Детерминанте

празна шема

$$\left\|\begin{matrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{matrix}\right\|$$

празна шема матрица

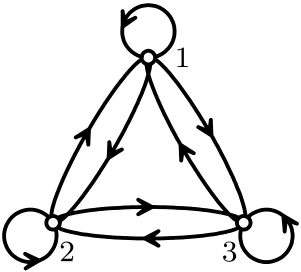
$$\left\| \begin{matrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{matrix} \right\| \xrightarrow{f} \left\| \begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix} \right\|$$

празна шема

$$\left\|\begin{matrix} \Box & \Box & \Box \\ \Box & \Box & \Box \\ \Box & \Box & \Box \end{matrix}\right\| \xrightarrow{f} \left\|\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}\right\|$$

матрица

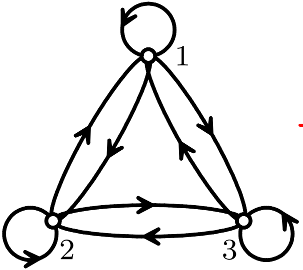
диграф



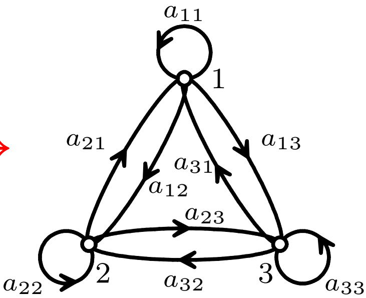
празна шема → матрица

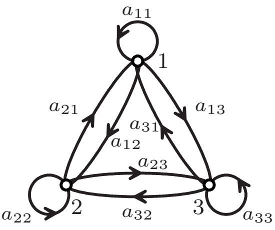
$$\begin{Vmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{Vmatrix} \xrightarrow{\;f\;} \begin{Vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{Vmatrix}$$

диграф → тежински диграф

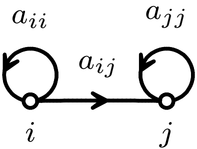


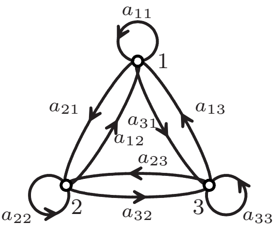
$$\xrightarrow{\;f\;}$$



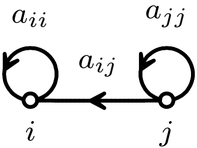

a11
a21
a13
a31
a12
a23
a22
2
a32
3
a33
1

G^A


aii
ajj
aij
i
j


a11
a21
a13
a31
a12
a23
a22
2
a32
3
a33
1

G_A


aii
ajj
aij
i
j

Дефиниција

Нека је $A = \|a_{ij}\|$ квадратна матрица реда n. *Детерминанта* матрице A је број $\det A$ дефинисан сумом

$$\det A = (-1)^n \sum_{F \in \mathcal{F}(A)} (-1)^{p(F)} C(F)$$

где сумирање иде по свим факторима F диграфа G_A.

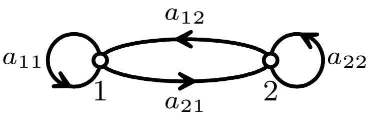
Пример

Израчунати детерминанту

$$\det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}.$$

$$A = \left\| \begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right\| \qquad |A| = \left| \begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right|$$

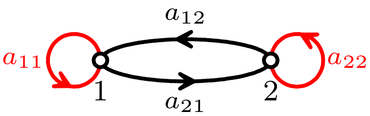
$$\begin{aligned} \det A &= (-1)^{2+2} a_{11} a_{22} + (-1)^{2+1} a_{12} a_{21} \\ &= a_{11} a_{22} - a_{12} a_{21} \end{aligned}$$



G_A

$$A = \left\| \begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right\| \qquad |A| = \left| \begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right|$$

$$\begin{aligned} \det A &= (-1)^{2+2} a_{11} a_{22} + (-1)^{2+1} a_{12} a_{21} \\ &= a_{11} a_{22} - a_{12} a_{21} \end{aligned}$$

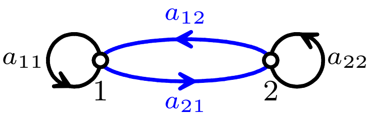


G_A

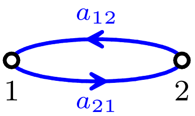
F_1

$$A = \left\| \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix} \right\| \qquad |A| = \left| \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix} \right|$$

$$\begin{aligned} \det A &= (-1)^{2+2} a_{11} a_{22} + (-1)^{2+1} a_{12} a_{21} \\ &= a_{11} a_{22} - a_{12} a_{21} \end{aligned}$$



G_A

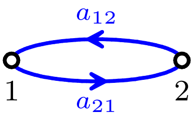


F_2

$$A = \left\| \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix} \right\| \qquad |A| = \left| \begin{matrix} \color{red}{a_{11}} & \color{blue}{a_{12}} \\ \color{blue}{a_{21}} & \color{red}{a_{22}} \end{matrix} \right|$$

$$\begin{aligned} \det A &= \color{red}{(-1)^{2+2} a_{11} a_{22}} + \color{blue}{(-1)^{2+1} a_{12} a_{21}} \\ &= \color{red}{a_{11} a_{22}} - \color{blue}{a_{12} a_{21}} \end{aligned}$$

F_1



F_2

Пример

Израчунати детерминанту

$$\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

помоћу

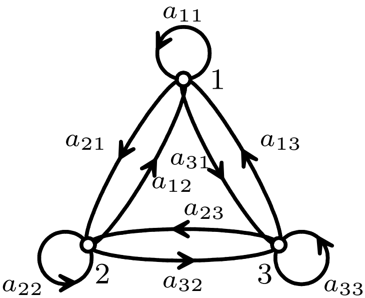
$$\det A = (-1)^n \sum_{F \in \mathcal{F}(A)} (-1)^{p(F)} C(F).$$

$$A = \left\| \begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix} \right\|$$

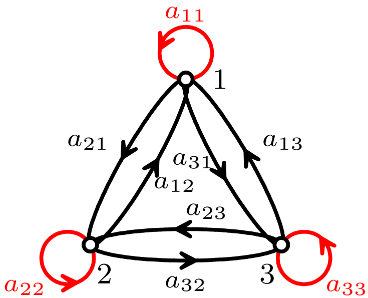
$$\begin{aligned} \det A \quad = \quad & (-1)^{3+3} a_{11} a_{22} a_{33} + (-1)^{3+1} a_{12} a_{31} a_{23} + \\ & (-1)^{3+1} a_{21} a_{32} a_{13} + (-1)^{3+2} a_{11} a_{23} a_{32} + \\ & (-1)^{3+2} a_{22} a_{13} a_{31} + (-1)^{3+2} a_{33} a_{12} a_{21} \end{aligned}$$

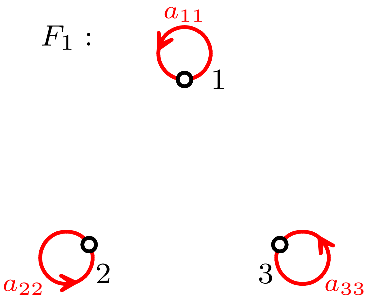
$$\begin{aligned} \det A \quad = \quad & a_{11} a_{22} a_{33} + a_{12} a_{31} a_{23} + a_{21} a_{32} a_{13} \\ = \quad & -a_{11} a_{23} a_{32} - a_{22} a_{13} a_{31} - a_{33} a_{12} a_{21} \end{aligned}$$

$$\det A = (-1)^{3+3}a_{11}a_{22}a_{33} + (-1)^{3+1}a_{12}a_{31}a_{23} + (-1)^{3+1}a_{21}a_{32}a_{13} + (-1)^{3+2}a_{11}a_{23}a_{32} + (-1)^{3+2}a_{22}a_{13}a_{31} + (-1)^{3+2}a_{33}a_{12}a_{21}$$

