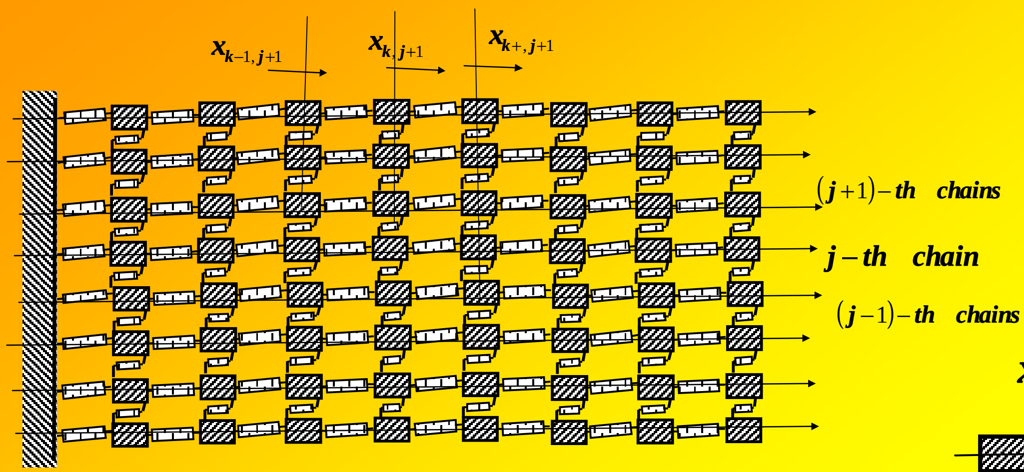


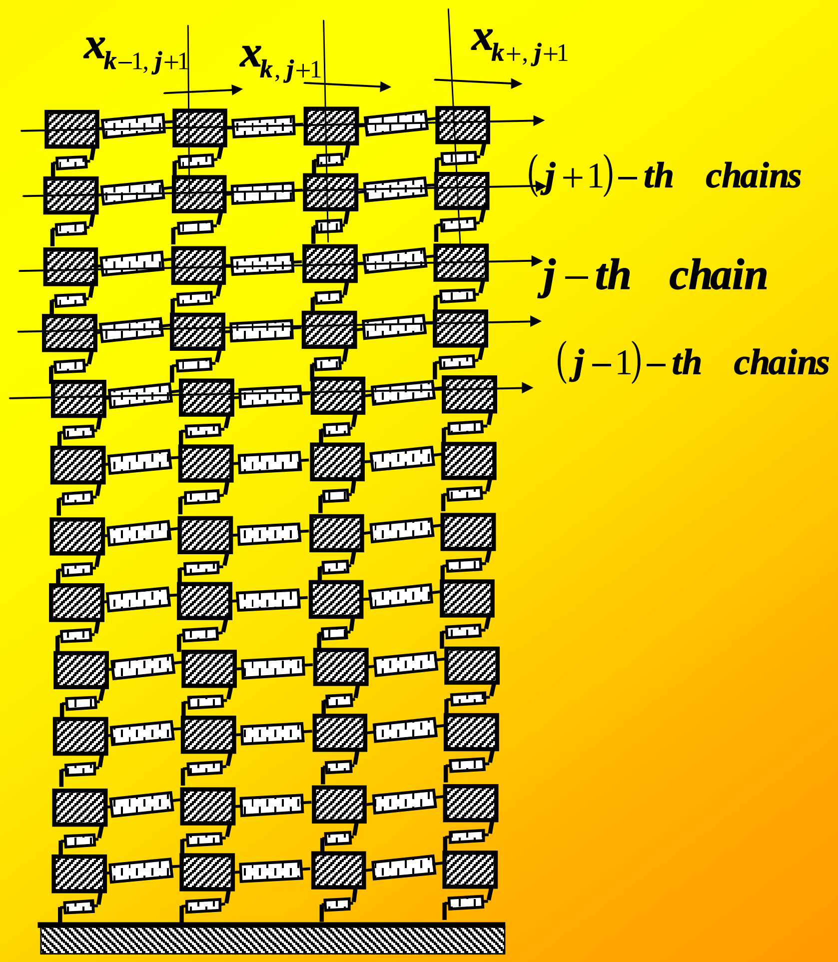
$c_{k-1,k-1(1,2)}$ $c_{k,k(1,2)}$ $c_{k+1,k+1(1,2)}$
$m_{k-1,2}$ $m_{k,2}$ $m_{k+,2}$
$m_{k-1,1}$ $m_{k,1}$ $m_{k+1,1}$
$(j-1,j,j+1)-th$ chains
$x_{k-1,j+1}$ $x_{k,j+1}$ $x_{k+,j+1}$
$(j+1)-th$ chains
$j-th$ chain
$(j-1)-th$ chains
$x_{k-1,j-1}$ $x_{k,j-1}$ $x_{k+1,j-1}$
$j-th$ chain
$x_{k-1,j}$ $x_{k,j}$ $x_{k+1,j}$
$\tilde{c}_{k-1,(j,j+1)}$ $\tilde{c}_{k,(j,j+1)}$ $\tilde{c}_{k+1,(j,j+1)}$
$c_{k-1,j}$ $c_{k,j}$ $c_{k+1,j}$
$m_{k-1,j}$ $m_{k,j}$ $m_{k+1,j}$
$\tilde{c}_{k-1,(j-1,j)}$ $\tilde{c}_{k,(j-1,j)}$ $\tilde{c}_{k+1,(j-1,j)}$
$\tilde{c}_{k-1,(j-1,j)}$ $\tilde{c}_{k,(j-1,j)}$ $\tilde{c}_{k+1,(j-1,j)}$ $c_{k-1,j}$ $c_{k,j}$ $c_{k+1,j}$
$\tilde{c}_{k-1,(j,j+1)}$ $\tilde{c}_{k,(j,j+1)}$ $\tilde{c}_{k+1,(j,j+1)}$

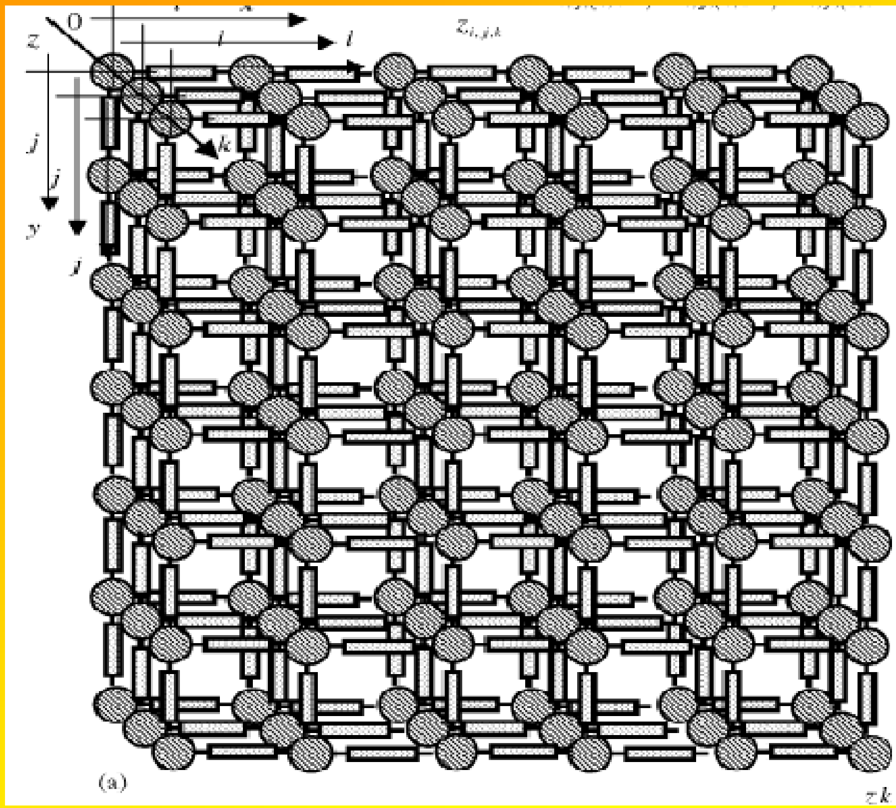
$$\ddot{}^{(s)(r)(p)} + \Omega^2_{(s)(r)(p)}\;{}^{(s)(r)(p)} + \Omega^2_{(s)(r)(p)}\mathfrak{D}_t\left[\;{}^{(s)(r)(p)}\right] = 0$$

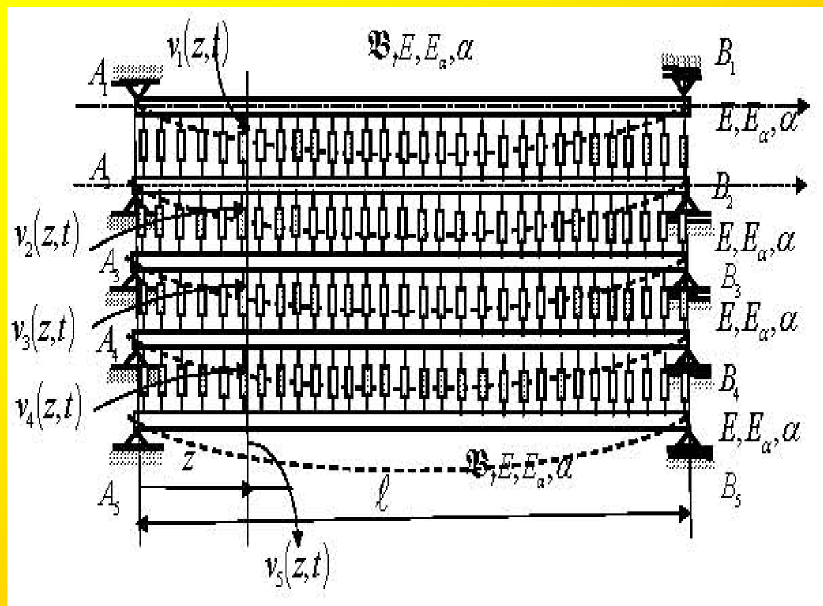





$x_{k-1,j+1}$
$x_{k,j+1}$
$x_{k+,j+1}$
$(j+1)-th$ chains
$j-th$ chain
$(j-1)-th$ chains


$x_{k-1,j+1}$
$x_{k,j+1}$
$x_{k+,j+1}$
$(j+1)-th$ chains
$j-th$ chain
$(j-1)-th$ chains


(a)


$v_1(z,t)$
$\mathfrak{B}, E, E_a, \alpha$
A_1
B_1
E, E_a, α
A_2
B_2
$v_2(z,t)$
E, E_a, α
A_3
B_3
$v_3(z,t)$
E, E_a, α
A_4
B_4
$v_4(z,t)$
E, E_a, α
z
$\mathfrak{B}, E, E_a, \alpha$
A_5
ℓ
B_5
$v_5(z,t)$

Theorem: Let fractional order system dynamics with finite nomber of degrees of freedom is defined by $\mathbf{A}$ matrix of coeficients of system inertia properties, $\boldsymbol{C}$ matrix of coeficients of system rigidity properties and $\mathbf{C}$ matrix of coeficients of system viscoelastic creep fractional order properties. In the case that modal matrix

$$\mathbf{R} = \left(\left\{\boldsymbol{K}_{nk}^{s}\right\}\right) = \left(\boldsymbol{K}_{nk}^{s}\right)_{\rightarrow\ s=1,2,3,\ldots,n}^{\downarrow\ k=1,2,3,\ldots,n}$$

of corresponding linear system produce diagonalization of matrix $\boldsymbol{C}$ of coeficients of system viscoelastic creep fractional order properties using product $\mathfrak{C}_s = \mathbf{R}'\mathbf{C}$ $\mathbf{R} = \left(c_{(\ )ss}\right)$ then this system possess eigen main independent fractional oreder modes Ξ_s $s = 1,2,3,\ldots,n$ governed by ordinary fractional order differential equations:

$$a_{ss}\ddot{\Xi}_s + c_{(\ )ss}\mathfrak{D}_t\{\Xi_s\} + c_{ss}\Xi_s = 0$$

or eigen nrmal modes $_s$ $s = 1,2,3,\ldots,n$ governed by $\ddot{}_s + \Omega^2_{(\ )s}\mathfrak{D}_t\{\ _s\} + \Omega^2_s\ _s = 0$ where

$\Omega^2_s = \dfrac{c_{ss}}{a_{ss}}$ and $\Omega^2_{(\ )s} = \dfrac{c_{(\ )ss}}{a_{ss}}$ are two sets of characteristic numbers of fractional order

Mathematical Institute SANU

THE 13th INTERNATIONAL CONFERENCE OF TENSOR SOCIETY ON DIFFERENTIAL GEOMETRY AND ITS APPLICATIONS, AND INFORMATICS BESIDES.

The 86th Anniversary of Radu MIRON'S Birth.

Faculty of Math. Al. I. Cuza Univ. Iași, Romania

テンソル学会 (TENSOR SOCIETY)

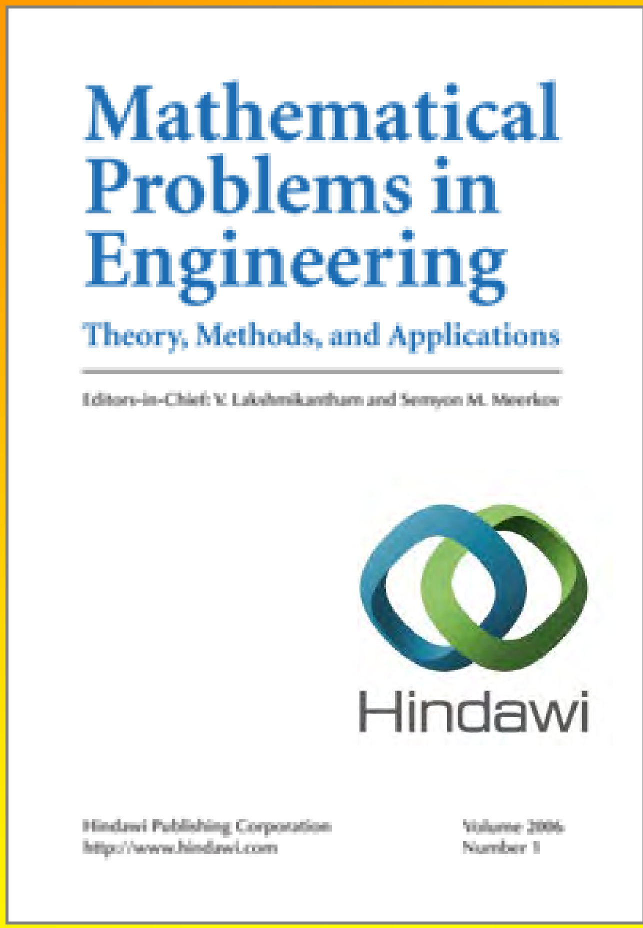
Inst. of Math. "Octav Mayer" Romanian Academy

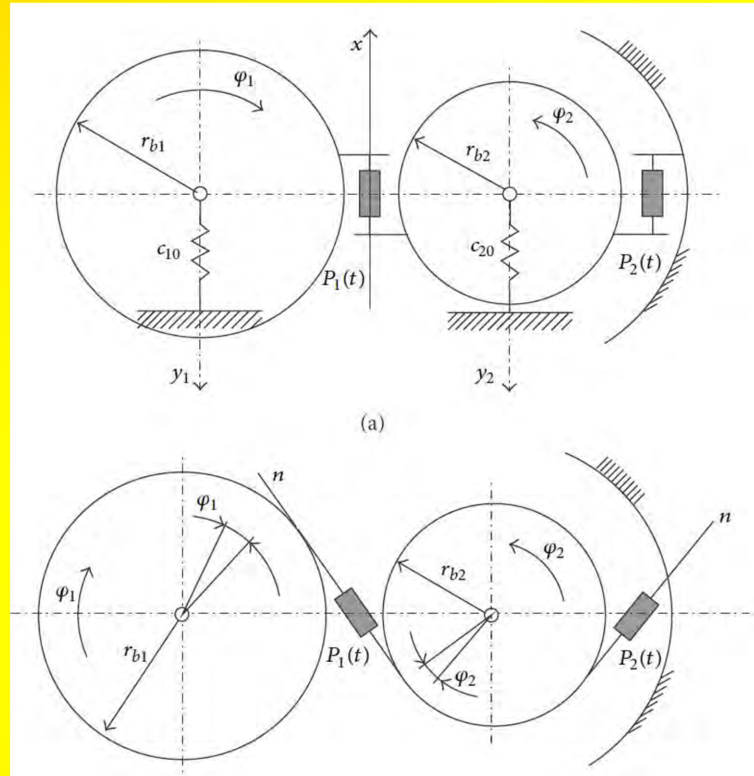
system oscillations, first set contan square of eigen circular frequencies same as for corresponding linear system, a second contan characteristic numbers ecxpresing fractional order system properties.

Elements of mathematical phenomenology in dynamics of multi-body system with fractional order discrete continuum layers

Dedicated to Centennial jubilee of Russian Academician Yury N. Rabotnov

Katica R. (Stevanović) Hedrih





Nikolic-Stanojevic Vera B Veljovic Ljiljana V Dolicanin Cemal B , A New Model of the Fractional Order Dynamics of the Planetary Gears (Article), MATHEMATICAL PROBLEMS IN ENGINEERING, (2013), vol. br. , str. -

- Hedrih (Stevanović) K., Milosavljević D., **Veljović Lj**.,**.:** *Multi-parameter Analysis of a Rigid Body Nonlinear Coupled Rotations around No Intersecting Axes Based on the Vector Method*, Adv. Theor. Appl. Mech., Vol. 6, 2013, no. 2, 49 - 70, HIKARI Ltd, ISSN 1313-6250 pp. 49-70, 2013
  - *Vector expressions of the kinetic parameters of the nonlinear dynamics of a rigid body coupled rotations around two no intersecting axes, on the basis of the three parametric analysis of the vector rotators, and transformations of the phase trajectories, show that vector method as well as applications of the mass moment vectors and vector rotators give a simplest way and expressions for analysis characteristic vector structures of coupled rotation kinetic properties, especially angular velocities of the vector rotators which are in directions of the kinetic pressures on shaft bearings or their reactions.*
  - *Vektorski izrazi kinetičkih parametaranelinearne dinamike spregnutih rotacija oko dve ose koje se mimoilaz, epokazuju da vektorska metoda zasnovana na vektorima momenata masa,predstavlja veoma pogodan način za proučavanje karakteristika kinetičkih parametara spregnutih rotacija.Posebno treba istaći direktnu zavisnost kinetičkih pritisaka od ugaone brzine i vektora rotatora.*
- **Lj. Veljovic.:**,*ANALYSIS OF A RIGID BODY ROTATION AROUND TWO NO INTERSECTING AXES – VECTOR METHOD AND PARAMETER ANALYSIS OF PHASE TRAJECTORIES,* SCIENTIFIC REVIEW (2013) Series: Scientific and Engineering - Special Issue Nonlinear Dynamics S2 (2013) pp. 319-324 YU ISSN 0350-2910
  - *Vector expressions for linear momentum and angular momentum and their corresponding derivatives with respect to time describe rigid body nonlinear dynamics with coupled rotations around axes without intersection**.** Analysis of rotation of a heavy gyro-rotor show us that in graphical presentations of the system kinetic parameters exits a set of the fixed points not depending of change of rigid body eccentricity or angle of inclination or of the orthogonal distance between axes of rigid body coupled rotations*

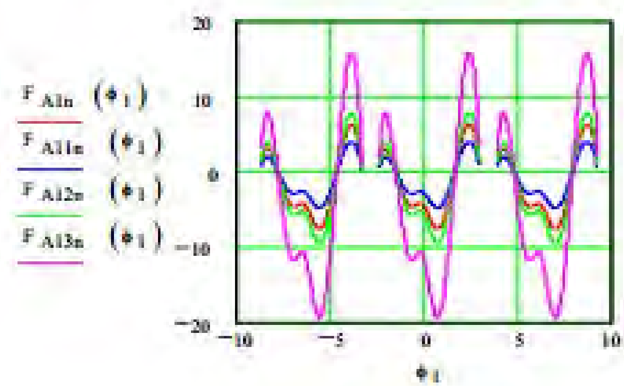
- Katica R. (Stevanović) Hedrih, **Ljiljana Veljović.:** *New Vector Description of Kinetic Pressures on Shaft Bearings of a Rigid Body Nonlinear Dynamics with Coupled Rotations around No Intersecting Axes* **,** Acya Polytechnica Hungarica, ISSN 1785-8860, Vol.10, No 7, pp 151-170, 2013
  – *New vector description of kinetic pressures on shaft bearings of a rigid body nonlinear dynamics with coupled rotations around no intersecting axes is first main result presented in this paper. As an example of defined dynamics, we take into consideration a heavy gyro-rotordisk with one degree of freedom and coupled rotations when one component of rotation is programmed by constant angular velocity.*
  – *Kinetički pritisci na ležišta vratila tela koje se obrće oko mimoilaznih osa predstavljeni su na nov vektorski način. Kao primer korišćen je model teškog rotora ...a stepena pokretljivosti tj. sa jednom*

$$\ddot{\varphi}_2 + \Omega^2(\lambda - \cos\varphi_2)\sin\varphi_2 + \Omega^2\psi\cos\varphi_2 = 0$$

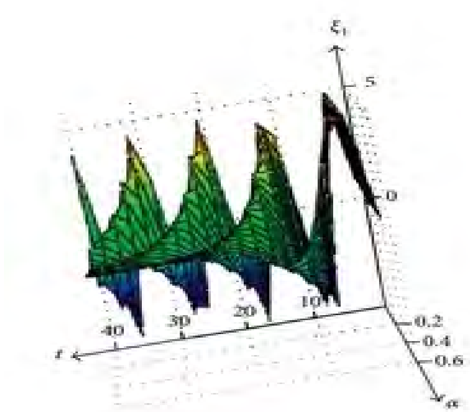
$$\vec{F}_{A\vec{n}_2} = \left[\left|\vec{S}^{(O)}_{\vec{n}_1}\right|\left(\vec{R}_1, \vec{n}_2\right) + \left|\vec{S}^{(O)}_{\vec{n}_2}\right|\left(\vec{R}_{21}, \vec{n}_2\right) - \left(\vec{G}, \vec{n}_2\right)\right]\vec{n}_2$$

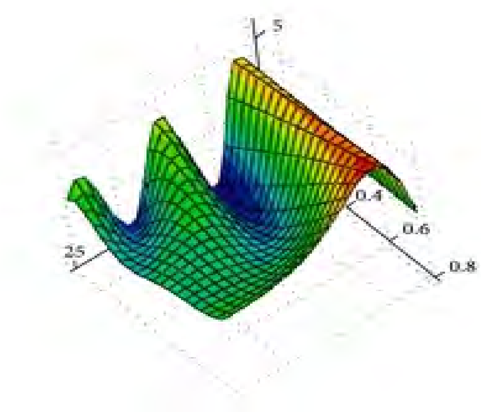
$$\begin{aligned}\vec{F}_{B2} = &\frac{1}{2}\left\{\left|\vec{S}^{(O_2)}_{\vec{n}_2}\right|\left[\vec{n}_2,\left[\vec{R}_2,\vec{n}_2\right]\right] + \left|\vec{S}^{(O_2)}_{\vec{n}_1}\right|\left[\vec{n}_2,\left[\vec{R}_1,\vec{n}_2\right]\right] + \left|\vec{S}^{(O_2)}_{\vec{n}_2}\right|\left[\vec{n}_2,\left[\vec{R}_{21},\vec{n}_2\right]\right] - \left[\vec{n}_2,\left[\vec{G},\vec{n}_2\right]\right]\right\} +\\ &+\frac{1}{2\ell}\left\{\left|\vec{D}^{(O_2)}_{\vec{n}_2}\right|\left[\vec{R}^{*}_{02},\vec{n}_2\right] + \left|\vec{D}^{(O_2)}_{\vec{n}_1}\right|\left[\vec{R}^{*}_{02},\vec{n}_2\right] + \left[\vec{n}_1,\vec{n}_2\right]\dot{\omega}_1\left(\vec{n}_1,\vec{J}^{(O_2)}_{\vec{n}_1}\right) - \left[\left[\vec{\rho}_C,\vec{G}\right],\vec{n}_2\right]\right\} +\\ &+\frac{1}{2\ell}\left[\left[\left\{\left[\vec{n}_2,\vec{J}^{(O_2)}_{n_1}\right] + \left[\vec{n}_1,\vec{J}^{(O_2)}_{n_2}\right] + J^{(O_2)}\left[\vec{n}_1,\vec{n}_2\right]\right\},\vec{n}_2\right],\vec{n}_1\right]\end{aligned}$$

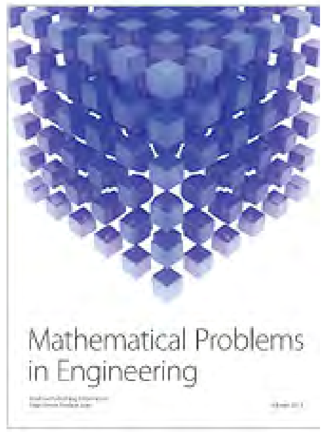
$$\begin{aligned}\vec{F}_{AT2} = &\frac{1}{2}\left\{\left|\vec{S}^{(O_2)}_{\vec{n}_2}\right|\left[\vec{n}_2,\left[\vec{R}_2,\vec{n}_2\right]\right] + \left|\vec{S}^{(O_2)}_{\vec{n}_1}\right|\left[\vec{n}_2,\left[\vec{R}_1,\vec{n}_2\right]\right] + \left|\vec{S}^{(O_2)}_{\vec{n}_2}\right|\left[\vec{n}_2,\left[\vec{R}_{21},\vec{n}_2\right]\right] - \left[\vec{n}_2,\left[\vec{G},\vec{n}_2\right]\right]\right\} -\\ &-\frac{1}{2\ell}\left\{\left|\vec{D}^{(O_2)}_{\vec{n}_2}\right|\left[\vec{R}^{*}_{02},\vec{n}_2\right] + \left|\vec{D}^{(O_2)}_{\vec{n}_1}\right|\left[\vec{R}^{*}_{01},\vec{n}_2\right] + \left[\vec{n}_1,\vec{n}_2\right]\dot{\omega}_1\left(\vec{n}_1,\vec{J}^{(O_2)}_{\vec{n}_1}\right) - \left[\left[\vec{\rho}_C,\vec{G}\right],\vec{n}_2\right]\right\} -\\ &-\frac{1}{2\ell}\left[\left[\left\{\left[\vec{n}_2,\vec{J}^{(O_2)}_{n_1}\right] + \left[\vec{n}_1,\vec{J}^{(O_2)}_{n_{21}}\right] + J^{(O_2)}\left[\vec{n}_1,\vec{n}_2\right]\right\},\vec{n}_2\right],\vec{n}_1\right]\end{aligned}$$



- Vera Nikolic-Stanojevic, **Ljiljana Veljovic**, Cemal Dolicanin,.: *A New Model of the Fractional Order Dynamics of the Planetary Gears,* Mathematical Problems in Engineering Volume 2013 (2013), Article ID 932150, 14 pages http://dx.doi.org/10.1155/2013/932150
  - *Dynamic model of the planetary gears with four degrees of freedomis is used. Applying the basic principles of analytical mechanics and taking the initial and boundary conditions into consideration, the system of equations representing physical meshing process between the two or more gears is obtained. This investigation was focused to a new model of the fractional order dynamics of the planetary gear.*
  - *Razmatra se dinamički model planetarnog prenosnika sa četiri stepena slobode. Primenom osnovnih principa mehanike dobijen je sistem diferencijalnih jednačina kojima je opisano sprezanje zupčanika. Prikazan je nov model frakcionih jednačina dinamike*







- Gordana Bogdanović, Dragan Milosavljević, **Ljiljana Veljović**, Aleksandar Radaković, Mirjana Lazić**.:** *COMPOSITE MATERIALS IN AUTOMOTIVE ENGINEERING – MECHANICAL BEHAVIOR OF ANISOTROPIC MEDIA***,** Mobility and Vehicle Mechanics, University of Kragujevac, Faculty of Engineering, ISBN 1450-5304 vol. 39, br. 1, str. 39-49, 2013.,
  - *In recent engineering practice composite materials are widely used. In such materials two or more materials are combined to obtain a new one with new properties, while individual properties of constituents remain distinguished. Such materials have notable feature that are anisotropic, having different mechanical properties in different directions. Here is special attention devoted to their mechanical behavior. Small changes of preferred direction have significant influence to stress strain relations in fibre reinforced layers.*
  - *U inženjerskoj praksi kompozitni materijali su u širokoj upotrebi.U takvim materijalima dva ili više materijala se kombinuju i daju posebne osobine novoformiranom materijalu. Posebna pažnja posvećena je mehaničkom ponašanju ovih anizitropnih materijala.*

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UDC 621 + 629(05)=802.0

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UNIVERSITY OF KRAGUJEVAC, FACULTY OF ENGINEERING

SERBIAN SOCIETY OF AUTOMOTIVE ENGINEERS

$$\rho \frac{\partial^2 u_i}{\partial t^2} = C_{ijkl} \frac{\partial^2 u_l}{\partial x_j \partial x_k} \qquad u_i = U_i e^{i(kn_j x_j - \omega t)} = U_i e^{i\varphi}$$

- Gordana Bogdanović, Dragan Milosavljević, **Ljiljana Veljović,** Aleksandar Radaković**.:** *COMPOSITE MATERIALS – MECHANICAL BEHAVIOR OF ANISOTROPIC*, Proceedings of papers 11 International Conference on Accomplishments in Electrical and Mechanical Engineering and Information Technology DEMI 2013, PP. 111-114

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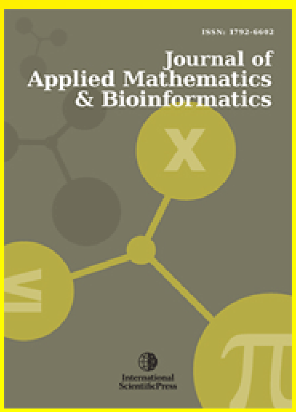

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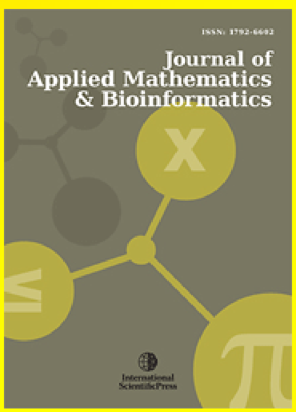

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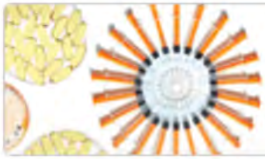

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Phenomenological mapping and dynamical absorptions in chain systems with multiple degrees of freedom

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Andjelka N Hedrih[3]

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Abstract

This Article

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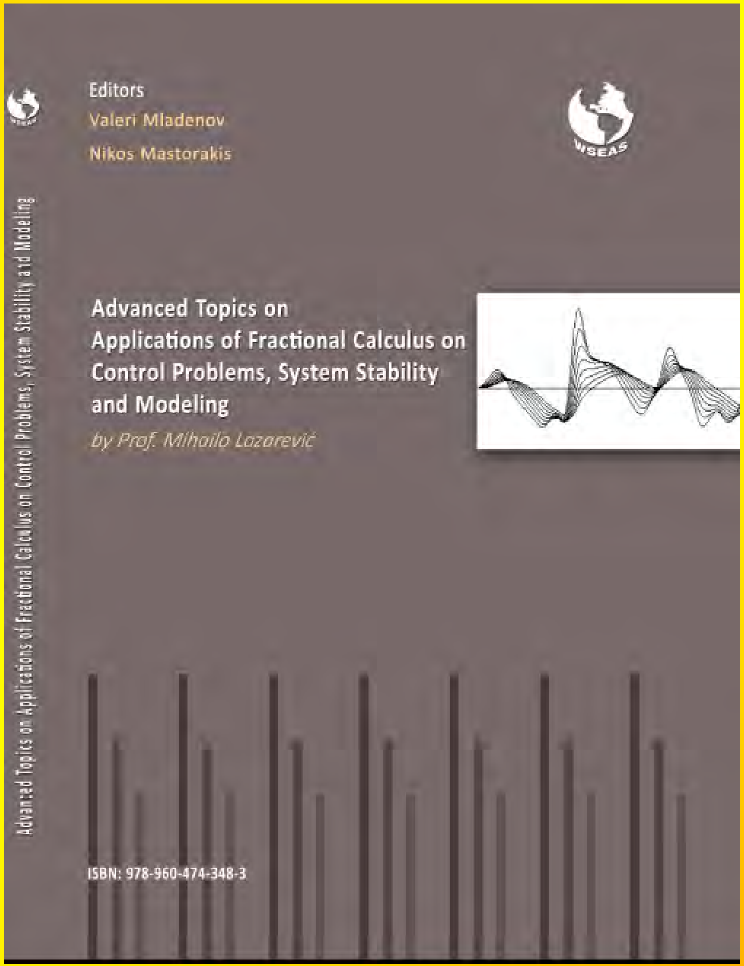

ENOC 2011
7th European Nonlinear Dynamics Conference
July 24-29, 2011 – Rome, ITALY

On the occasion of ENOC 2011, the EUROMECH Nonlinear Oscillations Conference Committee (ENOCC) has conferred an

ENOC 2011 Young Scientist Prize

to *Mrs. **Andjelka N. Hedrih***

for the best presentation given at the 7th European Nonlinear Dynamics Conference, Rome, 24-29 July 2011,

*within a Regular Session on **"Biosystems"**:*

"Modeling oscillations of Zona Pelucida before and after fertilization"

In addition to the requirements for the ENOC 2011 Proceedings the prize recipient is invited to write a contribution with respect to his (her) presentation, to be published in the EUROMECH Newsletter

Giuseppe Rega	*Dick H. van Campen*
Chairman of ENOC 2011	Chairman of ENOCC

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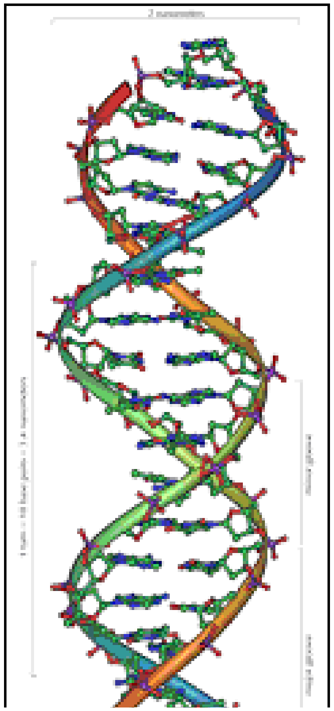
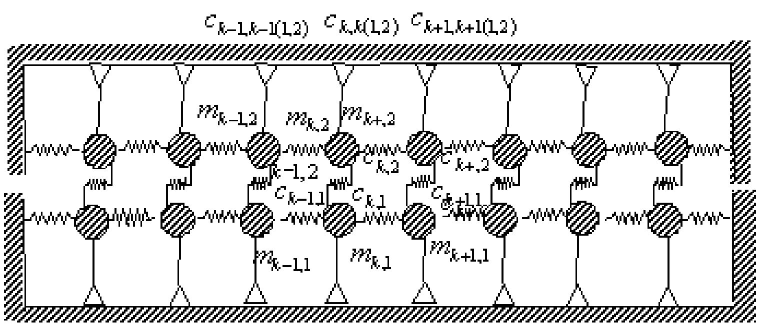
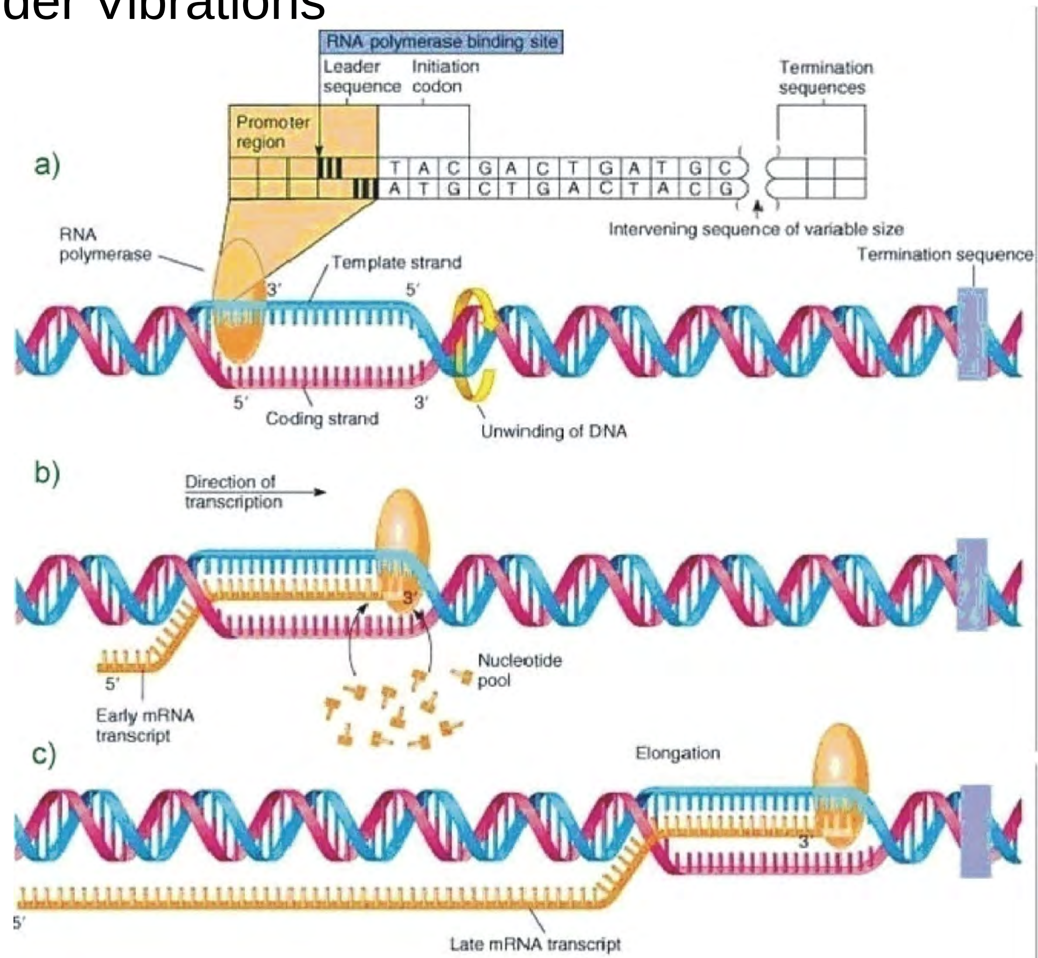
Modelling Double DNA Helix Main Chains of the Free and Forced Fractional Order Vibrations





