Spectral graph theory

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Spectral graph theory is a fast developing field in modern discrete mathematics with important applications in computer science, chemistry and operational research. By merging combinatorial techniques with algebraic and analytical methods it creates new approaches to hard discrete problems and gives new insights in classical Linear Algebra. The proposed minisymposium will bring together leading researchers on graph spectra to present their recent results and to discuss new achievements and problems. This meeting will further increase collaboration and boost the development of the field.

The structure of graphs with small M-indices

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Tue 17:10, Room C

In this talk we consider simple graphs and as graph matrices the adjacency matrix $A(G)$, the Laplacian matrix $L(G) = D(G) - A(G)$ and the signless Laplacian matrix $Q(G) = D(G) + A(G)$. Let M-index be the largest eigenvalue of G with respect to the graph matrix M.

By synthesizing the results of [1,2,3], we show that almost all graphs whose A-index does not exceed $\sqrt{2+\sqrt{5}}$ are graphs whose $\{L, Q\}$-index does not exceed $2+\epsilon$, where $2+\epsilon \approx 4.38298$ is the real root of $x^3 - 6x^2 + 8x - 4$. Furthermore we consider the analogy between the structure of graphs whose A-index does not exceed $\frac{3}{2}\sqrt{2}$ and the structure of graphs whose $\{L, Q\}$-index does not exceed 4.5.

Finally, we discuss the analogies between $\sqrt{2+\sqrt{5}}$ (or $\frac{3}{2}\sqrt{2}$) as limit point for the index in the A-theory and $2+\epsilon$ (resp. 4.5) as limit point for the index in the $\{L, Q\}$-theory of graph spectra.

[1] F. Belardo, E.M. Li Marzi, S.K. Simić, Ordering graphs with index in the interval $(2, \sqrt{2+\sqrt{5}})$, Discrete Applied Math., 156/10 (2008), pp. 1670–1682.
[2] J.F. Wang, Q.X. Huang, F. Belardo, E.M. Li Marzi, On graphs whose signless Laplacian index does not exceed 4.5, Linear Algebra Appl., 431 issues 1–2 (2009), pp. 162–178.
[3] J.F. Wang, F. Belardo, Q.X. Huang, E.M. Li Marzi, On graphs whose Laplacian index does not exceed 4.5, submitted.

Graphs of given order and size and minimal algebraic connectivity

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Wed 11:00, Room C

We investigate the structure of connected graphs that have minimal algebraic connectivity among all graphs with given number of vertices and edges. It has been conjectured [1] that such graphs are so called path-complete graphs. In this talk we show that the concept of *geometric nodal domains* can be used to derive some necessary conditions on the structure of graphs which have minimal algebraic connectivity. In particular we show that such extremal graphs consists of a chain of complete graphs which cannot have to many big cliques.

[1] S. Belhaiza et al., Variable neighborhood search for extremal graphs. XI. Bounds on Algebraic Connectivity, pp.1–16. In: D. Avis et al., Graph Theory and Combinatorial Optimization, New York, 2005. DOI: 10.1007/0-387-25592-3_1

Joint work with J. Leydold (WU Vienna, Austria)

Graph Eigenvalues in Combinatorial Optimization

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A number of remarkable spectral properties of graphs with applications in combinatorial optimization are surveyed and a few additional ones are introduced. Namely, several spectral bounds on the clique number, stability number, and chromatic number of graphs are analyzed and spectral graph tools for deciding about the existence of particular combinatorial structures (as it is the case of dominating induced matchings) are presented.

Decompositions of complete hypergraphs

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A classical result of Graham and Pollak states that the minimum number of complete bipartite subgraphs that partition the edges of a complete graph on n vertices is n-1. In this talk, I will describe our attempts at proving a natural hypergraph version of Graham-Pollak's theorem.

[1] S. M. Cioabă, A. Kündgen and J. Verstraëte, On decompositions of complete hypergraphs, *J. Combin. Theory, Series A* Volume 116, Issue 7, October 2009, Pages 1232-1234.

Joint work with Andre Kündgen (Cal State San Marcos) and Jacques Verstraëte(UC San Diego).

Some topics on integral graphs

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The M-spectrum of a graph is the spectrum of a graph matrix M (adjacency matrix A, Laplacian L, signless Laplacian Q, etc.). A graph is called *M-integral* if its M-spectrum consists entirely of integers. If the matrix M is fixed, we say, for short, *integral* instead of M-integral. A graph which is A-, L- and Q-integral is called *ALQ-integral*. A survey on integral graphs can be found in [1].

Integral graphs have recently found some applications in quantum computing, multiprocessor systems and chemistry.