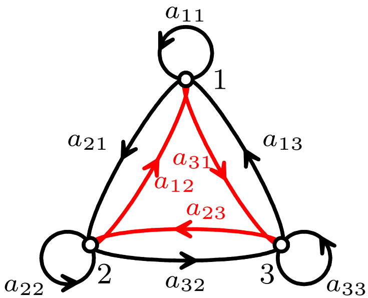


$$\det A = (-1)^{3+3}a_{11}a_{22}a_{33} + (-1)^{3+1}a_{12}a_{31}a_{23} + (-1)^{3+1}a_{21}a_{32}a_{13} + (-1)^{3+2}a_{11}a_{23}a_{32} + (-1)^{3+2}a_{22}a_{13}a_{31} + (-1)^{3+2}a_{33}a_{12}a_{21}$$

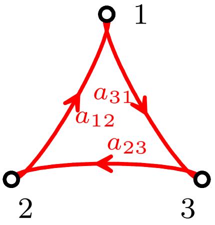




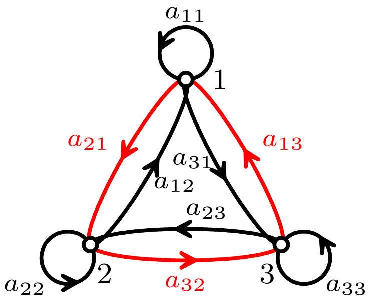
$$\det A = (-1)^{3+3}a_{11}a_{22}a_{33} + (-1)^{3+1}a_{12}a_{31}a_{23} + (-1)^{3+1}a_{21}a_{32}a_{13} + (-1)^{3+2}a_{11}a_{23}a_{32} + (-1)^{3+2}a_{22}a_{13}a_{31} + (-1)^{3+2}a_{33}a_{12}a_{21}$$



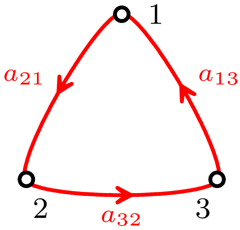
F_2 :



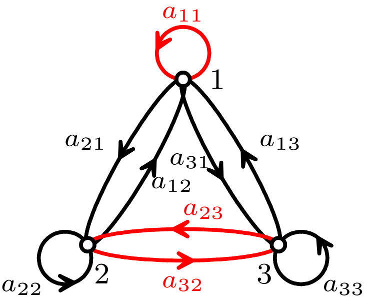
$$\begin{aligned}\det A \quad = \quad & (-1)^{3+3}a_{11}a_{22}a_{33} + (-1)^{3+1}a_{12}a_{31}a_{23} + \\ & (-1)^{3+1}a_{21}a_{32}a_{13} + (-1)^{3+2}a_{11}a_{23}a_{32} + \\ & (-1)^{3+2}a_{22}a_{13}a_{31} + (-1)^{3+2}a_{33}a_{12}a_{21}\end{aligned}$$

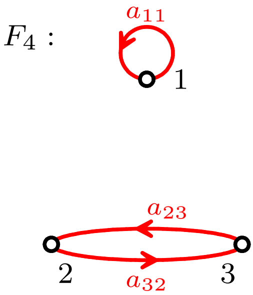


F_3 :

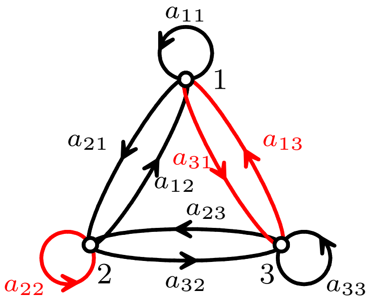


$$\begin{aligned}\det A \quad = \quad & (-1)^{3+3}a_{11}a_{22}a_{33} + (-1)^{3+1}a_{12}a_{31}a_{23} + \\ & (-1)^{3+1}a_{21}a_{32}a_{13} + (-1)^{3+2}a_{11}a_{23}a_{32} + \\ & (-1)^{3+2}a_{22}a_{13}a_{31} + (-1)^{3+2}a_{33}a_{12}a_{21}\end{aligned}$$

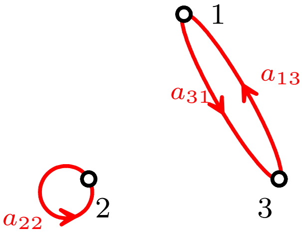




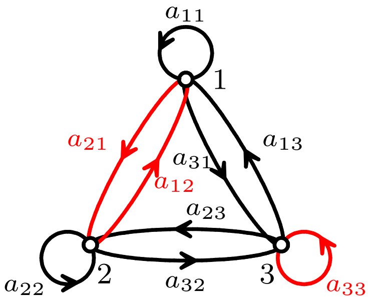
$$\begin{aligned}\det A \quad = \quad & (-1)^{3+3}a_{11}a_{22}a_{33} + (-1)^{3+1}a_{12}a_{31}a_{23} + \\ & (-1)^{3+1}a_{21}a_{32}a_{13} + (-1)^{3+2}a_{11}a_{23}a_{32} + \\ & {\color{red}(-1)^{3+2}a_{22}a_{13}a_{31}} + (-1)^{3+2}a_{33}a_{12}a_{21}\end{aligned}$$



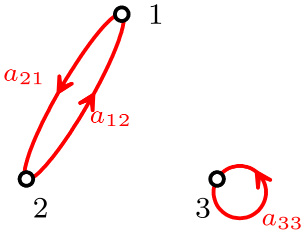
F_5 :



$$\det A = (-1)^{3+3}a_{11}a_{22}a_{33} + (-1)^{3+1}a_{12}a_{31}a_{23} + (-1)^{3+1}a_{21}a_{32}a_{13} + (-1)^{3+2}a_{11}a_{23}a_{32} + (-1)^{3+2}a_{22}a_{13}a_{31} + (-1)^{3+2}a_{33}a_{12}a_{21}$$



F_6 :



Теорема

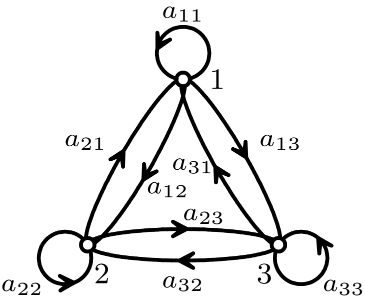
$$\det A^T = \det A.$$

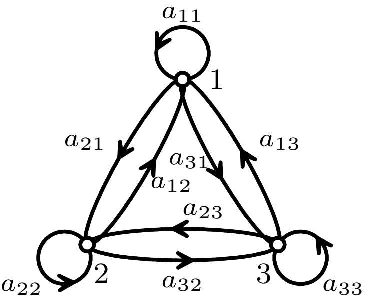
Теорема

$$\det A^T = \det A.$$

Ова теорема повлачи да свако тврђење које важи за врсте матрице важи и за колоне.