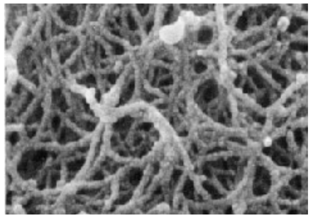


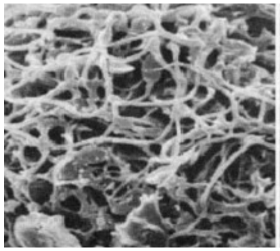
Transkripcija DNK

Andjelka N. Hedrih, Katica R. (Stevanović) Hedrih. (2014) Modeling Double DNA Helix Main Chains of the Free and Forced Fractional Order Vibrations. Ch 7 In: Advanced Topics on Applications of Fractional Calculus on Control Problems, System Stability and Modeling (eds: Mihailo Lazarevic, Nikos Mastorakis. (2014), pp. 145-183. and Appendix E pp. 192-200. WSEAS Press. ISBN: 978-960-474-348-3

Changes in 3D structure of mZP-SEM

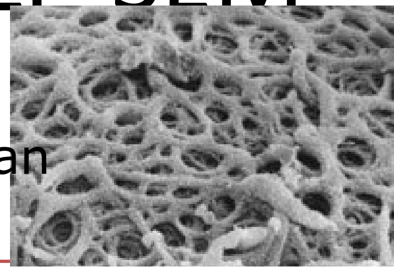
Mouse





human

human



Mouse mature oocyte

•ZP has mesh –like structure on scanning electron microscopy

•ZP has more pores with less diameter then in embryo

•ZP is penetrable for one certain spermatozoa

•Young module has 2.5 times higher value compare to ZP of mouse embryo

•Diameter of oocyte and its ZP is slightly lower than in embryo

Mouse embryo

•ZP is not penetrable for spermatozoa

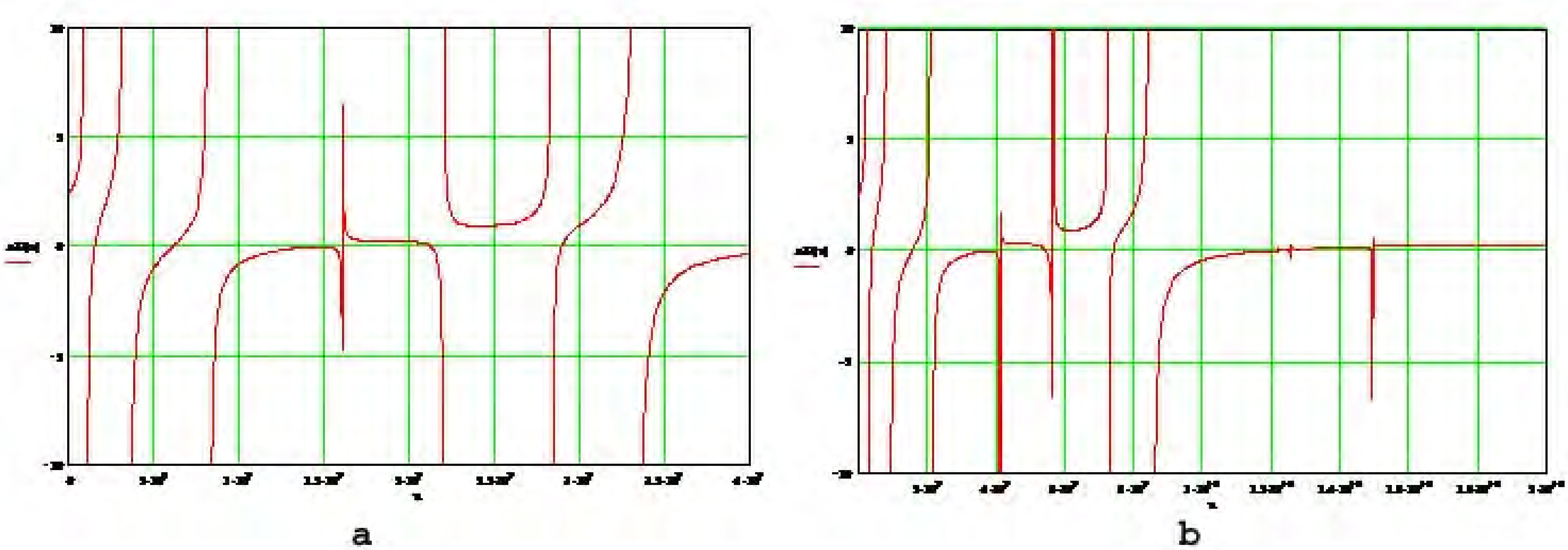
•ZP has les pores with larger diameter then in oocyte

•Young module has 2.5 times lower value compare to ZP of mouse oocyte

•Diameter of embryo and its ZP is slightly higher than in oocyte

Sorce:Familiari G, Relucenti M, Heyn R, Micara G, and Correr S. (2006) Three-Dimensional Structure of the Zona Pellucida at Ovulation. *Microscopy research and technique* 69:415–426

Transition in oscillatory behavior in mouse oocyte and mouse embryo trough oscillatory spherical net model of mouse Zona Pellucida



- Amplitude-frequency stationary forced regimes for forced oscillations of third material particle in chain excited by external excitation force with amplitude and frequency applied to third mass particle in chain; $x=\Omega^2$. a. oocyte b. embryo

Andjelka Hedrih. Transition in oscillatory behavior in mouse oocyte and mouse embryo trough oscillatory spherical net model of mouse Zona Pellucida" (LIF138) in Applied Non-Linear Dynamical Systems/DSTA 2013; Series title: Proceedings in Mathematics and Statics, Edited by Jan Awrejcewicz, Springer, in press.

Fertilization as a biomechanical oscillatory phenomenon in mammals

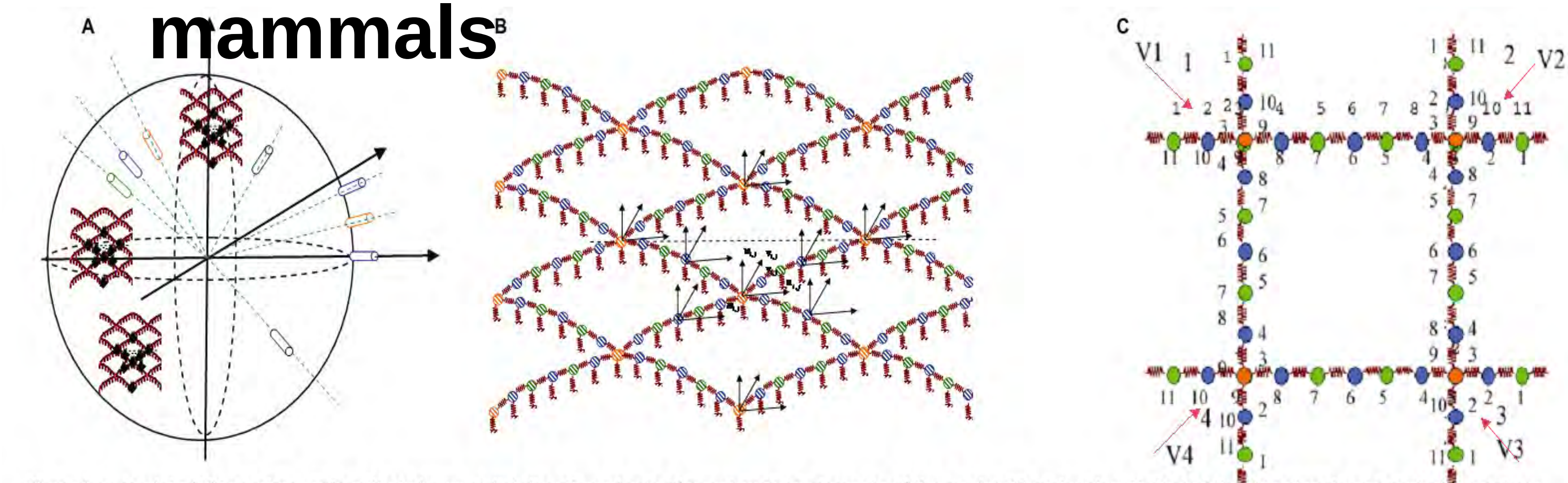
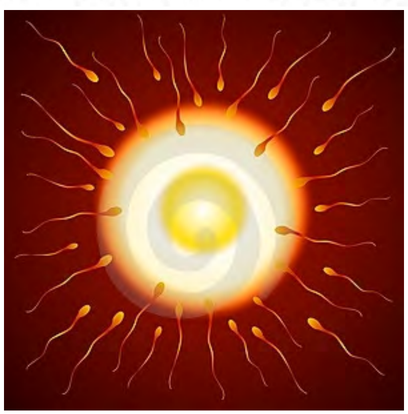
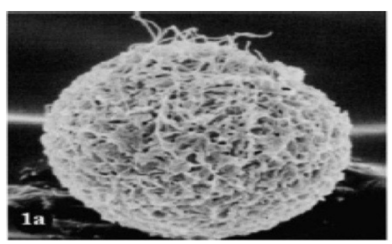
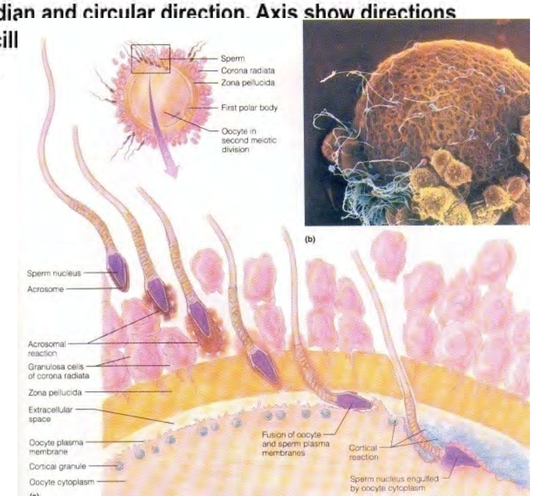


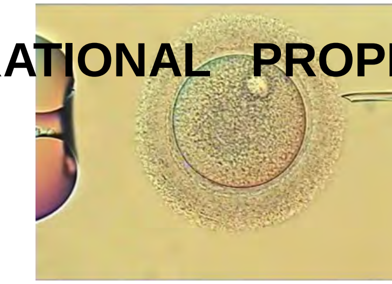
Figure1. A. Model of ZP sperical surface that shows radial direction of axis of constructives elements of the model-ZP molecules. B. Part of the ZP network on a part of the sphere-oocyte: orange (ZP1), blue (ZP2) and green (ZP3) represents molecules of ZP proteins. Chains of spherical net are identical in merdian and circular direction. Axis show directions of movements of molecules of ZP proteins. Each ZP protein in the model is connected to the sphere with elastic spring that can oscill surface net model of mZP: v1-v4 are sperm cells with effective velocities. Red arrows denote sperm cell impact on a knot molecule.







THE USE OF FINITE ELEMENTS METHOD IN VIBRATIONAL PROPERTIES CHARACTERIZATION OF MOUSE EMBRYO



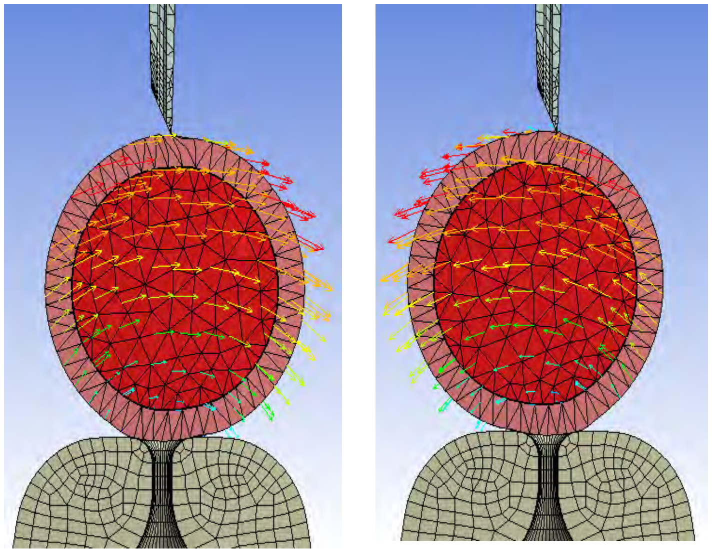


Figure 6. Shape and particle velocities distribution in extreme points of embryo vibrations in mode 2.

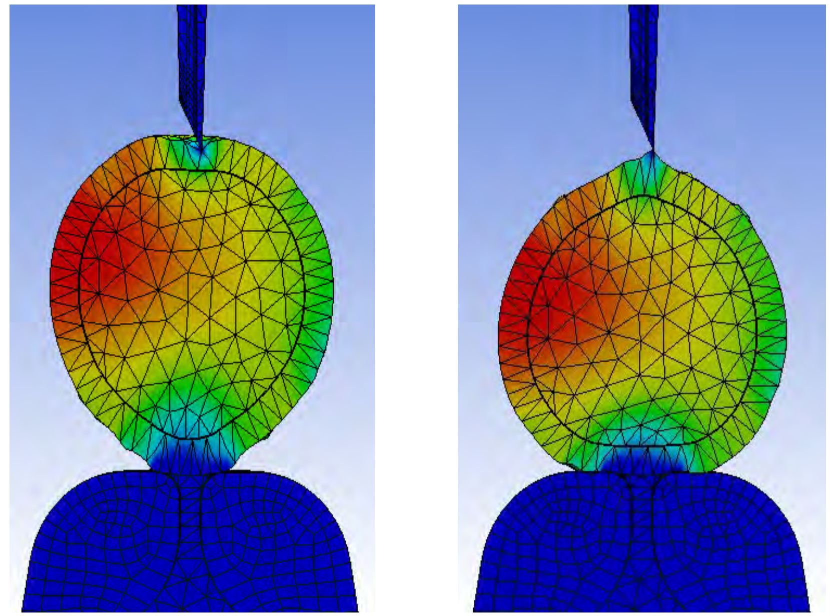


Figure 11. Shape and deformations distribution in extreme points of embryo oscillations in mode 2.

Hedrih A, Ugrčić M. The use of finite elements method in vibration properties characterization of mouse embryo**.** Symposium Non-linear Dynamics with Multy- and Interdisciplinary Applications (SNDMIA 2012), Belgrade, October 1-5 th 2012 (Eight Serbian Symposium in area of Non-linear Sciences) A-01:1-15. **SCIENTIFIC REVIEW.** Series: Scientific and Engineering Editor-in-Chief: Slobodan Perović. Serbian Scientific Society. 2013, 245-254. UDK 001 YU ISSN 0350-2910

Analysis of energy state of discrete fractional order spherical net of mouse *zona pellucida* before and after fertilization

Andjelka N. Hedrih and Katica (Stevanović) Hedrih

$$\Phi_{\mathrm{Rer}.Part,0<\alpha<1}=\sum_{\substack{j=1\\ j\neq 2}}^{j=3}\sum_{k=0}^{k=12}\frac{1}{2}c_{0<\alpha<1(k,k+1),j}\left\langle \mathfrak{D}_t^{\alpha}\left[u_{k+1,j}(t)-u_{k,j}(t)\right]\right\rangle^2+$$
$$+\sum_{\substack{j=2\\ j\neq 3}}^{j=4}\sum_{k=0}^{k=12}\frac{1}{2}c_{0<\alpha<1(k,k+1),j}\left\langle \mathfrak{D}_t^{\alpha}\left[v_{k+1,j}(t)-v_{k,j}(t)\right]\right\rangle^2+$$
$$+\sum_{k=1}^{k=11}\sum_{\substack{j=1\\ j\neq 3\\ j\neq 9}}^{j=11}\tilde{c}_{0<\alpha<1(k,k),j}\left\langle \mathfrak{D}_t^{\alpha}\left[w_{k,k}(t)\right]\right\rangle^2 \qquad (18)$$

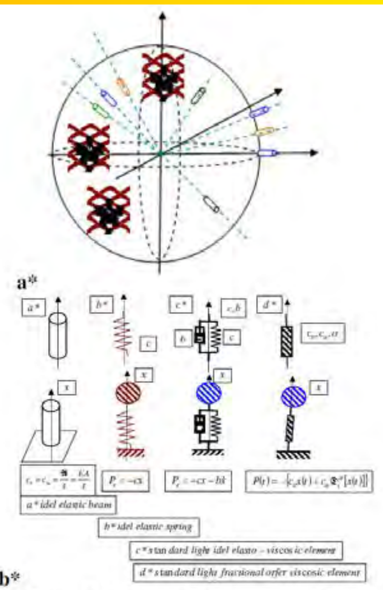
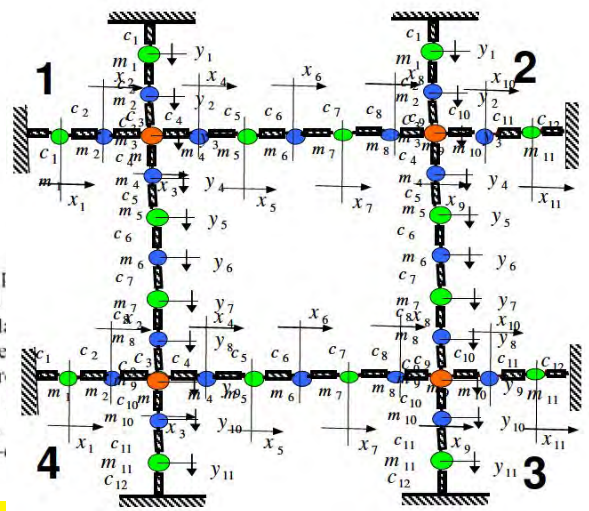


Figure 1. a* Part of the ZP spherical net model on a
the sphere (oocite). Orange (ZP1), blue (ZP2) and
(ZP3) represents ZP proteins. Net is identical in circul
meridian direction. Axis shows directions of moveme
ZP proteins. Each ZP protein is connected to the spher
standard light visco-elastic elements of fractional
(SLFOE) that can oscillate in radial direction.
Schematically presentation of fractional order visco-
element from part of Zona Pelucida to the model.



$$\frac{d}{dt}\left[\frac{\partial \mathbf{E}_{Kin,\mathrm{Re}\,pr.}}{\partial\langle \dot{u}_{k,j}(t)\rangle}\right]-\frac{\partial \mathbf{E}_{Kin,\mathrm{Re}\,pr.}}{\partial\langle u_{k,j}(t)\rangle}+\frac{\partial \mathbf{E}_{p,\mathrm{Re}\,pr.Part,\alpha=0}}{\partial\langle u_{k,j}(t)\rangle}+\frac{\partial \Phi_{\mathrm{Rer}.Part,0\le\alpha\le 1}}{\partial\left\langle \mathfrak{D}_t^{\alpha}\left[u_{k,j}(t)\right]\right\rangle}=0$$

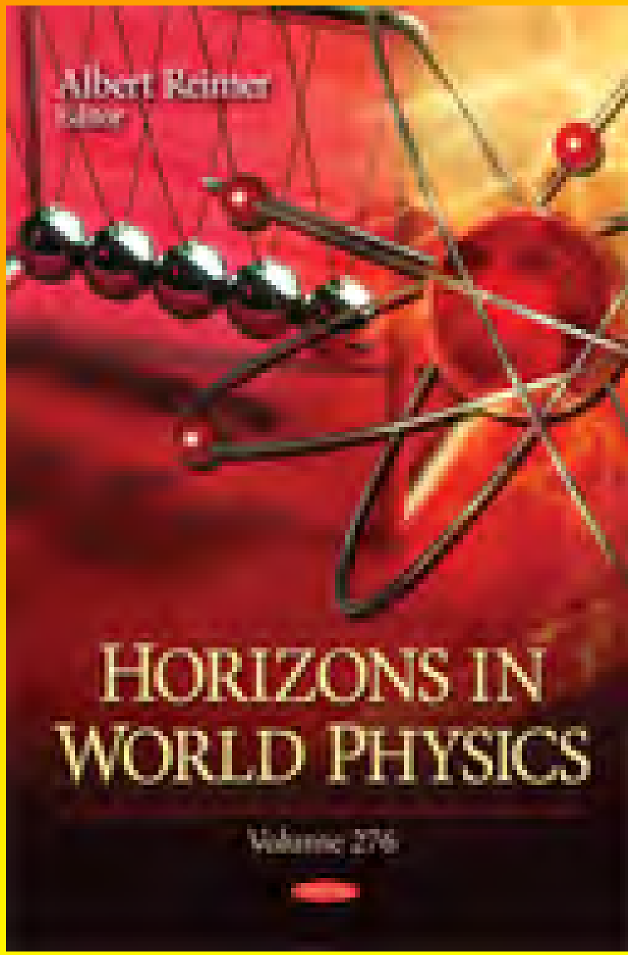
$$k=1,2,3,\ldots,11,\ j=1,3,\ 0\le\alpha\le 1 \qquad (23)$$

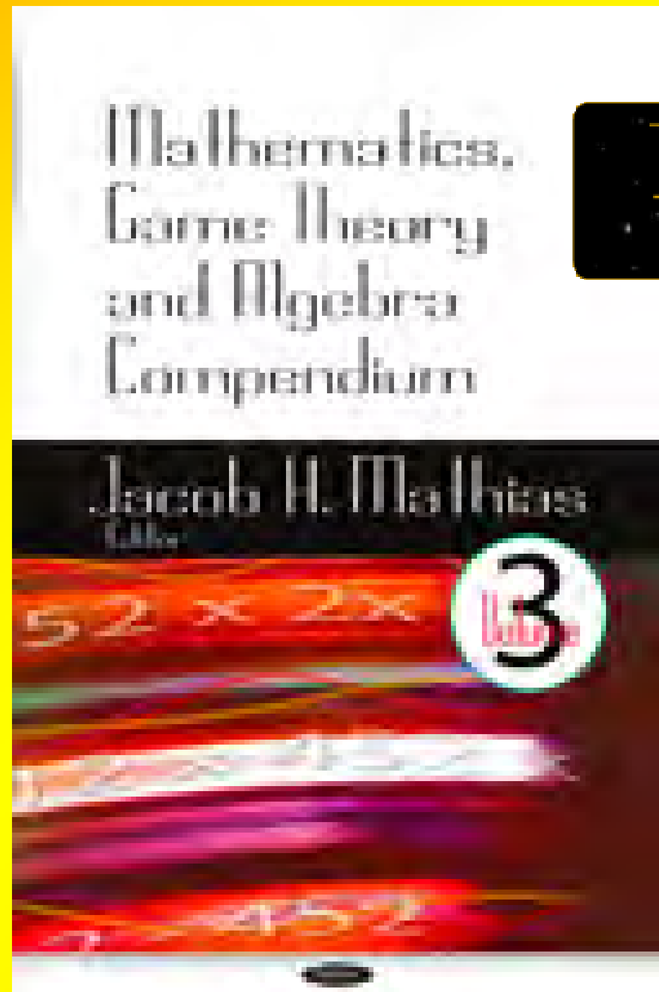
$$\frac{d}{dt}\left[\frac{\partial \mathbf{E}_{Kin,\mathrm{Re}\,pr.}}{\partial\langle \dot{v}_{k,j}(t)\rangle}\right]-\frac{\partial \mathbf{E}_{Kin,\mathrm{Re}\,pr.}}{\partial\langle v_{k,j}(t)\rangle}+\frac{\partial \mathbf{E}_{p,\mathrm{Re}\,pr.Part,\alpha=0}}{\partial\langle v_{k,j}(t)\rangle}+\frac{\partial \Phi_{\mathrm{Rer}.Part,0\le\alpha\le 1}}{\partial\left\langle \mathfrak{D}_t^{\alpha}\left[v_{k,j}(t)\right]\right\rangle}=0$$

$$k=1,2,3,\ldots,11,\ j=2,4,\ 0\le\alpha\le 1 \qquad (24)$$

$$\frac{d}{dt}\left[\frac{\partial \mathbf{E}_{Kin,\mathrm{Re}\,pr.}}{\partial\langle \dot{w}_{k,j}(t)\rangle}\right]-\frac{\partial \mathbf{E}_{Kin,\mathrm{Re}\,pr.}}{\partial\langle w_{k,j}(t)\rangle}+\frac{\partial \mathbf{E}_{p,\mathrm{Re}\,pr.Part,\alpha=0}}{\partial\langle w_{k,j}(t)\rangle}+\frac{\partial \Phi_{\mathrm{Rer}.Part,0\le\alpha\le 1}}{\partial\left\langle \mathfrak{D}_t^{\alpha}\left[w_{k,j}(t)\right]\right\rangle}=0$$

$$k,\ j=1,2,3,\ldots,11,\quad k\neq 3,\quad k\neq 9,\ 0\le\alpha\le 1 \qquad (25)$$





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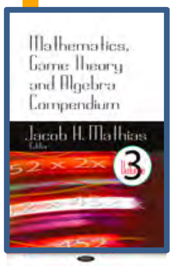
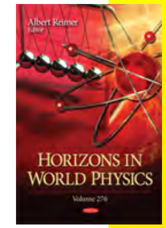
Rigorous Kinetic Analysis of the Racket Flick-Motion in Tennis for Generating Topspin and Backspin





Katica (Stevanović) Hedrih and Tijana T. Ivancevic*

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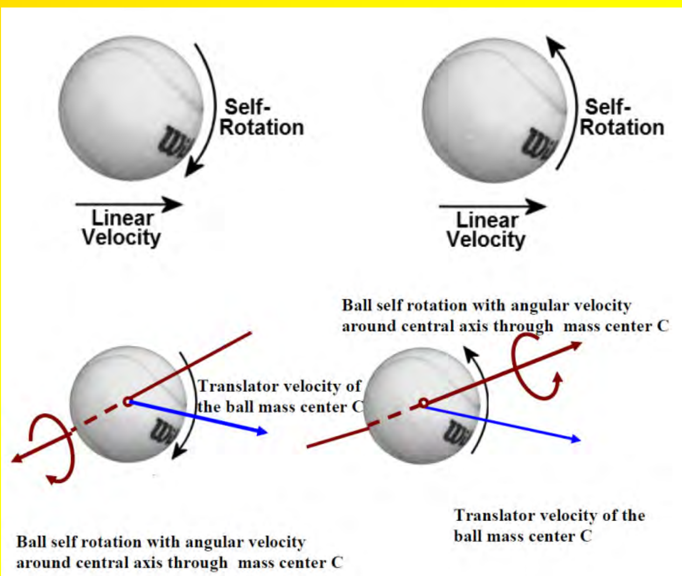

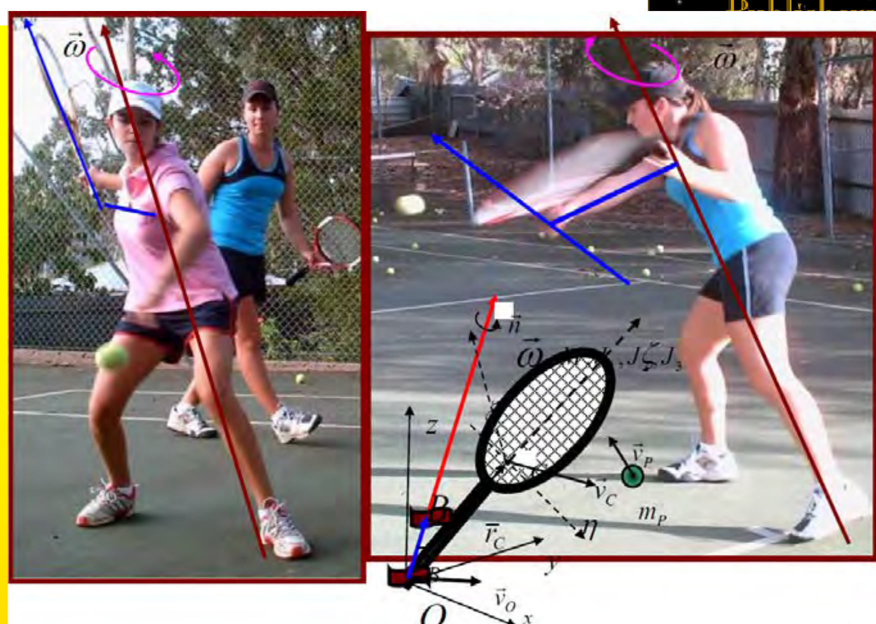

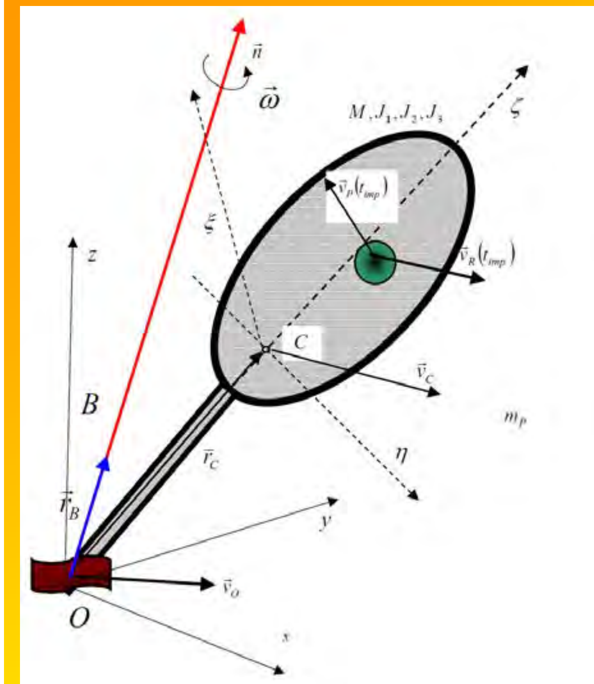
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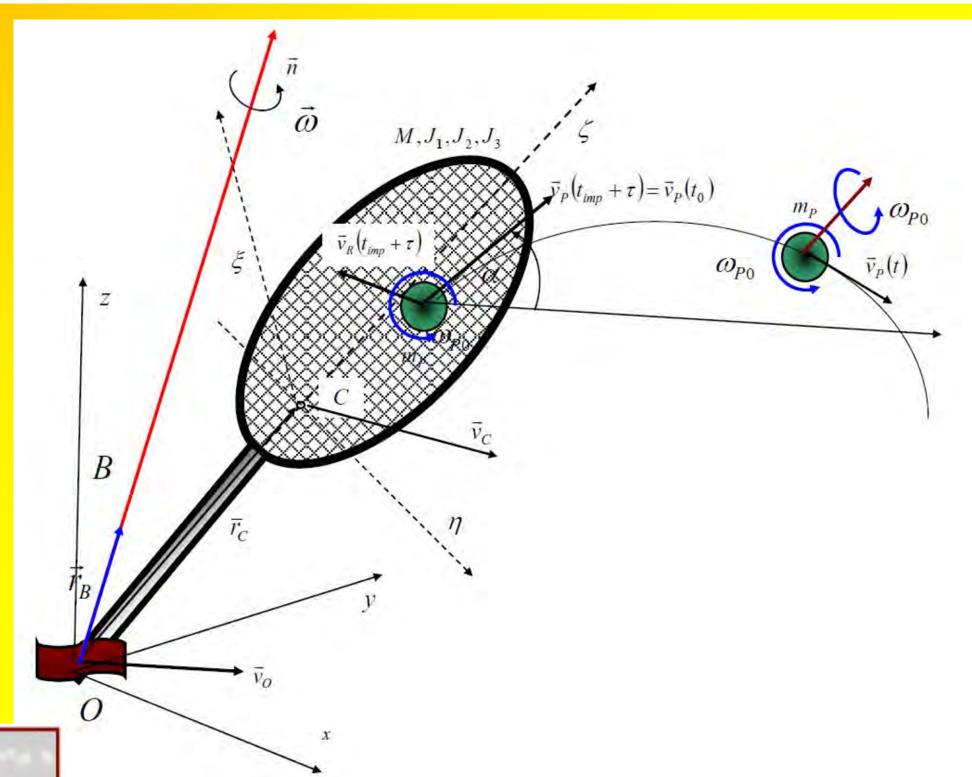
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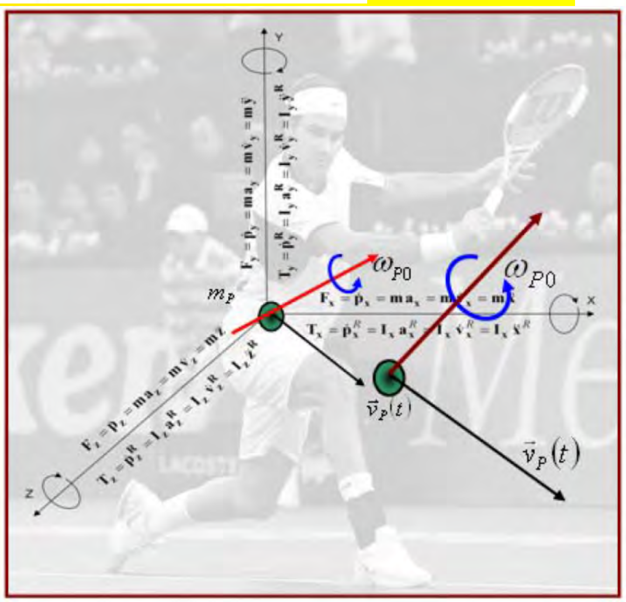
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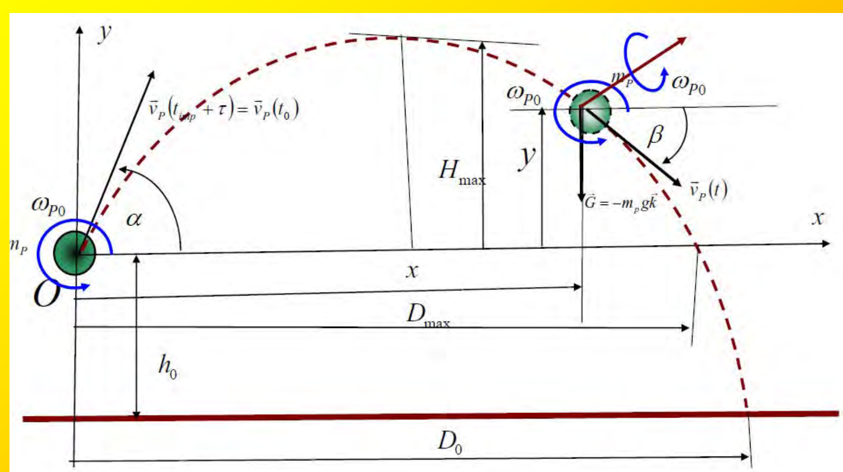
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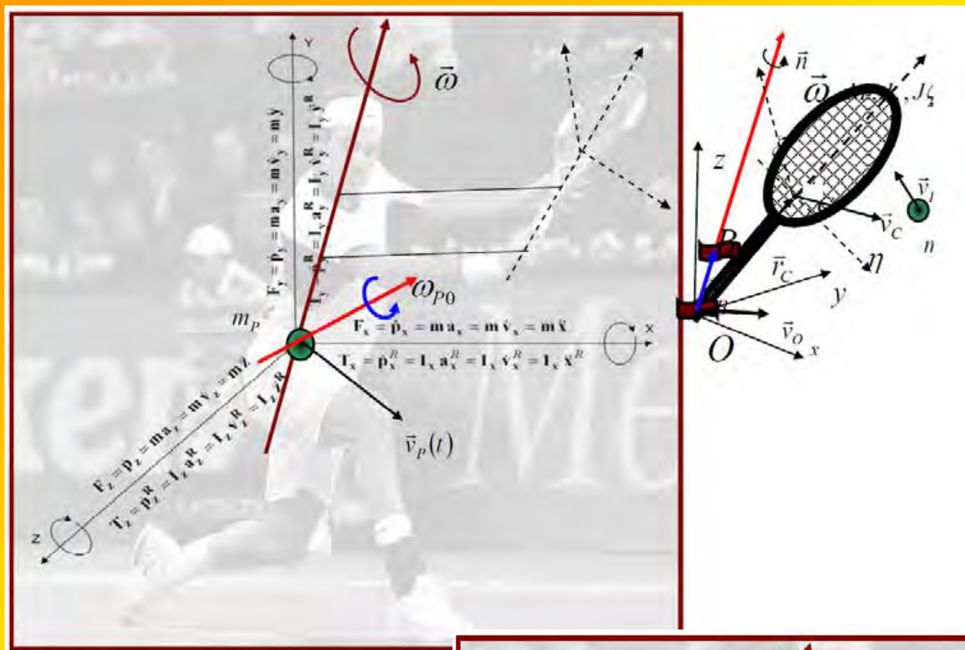
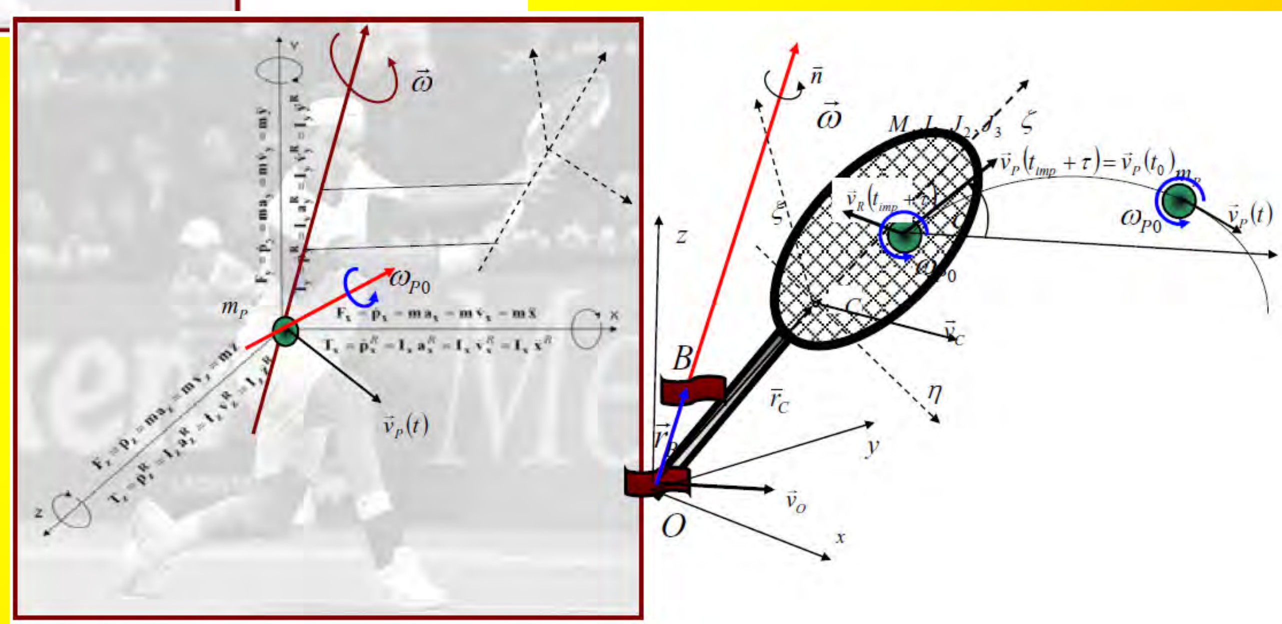
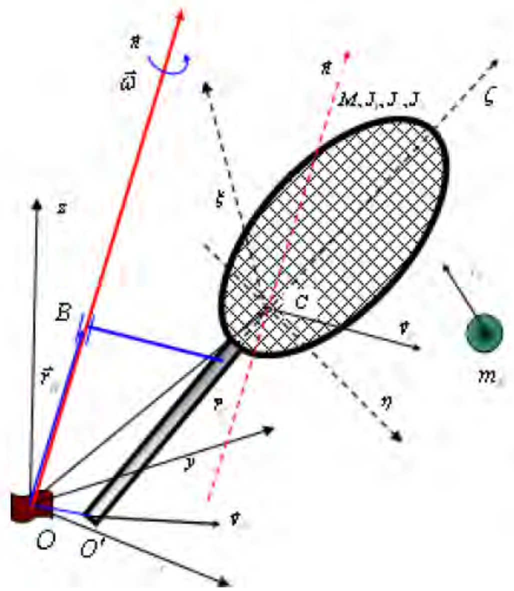
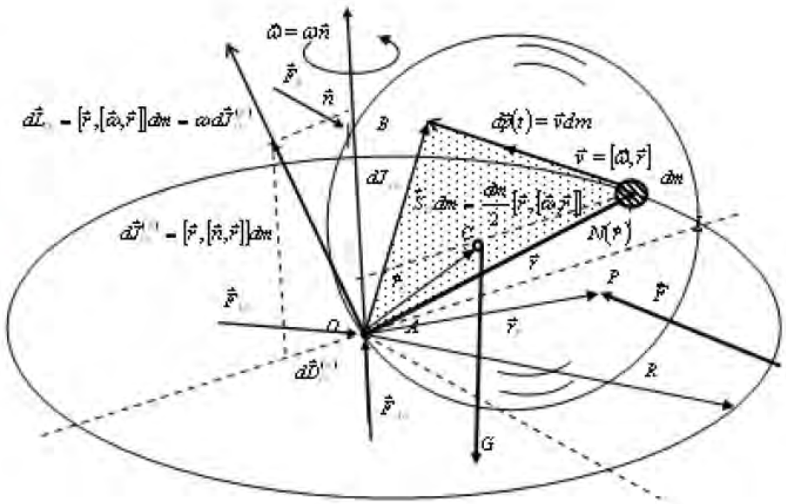
h_0
D_0