Let G be a graph with the largest A-eigenvalue λ_1 and the diameter D. The quantity $(D+1)\lambda_1$ is called the *tightness* of G and is denoted by $t(G)$. There are exactly 69 non-trivial connected graphs G with $t(G) \leq 9$ and among them 14 graphs are A-integral [2]. We present a classification of A-integral graphs G with $t(G) < 24$.

In integral graphs on n vertices there exist sets of n independent integral eigenvectors. Such sets can be constructed using star partitions of graphs and can be useful in treating the load balancing problems in multiprocessor systems and some problems in combinatorial optimization.

Some results on L- and ALQ-integral graphs are presented as well.

[1] K. Balińska, D. Cvetković, Z. Radosavljević, S. Simić, D. Stevanović, A survey on integral graphs, Univ. Beograd, Publ. Elektrotehn. Fak., Ser. Mat., **13**(2002), 42-65.
[2] D. Cvetković, T. Davidović, Multiprocessor interconnection networks with small tightness, Internat. J. Foundations Computer Sci., 20(2009), No. 5, 941-963.

Constructing infinite families of ALQ-integral graphs
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Tue 15:25, Room C

Let $G = (V, E)$ be a simple graph on n vertices and $D = diag(d_1, \ldots, d_n)$ be the diagonal matrix of its vertex degrees. Let A be the *adjacency*, $L = A - D$ the *Laplacian* and $Q = A + D$ the *signless Laplacian* matrices of G. Since 1974, when Harary and Schwenk posed the question *Which graphs have integral spectra?* [1], the search for graphs whose adjacency eigenvalues or Laplacian eigenvalues are all integers (here called *A-integral graphs* and *L-integral graphs*, respectively) has been on. More recently, *Q-integral graphs* (graphs whose signless Laplacian spectrum consists entirely of integers) were introduced in the literature [2–6]. It is known that these three concepts coincide for regular graphs. Also, for bipartite graphs, L-integral and Q-integral graphs are the same. A graph is called ALQ-integral graph if it is simultaneously an $A-$, $L-$ and $Q-$integral graph. Among all 172 connected Q-integral graphs up to 10 vertices, there are 42 ALQ-integral graphs, but only one of them is neither regular and nor bipartite [4]. Our aim is to show how to construct infinite families of non regular and non bipartite graphs but all of them ALQ-integral graphs.

[1] F. Harary, A.J. Schwenk, *Which graphs have integral spectra?*, in: R. Bari, F. Harary (Eds.), “Graphs and Combinatorics”, Lecture Notes in Mathematics, vol. 406, Springer, Berlin, 1974, pp. 45-51.
[2] D. Cvetković, P. Rowlinson, S. Simić, *Signless Laplacian of finite graphs*, Linear Algebra and its Applications 423 (2007) 155-171.
[3] S. Simić, Z. Stanić, *Q-integral graphs with edge-degrees at most five*, Discrete Math. 308 (2008) 4625-4634.
[4] Z. Stanić, *There are exactly 172 connected Q-integral graphs up to 10 vertices*, Novi Sad J. Math. 37 n. 2 (2007) 193-205.
[5] Z. Stanić, *Some results on Q-integral graphs*, Ars Combinatoria 90 (2009), 321-335.
[6] M.A.A. de Freitas *et al.*, *Infinite families of Q-integral graphs,* Linear Algebra Appl. (2009), doi:10.1016/j.laa.2009.06.029

Joint work with M.A.A. de Freitas (Federal University of Rio de Janeiro), R.R. Del-Vecchio (Fluminense Federal University) and C.T.M. Vinagre (Fluminense Federal University)

Distance spectral radius of trees
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Wed 11:50, Room C

Distance energy is a newly introduced molecular graph-based analog of the total π-electron energy, and it is defined as the sum of the absolute eigenvalues of the distance matrix. For trees and unicyclic graphs, the distance energy is equal to the doubled value of the distance spectral radius (the largest eigenvalue of the distance matrix).

We introduce two general transformations that strictly increase and decrease the distance spectral radius and provide an alternative proof that the path P_n has maximal distance spectral radius, while the star S_n has minimal distance spectral radius among trees on n vertices. We prove that a caterpillar $C_{n,d}$, obtained from the path P_d with all pendant vertices attached at the center vertex of P_d, has minimal spectral radius among trees with n vertices and diameter d. In addition, we characterize n-vertex trees with given matching number m which minimize the distance spectral radius. The extremal tree $A(n, m)$ is a spur, obtained from the star S_{n-m+1} by attaching a pendant edge to each of certain $m-1$ non-central vertices of S_{n-m+1}.