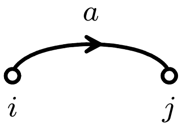
$$A = \left\| \begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix} \right\| \qquad A^T = \left\| \begin{matrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{matrix} \right\|$$





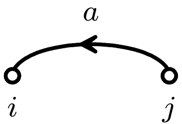
Дејство транспоновања

елемент a на позицији (i, j)



Дејство транспоновања

елемент a на позицији (j, i)

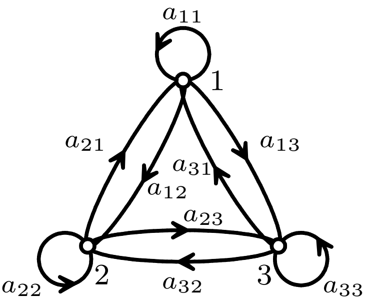


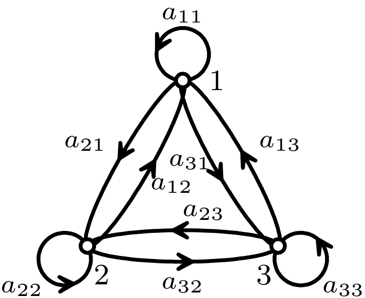
$$\det A = \qquad\qquad \det A^T =$$

$$= (-1)^{3+3} a_{11}a_{22}a_{33} + (-1)^{3+1} a_{12}a_{31}a_{23}$$

$$+(-1)^{3+1} a_{21}a_{32}a_{13} + (-1)^{3+2} a_{11}a_{23}a_{32}$$

$$+(-1)^{3+2} a_{22}a_{13}a_{31} + (-1)^{3+2} a_{33}a_{12}a_{21}$$



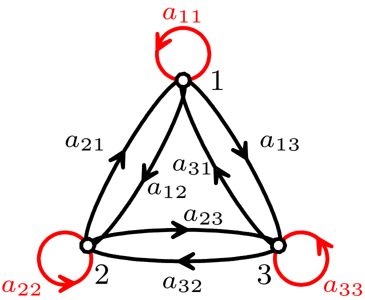


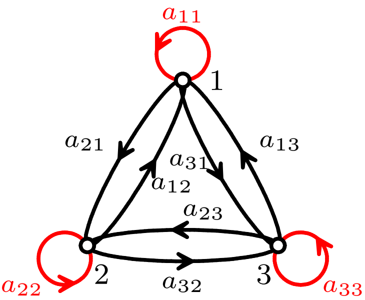
$$\det A = \qquad\qquad \det A^T =$$

$$= (-1)^{3+3} a_{11} a_{22} a_{33} + (-1)^{3+1} a_{12} a_{31} a_{23}$$

$$+(-1)^{3+1} a_{21} a_{32} a_{13} + (-1)^{3+2} a_{11} a_{23} a_{32}$$

$$+(-1)^{3+2} a_{22} a_{13} a_{31} + (-1)^{3+2} a_{33} a_{12} a_{21}$$



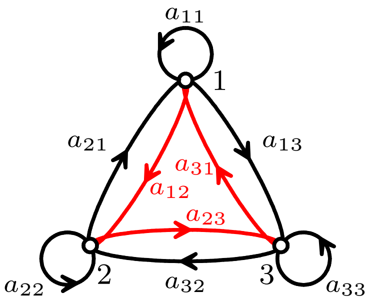


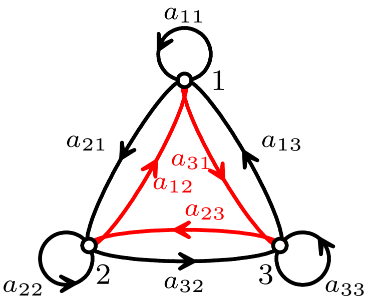
$$\det A = \qquad\qquad \det A^T =$$

$$= (-1)^{3+3}a_{11}a_{22}a_{33} + {\color{red}(-1)^{3+1}a_{12}a_{31}a_{23}}$$

$$+(-1)^{3+1}a_{21}a_{32}a_{13} + (-1)^{3+2}a_{11}a_{23}a_{32}$$

$$+(-1)^{3+2}a_{22}a_{13}a_{31} + (-1)^{3+2}a_{33}a_{12}a_{21}$$



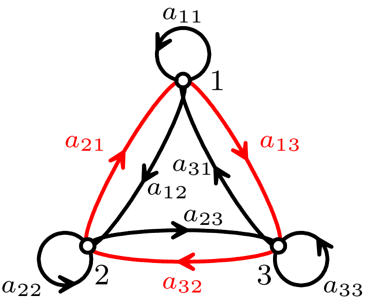


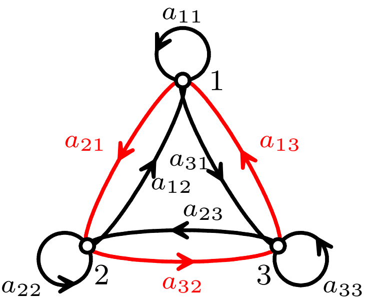
$$\det A = \qquad \det A^T =$$

$$= (-1)^{3+3} a_{11}a_{22}a_{33} + (-1)^{3+1} a_{12}a_{31}a_{23}$$

$$+ (-1)^{3+1} a_{21}a_{32}a_{13} + (-1)^{3+2} a_{11}a_{23}a_{32}$$

$$+ (-1)^{3+2} a_{22}a_{13}a_{31} + (-1)^{3+2} a_{33}a_{12}a_{21}$$



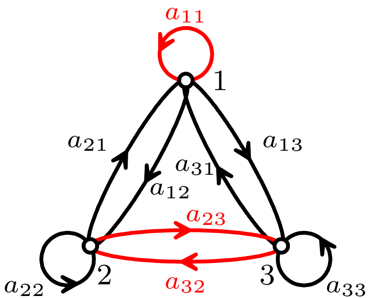


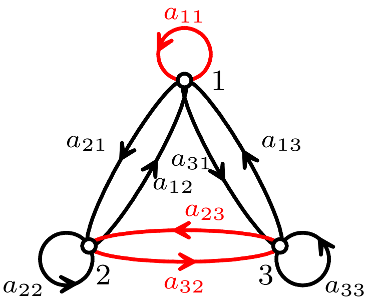
$$\det A = \qquad\qquad \det A^T =$$

$$= (-1)^{3+3} a_{11}a_{22}a_{33} + (-1)^{3+1} a_{12}a_{31}a_{23}$$

$$+(-1)^{3+1} a_{21}a_{32}a_{13} + {\color{red}(-1)^{3+2} a_{11}a_{23}a_{32}}$$

$$+(-1)^{3+2} a_{22}a_{13}a_{31} + (-1)^{3+2} a_{33}a_{12}a_{21}$$



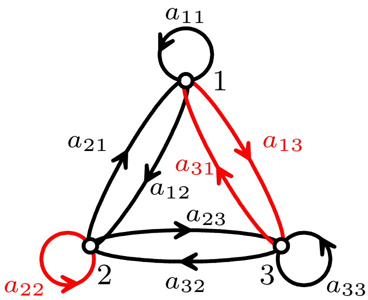


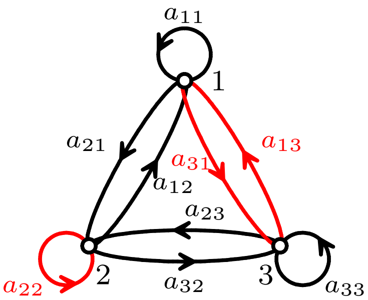
$$\det A = \qquad \det A^T =$$

$$= (-1)^{3+3} a_{11}a_{22}a_{33} + (-1)^{3+1} a_{12}a_{31}a_{23}$$

$$+(-1)^{3+1} a_{21}a_{32}a_{13} + (-1)^{3+2} a_{11}a_{23}a_{32}$$

$$+{\color{red}(-1)^{3+2} a_{22}a_{13}a_{31}} + (-1)^{3+2} a_{33}a_{12}a_{21}$$



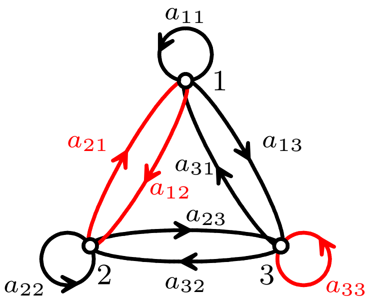


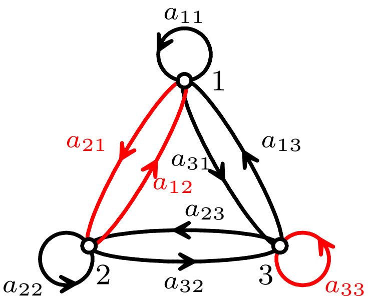
$$\det A = \qquad\qquad \det A^T =$$