$\vec{\omega}$
ω_{P0}
m_P
$\vec{v}_P(t)$
$\vec{n}$
$\vec{v}_C$
$\vec{r}_C$
$\vec{v}_O$
O

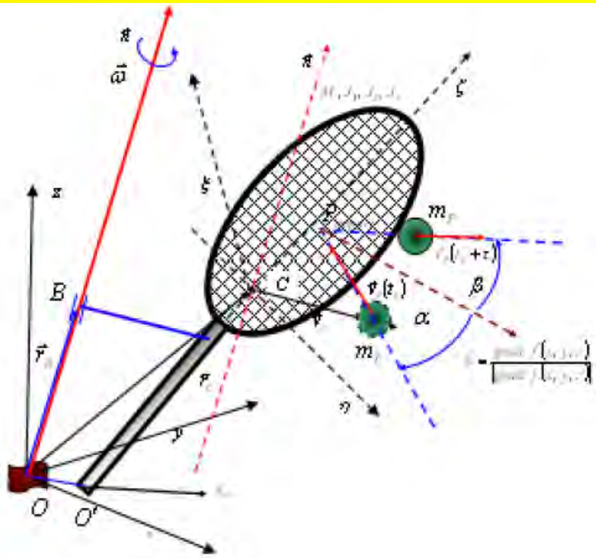

$\vec{\omega}$
ω_{P0}
m_P
$\vec{v}_P(t)$
$\vec{n}$
$\vec{v}_P(t_{imp}+\tau)=\vec{v}_P(t_0)$
$\vec{v}_C$
$\vec{r}_C$
B
$\vec{v}_O$
O



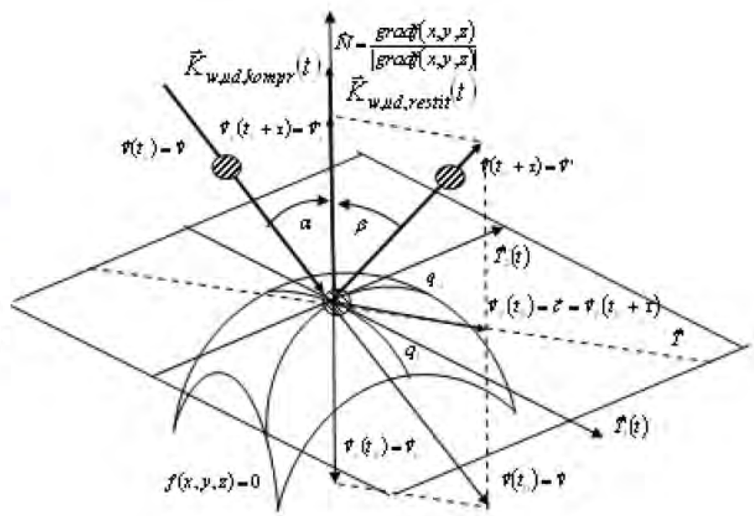
$\vec{\omega}$
$M, J_\xi, J_\eta, J_\zeta$
ζ
ξ
η
z
B
C
m_p
O
O'



$\vec{\omega} = \omega\vec{n}$
$d\vec{L}_O = [\vec{r},[\vec{\omega},\vec{r}]]dm = \omega d\vec{J}_n^{(O)}$
$d\vec{p}(t) = \vec{v}dm$
$\vec{v} = [\vec{\omega},\vec{r}]$
dm
$\vec{S}_r dm = \frac{dm}{2}[\vec{r},[\vec{\omega},\vec{r}]]$
$d\vec{J}_n^{(O)} = [\vec{r},[\vec{n},\vec{r}]]dm$
$N(\vec{r})$
$\vec{n}$
B
O
A
P
$\vec{F}$
R
G

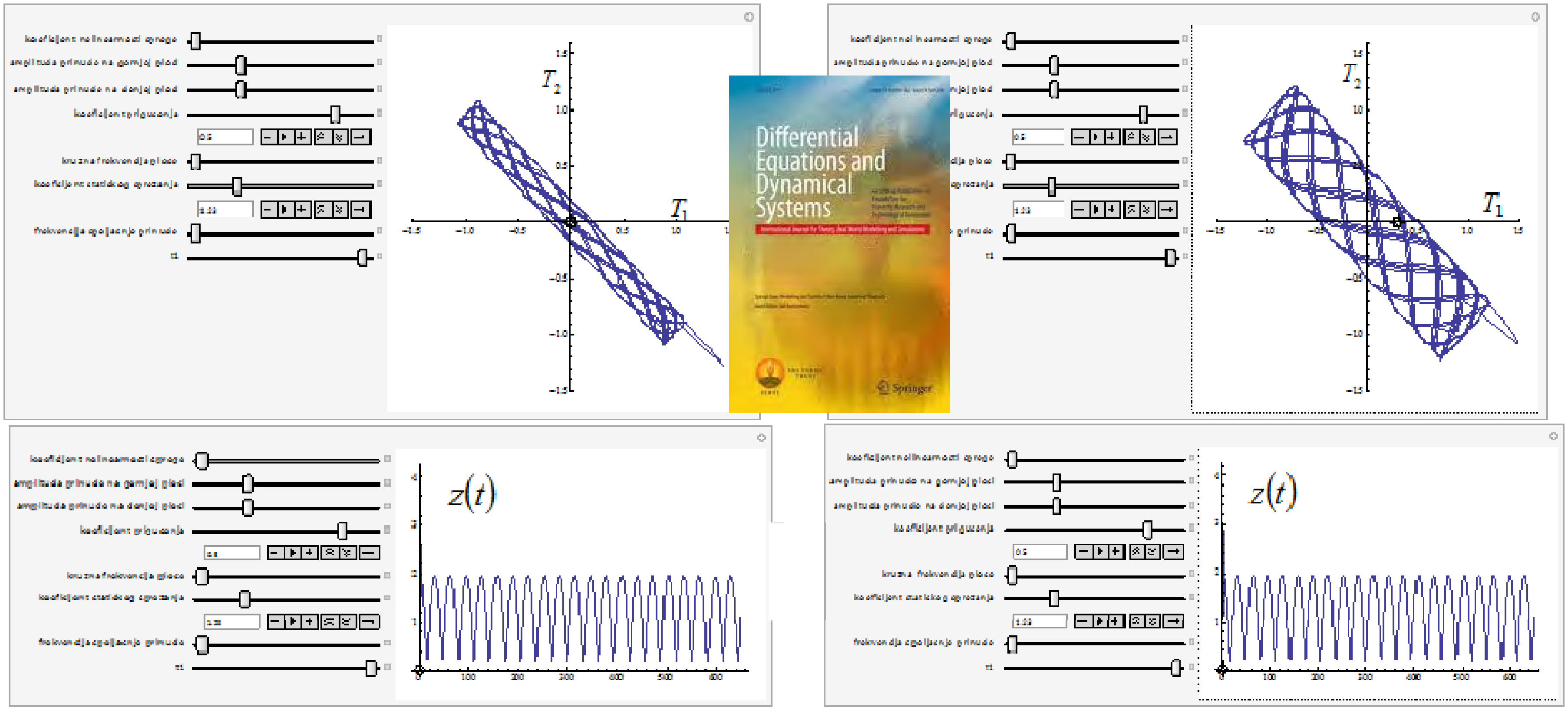


$\vec{\omega}$
ζ
ξ
η
z
B
C
m_p
α
β
O
O'



$\vec{N} = \frac{grad f(x,y,z)}{|grad f(x,y,z)|}$
$\vec{K}_{w,ud,kompr}(t)$
$\vec{K}_{w,ud,restit}(t)$
$\vec{v}(t) = \vec{v}$
$\vec{v}(t+\tau) = \vec{v}'$
α
β
$\vec{T}(t)$
$f(x,y,z) = 0$

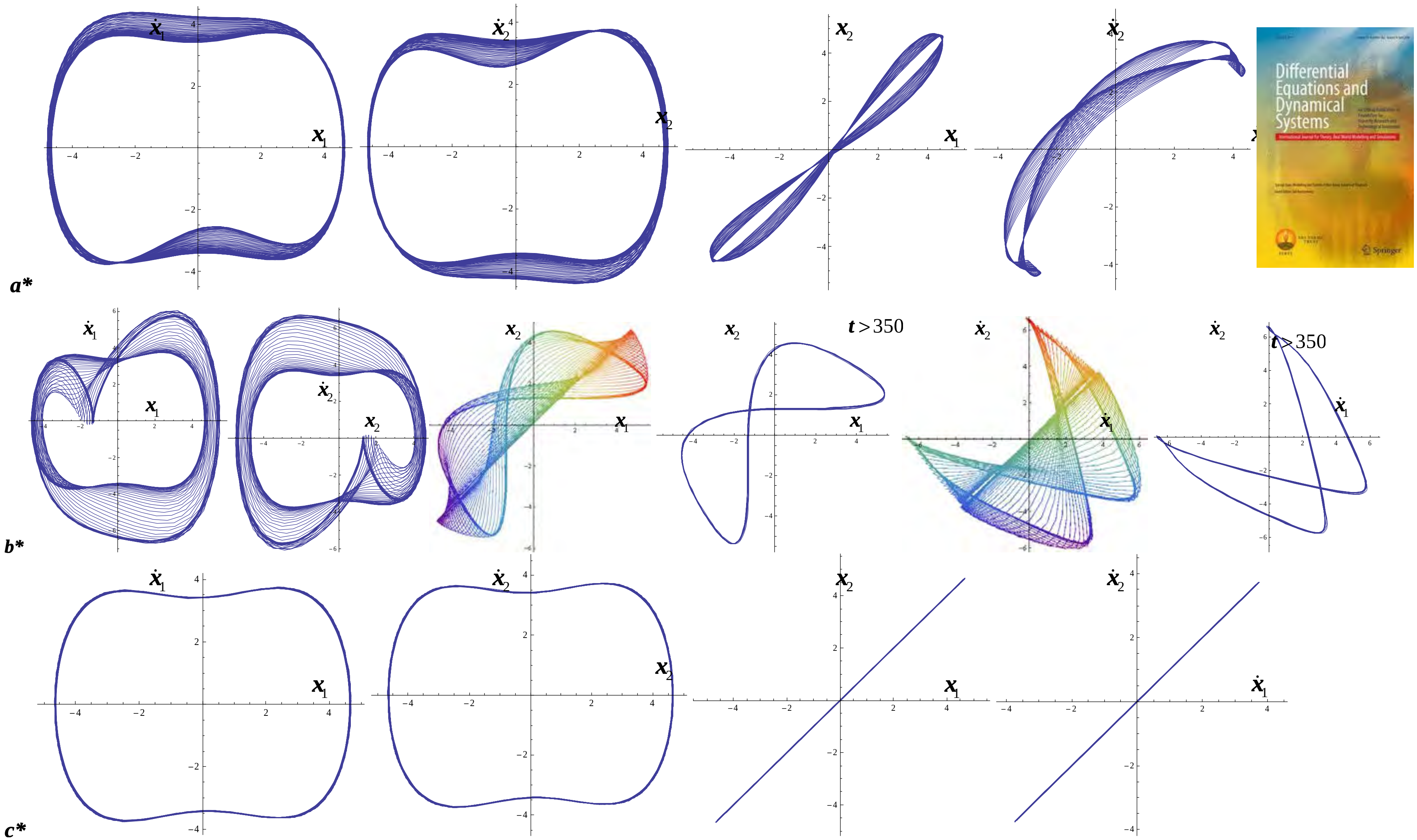
Simonović J., (2013), Synchronization in Coupled Systems with Different Type of Coupling Elements, Differential Equations and Dynamical Systems, Volume 21, Issue 1 (2013), Pp. 141-148, © Springer 2013.



a) b)

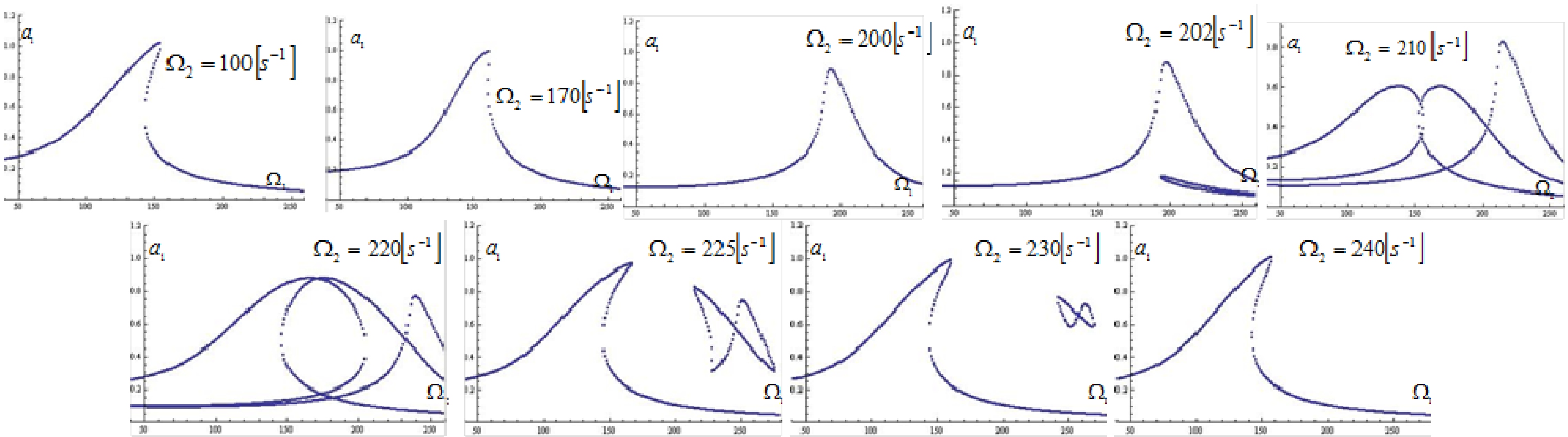
Karakteristični atraktori asinhronizacije u sistemu vremenskih funkcija oscilovanja dve ploče spregnute slojem visko-elastičnih nelinearnih elemenata sa prinudama istih amplituda za različite početne uslove i $a_{(i)}^2 = 1.23$ **i** $\delta_{(1)} = \delta_{(2)} = 0.25$ ***a)***

$T_1(t) = 0$; $T_2(t) = 0.2$; $\dot{T}_1(t) = \dot{T}_2(t) = 0$; ***b)*** $T_1(t) = 0.3$; $T_2(t) = 0.2$; $\dot{T}_1(t) = \dot{T}_2(t) = 0$

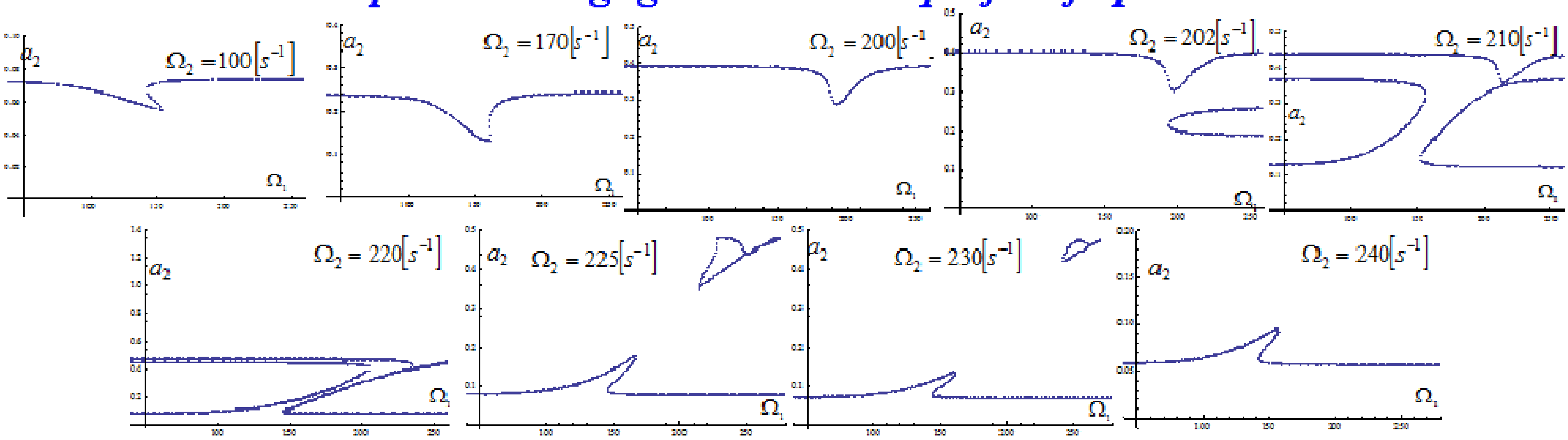


Fazni dijagrami hibridnih nelinearnih podsistema spregnutih statičkim vezama sa spoljašnjom pobudom. a* koeficijent krutosti statičke sprege je $\boldsymbol{a}_1^2 = 0.6$ ***, b**** $\boldsymbol{a}_1^2 = 0.8$ ***i c**** $\boldsymbol{a}_1^2 = 0.87$ ***a u sva tri slučaja početni uslovi su isti***

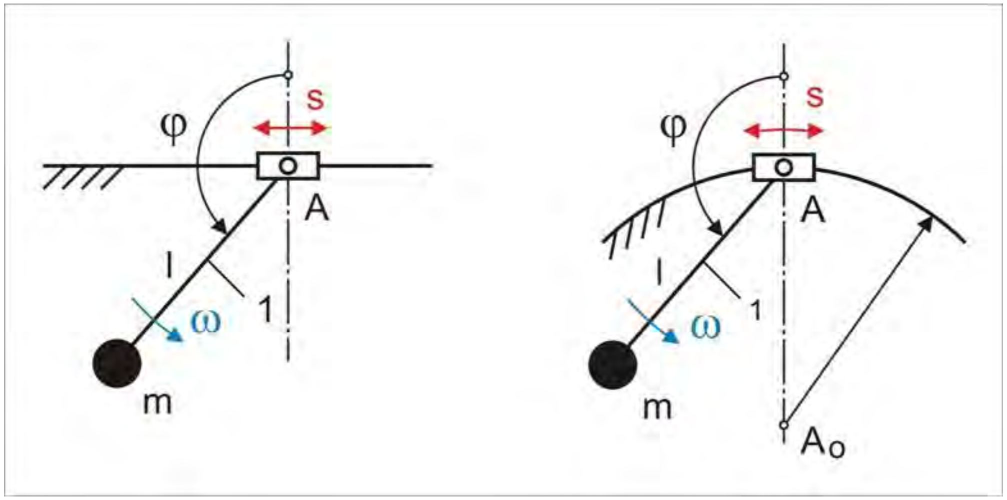
$\boldsymbol{x}_1(0) = 2.49, \dot{\boldsymbol{x}}_1(0) = -0.2$ ***i*** $\boldsymbol{x}_2(0) = 2.5, \dot{\boldsymbol{x}}_2(0) = -0.2$

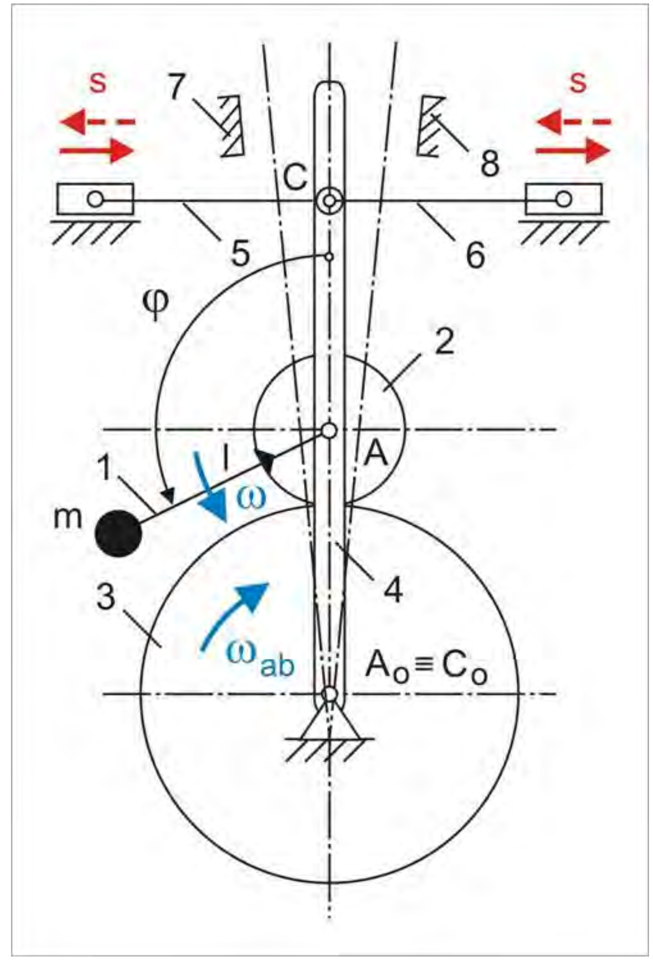


Amplitudno-frekventna kriva prvog harmonika $a_{1nm} = f_1(\Omega_{1nm})$ za fiksirane vrednosti druge kružne frekvencije prinudne sile Ω_{2nm}, i povećana vrednost amplitude drugog harmonika spoljašnje pobude



Amplitudno-frekventna kriva drugog harmonika $a_{2nm} = f_2(\Omega_{1nm})$





Tomislav Petrovic: Na slici 1. su dati analizirani modeli fizičkog **klatna sa pokretnim osloncem .** Klatno dužine l i mase m osciluje pod dejstvom sile u pokretnoj tački oslonca **A** .
Kretanje tačke A može biti pravolinijsko ili kružno .
Analizirajući mogućnosti praktične primene povoljniji je slučaj oscilovanja klatna čiji se pokretni oslonac (tačka A) kreće po kružnoj putanji oko nepokretne tačke Ao. Saglasno tome koncipirana je struktura i način funkcionisanja novog mehanizma za transformaciju oscilatornog u jednosmerno kružno kretanje slika 2.
U dosadašnjem radu su pored napred rečenog koncipirana nova konstrukciona rešenja ovog mehanizma koja bi u praksi mogla imati primenu.

Ostvarene teorijske rezultate nažalost nisam u mogućnasti da proverim eksperimentalnim putem zbog nedostatka finansiskih sredstava za realizaciju dosta složenog i prilično zahtevnog opitnog stola i skupe laboratorijske opreme.

Mathematical Institute SANU

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Phase Trajectory Portrait of the Vibro-impact Forced Dynamics of Two Heavy Mass Particles Motions along Rough Cyrcle

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Serbian Scientific Society

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7th EUROMECH Solid Mechanics Conference ESMC2009

September 7-11, 2009 Instituto Superior Técnico, Lisbon, Portugal

$$\vartheta(0)=\vartheta_0$$

$$\vartheta(0)=\vartheta_0$$

$$\vartheta_0<\Delta$$

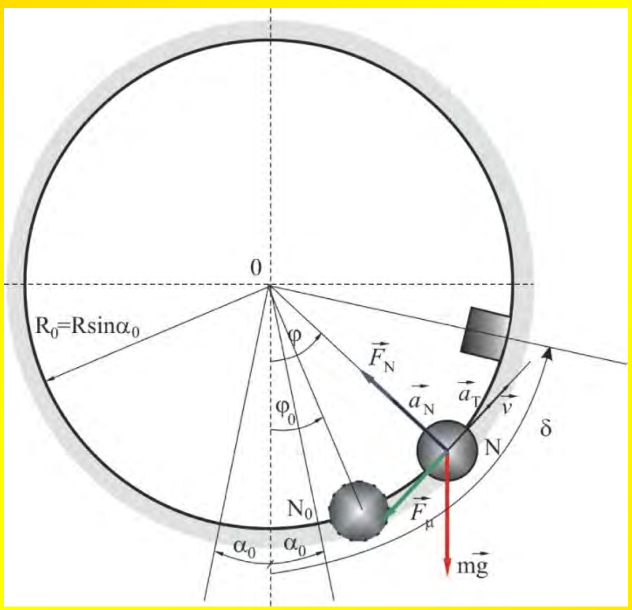
Figure 8. **Heavy material particle oscillations along rough circle with moving limiter in radial direction as a limiter wit double functions both sides impact limitation of the elongation; Positions of the limiter (a*) „on“ and (b*) „off“. Generalized coordinate ϑ and plan of the active and reactive forces; The „double relative“ equilibrium positions with properties of the altenations**

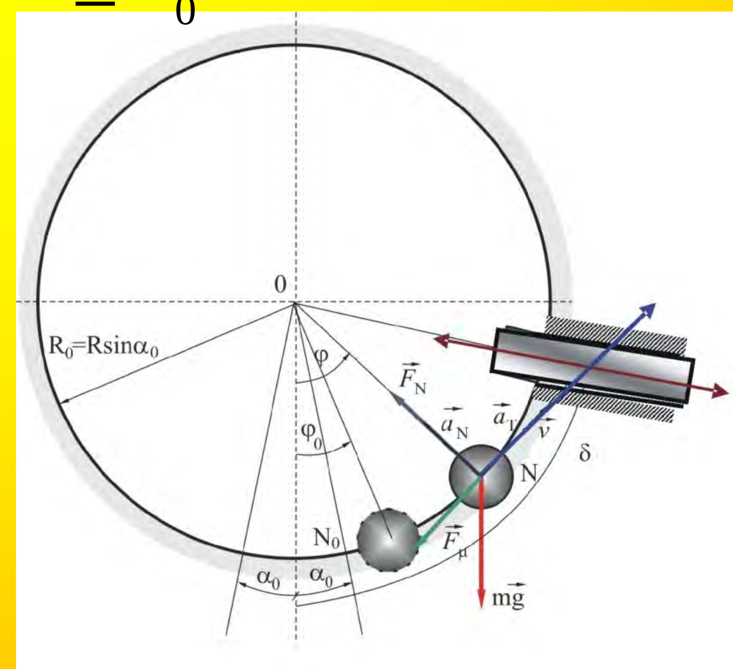
:

$\pm$ $\dot{0}$

$$\sqrt{2\frac{\boldsymbol{g}}{\boldsymbol{R}}(1-\cos\Delta)}<\vartheta_0<\sqrt{4\frac{\boldsymbol{g}}{\boldsymbol{R}}-2\frac{\boldsymbol{g}}{\boldsymbol{R}}(1-\cos\Delta)}$$

,





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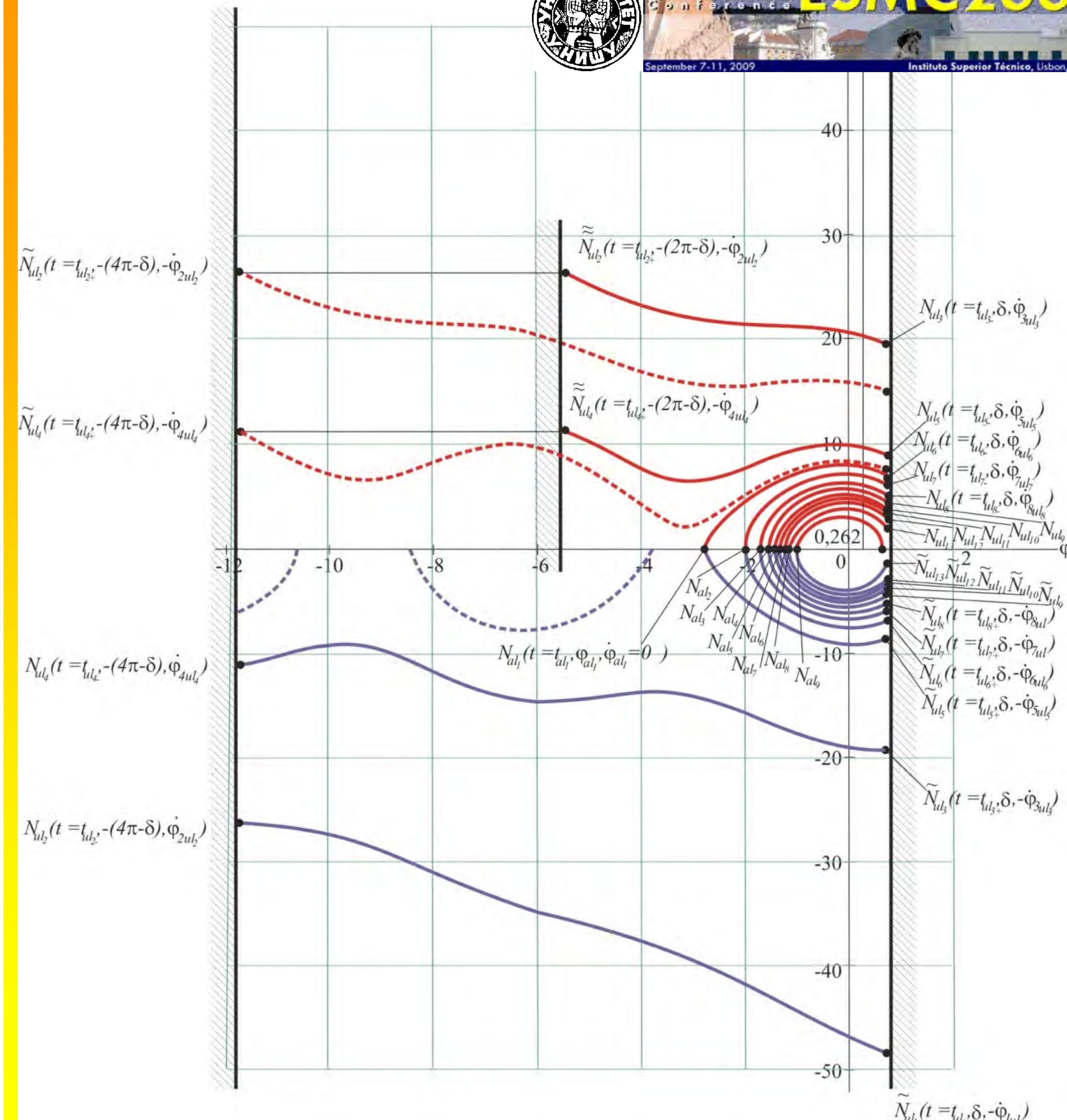
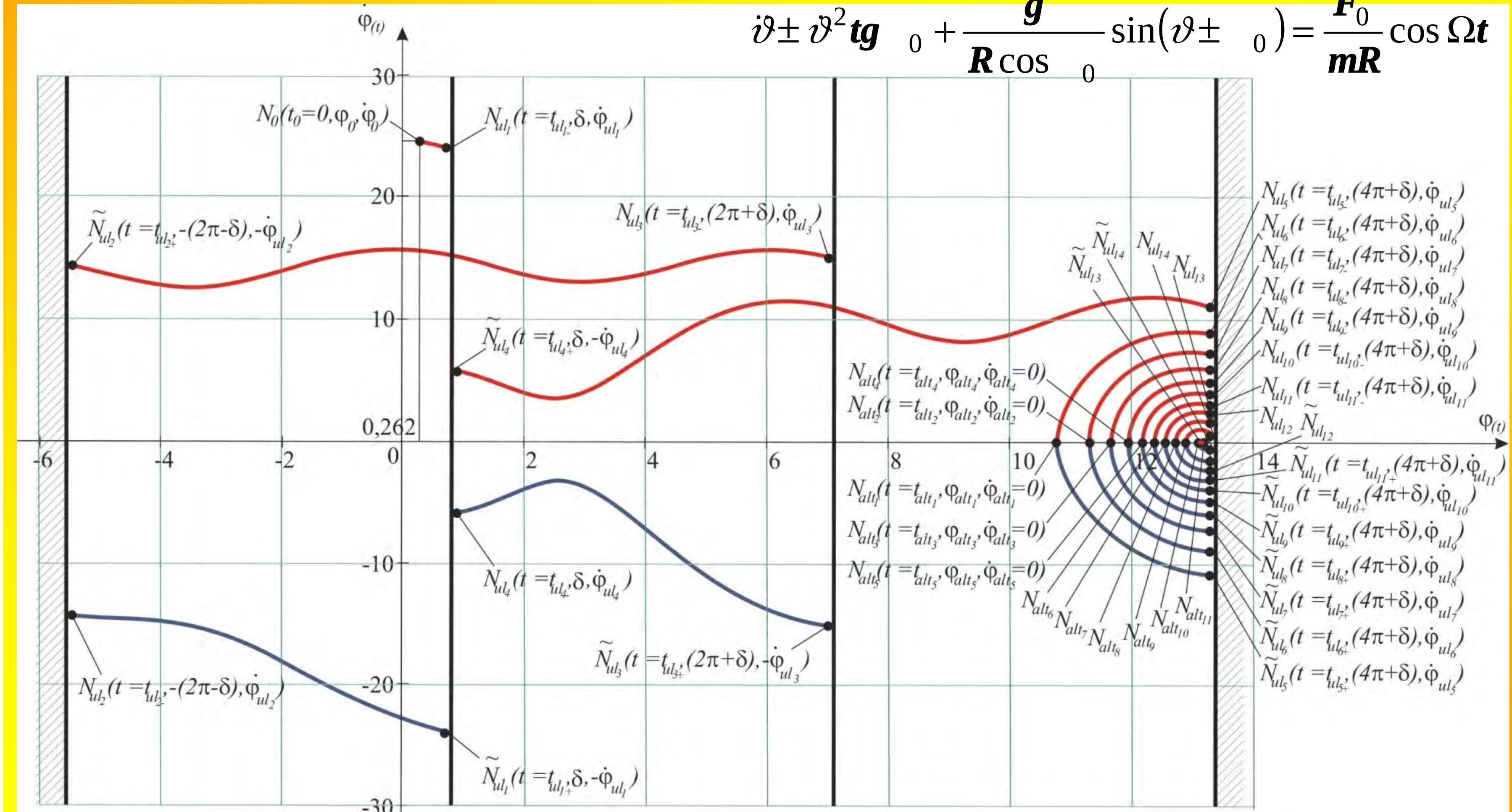


Figure 9. Phase trajectory branches $(\vartheta, \dot{\vartheta})$, of the vibroimpact system on the basis of the heavy material particle motions along rough circle with one radialy moving limiter and with both side limited angular elongation for the case that coefficient oft dray Coulomb's type friction . $= 0,05$.