In conclusion, we pose some conjectures concerning the extremal trees with minimum or maximum distance spectral radius based on the computer search among trees on $n \leq 24$ vertices.

Algebraic connectivity and vertex-deleted subgraphs
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Tue 16:45, Room C

Given an undirected graph G, its Laplacian matrix L can be written as $L = D - A$, where A is the $(0, 1)$ adjacency matrix for G, and D is the diagonal matrix of vertex degrees. The second smallest eigenvalue of L is known as the algebraic connectivity of G, and this quantity has been the subject of a good deal of work over the last several decades. In this talk, we will discuss some recent work relating the algebraic connectivity of a graph G to that of the graph formed from G by deleting a vertex and its incident edges.

Geometric nodal domains and extremal graphs with minimal k-th laplacian eigenvalue
J. LEYDOLD, WU Vienna, Austria
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A method for characterizing graphs that have smallest (or largest) Laplacian eigenvalue within a particular class of graphs works as following: Take a Perron vector, rearrange the edges of the graph and compare the respective Rayleigh quotients. By the Rayleigh-Ritz Theorem we can draw some conclusions about the change of the smallest eigenvalue. This approach, however, does not work for the k-th Laplacian eigenvalue, as now we have to use the Courant-Fisher Theorem that involves minimization of the Rayleigh quotients with respect to constraints that are hard to control. In this talk we show that sometimes we can get local properties of extremal graphs by means of the concept of *geometric nodal domains* and *Dirichlet matrices.* This is in particular the case for the algebraic connectivity.

Joint work with T. Bıyıkoğlu (Işık University, İstanbul)

A generalization of Fiedler's lemma and some applications
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In a previous paper, [1], a Fiedler's lemma introduced in [2] was used to obtain eigenspaces of graphs, and applied to graph energy. In this talk, this Fiedler's lemma is generalized and its generalization is applied to the determination of eigenvalues of graphs belonging to a particular family and also to the determinations of the graph energy (including lower and upper bounds).

[1] M. Robbiano, E. A. Martins and I. Gutman, Extending a theorem by Fiedler and applications to graph energy, MATCH Commun. Math. Comput. Chem. 64 (2010), 145-156.
[2] M. Fiedler, Eigenvalues of nonnegative symmetric matrices, Linear Algebra Appl. 9 (1974), 119-142.

Joint work with D.M. Cardoso (University of Aveiro), I. Gutman (University of Kragujevac) and Maria Robbiano (North Catholic University, Chile)

Forbidden subgraphs for some classes of treelike reflexive graphs

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Reflexive graphs are simple graphs whose second largest eigenvalue of $(0, 1)$ - adjacency matrix does not exceed 2. Treelike graph, or a cactus, is a graph in which any two cycles are edge disjoint. Several classes of treelike reflexive graphs have been characterized through sets of maximal graphs. This paper presents another possible approach to the characterization of such graphs, i.e. via corresponding sets of forbidden subgraphs, and gives such sets for some classes of treelike reflexive graphs.

[1] V. Brankov, D. Cvetković, S. Simić, D. Stevanović: Simultaneous editing and multilabelling of graphs in system newGRAPH, Univ. Beograd, Publ. Elektrotehn. Fak., Ser. Mat. 17, pp. 112-121, 2006.
[2] D. M. Cvetković, M. Doob, H. Sachs: Spectra of Graphs-Theory and Application. Deutscher Verlag der Wissenschaften-Academic Press, Berlin-New York, 1980; second edition 1982; third edition, Johann Ambrosius Barth Verlag, Heidelberg-Leipzig, 1995.
[3] D. Cvetković, L. Kraus, S. Simić: Discussing graph theory with a computer, Implementation of algorithms. Univ. Beograd, Publ. Elektrotehn. Fak., Ser. Mat. Fiz., No. 716-No. 734, pp. 100-104, 1981.
[4] B. Mihailović, Z. Radosavljević: On a class of tricyclic reflexive cactuses. Univ. Beograd, Publ. Elektrotehn. Fak., Ser. Mat., 16, pp. 55-63, 2005.
[5] A. Neumaier, J. J. Seidel: Discrete hyperbolic geometry. Combinatorica, 3, pp. 219-237, 1983.
[6] Z. Radosavljević, M. Rašajski: Multicyclic treelike reflexive graphs. Discrete Math, Vol. 296/1, pp. 43-57, 2005.
[7] Z. Radosavljević, S. Simić: Which bicyclic graphs are reflexive? Univ. Beograd, Publ. Elektroteh. Fak., Ser. Mat., 7, pp. 90-104, 1996.

Joint work with Z. Radosavljević, M. Rašajski (School of Electrical Engineering, University of Belgrade, Serbia)