$$= (-1)^{3+3}a_{11}a_{22}a_{33} + (-1)^{3+1}a_{12}a_{31}a_{23}$$

$$+(-1)^{3+1}a_{21}a_{32}a_{13} + (-1)^{3+2}a_{11}a_{23}a_{32}$$

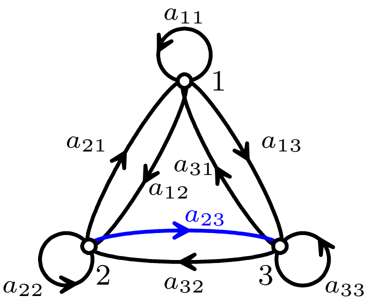
$$+(-1)^{3+2}a_{22}a_{13}a_{31} + {\color{red}(-1)^{3+2}a_{33}a_{12}a_{21}}$$





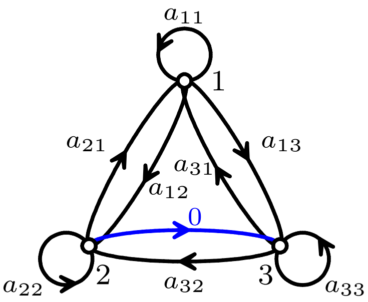
Дејство нула елемента

$$A = \begin{Vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & {\color{blue}a_{23}} \\ a_{31} & a_{32} & a_{33} \end{Vmatrix}$$



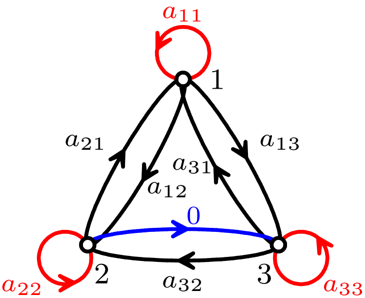
$$\begin{aligned} \det A \quad = \quad & (-1)^{3+3} a_{11}a_{22}a_{33} + (-1)^{3+1} a_{12}a_{31}{\color{blue}a_{23}} + \\ & (-1)^{3+1} a_{21}a_{32}a_{13} + (-1)^{3+2} a_{11}{\color{blue}a_{23}}a_{32} + \\ & (-1)^{3+2} a_{22}a_{13}a_{31} + (-1)^{3+2} a_{33}a_{12}a_{21} \end{aligned}$$

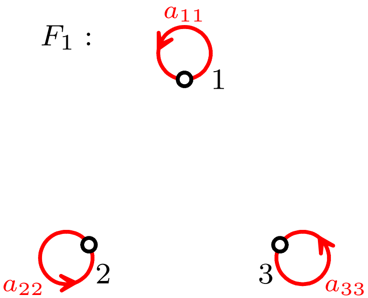
$$A = \left|\left|\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & {\color{blue}0} \\ a_{31} & a_{32} & a_{33} \end{matrix}\right|\right|$$



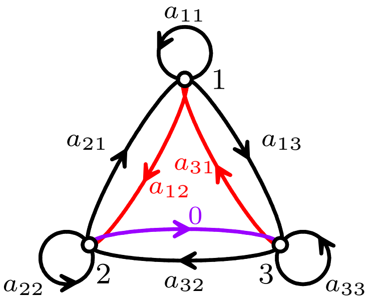
$$\begin{aligned} \det A \quad = \quad & (-1)^{3+3} a_{11}a_{22}a_{33} + (-1)^{3+1} a_{12}a_{31} \cdot {\color{blue}0} + \\ & (-1)^{3+1} a_{21}a_{32}a_{13} + (-1)^{3+2} a_{11} \cdot {\color{blue}0} \cdot a_{32} + \\ & (-1)^{3+2} a_{22}a_{13}a_{31} + (-1)^{3+2} a_{33}a_{12}a_{21} \end{aligned}$$

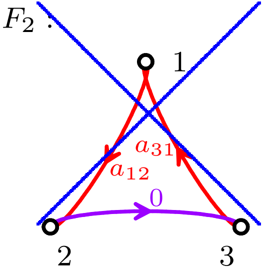
$$
\begin{aligned}
\det A \quad = \quad & {\color{red}(-1)^{3+3} a_{11} a_{22} a_{33}} + \qquad 0 \qquad + \\
& (-1)^{3+1} a_{21} a_{32} a_{13} + \qquad 0 \qquad + \\
& (-1)^{3+2} a_{22} a_{13} a_{31} + (-1)^{3+2} a_{33} a_{12} a_{21}
\end{aligned}
$$



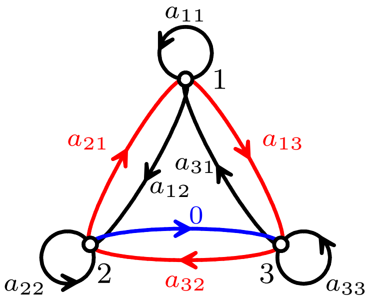


$$\begin{aligned}\det A \;=\;& (-1)^{3+3}a_{11}a_{22}a_{33} + \cancel{(-1)^{3+1}a_{12}a_{31}\cdot 0} \;+\\ & (-1)^{3+1}a_{21}a_{32}a_{13} + \qquad 0 \qquad +\\ & (-1)^{3+2}a_{22}a_{13}a_{31} + (-1)^{3+2}a_{33}a_{12}a_{21}\end{aligned}$$

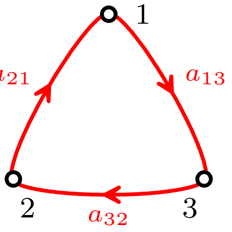




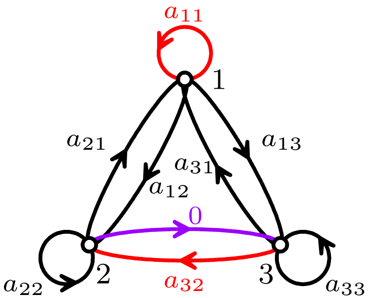
$$
\begin{aligned}
\det A \;=\;& (-1)^{3+3}a_{11}a_{22}a_{33} + \quad 0 \quad + \\
& (-1)^{3+1}a_{21}a_{32}a_{13} + \quad 0 \quad + \\
& (-1)^{3+2}a_{22}a_{13}a_{31} + (-1)^{3+2}a_{33}a_{12}a_{21}
\end{aligned}
$$



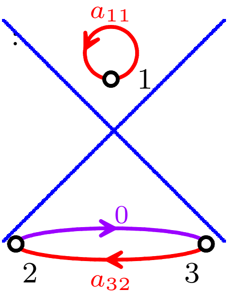
F_3 :



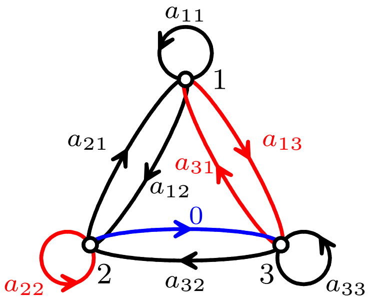
$$
\begin{aligned}
\det A \quad = \quad & (-1)^{3+3} a_{11}a_{22}a_{33} + \qquad 0 \qquad + \\
& (-1)^{3+1} a_{21}a_{32}a_{13} + \cancel{(-1)^{3+2} a_{11} \cdot 0 \cdot a_{32}} + \\
& (-1)^{3+2} a_{22}a_{13}a_{31} + (-1)^{3+2} a_{33}a_{12}a_{21}
\end{aligned}
$$