FORCED VIBROIMPACT SYSTEM DYNAMICS: HEAVY MATERIAL PARTICLE OSCILLATIONS ALONG ROUGH CIRCLE WITH TWO SIDE MOVING IMPACT LIMITERS

Katica R. (Stevanović) Hedrih*, Vladimir Raičević**, Srdjan Jovi ć**

$$\ddot{\vartheta} \pm \dot{\vartheta}^2 \boldsymbol{tg}\;{}_0 + \frac{\boldsymbol{g}}{\boldsymbol{R}\cos\;{}_0}\sin\left(\vartheta \pm\;{}_0\right) = \frac{\boldsymbol{F}_0}{\boldsymbol{mR}}\cos\Omega \boldsymbol{t}$$



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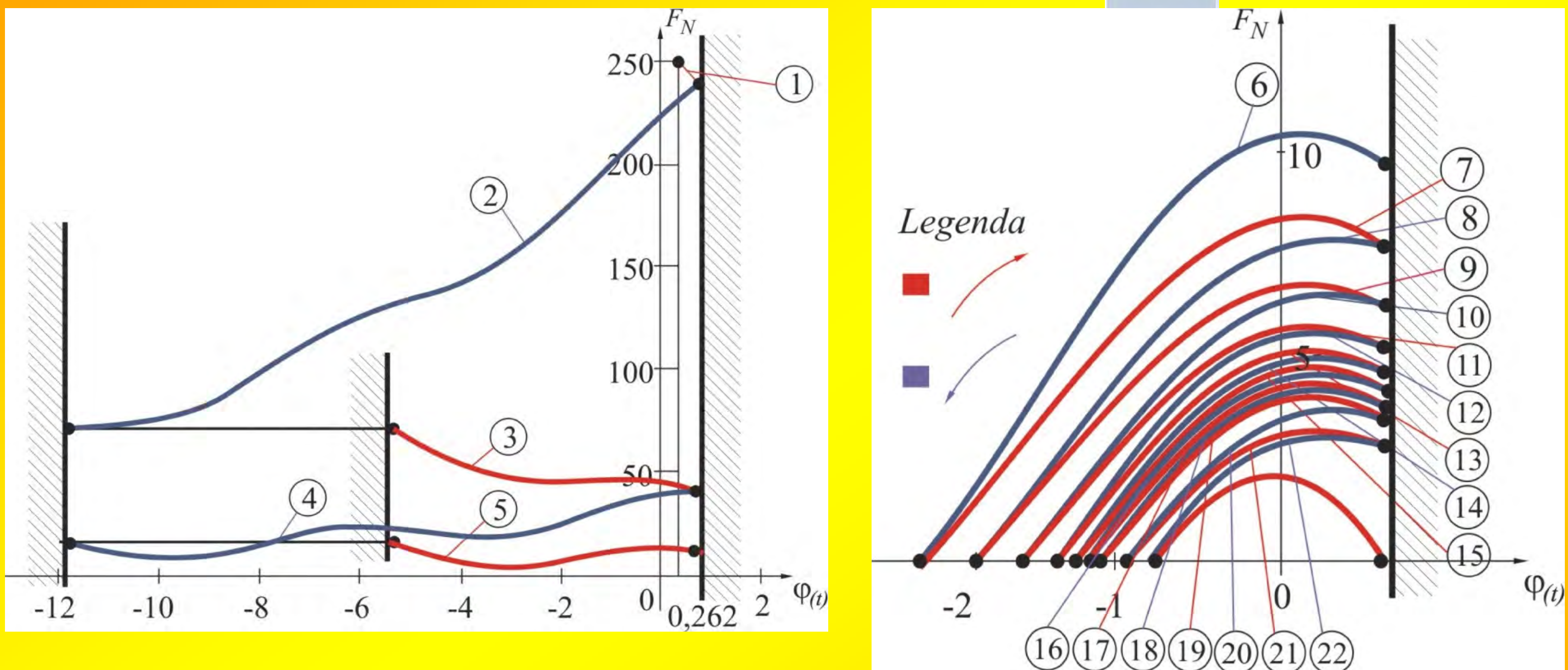


Figure 10. **Graphical presentation of the constraint normal reaction, (F_N, ϑ) , or preassure of the material particle on the rogh circle of the vibroimpact system on the basis of the heavy material particle motions along rough circle with one radialy moving limiter and with both side limited angular elongation for the case that coefficient oft dray Coulomb's type friction $=0,05$. b* is detail of the main graphical presentation in a*.**

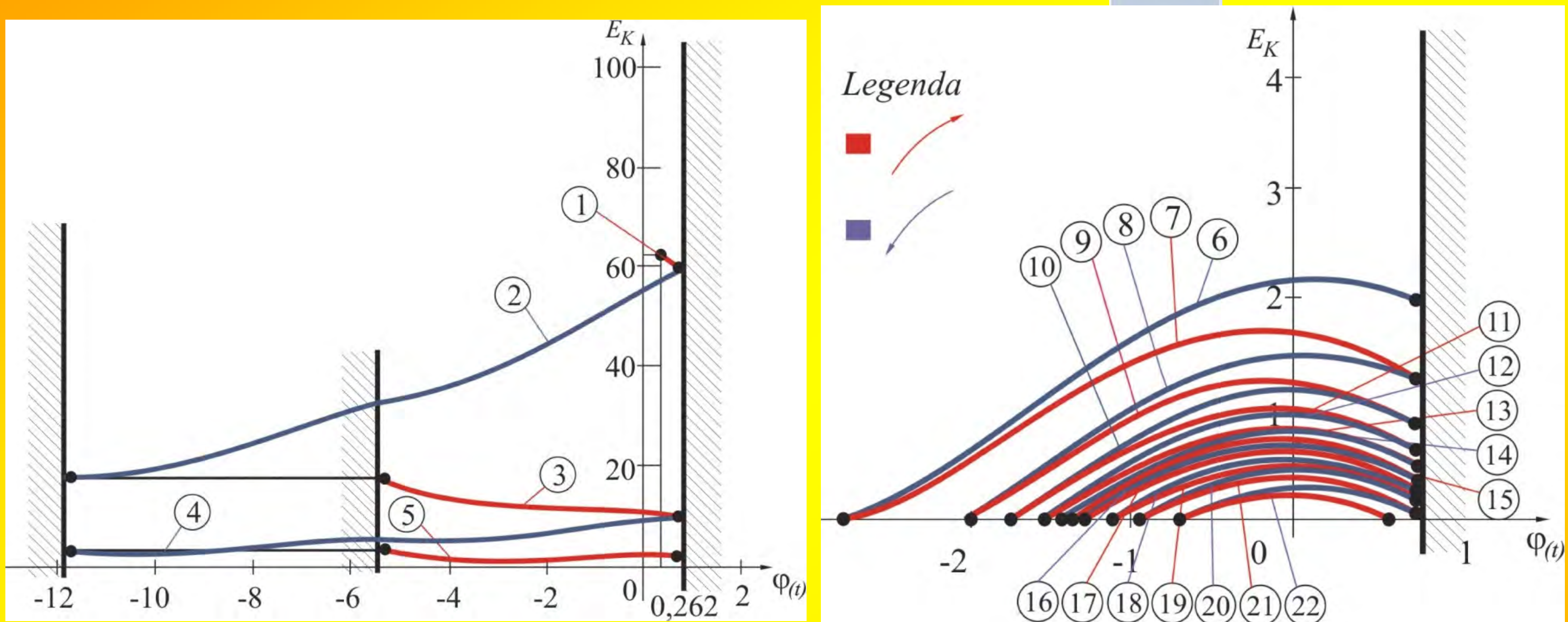


Figure 11. Graphical presentation of the kinetic energy (Ek, ϑ) of the material particle on the rogh circle of the vibroimpact system on the basis of the heavy material particle motions along rough circle with one radialy moving limiter and with both side limited angular elongation for the case that coefficient oft dray Coulomb's type friction $= 0{,}05$. b* is detail of the main graphical presentation in a*.

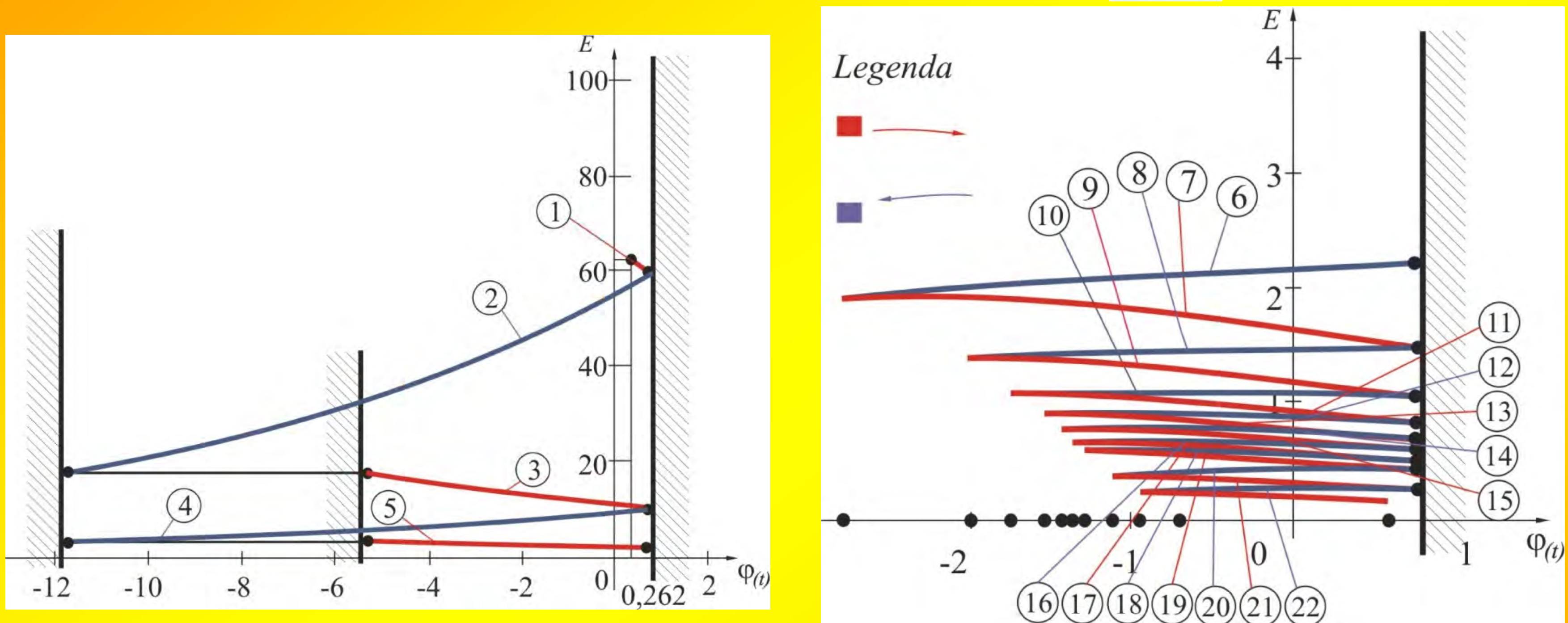


Figure 13. **Graphical presentation of the system total mechanical energy , $\left(E, \vartheta\right)$ of the material particle on the rogh circle of the vibroimpact system on the basis of the heavy material particle motions along rough circle with one radialy moving limiter and with both side limited angular elongation for the case that coefficient oft dray Coulomb's type friction $= 0{,}05$. b* is detail of the main graphical presentation in a*.**

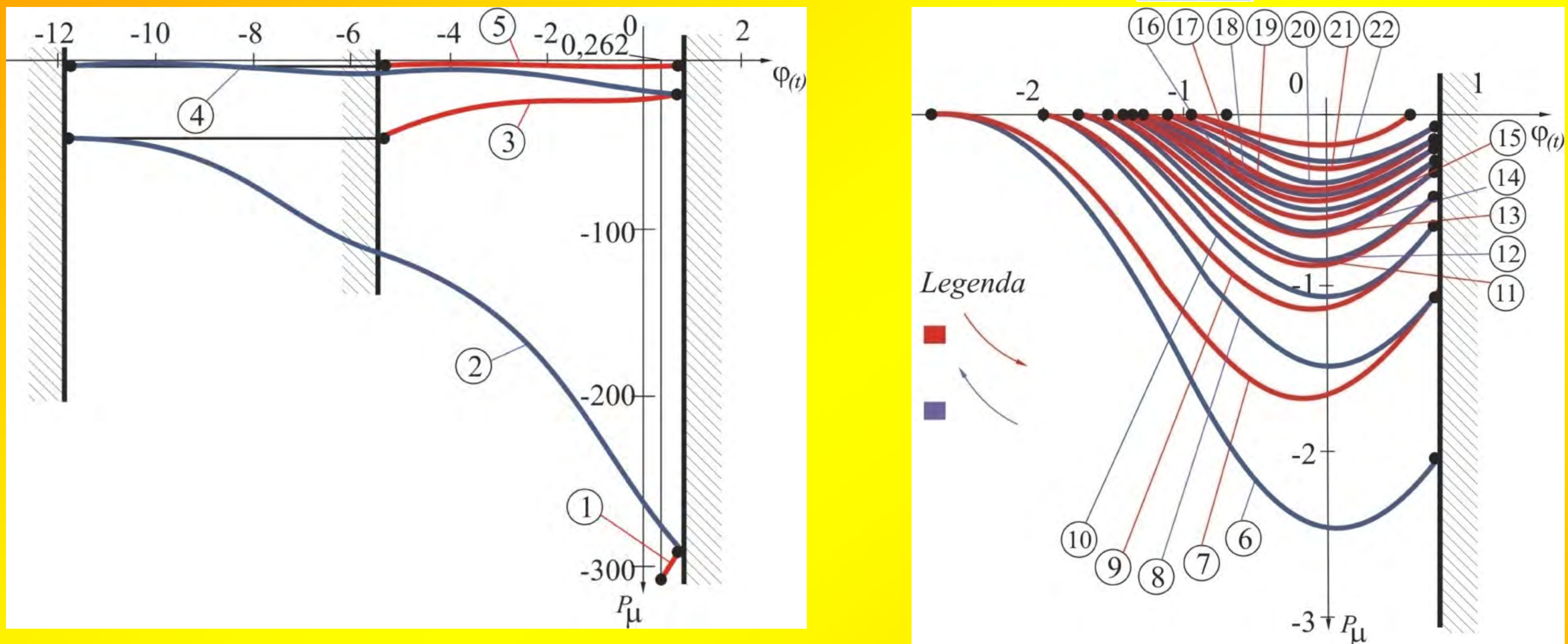
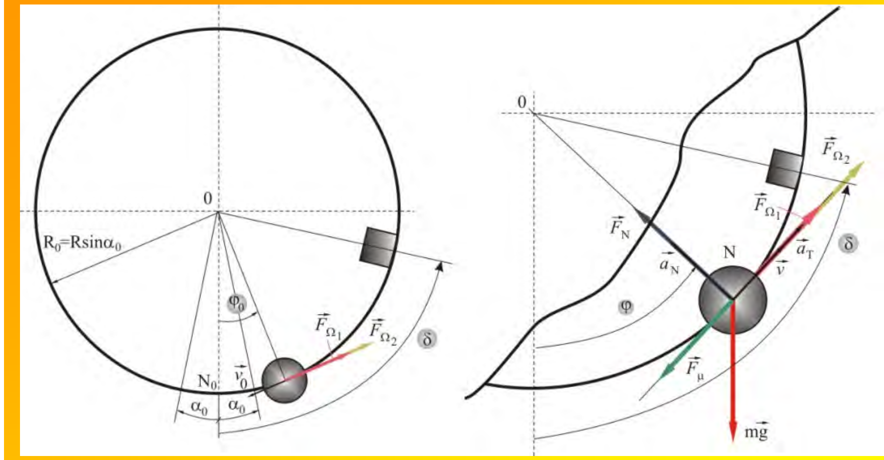
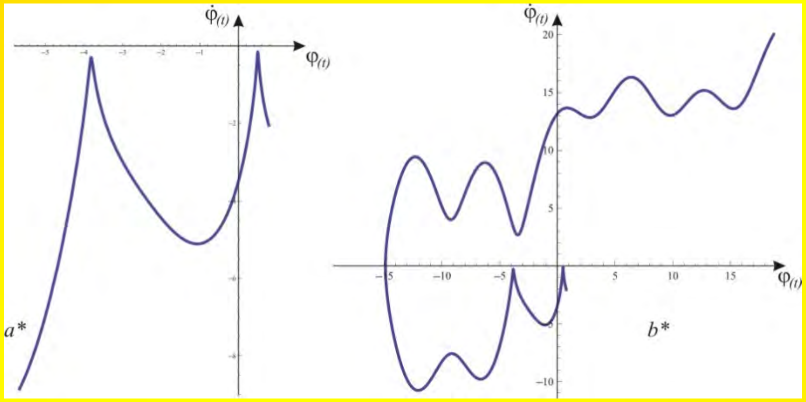
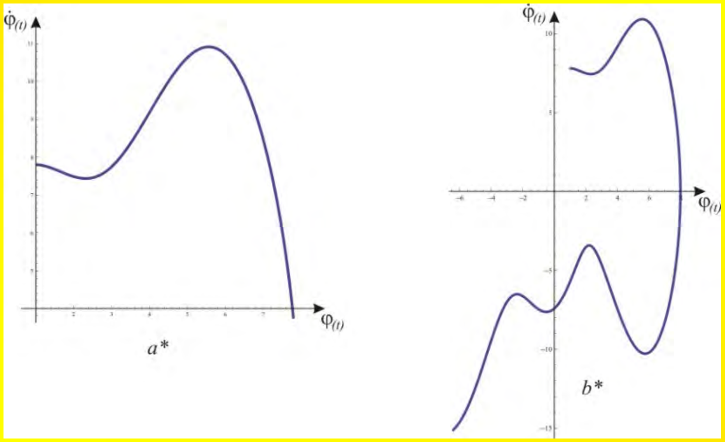


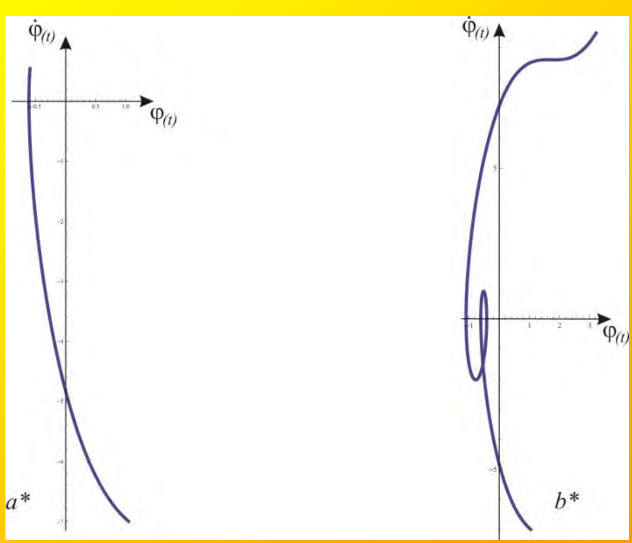
Figure 14. **Graphical presentation in the plane $(\boldsymbol{P}, \vartheta)$ power of the friction force of the material particle moyion reaction on the rogh circle of the vibroimpact system on the basis of the heavy material particle motions along rough circle with one radialy moving limiter and with both side limited angular elongation for the case that coefficient oft dray Coulomb's type friction $= 0{,}05$. b* is detail of the main graphical presentation in a*.**

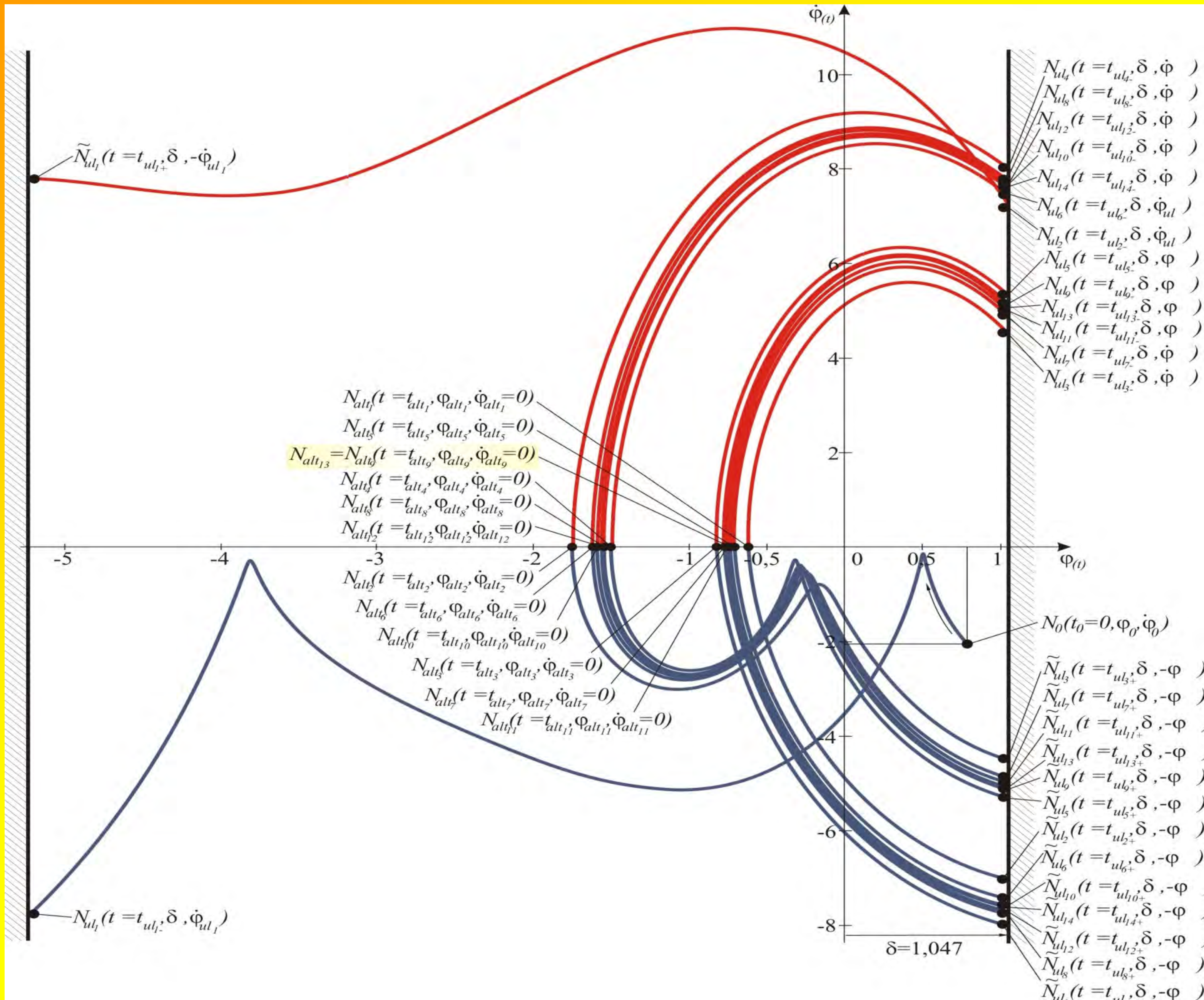




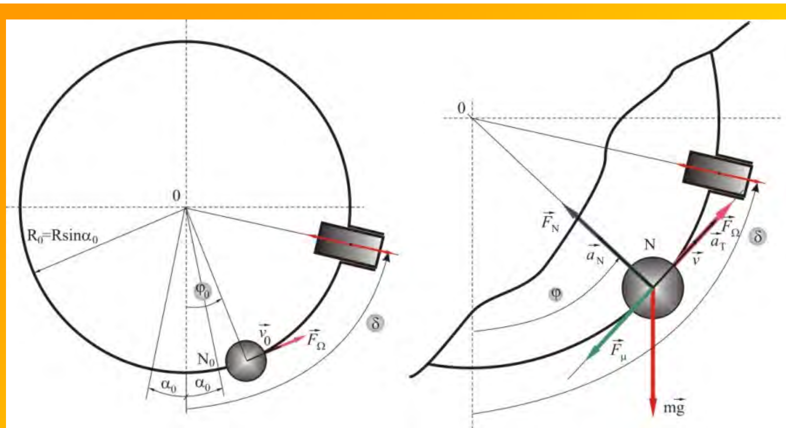
$$\ddot{\vartheta} \pm \vartheta^{2} \boldsymbol{tg} \quad {}_{0} + \frac{\boldsymbol{g}}{\boldsymbol{R}\cos \quad {}_{0}} \sin\left(\vartheta \pm \quad {}_{0}\right) = \frac{1}{\boldsymbol{mR}}\left(\boldsymbol{F}_{10}\cos\Omega_{1}\boldsymbol{t} + \boldsymbol{F}_{20}\cos\Omega_{2}\boldsymbol{t}\right)$$

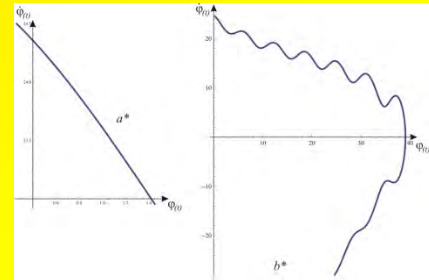


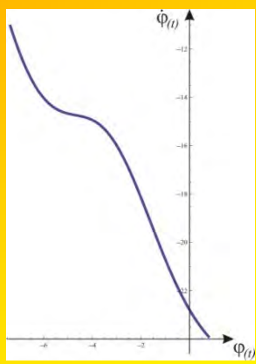




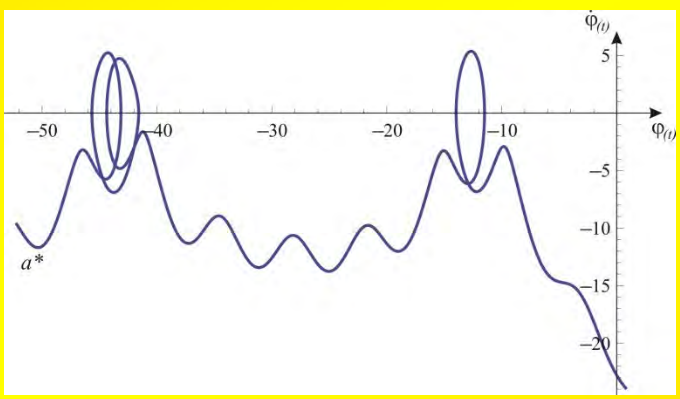
φ̇(t)
φ(t)
10
8
6
4
2
-2
-4
-6
-8
-5
-4
-3
-2
-1
-0,5
0
0,5
1
δ=1,047
Ñul1(t =tul1+,δ ,-φ̇ul1)
Nul1(t =tul1-,δ ,φ̇ul1)
Nul4(t =tul4-,δ ,φ̇)
Nul8(t =tul8-,δ ,φ̇)
Nul12(t =tul12-,δ ,φ̇)
Nul10(t =tul10-,δ ,φ̇)
Nul14(t =tul14-,δ ,φ̇)
Nul6(t =tul6-,δ ,φ̇ul)
Nul2(t =tul2-,δ ,φ̇ul)
Nul5(t =tul5-,δ ,φ)
Nul9(t =tul9-,δ ,φ)
Nul13(t =tul13-,δ ,φ)
Nul11(t =tul11-,δ ,φ)
Nul7(t =tul7-,δ ,φ̇)
Nul3(t =tul3-,δ ,φ̇)
Nalt1(t =talt1,φalt1,φ̇alt1=0)
Nalt5(t =talt5,φalt5,φ̇alt5=0)
Nalt13=Nalt9(t =talt9,φalt9,φ̇alt9=0)
Nalt4(t =talt4,φalt4,φ̇alt4=0)
Nalt8(t =talt8,φalt8,φ̇alt8=0)
Nalt12(t =talt12,φalt12,φ̇alt12=0)
Nalt2(t =talt2,φalt2,φ̇alt2=0)
Nalt6(t =talt6,φalt6,φ̇alt6=0)
Nalt10(t =talt10,φalt10,φ̇alt10=0)
Nalt3(t =talt3,φalt3,φ̇alt3=0)
Nalt7(t =talt7,φalt7,φ̇alt7=0)
Nalt11(t =talt11,φalt11,φ̇alt11=0)
N0(t0=0,φ0,φ̇0)
Ñul3(t =tul3+,δ ,-φ)
Ñul7(t =tul7+,δ ,-φ)
Ñul11(t =tul11+,δ ,-φ)
Ñul13(t =tul13+,δ ,-φ)
Ñul9(t =tul9+,δ ,-φ)
Ñul5(t =tul5+,δ ,-φ)
Ñul2(t =tul2+,δ ,-φ)
Ñul6(t =tul6+,δ ,-φ)
Ñul10(t =tul10+,δ ,-φ)
Ñul14(t =tul14+,δ ,-φ)
Ñul12(t =tul12+,δ ,-φ)
Ñul8(t =tul8+,δ ,-φ)
Ñul4(t =tul4+,δ ,-φ)

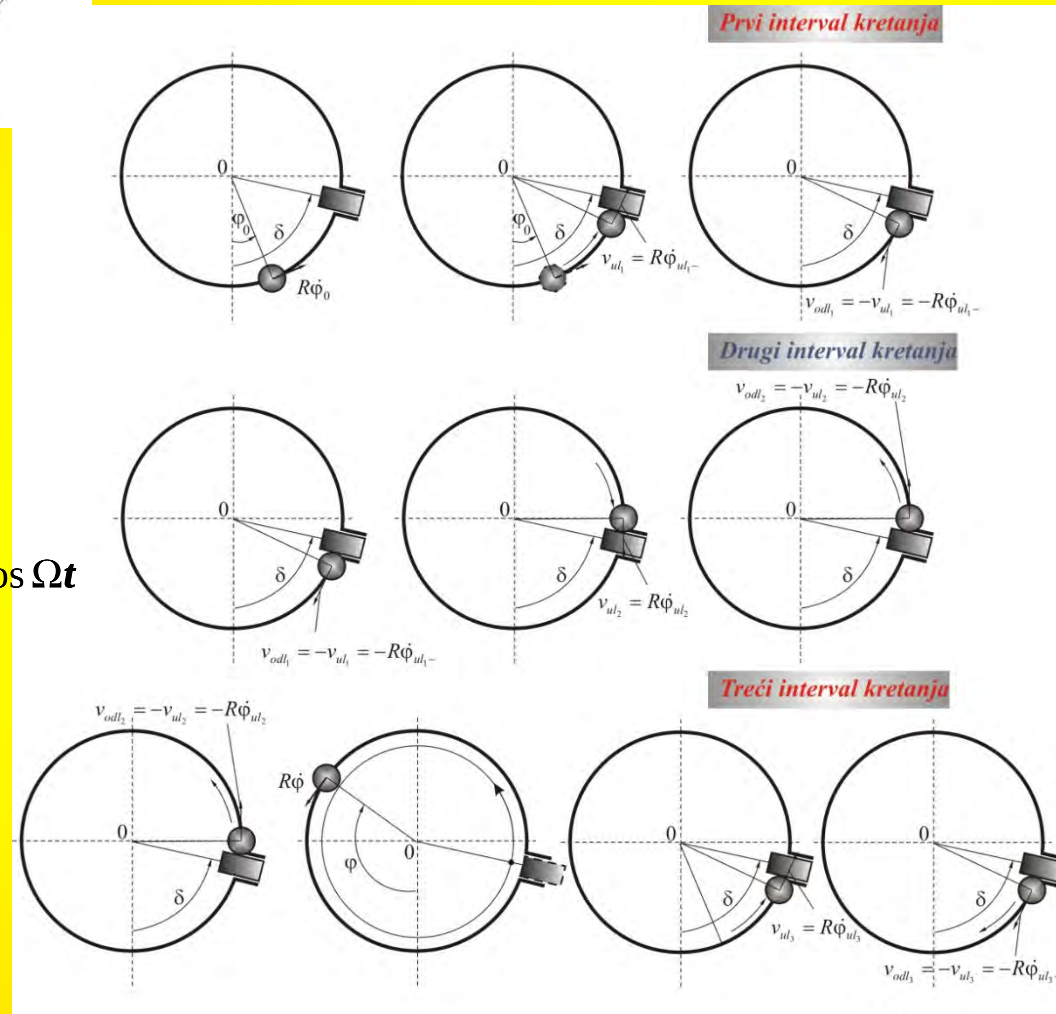


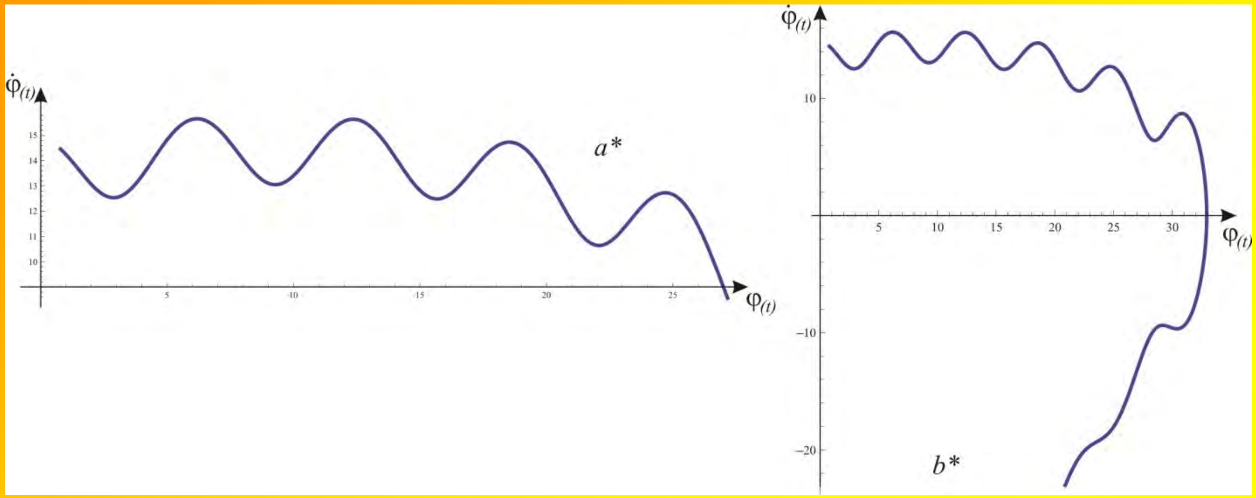




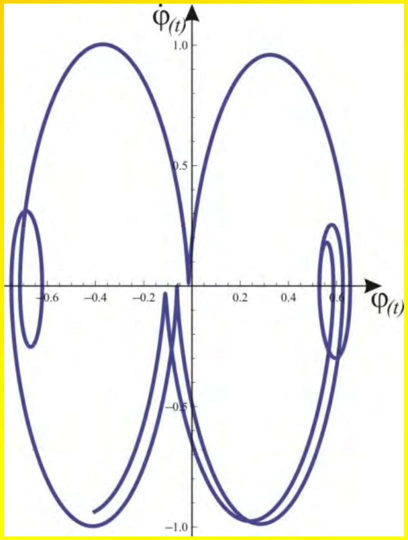
$$\ddot{\vartheta} \pm \dot{\vartheta}^2 \boldsymbol{tg}\ {}_0 + \frac{\boldsymbol{g}}{\boldsymbol{R}\cos\ {}_0}\sin\left(\vartheta \pm\ {}_0\right) = \frac{\boldsymbol{F}_0}{\boldsymbol{mR}}\cos\Omega \boldsymbol{t}$$

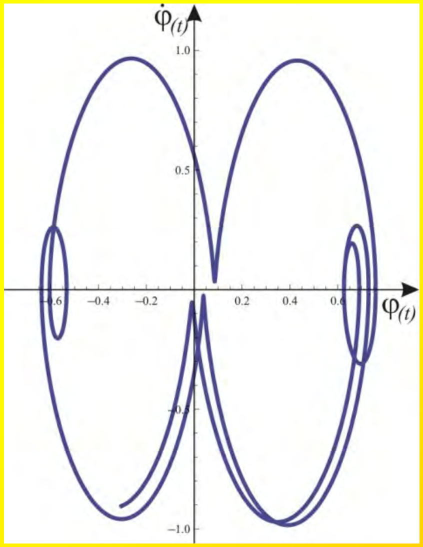






$$\ddot{\vartheta} \pm \vartheta^{2}\boldsymbol{tg}\quad_{0} + \frac{\boldsymbol{g}}{\boldsymbol{R}\cos\quad_{0}}\sin\left(\vartheta \pm\quad_{0}\right) = \frac{\boldsymbol{F}_{0}}{\boldsymbol{mR}}\cos\Omega\boldsymbol{t}$$

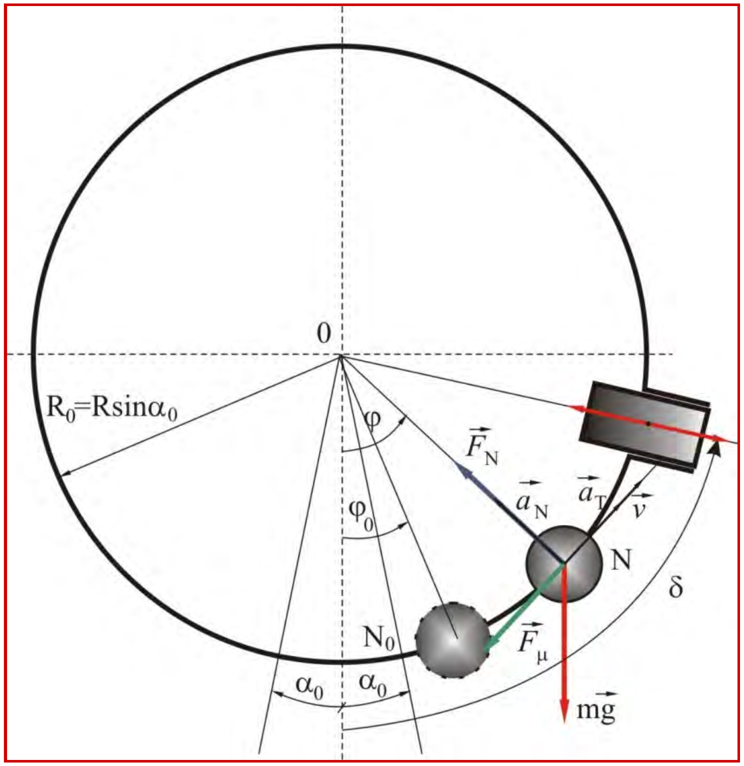




Katica R. (Stevanović) Hedrih and Srdjan V. Jović

VIBROIMPACT SYSTEM DYNAMICS: HEAVY MATERIAL PARTICLE OSCILLATIONS ALONG ROUGH CIRCLE WITH ONE SIDE IMPACT LIMIT

Introduction



Let us, to use principle of dynamical equilibrium and on the basis of this principle, we can write vector equation of the heavy material particle motion along rough line in the following form:

$$\boxed{\vec{I}_F + \vec{G} + \vec{F}_N + \vec{F} = 0}$$

Heavy material particle oscillations along rough circle with one side impact limit elongation

By use natural coordinate system:

$$\boxed{\left(-m\dot{v}\vec{T}\right)+\left(-m\frac{v^2}{R_k}\vec{N}\right)+mg\left(-\sin\vartheta\vec{T}-\cos\vartheta\vec{N}\right)+F_{N.}\vec{N}-\left|\vec{F}_N\right|\frac{\vec{v}}{\left|\vec{v}\right|}=0}$$

where R_k is the radius of the parth line curvature at the point of the material particle terminate position. $\vec{T}$ and $\vec{N}$ are unit vectors of the tangent and normal to the path line at terminate position.