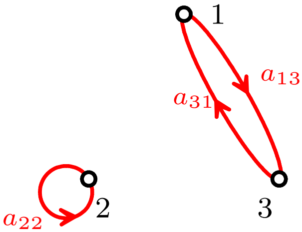
F_4 :



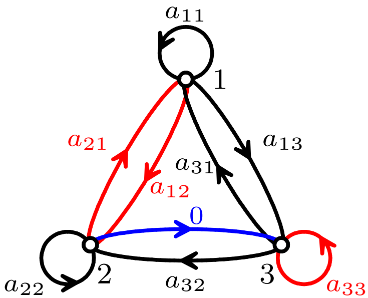
$$\det A = (-1)^{3+3}a_{11}a_{22}a_{33} + 0 + (-1)^{3+1}a_{21}a_{32}a_{13} + 0 + (-1)^{3+2}a_{22}a_{13}a_{31} + (-1)^{3+2}a_{33}a_{12}a_{21}$$



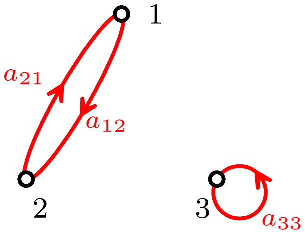
F_5 :



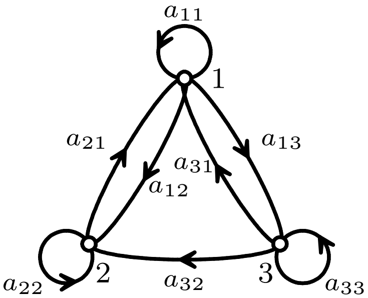
$$\begin{aligned}\det A \quad = \quad & (-1)^{3+3}a_{11}a_{22}a_{33} + \qquad 0 \qquad + \\ & (-1)^{3+1}a_{21}a_{32}a_{13} + \qquad 0 \qquad + \\ & (-1)^{3+2}a_{22}a_{13}a_{31} + {\color{red}(-1)^{3+2}a_{33}a_{12}a_{21}}\end{aligned}$$



F_6 :

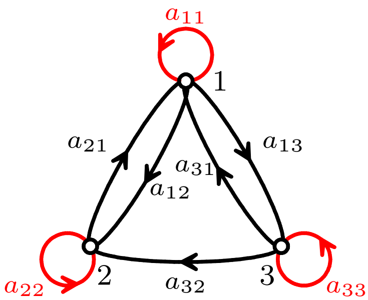


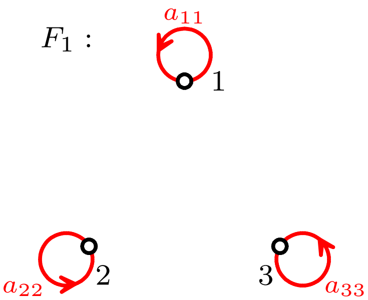
$$A = \left\| \begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{matrix} \right\|$$



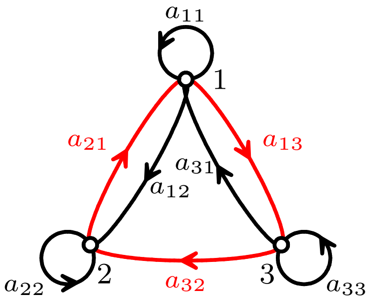
$$\begin{aligned} \det A \quad = \quad & (-1)^{3+3} a_{11}a_{22}a_{33} + \qquad 0 \qquad + \\ & (-1)^{3+1} a_{21}a_{32}a_{13} + \qquad 0 \qquad + \\ & (-1)^{3+2} a_{22}a_{13}a_{31} + (-1)^{3+2} a_{33}a_{12}a_{21} \end{aligned}$$

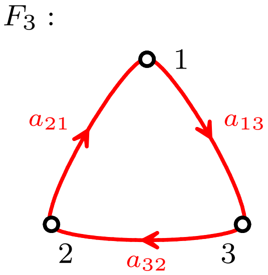
$$\begin{aligned}\det A \quad = \quad & {\color{red}(-1)^{3+3}a_{11}a_{22}a_{33}} + \qquad 0 \qquad + \\ & (-1)^{3+1}a_{21}a_{32}a_{13} + \qquad 0 \qquad + \\ & (-1)^{3+2}a_{22}a_{13}a_{31} + (-1)^{3+2}a_{33}a_{12}a_{21}\end{aligned}$$



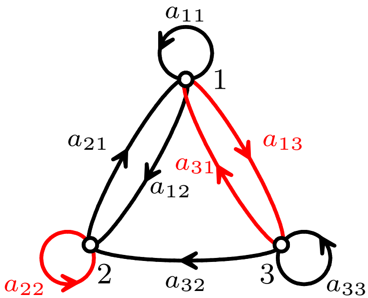


$$\begin{aligned}\det A \quad = \quad & (-1)^{3+3}a_{11}a_{22}a_{33} + \qquad 0 \qquad + \\ & {\color{red}(-1)^{3+1}a_{21}a_{32}a_{13}} + \qquad 0 \qquad + \\ & (-1)^{3+2}a_{22}a_{13}a_{31} + (-1)^{3+2}a_{33}a_{12}a_{21}\end{aligned}$$

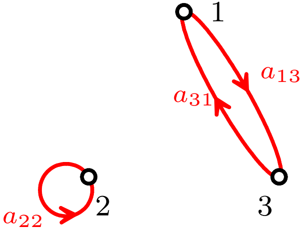




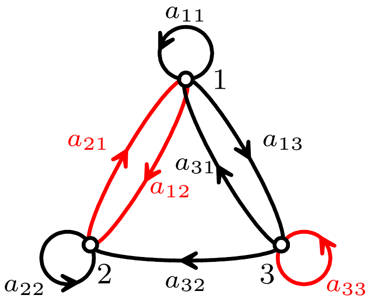
$$\begin{aligned}\det A \quad = \quad & (-1)^{3+3}a_{11}a_{22}a_{33} + \qquad 0 \qquad + \\ & (-1)^{3+1}a_{21}a_{32}a_{13} + \qquad 0 \qquad + \\ & {\color{red}(-1)^{3+2}a_{22}a_{13}a_{31}} + (-1)^{3+2}a_{33}a_{12}a_{21}\end{aligned}$$



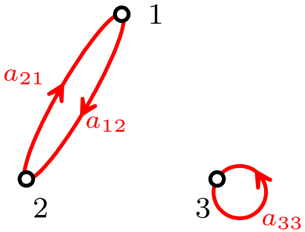
F_5 :



$$
\begin{aligned}
\det A \quad = \quad & (-1)^{3+3} a_{11}a_{22}a_{33} + \qquad 0 \qquad + \\
& (-1)^{3+1} a_{21}a_{32}a_{13} + \qquad 0 \qquad + \\
& (-1)^{3+2} a_{22}a_{13}a_{31} + \color{red}{(-1)^{3+2} a_{33}a_{12}a_{21}}
\end{aligned}
$$



F_6 :



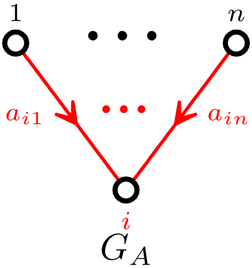
Теорема

Нека матрицу B добијамо тако што сваки елемент неке врсте (i-те врсте) матрице A помножимо са α. Тада је

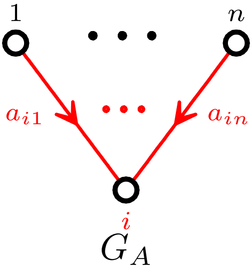
$$\det B = \alpha \cdot \det A.$$

$$A = \left\| \begin{matrix} \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \\ \end{matrix} \right\| \; i\text{-та врста}$$

$$A = \left\| \begin{matrix} \color{red}{a_{i1}} & \color{red}{a_{i2}} & \color{red}{\cdots} & \color{red}{a_{in}} \end{matrix} \right\| \; i\text{-та врста}$$

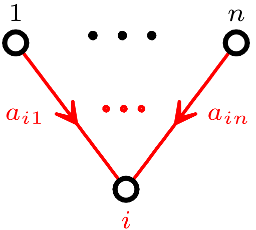


$$A = \left\| \begin{matrix} \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \\ \end{matrix} \right\| \quad i\text{-та врста}$$



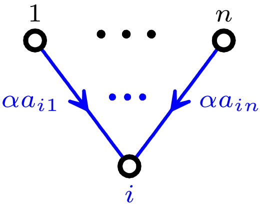
$$B = \left\| \begin{matrix} & \text{непромењено} & & \\ \alpha a_{i1} & \alpha a_{i2} & \cdots & \alpha a_{in} \\ & \text{непромењено} & & \end{matrix} \right\| \quad i\text{-та врста}$$

$$A = \left\| \begin{matrix} a_{i1} & a_{i2} & \cdots & a_{in} \end{matrix} \right\| \; i\text{-та врста}$$



G_A

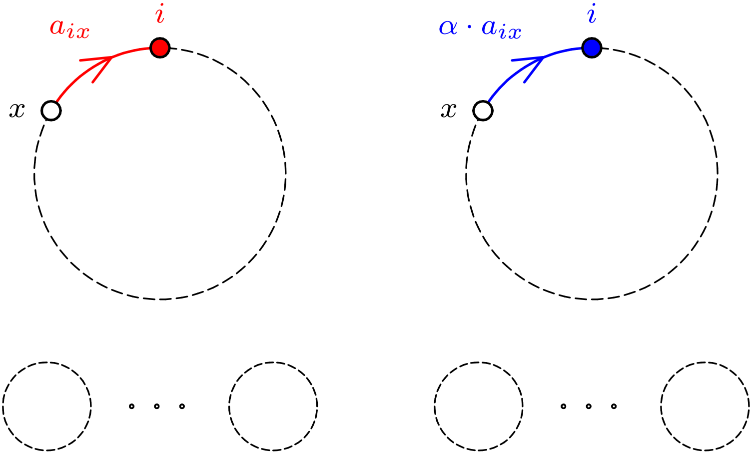
$$B = \left\| \begin{matrix} & \text{непромењено} & & \\ \alpha a_{i1} & \alpha a_{i2} & \cdots & \alpha a_{in} \\ & \text{непромењено} & & \end{matrix} \right\| \; i\text{-та врста}$$



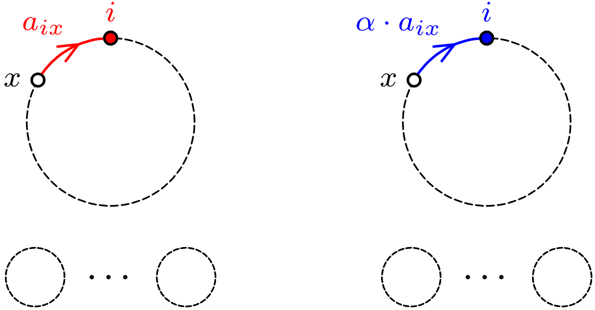
G_B

тежине осталих грана

су непромењене



$$F_A \in \mathcal{F}(A) \qquad\qquad F_B \in \mathcal{F}(B)$$



$$\alpha \cdot C(F_A) = C(F_B)$$

$$p(F_A) = p(F_B)$$

$$(-1)^{p(F_A)} = (-1)^{p(F_B)}$$

$$(-1)^n \sum_{F \in \mathcal{F}(A)} (-1)^{p(F)} \alpha \cdot C(F) = (-1)^n \sum_{F \in \mathcal{F}(B)} (-1)^{p(F)} C(F)$$

$$\alpha \cdot \det A = \det B$$

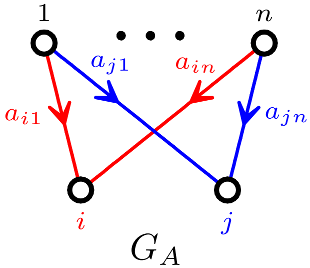
Теорема

Нека матрицу B добијамо тако што две врсте (i-та и j-та врста) матрице A замене места. Тада је

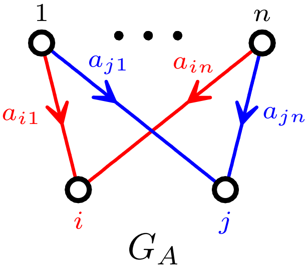
$$\det B = -\det A.$$

$$A = \begin{array}{c} \left\|\begin{matrix} {\color{red}a_{i1}} & {\color{red}a_{i2}} & {\color{red}\cdots} & {\color{red}a_{in}} \\ {\color{blue}a_{j1}} & {\color{blue}a_{j2}} & {\color{blue}\cdots} & {\color{blue}a_{jn}} \end{matrix}\right\| \end{array} \begin{array}{l} i\text{-та врста} \\ j\text{-та врста} \end{array}$$

$$A = \begin{array}{c} \\ \\ \end{array}\left\|\begin{matrix} \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \\ \end{matrix}\right\|\begin{matrix} \\ i\text{-та врста} \\ j\text{-та врста} \\ \\ \end{matrix}$$

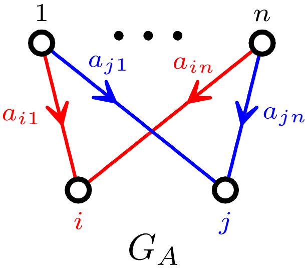


$$A = \begin{array}{c} \left\| \begin{array}{cccc} a_{i1} & a_{i2} & \cdots & a_{in} \\ & & & \\ a_{j1} & a_{j2} & \cdots & a_{jn} \end{array} \right\| \end{array} \begin{array}{l} i\text{-та врста} \\ \\ j\text{-та врста} \end{array}$$

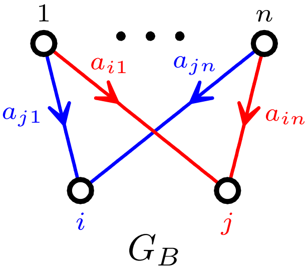


$$B = \left\| \begin{array}{cccc} & \text{непромењено} & & \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ & \text{непромењено} & & \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ & \text{непромењено} & & \end{array} \right\| \begin{array}{l} \\ i\text{-та врста} \\ \\ j\text{-та врста} \\ \\ \end{array}$$

$$A = \begin{Vmatrix} & & & \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ & & & \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ & & & \end{Vmatrix} \begin{array}{l} \\ i\text{-та врста} \\ \\ j\text{-та врста} \\ \\ \end{array}$$



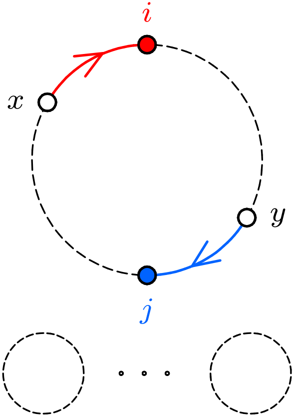
$$B = \begin{Vmatrix} & \text{непромењено} & & \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ & \text{непромењено} & & \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ & \text{непромењено} & & \end{Vmatrix} \begin{array}{l} \\ i\text{-та врста} \\ \\ j\text{-та врста} \\ \\ \end{array}$$