Governing differential equation of the heavy material particle oscillations along rough circle with one side impact limit elongation

Vector equation contains two scalar equations, and after elimination normal and tangential components of the rough circle line reaction, we obtain a ordinary nonlinear differential equation expressed by generalized coordinate ϑ in the following form:

$$\boxed{\vartheta \pm \vartheta^2 \boldsymbol{tg}\;{}_0 + \frac{\boldsymbol{g}}{\boldsymbol{R}\cos\;{}_0}\sin(\vartheta \pm \;{}_0) = 0}$$

in which upper sign is for $\vartheta > 0$ and lower sign for $\vartheta < 0$, according alternations of the friction force alternations, correspond to the opposite direction of the material particle motion. For complete conditions of the material particle motion it is necessary to add ***initial conditions***: $\vartheta(0) = \vartheta_0$ and $\vartheta(0) = \vartheta_0$, as well as ***one side impact limit condition***: $\vartheta(\boldsymbol{t}_{\boldsymbol{ul}.\boldsymbol{i}\mp}) = \Delta$ $\vartheta(\boldsymbol{t}_{\boldsymbol{ul}.\boldsymbol{i}-}) = \vartheta_{\boldsymbol{ul}.\boldsymbol{i}-}$ and $\vartheta(\boldsymbol{t}_{\boldsymbol{ul}.\boldsymbol{i}+}) = -\boldsymbol{k}\vartheta_{\boldsymbol{ul}.\boldsymbol{i}-}$, $\boldsymbol{i} = 1,2,3,\ldots.,\boldsymbol{n}$, where $\boldsymbol{n}$ is total number of the impacts before appear no impact oscillations or state of the rest of the material particle.

Phase trajectory equation of the heavy material particle oscillations along rough circle with one side impact limit elongation

By introducing the following $\vartheta^2 = u$ a transformation of the nonlinear differential equation give a ***first order differential equation with corresponding integral*** in the form:

$$[\vartheta(\vartheta)]^2 = \frac{2\boldsymbol{g}}{(1+4\boldsymbol{tg}^2\ {}_0)\boldsymbol{R}\cos\ {}_0}[\cos(\vartheta \pm\ {}_0) - 2\boldsymbol{tg}\ {}_0 \sin(\vartheta \pm\ {}_0)] + \boldsymbol{C}e^{\mp 2\vartheta \boldsymbol{tg}\ {}_0}$$

where $\boldsymbol{C}$ is integral constant depending of initial conditions for the corresponding interval of the material particle motion and in which upper sign is for $\vartheta > 0$ and lower sign for $\vartheta < 0$, according alternations of the friction force alternations. This previous equation is equation of the phase trajectory in the phase plane (ϑ, ϑ) containing representative phase point $\boldsymbol{N}(\boldsymbol{t}, \vartheta, \vartheta)$ presenting kinetic state of the material particle oscillations. For first phase trajectory branch, from initial kinetic state to the first impact moment, we use equation with upper sign, for the case $\vartheta > 0$:

$$\vartheta_1^2(\vartheta) = \frac{2\boldsymbol{g}}{(1+4\boldsymbol{tg}^2\ {}_0)\boldsymbol{R}\cos\ {}_0}[\cos(\vartheta +\ {}_0) - 2\boldsymbol{tg}\ {}_0 \sin(\vartheta +\ {}_0)] + \boldsymbol{C}_1(\vartheta_0, \vartheta_0)e^{-2\vartheta \boldsymbol{tg}\ {}_0}$$

Phase trajectory equation of the heavy material particle oscillations along rough circle with one side impact limit elongation

where $\boldsymbol{C}_1(\vartheta_0, \vartheta_0)$ integral constant defined by the following expression:

$$\boldsymbol{C}_1(\vartheta_0, \vartheta_0) = \boldsymbol{e}^{+2\vartheta_0 \boldsymbol{tg}\ _0} \left\{ \vartheta_{\boldsymbol{o}}^2 - \frac{2\boldsymbol{g}}{\left(1 + 4\boldsymbol{tg}^2\ _0\right)\boldsymbol{R}\cos\ _0}\left[\cos(\vartheta_0 + \ _0) - 2\boldsymbol{tg}\ _0 \sin(\vartheta_0 + \ _0)\right]\right\}$$

Firest impact appear at the moment: $\boldsymbol{t} = \boldsymbol{t}_{\boldsymbol{ul},1-}$, corresponding to one side impact limit $\vartheta(\boldsymbol{t}_{\boldsymbol{ul},1-}) = \Delta$ and corresponding impact angular velocity:

$$\vartheta(\boldsymbol{t}_{\boldsymbol{ul},1-}) = \vartheta_{\boldsymbol{ul},1-}$$

$$\vartheta_{\boldsymbol{ul},1} = \sqrt{\frac{2\boldsymbol{g}}{\left(1 + 4\boldsymbol{tg}^2\ _0\right)\boldsymbol{R}\cos\ _0}\left[\cos(\Delta + \ _0) - 2\boldsymbol{tg}\ _0 \sin(\Delta + \ _0)\right] + \boldsymbol{C}_1(\vartheta_0, \vartheta_0)\boldsymbol{e}^{-2\Delta \boldsymbol{tg}\ _0}}$$

at the moment $\boldsymbol{t} = \boldsymbol{t}_{\boldsymbol{ul},1-}$:

$$\boldsymbol{t}_{\boldsymbol{ul}_1} = \int_{\vartheta_0}^{\Delta} \frac{\boldsymbol{d}\vartheta}{\sqrt{\frac{2\boldsymbol{g}}{\left(1 + 4\boldsymbol{tg}^2\ _0\right)\boldsymbol{R}\cos\ _0}\left[\cos(\vartheta + \ _0) - 2\boldsymbol{tg}\ _0 \sin(\vartheta + \ _0)\right] + \boldsymbol{C}_1(\vartheta_0, \vartheta_0)\boldsymbol{e}^{-2\vartheta \boldsymbol{tg}\ _0}}}$$

Phase trajectory equation of the heavy material particle oscillations along rough circle with one side impact limit elongation

The normal constraint reaction $\boldsymbol{F_N}\left(\vartheta\right)$ in the function of the generalozed coordinate ϑ , as a normal preassure of the heavy material particle to the rough circle line in the frst time interval motion, before first impact, and corresponding for the force of friction $\boldsymbol{F}\ \left(\vartheta\right) = -\ \ \boldsymbol{F_N}\left(\vartheta\right)$ is defined by following expression:

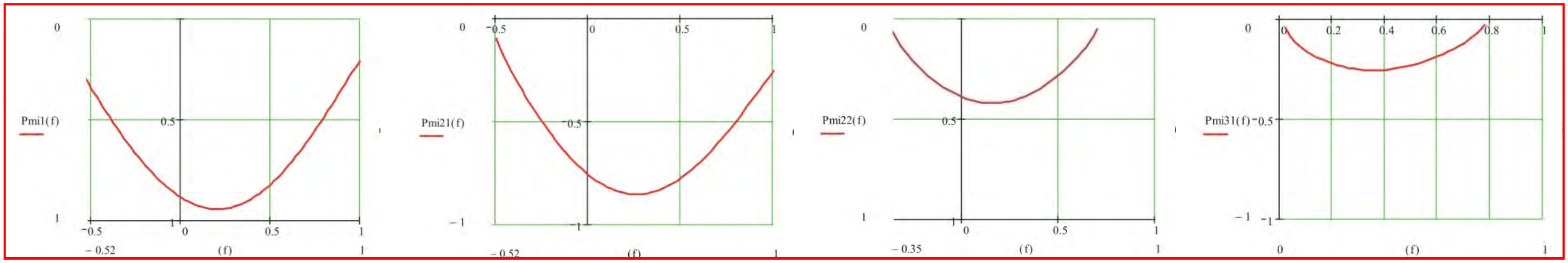
$$\boxed{\boldsymbol{F_N} = \boldsymbol{mg}\cos\vartheta + \boldsymbol{mR}\left(\frac{2\boldsymbol{g}}{\left(1+4\boldsymbol{tg}^2\ {}_0\right)\boldsymbol{R}\cos\ {}_0}\left[\cos\left(\vartheta+\ {}_0\right) - 2\boldsymbol{tg}\ {}_0\sin\left(\vartheta+\ {}_0\right)\right] + \boldsymbol{C}_1(\vartheta_0,\vartheta_0)\boldsymbol{e}^{-2\vartheta\boldsymbol{tg}\ {}_0}\right)}$$

Second interval of the material particle motion after first impact, and between first and second impacts, we splite to the two subinterval First subinterval is between first impact and first state of the alternation of the friction force, and second subinterval is between first state of the alternation of the friction force, and second impact.

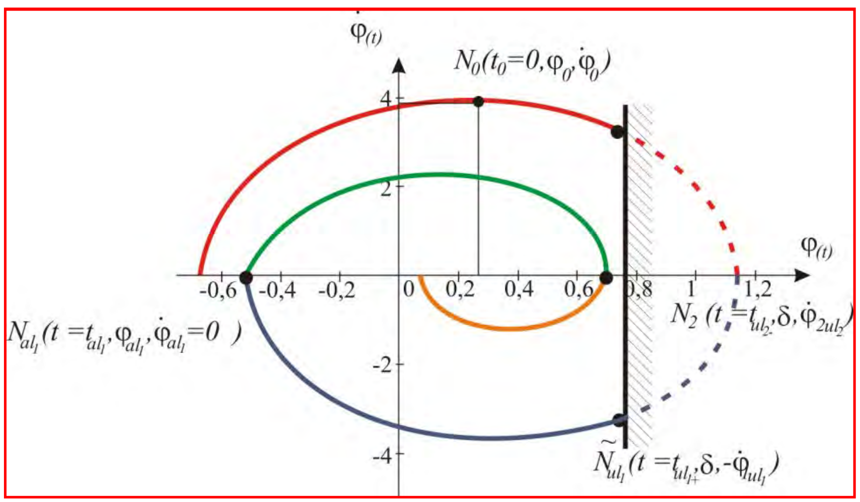
For second phase trajectory branch in the first subinterval, from first impact kinetic state to the first alternation of the direction of the friction force, we use equation with lower sign, for the case $\boxed{\vartheta < 0}$:

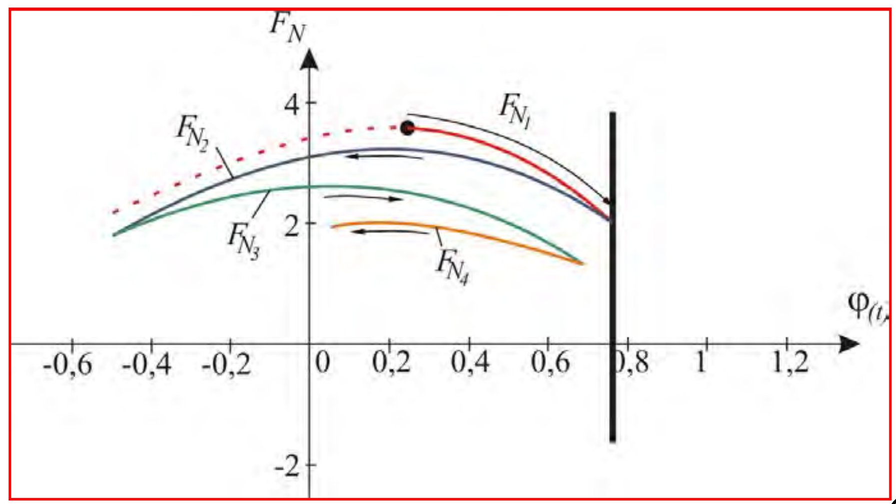
Phase trajectory equation of the heavy material particle oscillations along rough circle with one side impact limit elongation

Graphical presentation



Power of the work of the friction force between impact and alternations of the friction force directions





An example of the phase trajectory

The normal constraint reaction $F_N(\vartheta)$

Numerical example and visualization

By use ***MathCad program*** for the geometrical and kinetic parameters of the material particle motion along rough circle with one side impact limit elongation: $\boxed{\boldsymbol{m} = 0{,}2[\boldsymbol{kg}]}$, $\boxed{\boldsymbol{R} = 0{,}5[\boldsymbol{m}]}$, $\boxed{\quad = \boldsymbol{tg}\ \ _0 = 0.05}$ (dimension less),

$$\boxed{\Delta = \frac{\Gamma}{4}[\boldsymbol{rad}]},\ \boxed{\vartheta_0 = \frac{\Gamma}{12}[\boldsymbol{rad}]},\ \boxed{\dot{\vartheta}_0 = 7\left[\frac{\boldsymbol{rad}}{\boldsymbol{s}}\right]},\ \boxed{\boldsymbol{g} = 9{,}81\left[\frac{\boldsymbol{m}}{\boldsymbol{s}^2}\right]},\ \boxed{\boldsymbol{k} = 1},$$

we obtain a phase trajectory presentation of the impact system dynamics with initial conditions determining ten impacts. ***Thus phase trajectory is presented in Figure 1***.

After ten impacts osilations of the materijal particle is without impacts. For same example a graphical presentation of the ***normal constraint reaction in the functions of tis presented in Figure 2***.

Numerical example and visualization

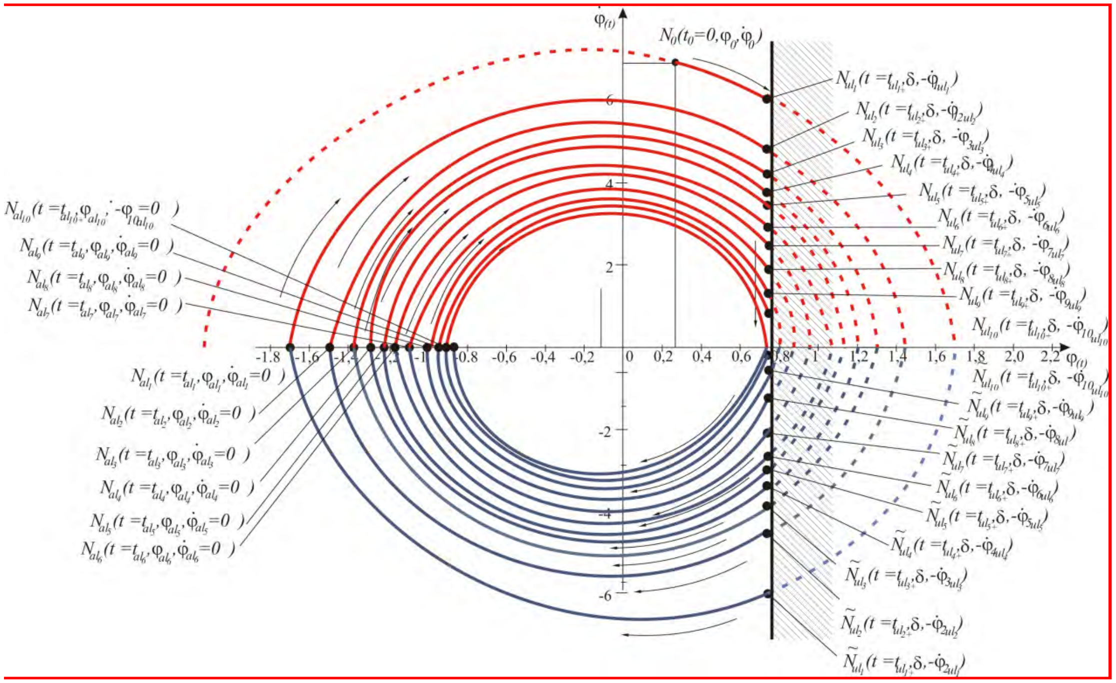


Figure 1. An example of the phase trajectory of the vibroimpact system with initial conditions determining ten impacts

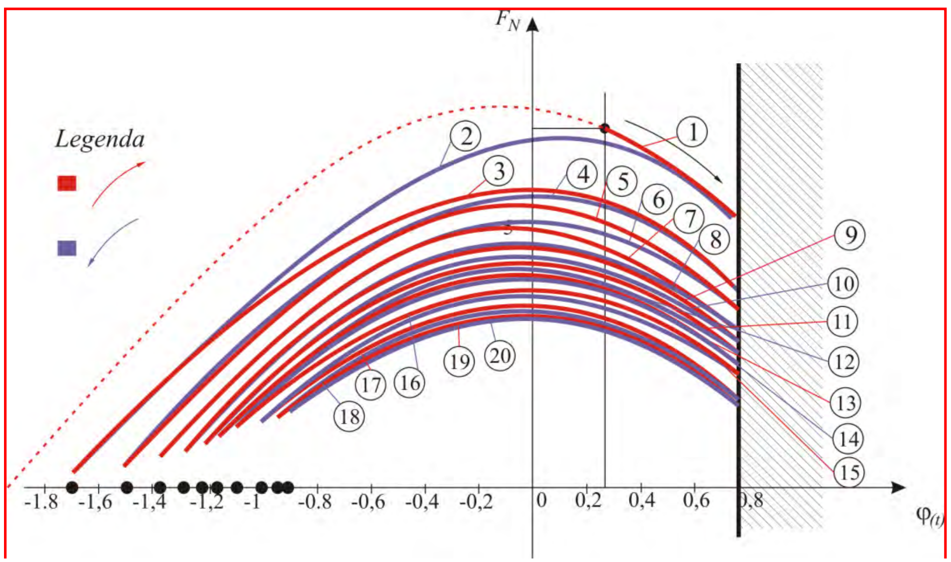
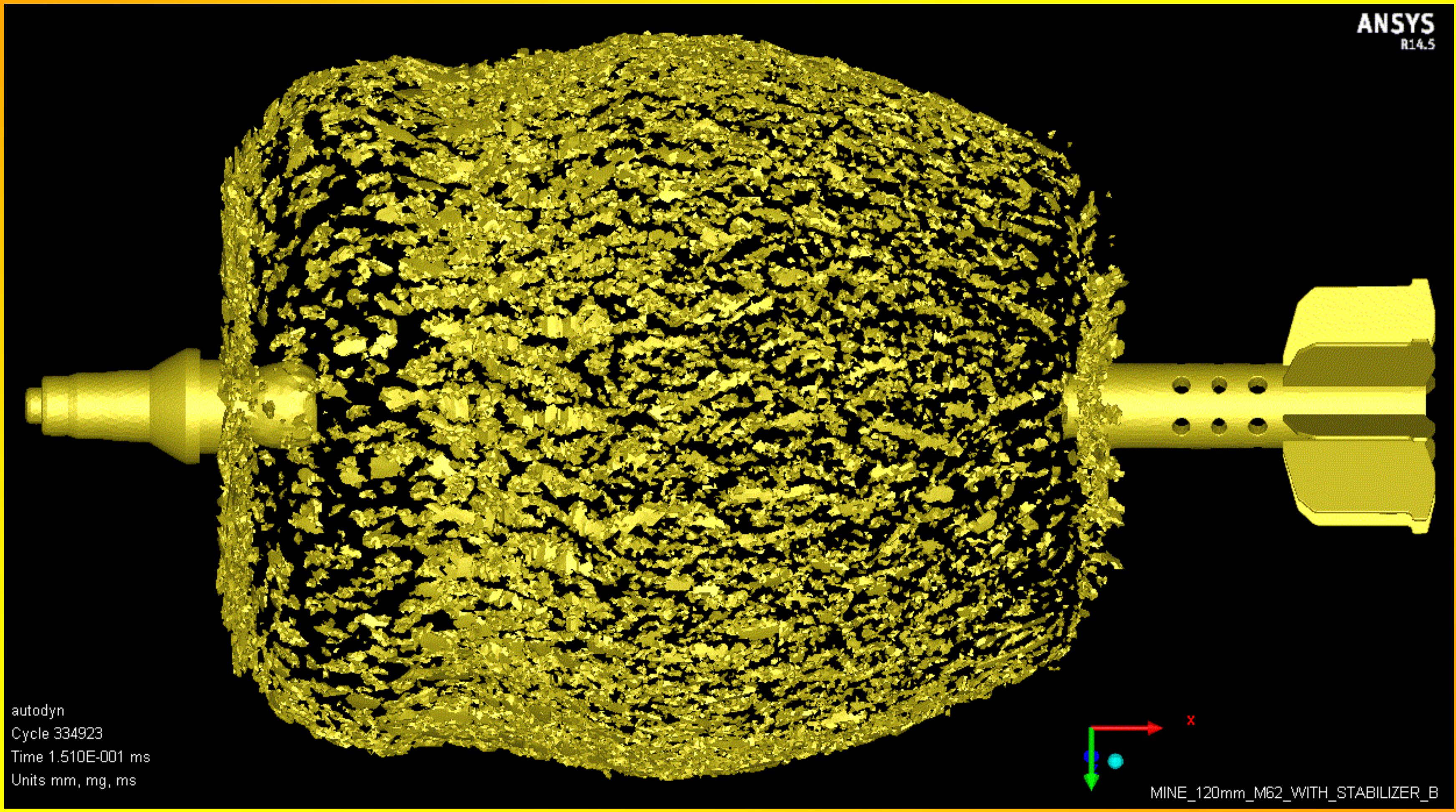
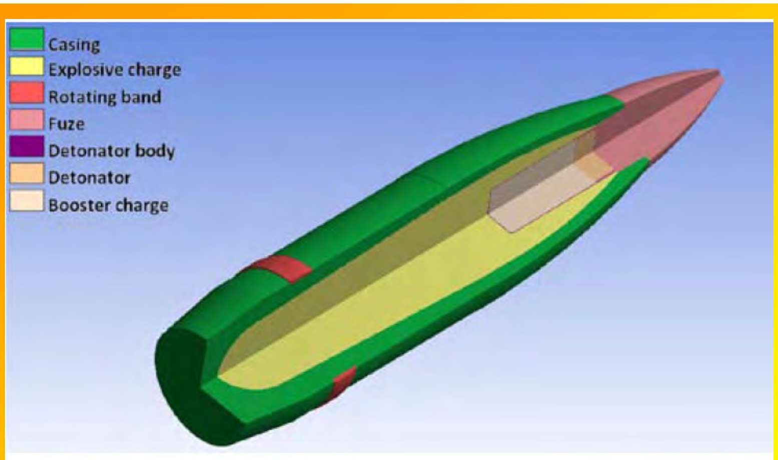
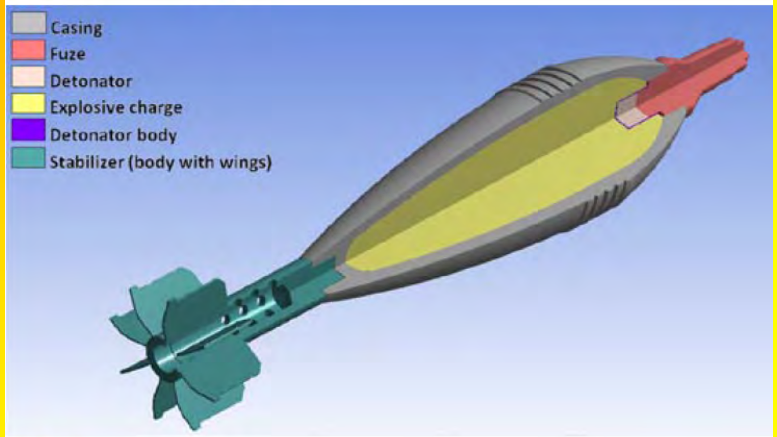


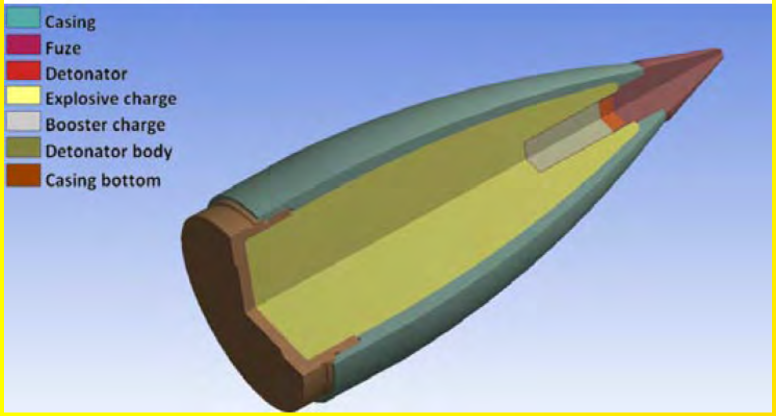
Figure 2. An example of the normal constraint reaction $F_N(\vartheta)$ in the functions of ϑ of the vibroimpact system with initial conditions determining ten impacts

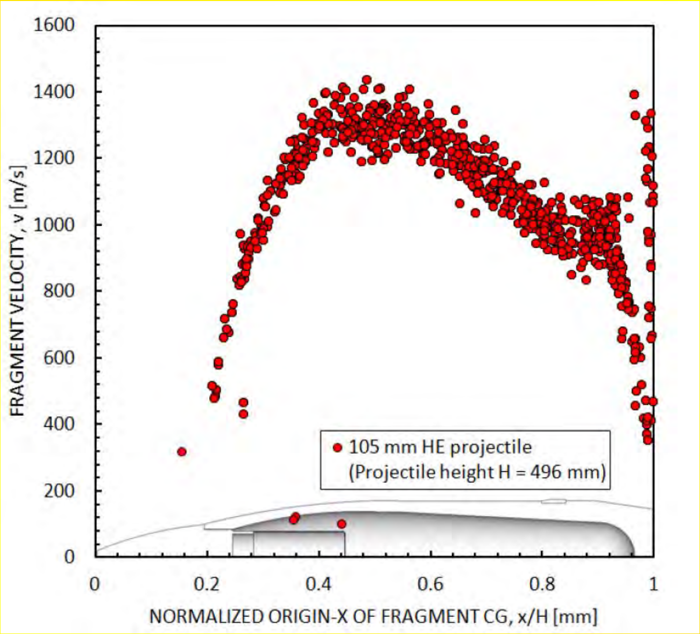
Marinko Ugrčić **amd** Miodrag Ivanišević, **Characterization of the Natural Fragmentation of High Explosive Projectiles Using the Numerical Techniques based on the FEM, (to appear)**

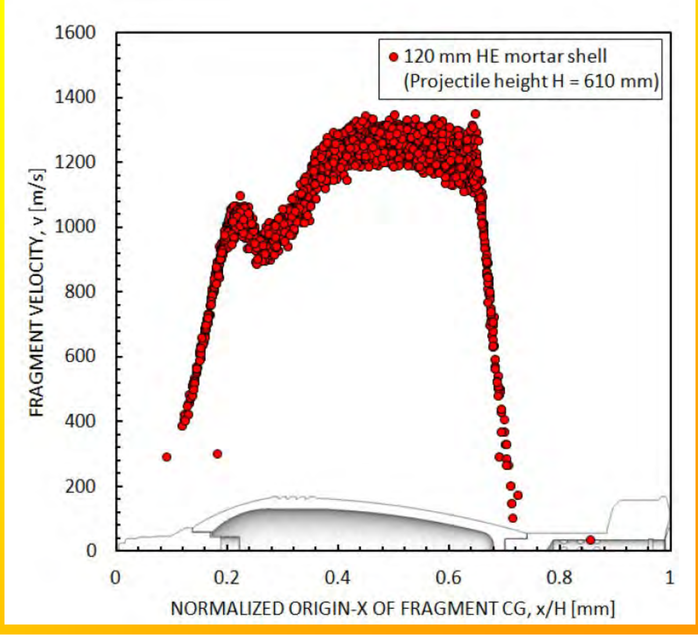



Casing
Explosive charge
Rotating band
Fuze
Detonator body
Detonator
Booster charge


Casing
Fuze
Detonator
Explosive charge
Detonator body
Stabilizer (body with wings)


Casing
Fuze
Detonator
Explosive charge
Booster charge
Detonator body
Casing bottom


105 mm HE projectile
(Projectile height H = 496 mm)
FRAGMENT VELOCITY, v [m/s]
NORMALIZED ORIGIN-X OF FRAGMENT CG, x/H [mm]


120 mm HE mortar shell
(Projectile height H = 610 mm)
FRAGMENT VELOCITY, v [m/s]
NORMALIZED ORIGIN-X OF FRAGMENT CG, x/H [mm]

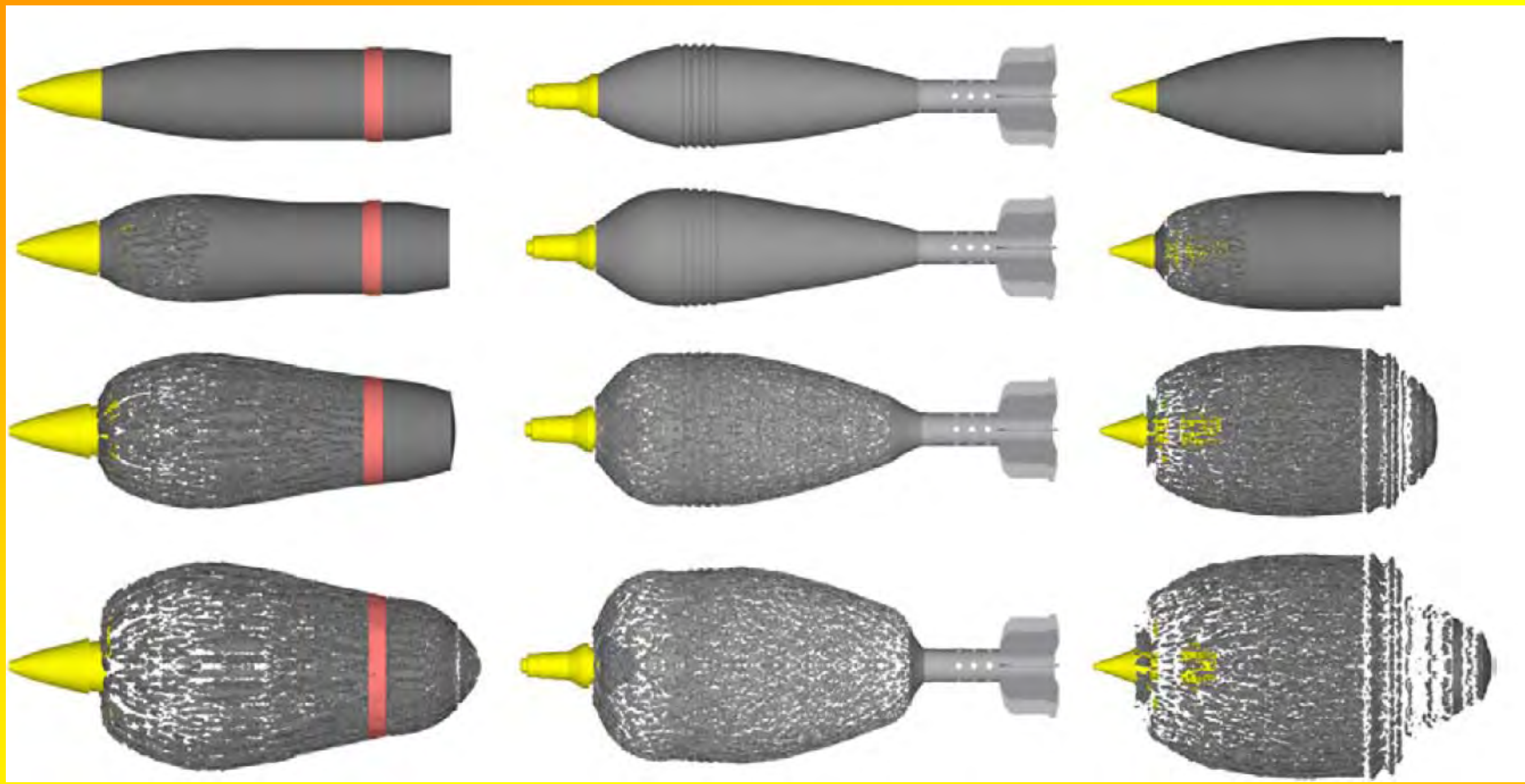
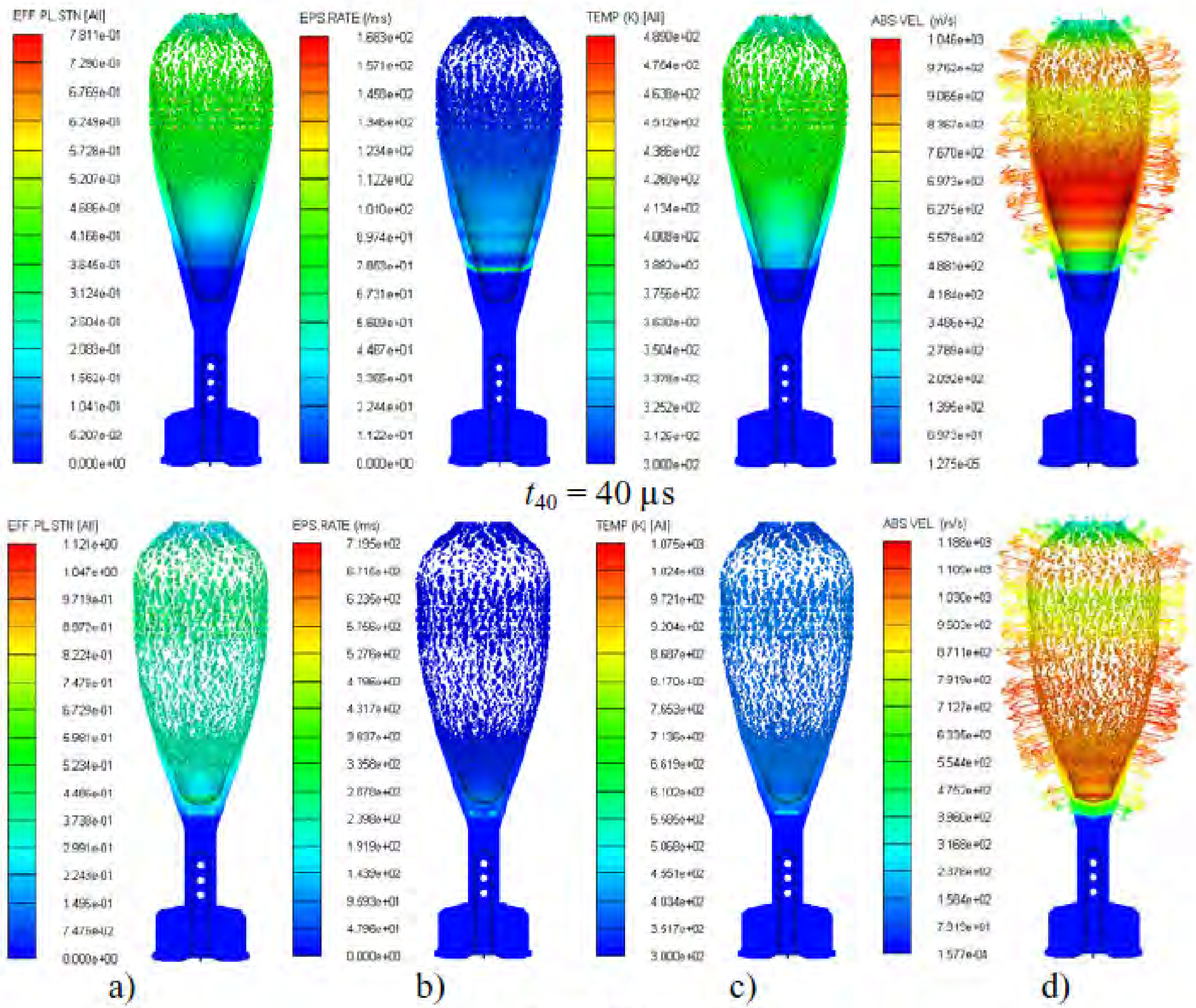


Figure 7. Fragmentation of the 105 mm HE projectile M1, 120 mm HE mortar shell M62 and 128 mm HE warhead M63: $t = 0$, 30, 60 and 90 μs



$t_{50} = 50\ \mu s$

Figure 21. Physical parameters of the 120 mm HE mortar shell fragmentation: a) Plastic strain, b) Strain rate, c) Temperature and d) Fragments velocity

PROJEKAT 174001

DINAMIKA HIBRIDNIH SISTEMA SLOŽENIH STRUKTURA. MEHANIKA MATERIJALA

PODTEMA:

Analiza stabilnosti i upravljanje sistemima sa čistim vremenskim kašnjenjem

ISTRAŽIVAČKI TIM

Prof. **Dr Dragutin Debeljković**, Mašinski fakultet u Beogradu

V. prof. **Dr Sreten Stojanović**, Tehnološki fakultet u Leskovcu

Dr **Nebojša Dimitrijević**, profesor, Visoka skola primenjenih strukovnih studija u Vranju

Dr **Goran Simeunović**, istraživač saradnik, Inovacioni centar Mašinskog fakulteta u Beogradu

KATEGORIJE RADOVA U OBJAVLJENIH U 2013.