тежине осталих грана

су непромењене

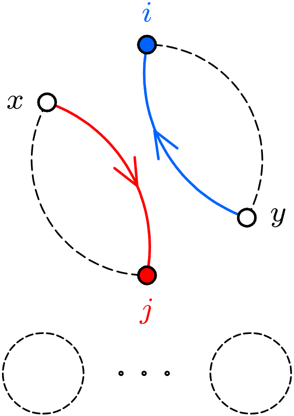
i и j су у истој контури



$$F_A \in \mathcal{F}(A)$$

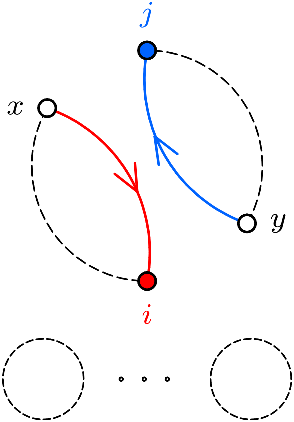
$\rightsquigarrow$

i и j нису у истој контури



$$F_B \in \mathcal{F}(B)$$

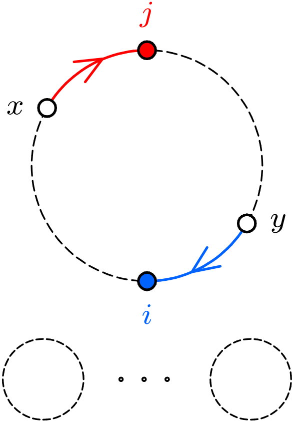
i и j нису у истој контури



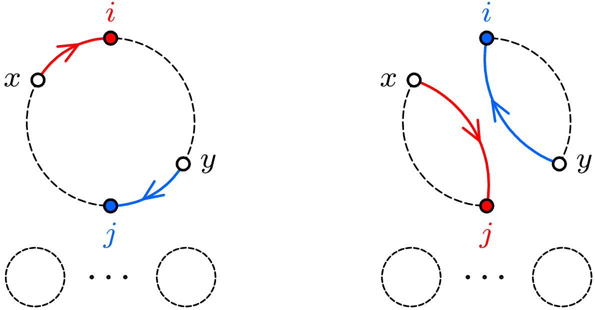
$F_A \in \mathcal{F}(A)$

$\rightsquigarrow$

i и j су у истој контури



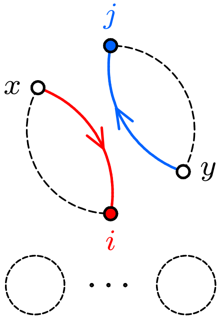
$F_B \in \mathcal{F}(B)$

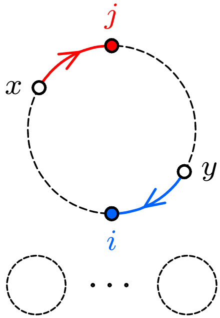


$$C(F_A) = C(F_B)$$

$$p(F_A) + 1 = p(F_B)$$

$$-(-1)^{p(F_A)} = (-1)^{p(F_B)}$$





$$C(F_A) = C(F_B)$$

$$p(F_A) - 1 = p(F_B)$$

$$-(-1)^{p(F_A)} = (-1)^{p(F_B)}$$

$$-(-1)^n \sum_{F \in \mathcal{F}(A)} (-1)^{p(F)} C(F) = (-1)^n \sum_{F \in \mathcal{F}(B)} (-1)^{p(F)} C(F)$$

$$-\det A = \det B$$

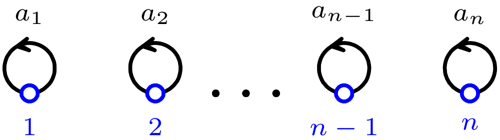
Пример

$$A = \begin{Vmatrix} a_1 & 0 & \cdots & 0 & 0 \\ 0 & a_2 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & a_{n-1} & 0 \\ 0 & 0 & \cdots & 0 & a_n \end{Vmatrix}$$

Израчунати детерминанту $\det A$.

$$A = \left\| \begin{matrix} a_1 & 0 & \cdots & 0 & 0 \\ 0 & a_2 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & a_{n-1} & 0 \\ 0 & 0 & \cdots & 0 & a_n \end{matrix} \right\|$$

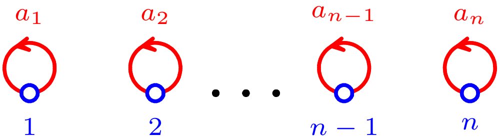
диграф G_A



$$\begin{aligned} \det A &= {\color{blue}(-1)^n}\Big((-1)^n a_1 a_2 \cdot \ldots \cdot a_n\Big) \\ &= a_1 a_2 \cdot \ldots \cdot a_n \end{aligned}$$

$$A = \left\| \begin{matrix} a_1 & 0 & \cdots & 0 & 0 \\ 0 & a_2 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & a_{n-1} & 0 \\ 0 & 0 & \cdots & 0 & a_n \end{matrix} \right\|$$

диграф G_A



$$\begin{aligned} \det A &= (-1)^n \Big((-1)^n a_1 a_2 \cdot \ldots \cdot a_n \Big) \\ &= a_1 a_2 \cdot \ldots \cdot a_n \end{aligned}$$

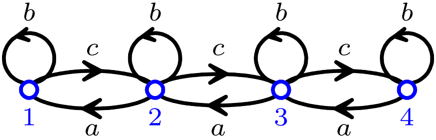
Пример

$$A = \left\| \begin{matrix} b & a & 0 & 0 \\ c & b & a & 0 \\ 0 & c & b & a \\ 0 & 0 & c & b \end{matrix} \right\|$$

Израчунати детерминанту $\det A$.

$$A = \begin{Vmatrix} b & a & 0 & 0 \\ c & b & a & 0 \\ 0 & c & b & a \\ 0 & 0 & c & b \end{Vmatrix}$$

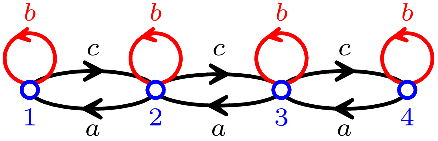
диграф G_A



$$\begin{aligned} \det A &= \color{blue}{(-1)^4}\Big((-1)^4 b^4 + 3(-1)^3 ab^2c + (-1)^2 a^2c^2\Big) \\ &= b^4 - ab^2c - ab^2c - ab^2c + a^2c^2 \\ &= b^4 - 3ab^2c + a^2c^2 \end{aligned}$$

$$A = \begin{Vmatrix} \color{red}{b} & a & 0 & 0 \\ c & \color{red}{b} & a & 0 \\ 0 & c & \color{red}{b} & a \\ 0 & 0 & c & \color{red}{b} \end{Vmatrix}$$

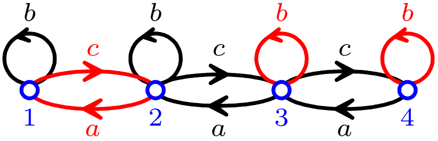
диграф G_A



$$\begin{aligned} \det A &= \color{blue}{(-1)^4}\left(\color{red}{(-1)^4 b^4} + 3(-1)^3 ab^2c + (-1)^2 a^2c^2\right) \\ &= \color{red}{b^4} - ab^2c - ab^2c - ab^2c + a^2c^2 \\ &= b^4 - 3ab^2c + a^2c^2 \end{aligned}$$

$$A = \begin{Vmatrix} b & a & 0 & 0 \\ c & b & a & 0 \\ 0 & c & b & a \\ 0 & 0 & c & b \end{Vmatrix}$$