Kategorija radova	Broj radova	Klasa sistema sa kašnjenjem	Koncept stabilnosti
M22	1	regularni	FTS
M23	3	regularni, singularni	FTS, praktična stabilnost
M24	1	regularni	FTS
M33	5	regularni, singularni	FTS, praktična stabilnost, FTB
M42	5	regularni, singularni	asimptotska stabilnost, FTS, praktična stabilnost, FTB

ANALIZA NAJZNAČAJNIJIH REZULTATA

- U navedenim radovima razmatrana je stabilnost na konačnom i beskonačnom vremenskom intervalu sistema sa kašnjenjem sa ili bez prisustva neodređenosti u modelu.
- Pored regularne klase sistema, razmatrana je i klasa singularnih (deskriptivnih) sistema sa kašnjenjem.
- Problem upravljanja ovim sistemima realizovan je u povratnoj sprezi koristeći metodu stabilizacije sistema.

M22

- Na osnovu kvazi Ljapunove metode i svojstva matričnih nejednakosti, izvedeni su dovoljni uslovi robusne stabilnosti i stabilizacije na konačnom vremenskom intervalu sistema sa kašnjenjem i prisutnim neodređenostima.

Theorem 2. There exists a memoryless state feedback controller such that the closed-loop system is FTS with respect to $(\boldsymbol{c}_1, \boldsymbol{c}_2, \boldsymbol{T})$, $\boldsymbol{c}_1 < \boldsymbol{c}_2$, if there exist a nonnegative scalar , positive scalars , Δ, , $_1$, $_2$, $_3$, positive-define symmetric matrices $\boldsymbol{X}$, $\boldsymbol{Y}$ and matrix $\boldsymbol{Z}$ such that the conditions hold:

$$\Omega = \begin{bmatrix} \begin{matrix} \boldsymbol{A}_0\boldsymbol{X} + \boldsymbol{X}\boldsymbol{A}_0^T + \boldsymbol{B}\boldsymbol{Z} + \boldsymbol{Z}^T\boldsymbol{B}^T + \boldsymbol{Y} - \quad \boldsymbol{X} \\ + \quad \boldsymbol{D}_0\boldsymbol{D}_0^T + \Delta\boldsymbol{D}_1\boldsymbol{D}_1^T + \quad \boldsymbol{D}_B\boldsymbol{D}_B^T \end{matrix} & \boldsymbol{A}_1\boldsymbol{X} & \boldsymbol{X}\boldsymbol{E}_0^T & 0 & \boldsymbol{Z}^T\boldsymbol{E}_B^T \\ * & -\boldsymbol{Y} & 0 & \boldsymbol{X}\boldsymbol{E}_1^T & 0 \\ * & * & - \quad \boldsymbol{I} & 0 & 0 \\ * & * & * & -\Delta\boldsymbol{I} & 0 \\ * & * & * & * & - \quad \boldsymbol{I} \end{bmatrix} < 0$$

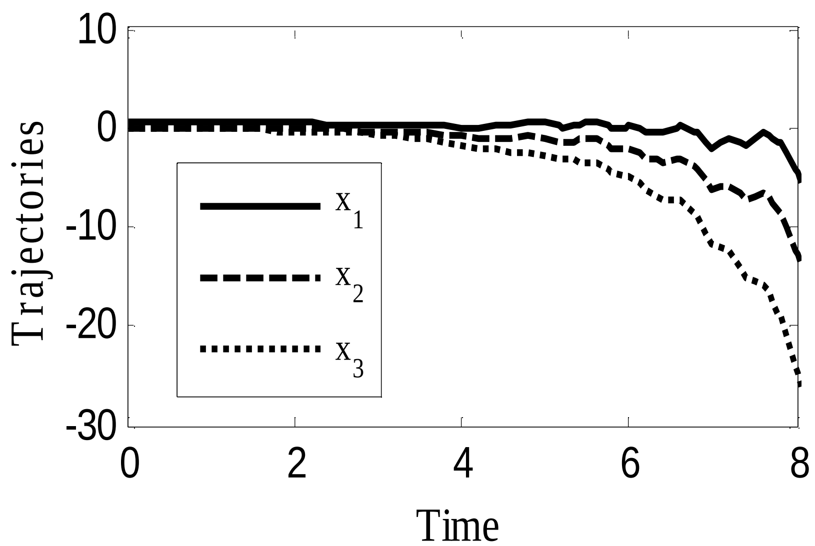
$$-\boldsymbol{c}_2\boldsymbol{e}^{- \quad \boldsymbol{T}} \quad _1 + \boldsymbol{c}_1 \quad _2 + \quad \boldsymbol{c}_1 \quad _3 < 0$$

$$\begin{bmatrix} \boldsymbol{X} & \boldsymbol{I} \\ \boldsymbol{I} & _2\boldsymbol{I} \end{bmatrix} > 0, \quad \begin{bmatrix} _1^{-1}\boldsymbol{I} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{X}^{-1} \end{bmatrix} > 0, \quad \begin{bmatrix} \boldsymbol{Y}^{-1} & \boldsymbol{X}^{-1} \\ \boldsymbol{X}^{-1} & _3\boldsymbol{I} \end{bmatrix} > 0$$

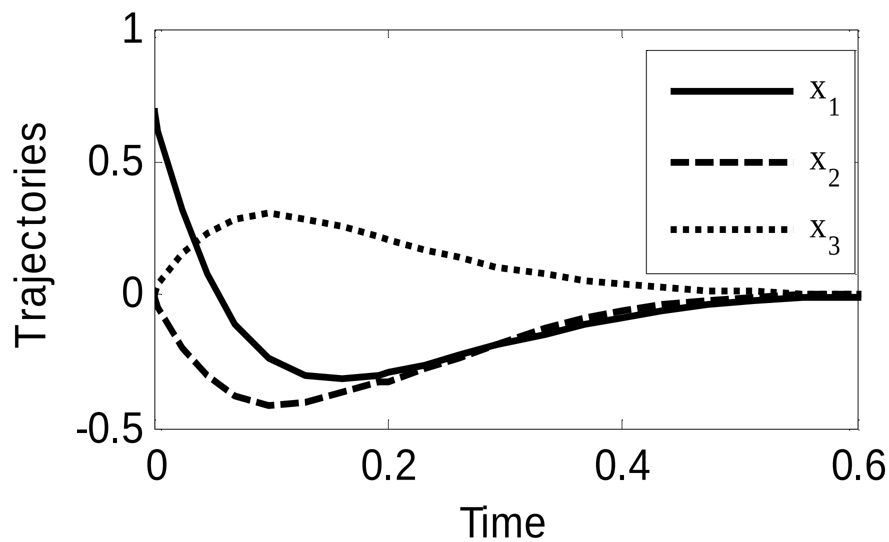
The memoryless state finite-time controller gain is given by $\boldsymbol{K} = \boldsymbol{Z}\boldsymbol{X}^{-1}$.

- Problem projektovanja kontrolera u povratnoj sprezi rešen je pomoću linearnih matričnih nejednačina i uz primenu CCLI algoritma.
- Simulacija projektovanog sistema upravljanja

otvoreni sistem



zatvoreni sistem



M23

- U radu je razmatrana stabilnost na konačnom vremenskom intervalu za klasu linearnih kontinualnih sistema sa kašnjenjem. Koristeći podesan kvazi Ljapunov funkcional, kao i Jensenovu i Kopelovu nejednakost, izveden je uslov stabilnosti na konačnom vremenskom intervalu u obliku skupa algebarskih nejednakosti.

Theorem 1. The time-delayed system with is finite-time stable with respect to $\{\ ,\ ,\boldsymbol{T}\}$ if there exists a positive scalar $\wp$ such that:

$$\boldsymbol{x}^{\boldsymbol{T}}(\boldsymbol{t}-\vartheta)\,\boldsymbol{x}(\boldsymbol{t}-\vartheta) < \boldsymbol{q}\,\boldsymbol{x}^{\boldsymbol{T}}(\boldsymbol{t})\,\boldsymbol{x}(\boldsymbol{t}), \quad \boldsymbol{q}>0, \quad \vartheta\in[-\ ,\ 0], \quad \forall \boldsymbol{t}\in[0,\boldsymbol{T}]$$

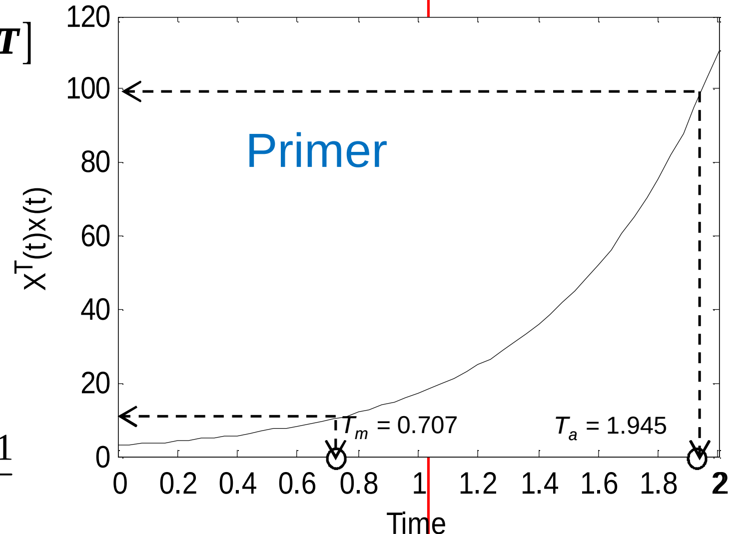
$$(1+\ )(1+\ )\left(1-\wp\ \ -\frac{\boldsymbol{q}}{\wp}\right)^{-1}\boldsymbol{e}^{\ _{\max}(\Xi)\boldsymbol{T}} < \frac{\ }{\ }$$

$$\wp\in\left(\max\{\wp_1,0\},\ \ \wp_2\right), \quad \wp_{1,2}=\frac{1\pm\sqrt{1-4\ \ \boldsymbol{q}}}{2}, \quad 4\ \ \boldsymbol{q}<1$$

where:

$$\boldsymbol{R}=\boldsymbol{A}_0+\boldsymbol{Q}(0), \quad \Xi=\boldsymbol{R}^{\boldsymbol{T}}+\boldsymbol{R}, \quad\ = \ _{\max}\left(\boldsymbol{Q}(0)\boldsymbol{Q}^{\boldsymbol{T}}(0)\right)\frac{\boldsymbol{e}^{2\ \ _2(\boldsymbol{R})}-1}{2\ \ (\boldsymbol{R})}$$

$_2(\boldsymbol{R})$ being matrix measure of matrix $\boldsymbol{R}$ and $\boldsymbol{Q}(0)$ is any solution of the following nonlinear transcendental matrix equation: $\boldsymbol{e}^{\boldsymbol{A}_0+\boldsymbol{Q}(0)}\,\boldsymbol{Q}(0)=\boldsymbol{A}_1$.



M23

- Koristeći kvazi Ljapunove funkcije izvedeni su novi, dovoljni uslovi stabilnosti, od kojih neki zavise od kašnjenja, a drugi su nezavisni. Pri analizi koncepta praktične stabilnosti, prethodno pomenuti prilaz kombinovan je sa klasičnom Ljapunovom metodom kako bi se obezbedila atraktivna praktična stabilnost razmatranog dinamičkog ponašanja sistema. Prilaz sa stanovišta LMI metode je takođe primenjen sa ciljem da se oslabe neki od ograničavajućih uslova iz prethodnih rezultata.

Theorem 3. Singular time delayed ***s***ystem is regular, impulse free and finite time stable with respect to $\{\ ,\ ,\boldsymbol{T}\}$, $\ <\ $, if for some fixed nonnegative scalar $\wp$ there exist positive scalars $_1$, $_2$ and $_3$, nonsingular matrix $\boldsymbol{P}$, positive definite matrices Π and $\boldsymbol{Q}$, such that the following conditions hold:

$$\boldsymbol{PE} = \boldsymbol{E}^{\boldsymbol{T}}\boldsymbol{P}^{\boldsymbol{T}} \geq 0, \quad \boldsymbol{PE} = \boldsymbol{E}^{\boldsymbol{T}}\Pi\boldsymbol{E}$$

$$\begin{bmatrix} \boldsymbol{A}_0^{\boldsymbol{T}}\boldsymbol{P}^{\boldsymbol{T}} + \boldsymbol{PA}_0 + \boldsymbol{Q} - \wp\boldsymbol{PE} & \boldsymbol{PA}_1 \\ * & -\boldsymbol{Q} \end{bmatrix} < 0$$

$$_1\boldsymbol{I} < \Pi, \quad _2\boldsymbol{I} > \boldsymbol{PE}, \quad _3\boldsymbol{I} > \boldsymbol{Q}, \quad - \quad \boldsymbol{e}^{-\ \boldsymbol{T}}\ _1 + \ _2 + \ _3 < 0$$

PROJEKAT 174001
DINAMIKA HIBRIDNIH SISTEMA SLOŽENIH STRUKTURA. MEHANIKA MATERIJALA

- Radovi (2013)
- Istraživač Nataša Trišović, Mašinski fakultet u Beogradu

- **Ужа категорија М23: Научни радови у међународним часописима (1 рад)**
-
- Ezedine Allaboudi, Tasko Maneski, Natasa Trisovic, Todor Ergić, „Improving structure dynamic behaviour using a reanalysis procedures technique“, Journal: Tehnički vjesnik, Vol.20, No.2, Str. 297 - 304, April, 2013, ISSN 1330-3651, (ISI-SCI list, IF: 0,347)
-

http://hrcak.srce.hr/index.php?show=clanak

- **Ужа категорија М33: Радови саопштени на скупу међународног значаја, штампани у целини (1 радова)**

- N. Trišović, Wavelet Families – A Primer, the Forth (29th Yu) International
Congress of Serbian Society of Mechanics held in Vrnjačka Banja, 4th – 7th June, 2013. Pp.1005-1011.ISBN 978-86-909973-5-0

- **Ужа категорија M24: Рад у часопису међународног значаја верификованог посебном одлуком (1 рад)**

- Trišović Nataša, Maneski Taško, Golubović Zorana, Segla Štefan, **Elements of Dynamic Parameters Modification and Sensitivity**, FME Transactions, Volume 41, No 2, pp. 146-152, Beograd, 2013, (ISSN, 1451-2092)

- http://www.mas.bg.ac.rs/transactions/Vol_41

Ужа категорија М51: Рад у часопису националног значаја (1 рад)

Nataša Trišović, Wei Li, Taško Maneski, Mirjana Misita, Ljubica Milović,
Elements of Dynamic Modifications and Sensitivity Considering the Effect of Structural Parameters Uncertainty

SCIENTIFIC REVIEW, ISSN 0350-2910, BELGRADE (2013), Serbian Scientific Society, p.p. 389-404

http://afrodita.rcub.bg.ac.rs/~nds/
http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Editor-in-Chief: Slobodan Perović
Series: Scientific and Engineering
Special Issue Nonlinear Dynamics S2 (2013)
Dedicated to Milutin Milanković (1879- 1958)
Guest Editors:
Katica R. (Stevanović) Hedrih and Žarko Mijajlović

- **Ужа категорија (М36): Уређивање зборника саопштења међународног научног скупа**
-
- Proceedings, 1st International Congress of Serbian Society of Mechanics, Vrnjačka Banja, June 3-7, 2013, ISBN 978-86-909973-5-0, Editors: S. Maksimović, T. Igić, N. Trišović;

Napomena uz radove

- Godine 2013 napravljen je pomak uvođenjem teorije verovatnoće u predhodna istraživanja koja su se odnosila na reanalizu konstrukcija. Analizirana su moguća odstupanja ulaznih parametara, i pri tome je korišćena Monte Karlo metoda. U nastavku autor će se baviti daljim istraživanjima iz ove oblasti.
- Osim toga dat je jedan pregledni rad iz oblasti Wavelets na kongresu mehanike, što takođe

PROJEKAT 174001
DINAMIKA HIBRIDNIH SISTEMA SLOŽENIH STRUKTURA.
MEHANIKA MATERIJALA

Radulović Radoslav, Mašinski fakultet u Beogradu

Spisak položenih ispita u 2013. god

Re. Br.	Predmet	Profesor	Ocena
1	O metodana naučno istraživačkog rada i komunikacije	prof. Dr Miloš Nedeljković	10
2	Viši kurs matematike	prof. Dr Slobodan Radojević	10
3	Numeričke metode	prof. Dr Miodrag Spalević	10
4	Odabrana poglavlja iz mehanike	prof. Dr Zoran Mitrović	10
5	Tenzorski račun	prof. Dr Zoran Stokić	10
6	Analitička Mehanika	prof. Dr Olivera Jeremić	10
7	Epistemologija nauke i tehnike	prof. Dr Zoran Stokić	10
8	Oscilacije mehaničkih sistema-linearne	prof. Dr Aleksandar Obradović	10
9	Oscilacije mehaničkih sistema-nelinearne	prof. Dr Zoran Mitrović	10
10	Stabilnost kretanja sistema	prof. Dr Zoran Mitrović	10
11	Dinamika sistema krutih tela	prof. Dr. Mihailo Lazarević	10
12	Upravljanje kretanjem mehaničkih sistema	prof. Dr Aleksandar Obradović	10
13	Mehanika kontinuuma	prof. Dr Zoran Stokić	10
14	Mehanika sistema promenljive mase	prof. Dr Olivera Jeremić	10
15	Mehanika neholonomnih sistema	prof. Dr Dragomir Zeković	10
16	Mehanika udara	prof. Dr Mirko Pavišić	10

Objavljeni radovi i nagrade u 2013. godini

Spisak prezentovanih radova na kongresima i simpozijumima:

[1] Radulović, R.: *Shooting method in determining global minimum time of brachistochronic motion*, in: *Proceedings of the 4th International Congress of Serbian Society of Mechanics,*04-07.06.2013, Vrnjačka Banja, pp. 159-164.

[2] Radulović, R., Obradović, A. and Jeremić, B.: *Brachistochronic Motion of a Nonholonomic Mechanical System with Limited Reactions of Constraints*, in: *Proceedings of the 4th International Congress of Serbian Society of Mechanics,*04-07.06.2013, Vrnjačka Banja, pp. 903-908.

Spisak radova u časopisima:

[3] Radulović, R., Obradović, A. and Jeremić, B.: *Analysis of the minimum required coefficient of sliding friction at brachistochronic motion of a nonholonomic mechanical system*, FME Transactions, 2013.

Istraživač Radulović Radoslav je nagrađen prestižnom nagradom *„Rastko Stojanović“* za rad [1] kao samostalni autor na međunarodnom kongresu Srpskog društva za mehaniku koji je održan u Vrnjačkoj Banji od 04.-07. juna 2013. godine.

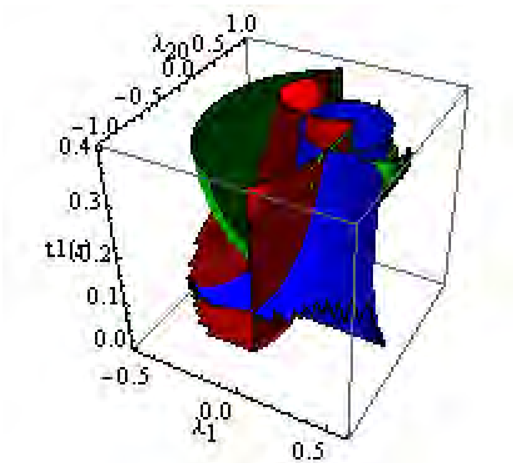
Analiza najznačajnijih rezultata

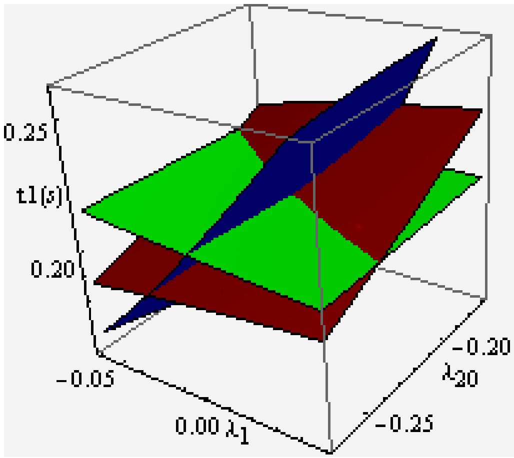
➢ U radu [1] razmatra se problem brahistohronog kretanja za opšti slučaj holonomnog skleronomnog mehaničkog sistema.

➢ Daje se postupak određivanja globalnog minimuma za sisteme sa 3 DOF , gde je moguće dati i grafičke prezentacije u trodimenzionom prostoru, gde je treća dimenzija krajnji trenutak.

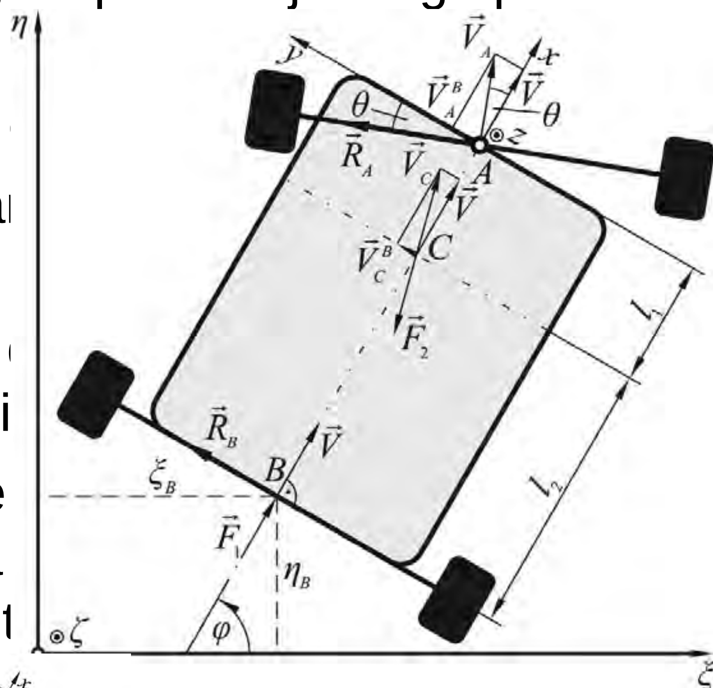
➢ Globalni minimum vremena kod brahistohronog kretanja krutog tela može se dati na osnovu grafičkog prikaza rešenja sistema nelinearnih jednačina, kao i globalne procene koordinata spregnutog vektora.

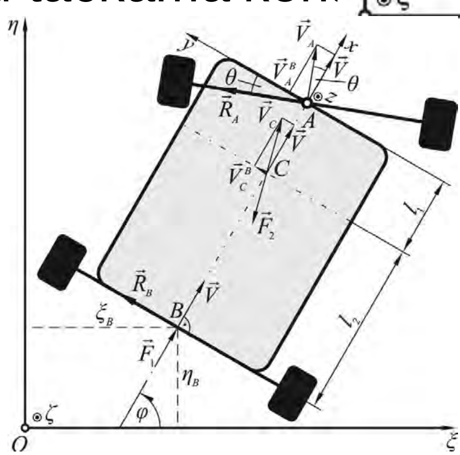
$$\psi_1 = \boldsymbol{f}_\psi\left(\lambda_1, \lambda_{20}, \boldsymbol{t}_1\right), \theta_1 = \boldsymbol{f}_\theta\left(\lambda_1, \lambda_{20}, \boldsymbol{t}_1\right) \text{ i } \varphi_1 = \boldsymbol{f}_\varphi\left(\lambda_1, \lambda_{20}, \boldsymbol{t}_1\right)$$

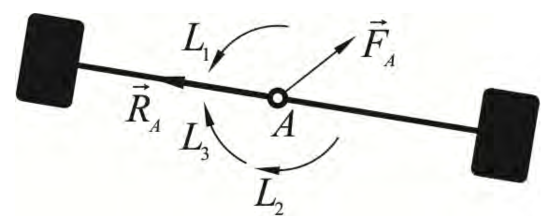




- U radovima [2] i [3] analizira se problem brahistohronog kretanja mehaničkog sistema, na primeru jednog uprošćenog modela vozila.
- Sistem se kreće … ja pri neizmenjenoj vrednosti mehaničke …
- Numeričko rešava… problema vrši se metodom šutinga.
- Na osnovu tako … etanja određuju se aktivne upravljačke si…
- Koristeći Kulonove … eđuje se minimalna vrednost koeficijenta … šlo do proklizavanja vozila u tačkama kont…







PROJEKAT 174001
DINAMIKA HIBRIDNIH SISTEMA SLOŽENIH STRUKTURA. MEHANIKA MATERIJALA

Prof dr Zeković Dragomir, Mašinski fakultet u Beogradu

DRAGOMIR N. ZEKOVIĆ, DIFFERENTAIL EQUATIONS OF MOTION FOR MECHANICAL SYSTEMS WITH NONLINEAR NONHOLONOMIC CONSTRAINTS - VARIOUS FORMS AND THEIR EQUIVALENCE, SCIENTIFIC REVIEW, ISSN 0350-2910, BELGRADE (2013), Serbian Scientific Society, p.p. 179-196

http://afrodita.rcub.bg.ac.rs/~nds/
http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Editor-in-Chief: Slobodan Perović
Series: Scientific and Engineering
Special Issue Nonlinear Dynamics S2 (2013)
Dedicated to Milutin Milanković (1879- 1958)
Guest Editors:
Katica R. (Stevanović) Hedrih and Žarko Mijajlović

PROJEKAT 174001
DINAMIKA HIBRIDNIH SISTEMA SLOŽENIH STRUKTURA. MEHANIKA MATERIJALA

Veljić Vladimir, Mašinski fakultet u Beogradu

Objavljeni radovi i nagrade u 2013. godini

Uža kategorija M33: Radovi saopšteni na skupu međunarodnog značaja, štampani u celini

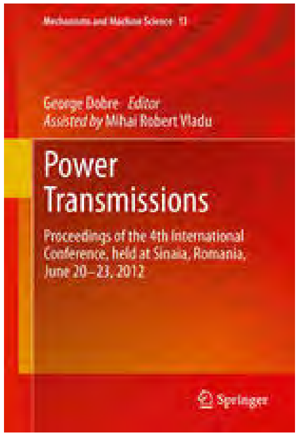
MECHANICAL PROPERTIES INVESTIGATION OF COMMERCIAL AND AND NANOPHOTONICS SOFT CONTACT LENSES	4th (29th Yu) International Congress of Serbian Society of Mechanics held in Vrnjačka Banja, 4th – 7th June, 2013.	V. Veljić, A. Debeljković, Đ.Koruga	2013	Serbian Society of Mechanics	pp. 591-597	ISBN 978-86-909973-5-0
STUDY OF MECHANICAL PROPERTIES OF COMMERCIAL AND NANOPHOTONICS MATERIALS FOR SOFT CONTACT LENSES BY OPTOMAGNETIC SPECTROSCOPY	4th (29th Yu) International Congress of Serbian Society of Mechanics held in Vrnjačka Banja, 4th – 7th June, 2013.	V. Veljić, A. Debeljković, Đ.Koruga	2013	Serbian Society of Mechanics	pp. 961-967	ISBN 978-86-909973-5-0

Uža kategorija M24: Rad u časopisu međunarodnog značaja verifikovanog posebnmom odlukom

Characterization of materials for commercial and new nanophotonic soft contact lenses by Optomagnetic Spectroscopy	Journal: FME Transactions, Volume 42, No 2,	A. Debeljkovic, V. Veljic, V. Sijacki-Zeravcic, L. Matija,Dj. Koruga	2014	Faculty of Mechanical Engineering, Belgrade, Serbia	pp. 141-146,	

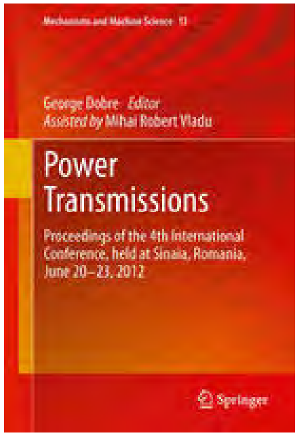
U radu „MECHANICAL PROPERTIES INVESTIGATION OF COMMERCIAL AND AND NANOPHOTONICS SOFT CONTACT LENSES“ korišćen je AFM skenirajuća tehnikom kojaom je potvrđeno da su mehaničke osobine mekih kontaktnih sočiva bolje kada se u komercijalna meka kontaktna sočiva implementiraju nanočestice

U radovima „ STUDY OF MECHANICAL PROPERTIES OF COMMERCIAL AND
NANOPHOTONICS MATERIALS FOR SOFT CONTACT LENSES
BY OPTOMAGNETIC SPECTROSCOPY“ i „ Characterization of materials for commercial and new nanophotonic soft contact lenses by Optomagnetic Spectroscopy“ korišćena je metoda optomegnetne spektorskopije, koja je dala značajne kvalitativne rezultate u pogledu mehaničkih i optičkih karakteristika kontaktnih sočiva napravljenih od nanomaterijala



M. Jelić, **I. Atanasovska**: THE NEW APPROACH FOR CALCULATION OF TOTAL MESH STIFFNESS AND NONLINEAR LOAD DISTRIBUTION FOR HELICAL GEARS, **Mechanisms and Machine Science (Book Series), Series Ed.: Ceccarelli Marco, ISSN: 2211-0984, *Vol. 13: Power Transmissions (Proceedings of The 4th International Conference on Power Transmissions- PT 12, June 20 -23, 2012, Sinaia, Romania),*** *Editor: G.Dobre, ISBN: 978-94-007-6557-3 (Print) 978-94-007-6558-0 (Online), Publisher:* ***Springer Science*** + *Business Media Dordrecht 2013, doi: 10.1007/978-94-007-6558-0_52, pp. 645-654*
http://link.springer.com/chapter/10.1007/978-94-007-6558-0_52#

I. Atanasovska, M. Vukšić Popović: DYNAMICS OF GEAR-PAIR SYSTEMS WITH PERIODIC VARYING MESH STIFFNESS - SPUR GEARS VS HELICAL GEARS, Series: Scientific Review, Scientific and Engineering - Special Issue - Nonlinear Dynamics S2 (2013) Dedicated to Milutin Milanković (1879-1958), *Guest Editors: Katica R (Stevanović)Hedrih and Željko Mijajlović, YU ISSN:0350-2910, UDK 001, Publisher: Serbian Scientific Society,* 2013., *pp.* 373-388*.*
http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

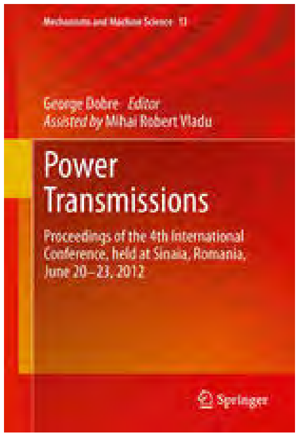


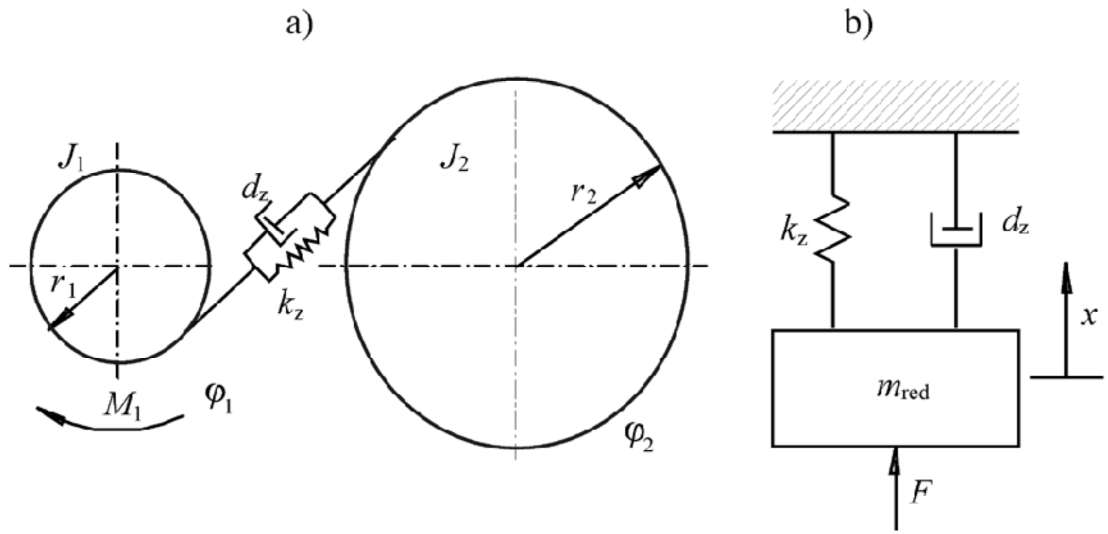
Abstrakt.

Opisan je novi pristup rešavanju krutosti spregnutih zubaca i raspodele opterećenja za zupčanike sa kosim zupcima. Krutost para spregnutih zubaca je parameter koji varira u toku perioda sprezanja i takođe duž kontaktne linije spregnutih zubaca, pa se može precizno izračunati samo istovremenim rešavanjem oba navedena zadatka. Metoda konačnih elemenata korišćena je za proračun promene ukupne deformacije I raspodele opterećenja u vremenu I duž kontaktne linije.

Korišćen je model dinamičkog ponašanja zupčastog para u komparativnoj analizi zupčanika sa pravim i kosim zupcima sa aspekta njihove stabilnosti. Dobijeni rezultati prikazani kao fazni portreti potvrđuju da zupčanici sa kosim zupcima imaju stabilniji rad od zupčanika sa pravim zupcima iste geometrije i pri istim opterećenjem.

Pojednostavljeni dinamički model zupčastog para $\rightarrow$ zupčasti par se simulira sa dva diska između kojih se kontakt simulira nelinearno promenljivom krutosti sprege i prigušenjem sprege





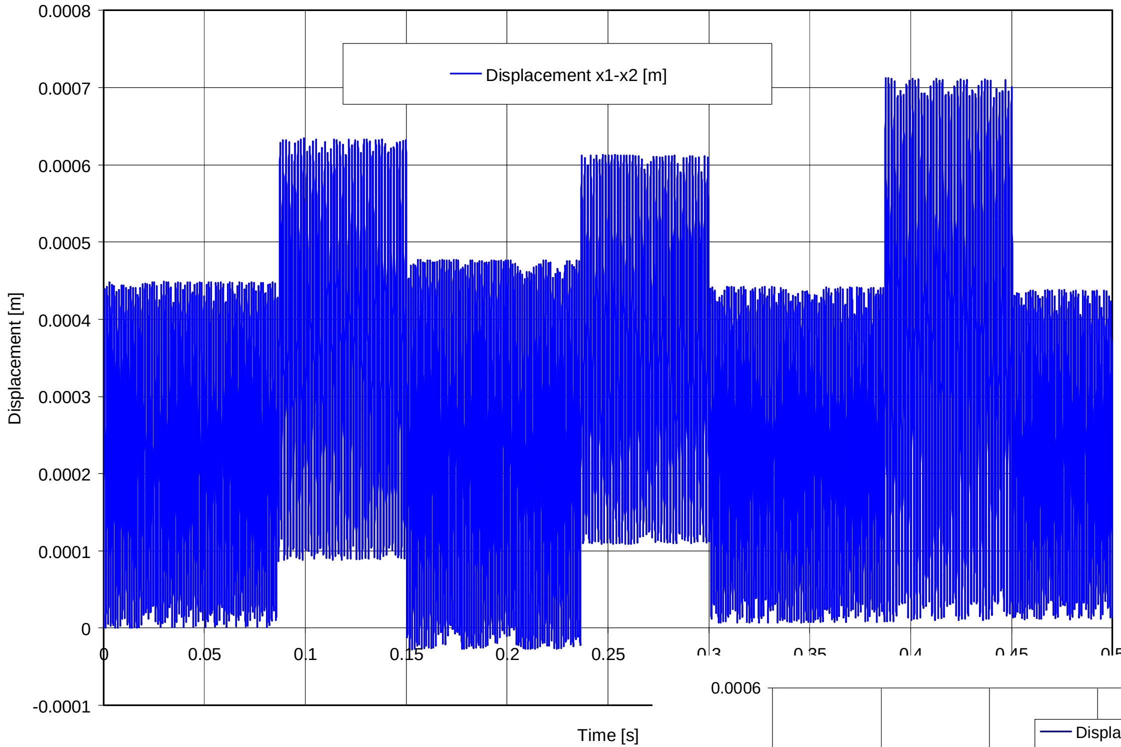
a) model II – sa dva stepena slobode, b) model I – sa jednim stepenom slobode

$$m_1\ddot{x}_1 + d_z(t)(\dot{x}_1 - \dot{x}_2) + k_z(t)(x_1 - x_2) = F_n(t)$$

$$m_{red}\ddot{x} + d_z(t)\dot{x} + k_z(t)x = F_n(t)$$

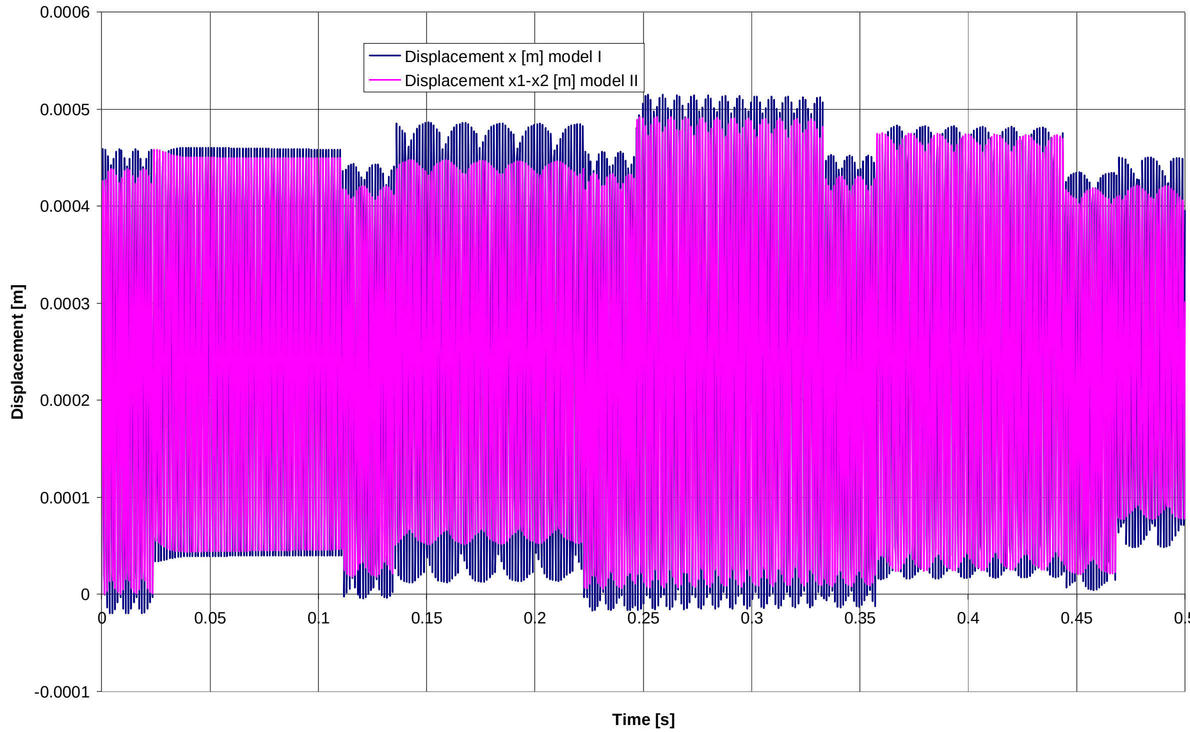
$$m_2\ddot{x}_2 - d_z(t)(\dot{x}_1 - \dot{x}_2) - k_z(t)(x_1 - x_2) = -F_n(t)$$

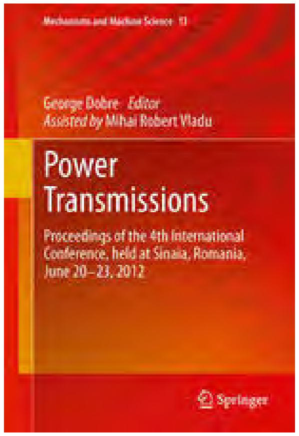
mi (*i*=1,2) – ekvivalentna masa zupčanika 1 i 2; *mred* – redukovana masa zupčanika; kz(*t*) – krutost sprege; *dz*(*t*) – prigušenje sprege; *Fn*(*t*) – funkcija raspodele spoljašnjeg opterećenja

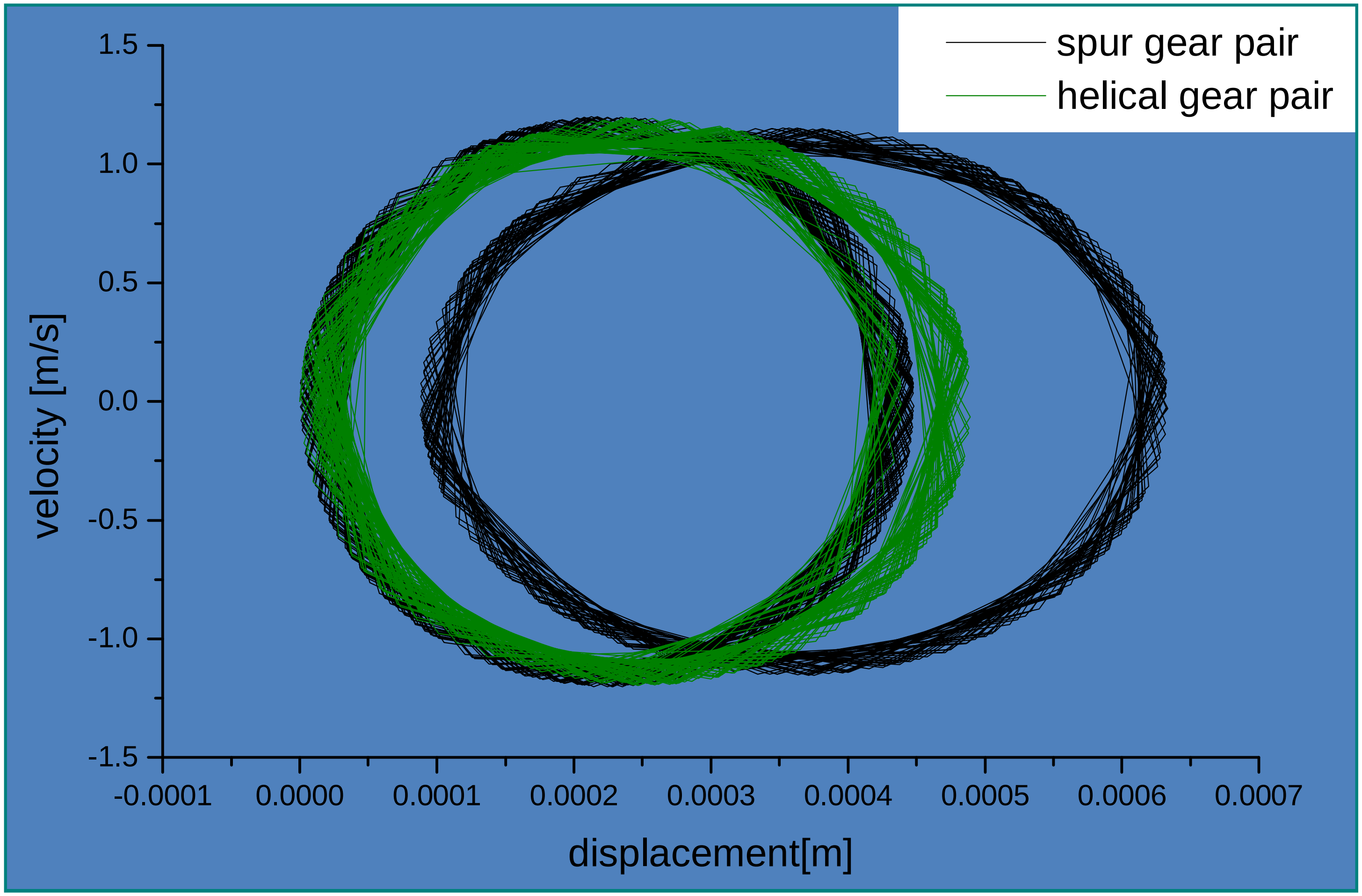


Zupčasti par sa pravim zupcima

Zupčasti par sa kosim zupcima




George Dobre Editor
Assisted by Mihai Robert Vladu
Power Transmissions
Proceedings of the 4th International Conference, held at Sinaia, Romania, June 20–23, 2012
Springer

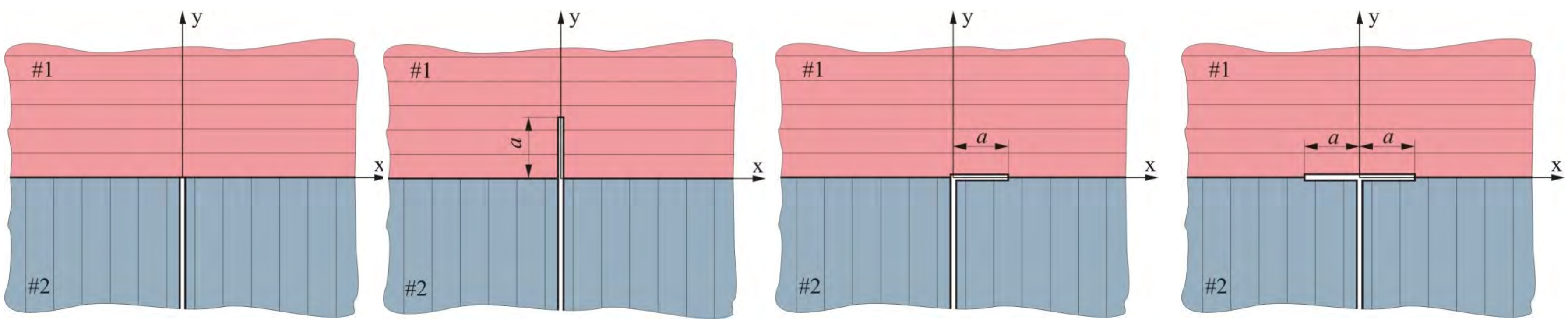

spur gear pair
helical gear pair
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
velocity [m/s]
-0.0001
0.0000
0.0001
0.0002
0.0003
0.0004
0.0005
0.0006
0.0007
displacement[m]

Jelena M. Djoković, Ružica R. Nikolić,

" CRACK DEFLECTION AT AN INTERFACE BETWEEN THE TWO ORTHOTROPIC MATERIALS",

Proceedings Fourth Serbian (29th Yu) Congress on Theoretical and Applied Mechanics,
Vrnjačka Banja, Serbia, 4-7 June 2013, C43, pp.535-540, ISBN 978-86-909973-5-0

In this paper is presented studying of the problem of a crack that approaches interface between the two orthotropic materials at the right angle. The three possible cases of the crack attacking the interface were considered: crack penetrates the interface and continues to grow in the material above it; the crack deflects into the interface as a single branch and crack deflects into the interface in two branches (double deflection).

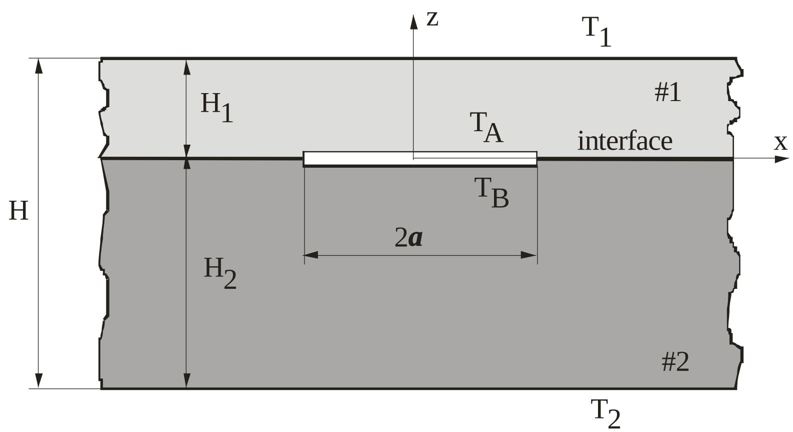


Based on results presented in this paper, it can be concluded that the ratio of energy release rates depends on variation of the anisotropic parameters λ_1, λ_2, ρ_1 and ρ_2. It is noticed that the value of the G_d/G_p ratio changes within interval 0.2 to 5. When the value of this ratio exceeds this means that crack approaching interface at the right angle will deflect into it even if the interface toughness is higher than the toughness of the base material.

Results presented in this paper provide the possibility for comparison of the interface and base material (substrate) tough nesses, for the purpose of determination whether the incoming crack would deflect into the interface or would it penetrate the interface and continue to propagate in the second material above it. If the ratio of the interface fracture toughness to fracture toughness of the material into which the crack continues to propagate is less than the ratio of the energy release rate for the crack that deflects into the interface and the energy release rate for the crack that penetrates the interface, the crack will deflect into the interface. If this relation is reversed the crack will penetrate the interface.

Jelena M. Djoković, Ružica R. Nikolić, Katarina Z. Živković,
"INTERFACIAL CRACK BEHAVIOR IN THE STATIONARY TEMPERATURE FIELD CONDITIONS" ,
Thermal Science, 2013, Vol. 17, No.17, Suppl. 1, pp. S169-S178, ISSN 0354-9836.
DOI information: 10.2298/TSCI120828113D

In this paper the theoretical basis for determining the driving forces of interfacial crack propagation in a two-layer bimaterial specimen is presented, under conditions when the temperatures of the outer layers' surfaces are different.
The analysis is limited to the fact that the two-layer bi-material sample is exposed to a stationary temperature field.



The driving force of the interfacial fracture, in this case, is the energy release rate *G*, which is determined as a function of the temperature loading. It was noticed that the energy release rate tends to increase with increasing temperature difference. This relation can be used to predict the maximum temperature differences the two-layer sample can sustain without delamination.

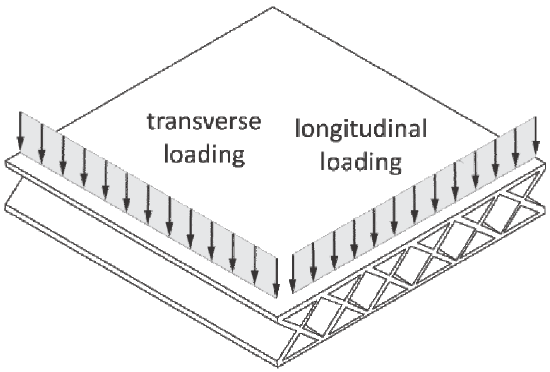
The highest value of the energy release rate is for a crack located at approximately one fifth of the sample thickness, what means that it is the most likely that the crack, which causes the sample delamination lies at this distance.

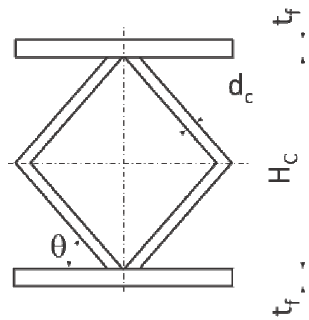
Variation of the Biot's number imposes significantly higher influence on Mode II stress intensity factor and in the case of the mixed mode crack propagation the value of dimensionless stress intensity factor for Mode I is much smaller than the value of this factor for Mode II. The Biot's number B_c plays an important role in heat flow through the crack.

For large values of the Biot's number, the energy release rate *G* has a relatively lower value, which means that the crack opening is small. However, when B_c is small, the energy release rate is large, as well as the crack opening, namely crack is completely and totally isolated, and suitable for delamination of the sample if the temperature gradient is sufficiently large.

The threshold of the temperature difference also increases with temperature. If the threshold exceeds the imposed temperature difference, delamination of the sample should not be expected. The heat flow through the crack can be significant and the assumption of a perfectly isolated crack would be wrong.

Jelena M. Djoković, Ružica R. Nikolić, Katarina M. Veljković,
"OPTIMIZATION OF PLATES WITH PRISMATIC CORES",
Proceedings of ICET 2013 – International Congress on Engineering and Technology,
25th – 27th June, 2013, Dubrovnik – Croatia, pp. 159-166. ISBN: 978-8-87670-08-8





In this paper are analyzed multifunctional sandwich plates with prismatic cores. The comparison of those plates to plates with honeycomb cores was performed. An estimate was done of optimal dimensions and minimal mass of plates with prismatic cores. The two loading directions were considered, longitudinal and transversal. The fracture mechanisms are derived that take into account mutual interaction of the core and the basic plate during buckling for each loading direction. The dimensionless expressions were derived for the loading combination of transversal force and bending moment for each fracture mechanism. Four variables were used in dimensions optimization, which reduces the sandwich plate mass. The plates with prismatic cores have best performances when loaded longitudinally, since the plate characteristics are restricted by buckling of a plate and not of a beam. Plates with the honeycomb cores are more efficient related to weight, than the plates with the prismatic cores for smaller loads. This advantage vanishes with load increase. Large plastic deformations of material used in manufacturing of the sandwich plates produce better performances for the plates with the prismatic cores. To emphasize advantages of those plates the comparison to sandwich plates with the truss cores was also performed

Jelena M. Djoković and Ružica R. Nikolić,
"COMPARATIVE OPTIMIZATION OF SANDWICH PLATES WITH PRISMATIC AND TRUSS CORES",
Proceedings 10-th European Conference of Young Researchers and Scientists, TRANSCOM 2013,
24-26 June 2013, University of Žilina, Žilina, Slovak Republic, pp.71-74, ISBN 978-80-554-0695-4

In this paper have been analyzed multifunctional sandwich panels with prismatic and truss cores. Their behavior have been compared with panels designed using honeycomb cores. The optimal dimensions and the minimum weight of sandwich panels with prismatic and truss cores have been evaluated and their mutual comparison. Non-dimensional expressions are obtained for combination of bending moment and shear force for both coress. A quadratic optimizer is used to ascertain the dimensions that minimize the panel weight. Honeycomb core panels are more weight efficient than prismatic and truss core panels at low load capacity. The benefit diminishes as the load increases. The large the yield strain of the material used to manufacture the panel, the grater the performance, and the larger the benefits of the prismatic core

Jelena M. Djoković, Ružica R. Nikolić and Jan Bujnak,
"FUNDAMENTAL PROBLEMS OF MODELING THE FRACTURE PROCESSES IN CONCRETE I: MICROMECHANICS AND LOCALIZATION OF DAMAGES",
Procedia Engineering (2013), pp. 186-195 ISSN: 1877-7058.
DOI information: 10.1016/j.proeng.2013.09.029

This paper presents an attempt to review some of the most important works in the area of modeling the fracture processes in concrete. Here are considered two out of several major issues the researchers are confronted with when studying this field: micromechanics and localization of damages leading to concrete fracture. From the very first works of Vile and Bažant, numerous scientists have studied the mentioned problems. Plethora of papers was published on defining the constitutive law for concrete, with more or less success. Works of Bažant, Stroeven, Ferretti and others are major contributions in understanding the fracture process initiation and development in concrete. The problem of damage localization-the strain softening is also a very important one. This process starts from the beginning of loading and progresses with load increase. The interactions of the preexisting cracks due to acting load are yet to be fully explained and understood. All the works published on those subjects until now are explaining some of the aspects in searching for the solution how to explain, understand and prevent the concrete fracture. The complete and final answer is not in sight. There are also other major issues to be discussed, like the size effect of concrete particles on the fracture process.

Jelena M. Djoković, Ružica R. Nikolić and Jan Bujnak,
" FUNDAMENTAL PROBLEMS OF MODELING THE FRACTURE PROCESSES IN CONCRETE II: SIZE EFFECT AND SELECTION OF THE SOLUTION APPROACH",
Procedia Engineering (2013), pp. 196-205, ISSN: 1877-7058.
DOI information: 10.1016/j.proeng.2013.09.030

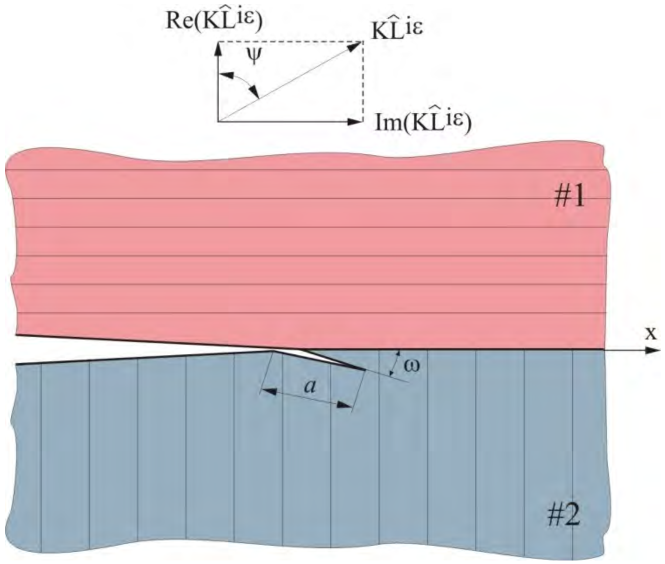
This paper presents the second part of the review of the most important works in the area of modeling the fracture processes in concrete. Here are considered two out of several major issues the researchers are confronted with when studying this field: size effect and scaling laws and phenomenological versus micromechanical approach to solving the fracture processes in concrete. Works of Vile, Bažant, Stroeven, Ferretti and others represent major contributions in understanding the fracture process initiation and development in concrete. All the works published on those subjects until now are explaining some of the aspects in searching for the solution how to explain, understand and prevent the concrete fracture. The complete and final answer is still not in sight.

Jelena M. Djokovic,

"CRACK KINKING IN MATERIALS WITH THE PERPENDICULAR ANISOTROPY",

Proceedings 45-th International October Conference on Mining and Metallurgy, IOC 2013,
16-19 October 2013, Bor Lake, Bor (Serbia), pp.45-48, ISBN 978-86-6305-012-9

In this work an attempt was made to analyze the crack kinking away from the interface between the two different anisotropic materials. The attention was focused on the kinking initiation and on the condition that the length of the segment, which is leaving the interface, is small with respect to the crack segment that remains on the interface. Based on this analysis, the stress intensity factors were obtained as well as the energy release rate for the kinking crack in terms of the corresponding variables for the crack prior to kinking



The practical use of this analysis is for the interface between the glued layers and also in application of the various protective coatings on metals. Certain special cases were considered like the crack kinking in orthotropic materials with mutually perpendicular main axes. The competition between the crack kinking away from the interface or propagating along it is measured by the ratio of the corresponding "driving forces", G^S and G.

The average energy needed for the crack to kink away from the interface increa-ses abruptly with the change of the load phase angle and then it drops. The influence of the anisotropy, present within the two dimensionless parameters λ and ρ, is that the kinking will be easier, i.e., it is easier for the crack to kink away from the interface into the "softer" of the two materials.

The role of parameters α and β, which reflect the mismatch of the elastic characteristics of the two materials, is shown in this paper. The parameter α measures the relative stiffness of the two materials, while the parameter β defines the oscillatory nature of the field around the crack tip. For the majority of the practical problems, the coefficient β ranges between -0.25 and +0.25 and its influence on the crack behavior can be neglected. The influence of the parameter α, on the other hand, is far more prominent. For the bimaterial combinations shown in this paper the highest probability that the crack would kink away from the interface is for the parameters combination α = 0.5 and β = 0, while the lowest probability of kinking is for the case α = -0.5 and b = 0. The load phase angle is based on the reference length which is being chosen in such a manner that its value with respect to kinking length a. The ratio G^S / G versus the load phase angle decreases with decrease of α, with β being constant.

NUMERICKA PROCENA VEKA ELEMENATA KONSTRUKCIJA

- analiza ponašanja konstrukcija pod dejstvom cikličnih opterećenja
- primena numeričkih metoda u razvoju aplikativnog softvera za procenu veka elemenata konstrukcija
- poređenje rezultata proračuna sa eksperimentalno dobijenim rezultatima

A1 Boljanović S., Maksimović S. ***Mixed mode crack growth simulation with/without overloads*****, International Journal of Fatigue ISSN 0142-1123. DOI:http://dx.doi.org/10.1016/j.ijfatigue.2013.11.011. (IF = 1.976 (1.974)). M_{21}= 8**

A2 Boljanović S., Maksimović S. ***Fatigue crack growth modeling of attachment lugs*****,**
International Journal of Fatigue 58(1), 2014, ISSN 0142-1123, pp. 66-74. (IF = 1.976 (1.974)). M_{21}= 8

A3 Boljanović S., Maksimović S. ***Analysis of the crack growth propagation process under mixed mode loading*****. Engineering Fracture Mechanics 78(8), May 2011, ISSN 0013-7944, pp.1565-1576. (2011-IF = 1.353 (1.776)).**
M_{21}= 8

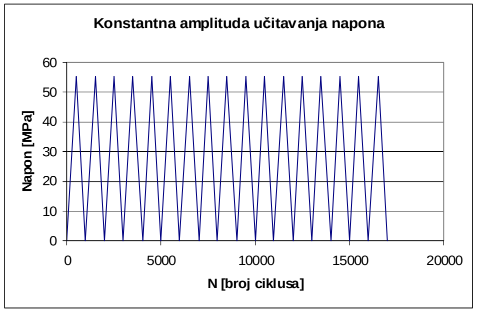
Radovi štanpani u vodećim časopisima nacionalnog značaja (M_{51})

B1 Boljanović S., Maksimović S., Carpinteri A. *Fatigue life evaluation of damaged aircraft lugs*. Scientific Technical Review 63(4), 2013, ISSN 1820- 0206, pp.3-9. $M_{51} = 2$

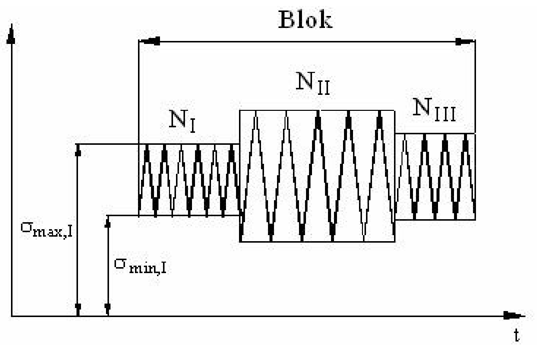
B2 Boljanović S. *Fatigue strength analysis of a semi-elliptical surface crack*, Scientific Technical Review 62(1), 2012, ISSN 1820- 0206, pp. 10-16. $M_{51} = 2$

B3 Boljanović S. Maksimović, S., Carpinteri A. *An analysis of crack propagation and a plasticity-induced closure effect*. Scientific Technical Review 60(2), 2010, ISSN 1820- 0206, pp. 14-19. $M_{51} = 2$

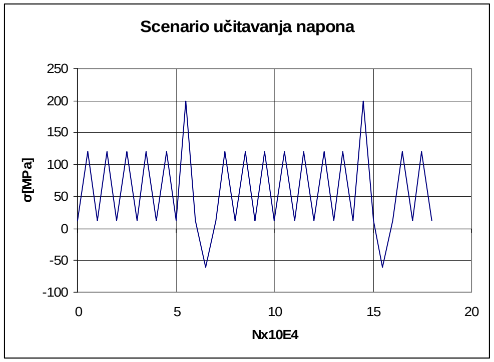
Po svojoj prirodi ciklična opterećenja, bar kada se radi o opisivanju ponašanja aviona u letu, najčešće mogu biti razmatrana kao:



- opterećenja konstantne amplitude



- opterećenja promenljive amplitude u vidu "stepenastog" spektra opterećenja



- oprerećenja promenljive amplitude sa pojedinačnim "pikovima",tj. ciklusima sa povećanim ili smanjenim intezitetima opterećenja

NUMERIČKE METODE ZA PROCENU VEKA ELEMENATA KONSTRUKCIJA DO POJAVE INICIJALNOG OŠTEĆENJA

Krive malociklusnog zamora

- Morrow-ova kriva malociklusnog zamora je oblika

$$\frac{\Delta}{2} = \frac{{}'_{f} - {}_{m}}{E} N_f^b + {}'_{f}\, N_f^c$$

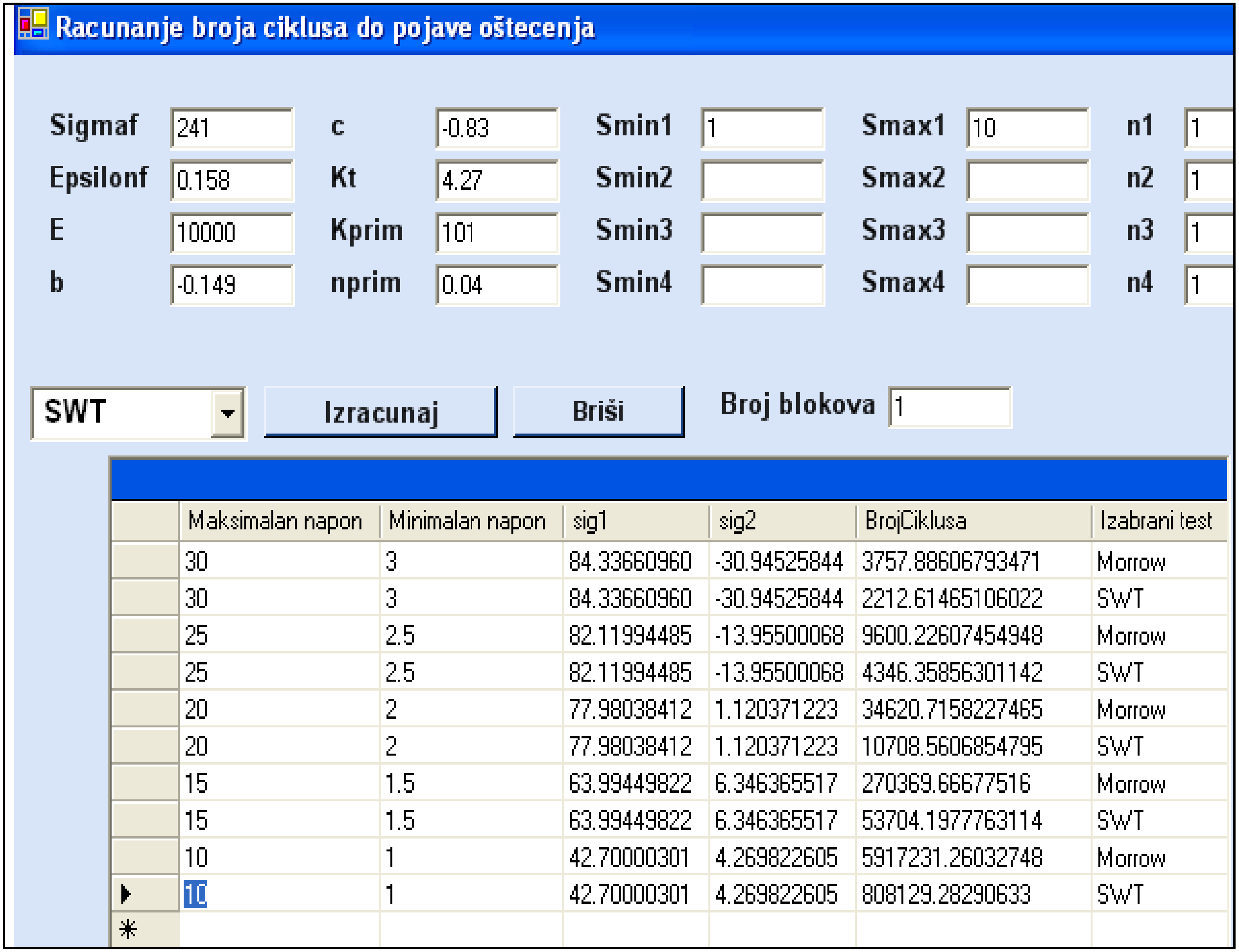
- Manson-Halford-ova kriva malociklusnog zamora:

$$\frac{\Delta}{2} = \frac{{}'_{f} - {}_{m}}{E} N_f^b + \left(\frac{{}'_{f} - {}_{m}}{{}'_{f}}\right)^{\frac{c}{b}} {}'_{f}\, N_f^c$$

- Smith-Watson-Topper -ova kriva malociklusnog zamora

$$P_{SWT} = \sqrt{{}_{max}\frac{\Delta}{2}E} = \sqrt{\left({}'_{f}\right)^2 \left(N_f\right)^{2b} + E\, {}'_{f}\, {}'_{f} \left(N_f\right)^{b+c}}$$

$${}_{max} = {}_{m} + \frac{\Delta}{2}$$



Maksimalan napon	Minimalan napon	sig1	sig2	BrojCiklusa	Izabrani test
30	3	84.33660960	-30.94525844	3757.88606793471	Morrow
30	3	84.33660960	-30.94525844	2212.61465106022	SWT
25	2.5	82.11994485	-13.95500068	9600.22607454948	Morrow
25	2.5	82.11994485	-13.95500068	4346.35856301142	SWT
20	2	77.98038412	1.120371223	34620.7158227465	Morrow
20	2	77.98038412	1.120371223	10708.5606854795	SWT
15	1.5	63.99449822	6.346365517	270369.66677516	Morrow
15	1.5	63.99449822	6.346365517	53704.1977763114	SWT
10	1	42.70000301	4.269822605	5917231.26032748	Morrow
10	1	42.70000301	4.269822605	808129.28290633	SWT

Program za računanje broja ciklusa do pojave oštećenja omogućava korisniku izbor kriterijuma za procenu veka. Ponuđeni su Morrow-ov I SWT kriterijum.

Rezultati računanja broja ciklusa pri širenju prskotine na epruveti **P1-CCT-2219** sa otvorom koji ima jednu prskotinu

Direktno rešavanje diferencijalne jednačine

N	DeltaSigma	DeltaK	Kmax	a
800	55.16	140.38106221966	140.38106221966	2.06166767103115
1600	55.16	142.5355700402	142.5355700402	2.12543645963429
2400	55.16	144.727144432275	144.72714443227	
3200	55.16	146.956426708168	146.95642670816	
4000	55.16	149.224069233573	149.22406923357	
4800	55.16	151.530735618326	151.53073561832	
5600	55.16	153.877100910438	153.87710091043	
6400				
7200				
8000				
8800				
9600				
10400				
11200				
12000				
12800				
13600				
14400				
15200				
16000				
16800				
17600				

NUkupno	a0	a1
1282.95554721385	2	2.1
2502.7314072725	2.1	2.2
3665.18401541138	2.2	2.3
4775.39612887147	2.3	2.4
5837.80691121492	2.4	2.5
6856.31583815177	2.5	2.6
7834.36649258534	2.6	2.7
8775.01473865834	2.7	2.8
9680.98463842215	2.8	2.9
10554.7146599741	2.9	3
11398.3961289534	3	3.1
12214.0054328053	3.1	3.2
13003.3311556866	3.2	3.3
13767.9970709697	3.3	3.4
14509.4817266175	3.4	3.5
15229.1352109794	3.5	3.6
15928.1935717826	3.6	3.7
16607.7912712171	3.7	3.8
17268.9719891417	3.8	3.9
17912.698030147	3.9	4

P1-CCT-2219-T851-Njutnovo pravilo za integraciju

Ulazni podaci

E	71000	In'	3.067	W	26
SigmaF	613	Psi	0.95152	a0	2
EpsilonF'	0.35	DeltaKth0	8	SigmaMax	55.16
n'	0.121	Kc	380	SigmaMin	0
Korak:	0.1	Izracunaj integral	Brisi	b	6

Rezultati

DeltaSigma	55.16
R	0
ac	4
C1	4.47554740288410722 69E-09
Konstanta	8

NUkupno	a0	a1	a/b	KI	KImax	Alfa
0	2	2.1	0.333333333333333	138.26561562864	138.265615628646	1.04104115226337
1316.72246764456	2.1	2.2	0.35	141.68009559718	141.680095597183	1.0428381729375
2567.04006539822	2.2	2.3	0.366666666666666	145.01420106901	145.014201069017	1.04411495130453
3757.24753992615	2.3	2.4	0.383333333333333	148.27335409622	148.273354096222	1.04509312497248
4892.79025502987	2.4	2.5	0.4	151.46239321429	151.462393214296	1.045981968
5978.40990586784	2.5	2.6	0.416666666666666	154.58565774825	154.585657748255	1.04697600003215
7018.26034244001	2.6	2.7	0.433333333333333	157.64705707598	157.647057075985	1.04825259543621
8016.00055821112	2.7	2.8	0.45	160.65012800485	160.650128004859	1.0499695924375
8974.87003251481	2.8	2.9	0.466666666666666	163.59808266369	163.598082663691	1.05226290225514
9897.75029176301	2.9	3	0.483333333333333	166.49384875872	166.493848758728	1.05524411823791
10787.2156028216	3	3.1	0.5	169.34010363098	169.340103630985	1.058998125
11645.5750186955	3.1	3.2	0.516666666666666	172.13930324307	172.139303243074	1.06358070755684
12474.907485585	3.2	3.3	0.533333333333333	174.89370698875	174.893706988758	1.06901616046091
13277.0913393467	3.3	3.4	0.55	177.60539903822	177.605399038227	1.0752948969375
14053.8292323637	3.4	3.5	0.566666666666666	180.27630679261	180.276306792619	1.08237105802058
14806.6693135058	3.5	3.6	0.583333333333333	182.90821691232	182.908216912321	1.09016012168853
15537.0233163017	3.6	3.7	0.6	185.50278929790	185.502789297906	1.098536512
16246.1820807452	3.7	3.8	0.616666666666666	188.06156933451	188.061569334518	1.10733120822968
16935.3289329735	3.8	3.9	0.633333333333333	190.58599865630	190.585998656302	1.11632935400412
17605.5512675243	3.9	4	0.65	193.07742464379	193.07742464379	1.1252678664375

$$\int_{x_0}^{x_3} f(x)dx = \frac{3h}{8}(f(x_0)+3f(x_1)+3f(x_2)+f(x_3)) - \frac{3h^5}{80} f^{(4)}(\Xi);(x_0 < \Xi < x_3)$$

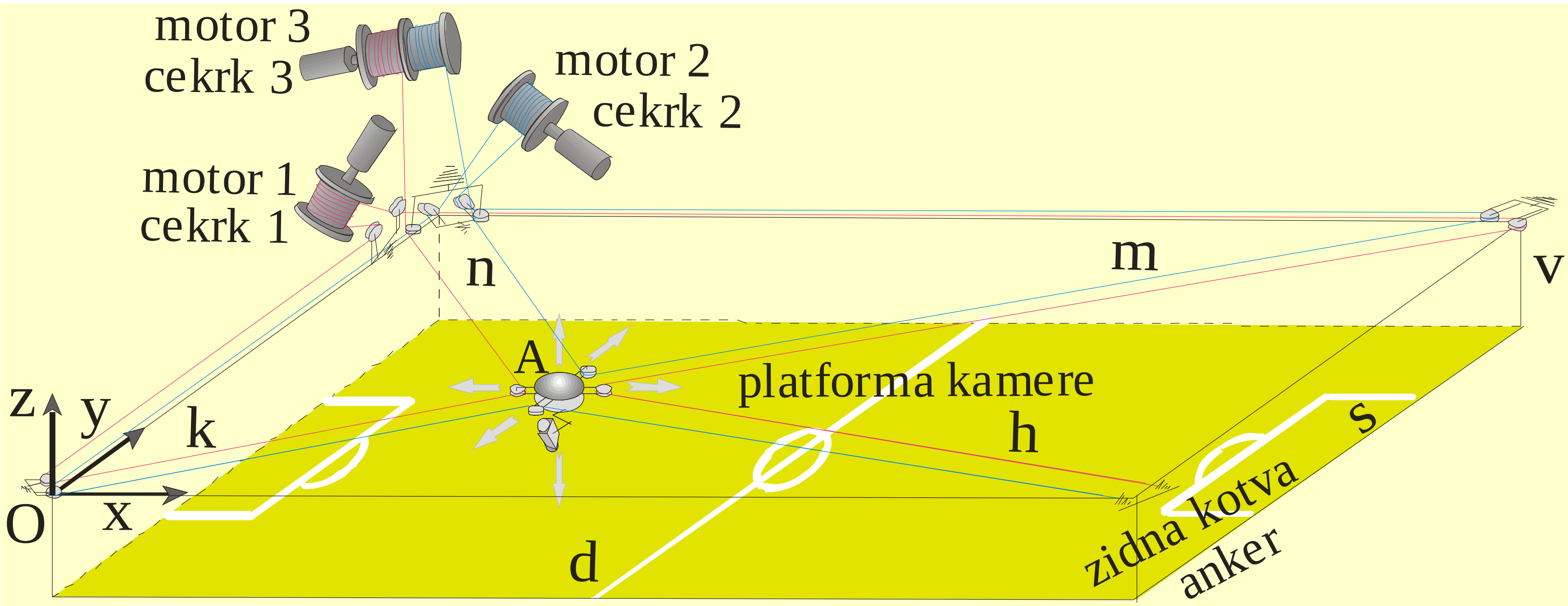
Gauss-Legendre-ova formula

Primer kablovski vođenog robotskog sistema: CABLE-SUSPENDED PARALLEL ROBOT TYPE C (CPR-C System)

www.zl50.com/Article.aspx?wd=Off...

Ljubinko Kevac, Innovation Center of School of Electrical Engineering, Bulevar Kralja Aleksandra 73, 11000 Belgrade, Serbia, ljubinko.kevac@ic.etf.rs, ljubinko.kevac@gmail.com,

Mirjana Filipovic, Mihajlo Pupin Institute, The University of Belgrade, Volgina 15, 11060 Belgrade, Serbia, mira@robot.imp.bg.ac.rs



PROSTORNI IZGLED

Matematički model sistema CPR-C

$\boldsymbol{p} = \begin{bmatrix} \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} \end{bmatrix}^{\boldsymbol{T}}$ spoljašnje koordinate, pozicije kamere u 3D prostoru

$\boldsymbol{\Phi} = \begin{bmatrix} {}_{1} & {}_{2} & {}_{3} \end{bmatrix}^{\boldsymbol{T}}$ unutrašnje koordinate, angularne pozicije motora

- potrebno je naći vezu između unutr. i spolj. koord. što daje Jakobijeva matrica – vidi se da je matrica puna, što znači da postoji veliko kuplovanje između unutr. i spolj. koordinata

$$\dot{\boldsymbol{\Phi}} = \boldsymbol{J_c} \cdot \dot{\boldsymbol{p}}$$

$$\boldsymbol{J_c} = \begin{bmatrix} \boldsymbol{J}_{\boldsymbol{c}11} & \boldsymbol{J}_{\boldsymbol{c}12} & \boldsymbol{J}_{\boldsymbol{c}13} \\ \boldsymbol{J}_{\boldsymbol{c}21} & \boldsymbol{J}_{\boldsymbol{c}22} & \boldsymbol{J}_{\boldsymbol{c}23} \\ \boldsymbol{J}_{\boldsymbol{c}31} & \boldsymbol{J}_{\boldsymbol{c}32} & \boldsymbol{J}_{\boldsymbol{c}33} \end{bmatrix}$$

Matematički model sistema CPR-C

$$E_k = \frac{1}{2}\cdot m\cdot \dot{x}^2 + \frac{1}{2}\cdot m\cdot \dot{y}^2 + \frac{1}{2}\cdot m\cdot \dot{z}^2 \quad E_p = m\cdot g\cdot z$$

- Kinetička i potencijalna energija

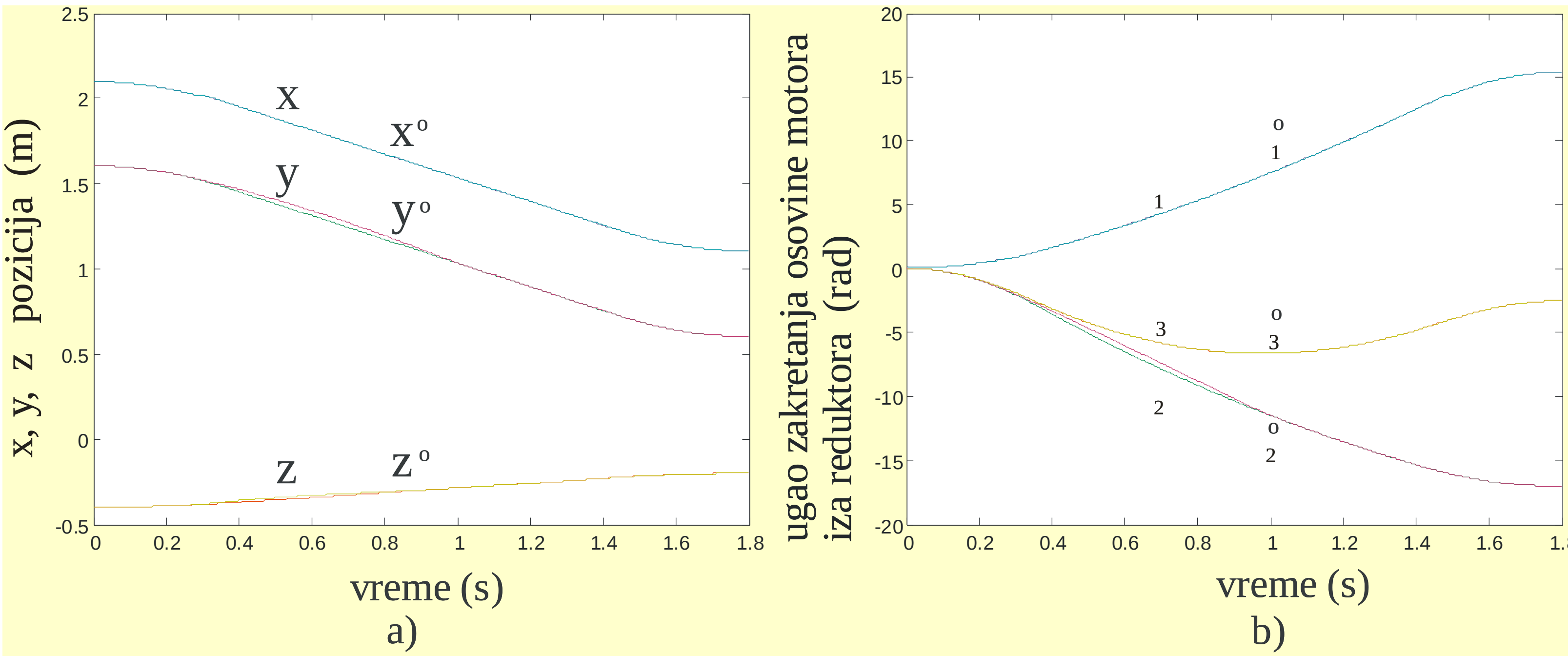
$$u = G_v \cdot \ddot{\Phi} + L_v \cdot \dot{\Phi} + S_v \cdot R \cdot C_c \cdot F$$

- Dinamički model sistema, gde su: u-vektor napona na motorima (upravljanja), Gv – dijagonalna matrica inercije tri motora, Lv – dijagonalna matrica prigušenja tri motora, Sv – dijagonalna matrica geometrijski karakteristika, F – spoljašnje sile, R – vektor poluprečnika tri čekrka, Cc – matrica koja povezuje spoljašnje i rezultantne sile.

$$u_i = K_{lpi}\cdot(\ {}^{o}_{i} - \ {}_{i}) + K_{lvi}\cdot(\dot{\ }{}^{o}_{i} - \dot{\ }{}_{i})$$

- Za proračun upravljačkih signala se koriste PD regualtori

Simulacioni rezultati



referentna i realna trajektorija

$$\boldsymbol{p}^{o}_{start} = [2.1 \quad 1.6 \quad -0.4](\boldsymbol{m})$$

početna tačka

$$\boldsymbol{p}^{o}_{end} = [1.1 \quad 0.6 \quad -0.2](\boldsymbol{m})$$

ciljana tačka

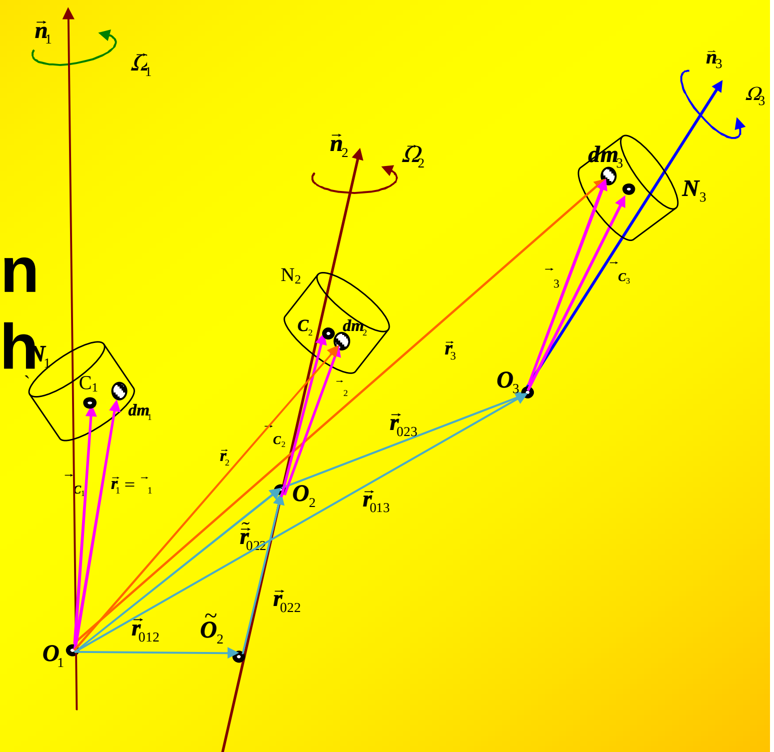
Katica R. (Stevanović) Hedrih,

Marija B. Stamenković ,

Mass moment vector application to the multi body dynamics with coupled rotations about no intersecting axes

GAMM 2013 NOVI SAD

March 18-22

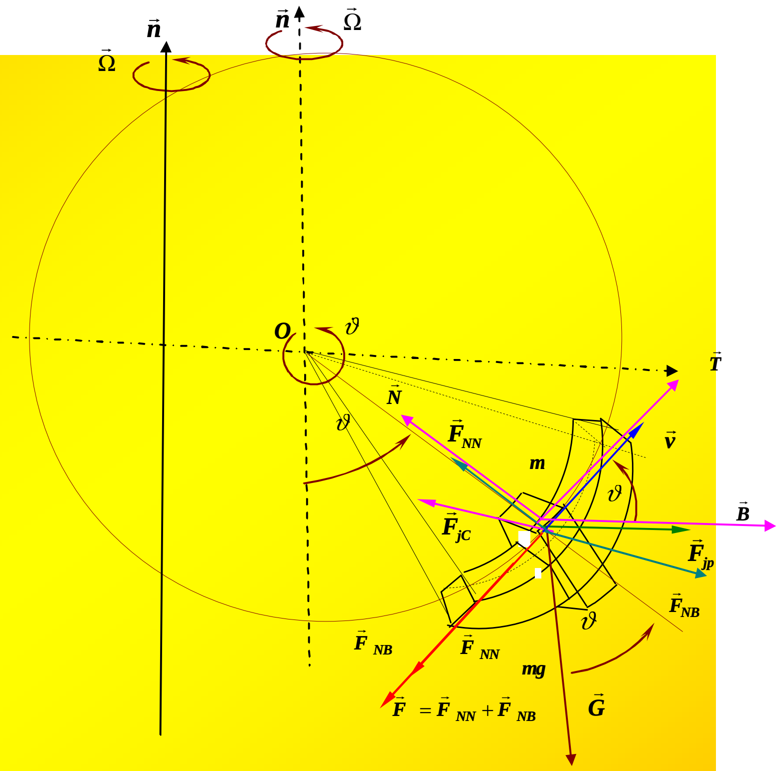


- Систем са коначним бројем крутих тела налазе се на осама које се не секу у односу једна на другу.

- Изводи по времену количине кретања и момента количине кретања, коришћењем вектора момента масе и вектор ротаторе за пол и осу су одређени.

- Дефинисан је број теорема.

Marija B. Stamenković , Marija D. Mikić



4th International Congress of Serbian Society of Mechanics, IconSSM 2013

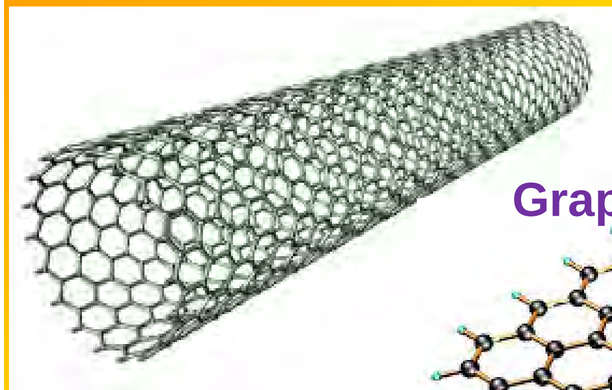
Testing of singularity and position of non-linear dynamics relative equilibrium of heavy material particle on eccentrically rotating rough circle line, with constant angular velocity

- Рад садржи аналитичко описивање материјалне честице која се креће по ротирајућој кружној храпавој линији, која ротира око вертикалне осе екцентрично постављене у односу на центар кружне линије на растојању е, угаоне брзине Ω.

- У раду се користи софтвер GeoGebra и врши се тестирање сингуларитета и положаја нелинеарне динамике релативне равнотеже тешке материјалне тачке на ексцентрично ротирајућој храпавој кружној линији.

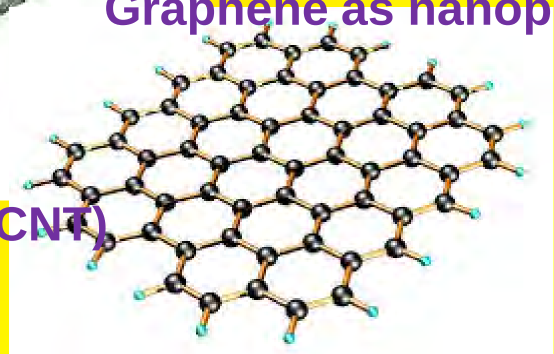
Истраживачи-сарадници **Данило Карличић**, **Милан Цајић**

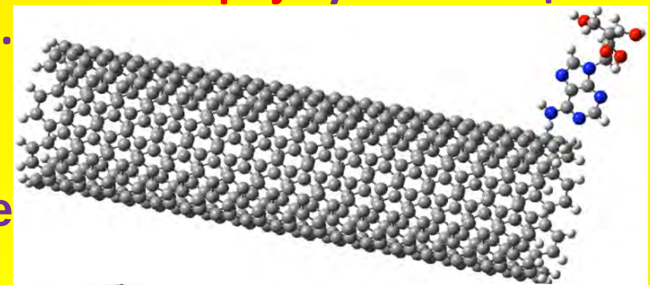
•**Истраживања у области примене аналитичких метода линеарне и нелинеарне механике и Ерингенове нелокалне теорије у механици нано-структура.**

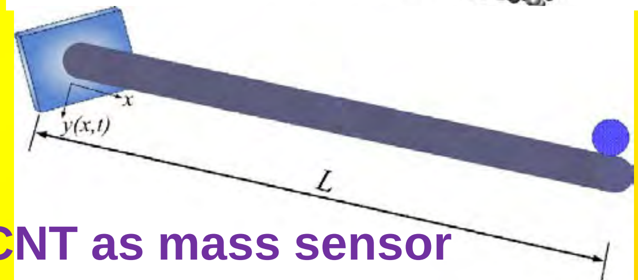


Carbon nanotube (CNT) as nanobeam

Graphene as nanoplate







CNT as mass sensor

•Прихваћена два коауторска рада за презентацију на 8th European Nonlinear Dynamics Conference, 6-11 jula, Beč.

•У првом раду под насловом „**Nonlocal axial vibration of a fractional order viscoelastic nanorod**“ примењује се фракциони модел вискоеластичности и нелокална теорија за испитивање слободних лонгитудиналних осцилација **једнозидне угљеничне наноцеви** (**SWCNT**)

•У другом раду „**Nonlinear vibration of nonlocal Kelvin-Voigt viscoelastic nanobeam embedded in elastic medium**“ испитују се слободне и принудне нелинеарне осцилације модела **наногреде** у медијуму са нелинеарним еластичним својствима.

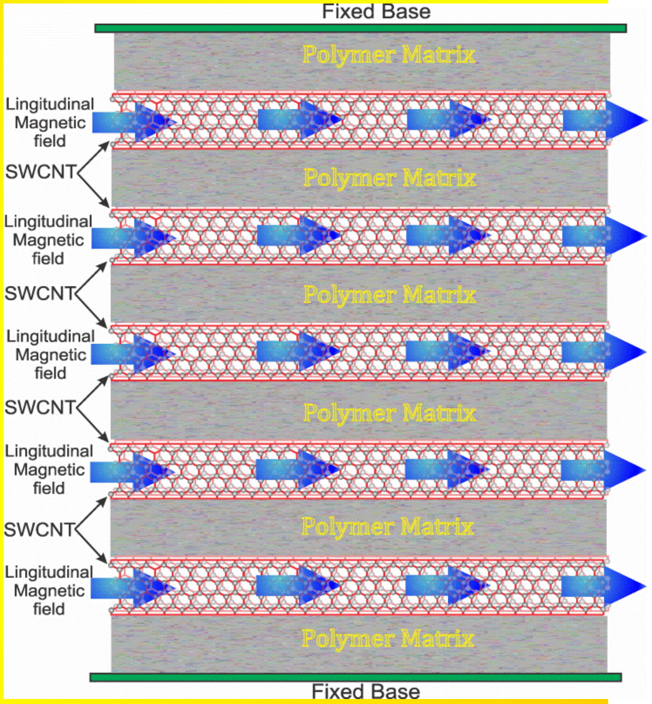
Данило Карличић, Милан Цајић

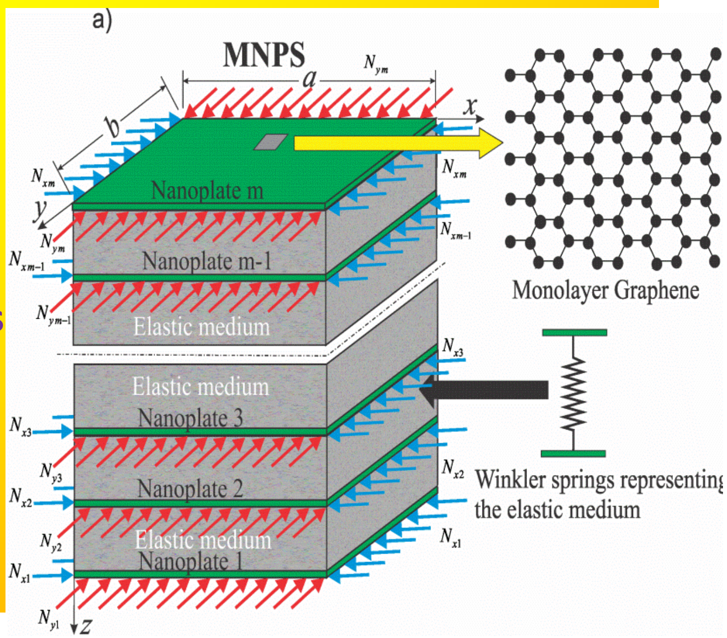
Међународна сарадња са проф. др. Sondipon Adhikari, Swansea University, др. Tony Murmu, University of the West of Scotland у 2013. и 2014. години и Xiao-Jun Yang, China University of Mining and Technology, Xuzhou у 2013. години.

Коауторски радови на рецензији:

- •Longitudinal vibration of a nonlocal viscoelastic double nanorod system coupled by a light viscoelastic layer
- •Nonlocal longitudinal vibration of a complex multi-nanorod system
- •Exact closed-form solution for non-local vibration and buckling of bonded multi-nanoplate-system
- •Dynamics of multiple viscoelastic carbon nanotube based nanocomposites with axial magnetic field
- •Fractional order spring-damper/actuator element in a multibody system: application of an expansion formula
- •Free transverse vibration of nonlocal viscoelastic orthotropic multi-nanoplate system embedded in a viscoelastic medium
- •Axial vibrations of a nonlocal viscoelastic coupled multi-nanorod system
- •Flexural vibration and buckling of single-walled carbon nanotubes using different gradient elasticity theories for Reddy and Huu-Thai beam theories

Часописи: European Journal of Mechanics, Journal of Applied Physics, Composites Part B, Composite Structures, Journal of Vibration and Control, International Journal of Engineering Science, Mechanics Research Communications





Публиковани радови и планови за будућа истраживања

Радови публиковани у **2013.** години у часописима **међународног значачаја**:

- Yang, X. J., Baleanub, D., Lazarević, M. P., & **Cajić, M. S.** (2013). Fractal boundary value problems for integral and differential equations with local fractional operators. Thermal Science, (00), 103-103. **M23**
- Kozić, P., Pavlović, R., & **Karličić, D.** (2014). The flexural vibration and buckling of the elastically connected parallel-beams with a Kerr-type layer in between. Mechanics Research Communications, 56, 83-89. **M22**

Радови публиковани у **2013.** години у часописима **националног значаја**:

- **Cajić, M. S.**, D., Lazarević, M. P. (2014). Determination of joint reaction forces in rigid multibody system, two different approaches. Accepted for publication in FME Transactions, 42, 141-150. **M51**

Планови за даља истраживања:

Примена аналитичних и нумеричких метода у истраживању нелинеарних (осцилација) и мултифизичких феномена (утицај магнетноg поља, провођење топлоте, лом итд.) и механичких својстава нано-структура. Примена фракционог рачуна и нелокалне теорије у моделирању комплексних нано-структура. Нумеричка симулација стохастичких процеса и молекуларне динамике нано-структура.

ENOC 2014

8th European Nonlinear Dynamics Conference

July 6 – 11, 2014, Vienna, Austria

TU WIEN
TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

EUROPEAN MECHANICS SOCIETY

EUROPEAN MECHANICS SOCIETY

Spisak radova prihvacenih za ENOC 2014

ID: 326 Plenary session
Title: **Elements of mathematical phenomenology and qualitative /mathematical analogies on the basis of generalized Lissajous curves**
Katica (Stevanović) Hedrih, Juliajna Simonović, Ana Ivanvić-Šesić, Ljiljana Kolar Anić, Željko Cupić and Andjelka N Hedrih
Presenting Author: Katica (Stevanović) Hedrih

ID: 325 MS-09 Nonlinear Dynamics of Structural and Machine Elements
Title: **Petrović's theory of elements of mathematical phenomenology and phenomenological mapping applied to system nonlinear dynamics**
Authors: Katica (Stevanović) Hedrih and Andjelka N Hedrih, Presenting Author: Andjelka N Hedrih

ID: 237 MS-13 Nonlinear Dynamics in Biological Systems
Title: **Synchronization in oscillatory model of embryo's ZP molecules in context of polyspermy block**
Author(s) : Simonović Julijana; Hedrih, Andjelka , Presenting Author : Simonović Julijana

Contribution ID: 438 Type: MS-09 Nonlinear Dynamics of Structural and Machine Elements
Title: **Rigid Body Coupled Rotation around Axes without Intersection**
Author(s): Veljović, Ljiljana , Presenting Author : Veljović, Ljiljana

Contribution ID: 223 Type : GT General Track Contribution
Title : **Nonlinear vibration of nonlocal Kelvin-Voigt viscoelastic nanobeam embedded in elastic medium**
Author(s): Karličić, Danilo Z.; Cajić, Milan S.; Stamenković, Marija, Presenting Author: Karličić Danilo

Contribution ID: 271 Type: MS-06 Fractional Derivatives
Title: "**Nonlocal axial vibration of a fractional order viscoelastic nanorod**"
Authors: Milan S. Cajić, Danilo Z. Karličić and Mihailo P. Lazarević , Presenting Author: Milan S. Cajić,

ID: 199
Title: **The complex motion of Cable-suspended parallel robot under the influence of the disturbance**
Author(s): Kevac :jubomlp and Mprjana Filipović
Presenting Author: Kevac Ljubinko

ID: 386 Poster
Title: **Three parametric testing of singularity and position of non-linear dynamics relative equilibrium of heavy material particle on eccentrically rotating rough circle line, with constant angular velocity**
Authors: Marija Mikić and Marija Stamenković

8th European Nonlinear Dynamics Conference
July 6 – 11, 2014, Vienna, Austria

ENOC 2014

TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

Spisak radova prihvacenih za ENOC 2014

ID: 326 **Plenary session**
Title: **Elements of mathematical phenomenology and qualitative /mathematical analogies on the basis of generalized Lissajous curves**
Katica (Stevanović) Hedrih, Juliajna Simonović, Ana Ivanvić-Šesić, Ljiljana Kolar Anić, Željko Cupić and Andjelka N Hedrih
Presenting Author: Katica (Stevanović) Hedrih

ID: 325 MS-09 Nonlinear Dynamics of Structural and Machine Elements
Title: **Petroviċ's theory of elements of mathematical phenomenology and phenomenological mapping applied to system nonlinear dynamics**
Authors: Katica (Stevanović) Hedrih and Andjelka N Hedrih
Presenting Author: Andjelka N Hedrih

Spisak radova prihvacenih za ENOC 2014

ID: 237 MS-13 Nonlinear Dynamics in Biological Systems
Title: **Synchronization in oscillatory model of embryo's ZP molecules in context of polyspermy block**
Author(s) : Simonović Julijana; Hedrih, Andjelka
Presenting Author : Simonović Julijana

Contribution ID: 438 Type: MS-09 Nonlinear Dynamics of Structural and Machine Elements
Title: **Rigid Body Coupled Rotation around Axes without Intersection**
Author(s): Veljović, Ljiljana
Presenting Author : Veljović, Ljiljana

Contribution ID: 223 Type : GT General Track Contribution
Title : **Nonlinear vibration of nonlocal Kelvin-Voigt viscoelastic nanobeam embedded in elastic medium**
Author(s): Karličić, Danilo Z.; Cajić, Milan S.; Stamenković, Marija
Presenting Author: Karličić Danilo

Spisak radova prihvacenih za ENOC 2014

Contribution ID: 271 Type: MS-06 Fractional Derivatives
Title: "**Nonlocal axial vibration of a fractional order viscoelastic nanorod**"
Authors: Milan S. Cajić, Danilo Z. Karličić and Mihailo P. Lazarević
Presenting Author: Milan S. Cajić,

ID: 199
Title: **The complex motion of Cable-suspended parallel robot under the influence of the disturbance**
Author(s): Kevac :jubomlp and Mprjana Filipović
Presenting Author: Kevac Ljubinko

ID: 386 Poster
Title: **Three parametric testing of singularity and position of non-linear dynamics relative equilibrium of heavy material particle on eccentrically rotating rough circle line, with constant angular velocity**
Authors: Marija Mikić and Marija Stamenković

Mathematical Institute SANU

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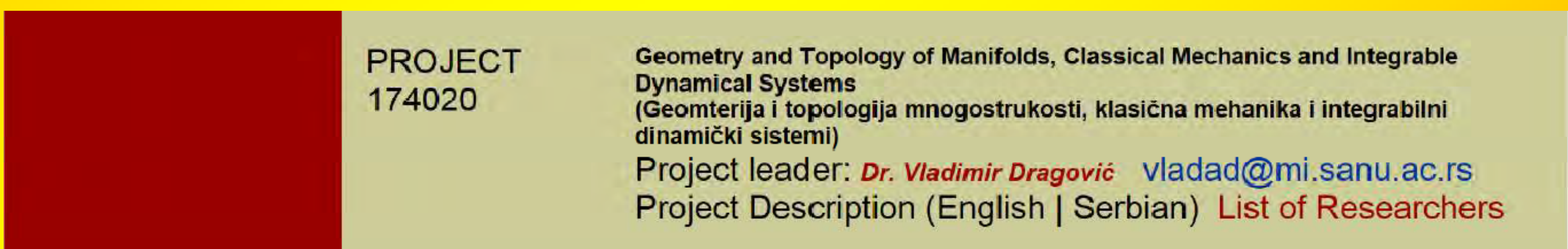
Current Projects (2011–2014)
supported by the Ministry of Science, Technology and Development, Republic of Serbia

Fundamental Research | Interdisciplinary | Technological Development | Past Projects

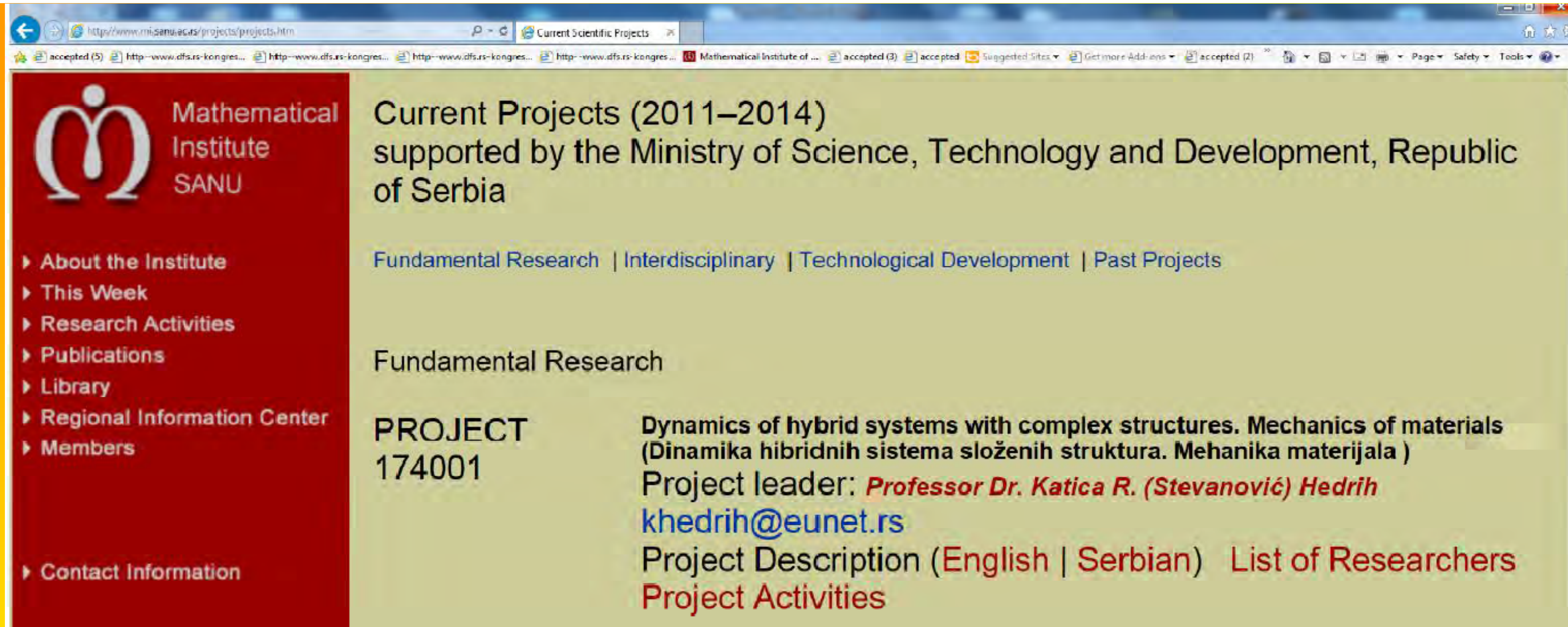
Fundamental Research

PROJECT 174001	**Dynamics of hybrid systems with complex structures. Mechanics of materials (Dinamika hibridnih sistema složenih struktura. Mehanika materijala)** Project leader: ***Professor Dr. Katica R. (Stevanović) Hedrih*** khedrih@eunet.rs Project Description (English | Serbian) List of Researchers Project Activities



PROJECT 174020	**Geometry and Topology of Manifolds, Classical Mechanics and Integrable Dynamical Systems (Geomterija i topologija mnogostrukosti, klasična mehanika i integrabilni dinamički sistemi)** Project leader: ***Dr. Vladimir Dragović*** vladad@mi.sanu.ac.rs Project Description (English | Serbian) List of Researchers

Projekat ON 174005 **Viskoelasticnost frakcionog tipa I optimizacija oblika u teorji stapova**
Viscoelasticity fractional type and optimization in theory of rods
Project Leader Teodor Atanackovic in 2013
Project Leader in 2014 Dusan Zorica



Radovi u 2013 vidljiva na KoBSON-u

Autori Hedrih (Stevanovic) Katica R

44 rada

M23=3 plus M24=3/2 Plus M21=8/2 Plus 3 rada u medjunarodnim casopisim koji nisu vidljivi na KOBSONU 2 troautorska I jedan I dva u PAMM

Suma 14(8,5) u 2013 na KoBSON-u

Plus chapter in Book , [;us drugi nevidljivi na Kobsonu

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Acknowledgement

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Grant OI174001 "Dynamics of hybrid systems with complex structures. Mechanics of Materials"

and Faculty of Mechanical Engineering University of Niš.

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