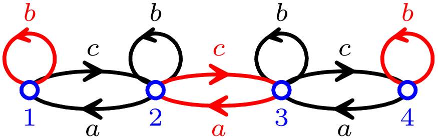
диграф G_A



$$\begin{aligned} \det A &= (-1)^4\Big((-1)^4 b^4 + 3(-1)^3 ab^2c + (-1)^2 a^2c^2\Big) \\ &= b^4 - ab^2c - ab^2c - ab^2c + a^2c^2 \\ &= b^4 - 3ab^2c + a^2c^2 \end{aligned}$$

$$A = \begin{Vmatrix} b & a & 0 & 0 \\ c & b & a & 0 \\ 0 & c & b & a \\ 0 & 0 & c & b \end{Vmatrix}$$

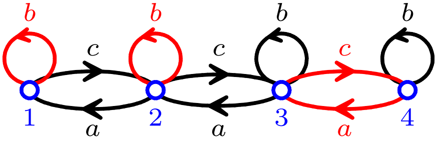
диграф G_A



$$\begin{aligned} \det A &= (-1)^4\Big((-1)^4 b^4 + 3(-1)^3 ab^2c + (-1)^2 a^2c^2\Big) \\ &= b^4 - ab^2c - ab^2c - ab^2c + a^2c^2 \\ &= b^4 - 3ab^2c + a^2c^2 \end{aligned}$$

$$A = \begin{Vmatrix} b & a & 0 & 0 \\ c & b & a & 0 \\ 0 & c & b & a \\ 0 & 0 & c & b \end{Vmatrix}$$

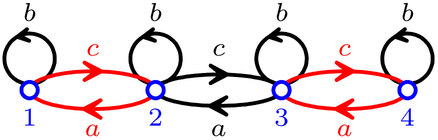
диграф G_A



$$\begin{aligned} \det A &= (-1)^4\Big((-1)^4 b^4 + 3(-1)^3 ab^2c + (-1)^2 a^2c^2\Big) \\ &= b^4 - ab^2c - ab^2c - ab^2c + a^2c^2 \\ &= b^4 - 3ab^2c + a^2c^2 \end{aligned}$$

$$A = \begin{Vmatrix} b & \color{red}{a} & 0 & 0 \\ \color{red}{c} & b & a & 0 \\ 0 & c & b & \color{red}{a} \\ 0 & 0 & \color{red}{c} & b \end{Vmatrix}$$

диграф G_A



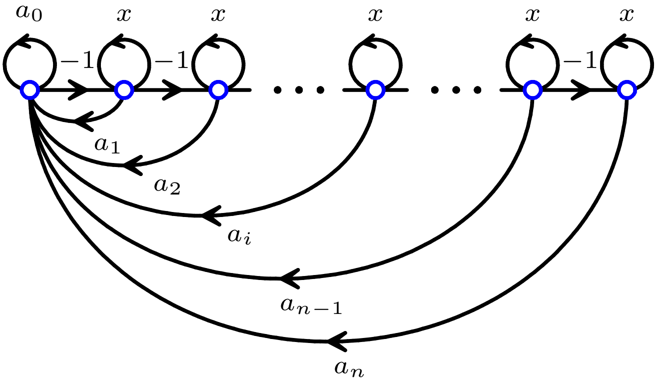
$$\begin{aligned} \det A &= (-1)^4\Big((-1)^4 b^4 + 3(-1)^3 ab^2c + (-1)^2 a^2c^2\Big) \\ &= b^4 - ab^2c - ab^2c - ab^2c + a^2c^2 \\ &= b^4 - 3ab^2c + a^2c^2 \end{aligned}$$

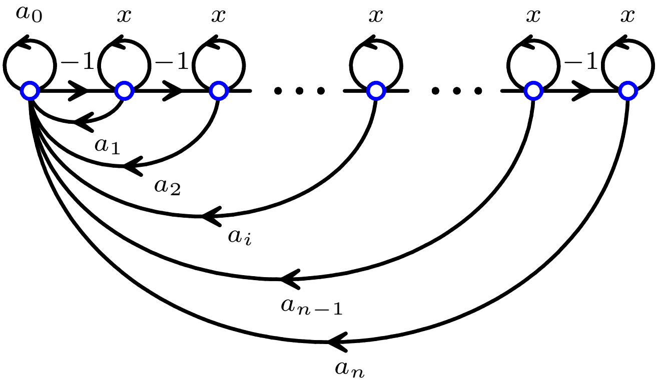
Пример

Доказати да је

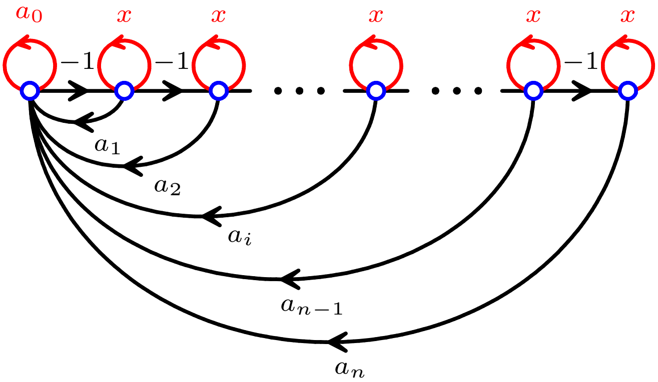
$$\begin{vmatrix} a_0 & a_1 & a_2 & \cdots & a_{n-1} & a_n \\ -1 & x & 0 & \cdots & 0 & 0 \\ 0 & -1 & x & & 0 & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & \ddots & x & 0 \\ 0 & 0 & 0 & & -1 & x \end{vmatrix} = \sum_{i=0}^{n} a_i x^{n-i}.$$

$$A = \left\| \begin{matrix} a_0 & a_1 & a_2 & \cdots & a_{n-1} & a_n \\ -1 & x & 0 & \cdots & 0 & 0 \\ 0 & -1 & x & & 0 & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & \ddots & x & 0 \\ 0 & 0 & 0 & & -1 & x \end{matrix} \right\|$$

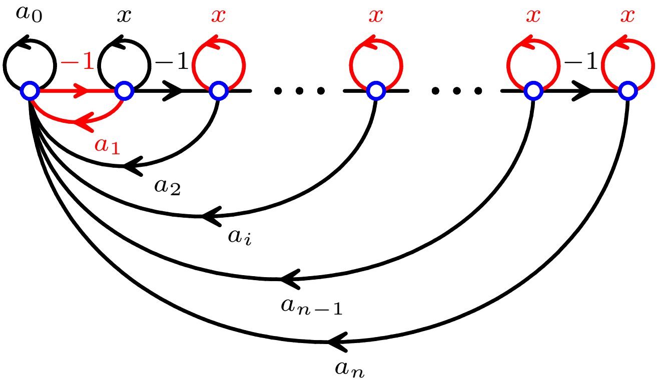




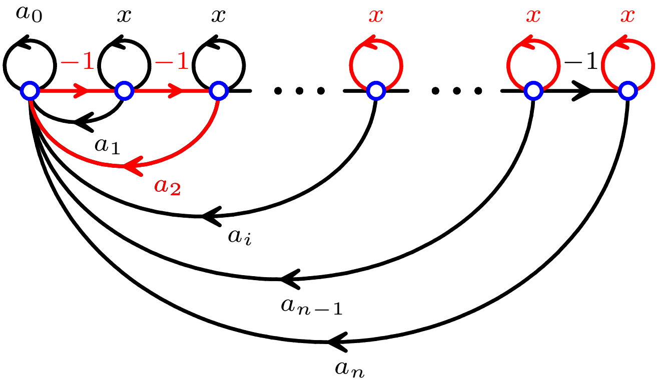
$$\det A = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_ix^{n-i} + \ldots + a_{n-1}x + a_n = \sum_{i=0}^{n} a_ix^{n-i}$$



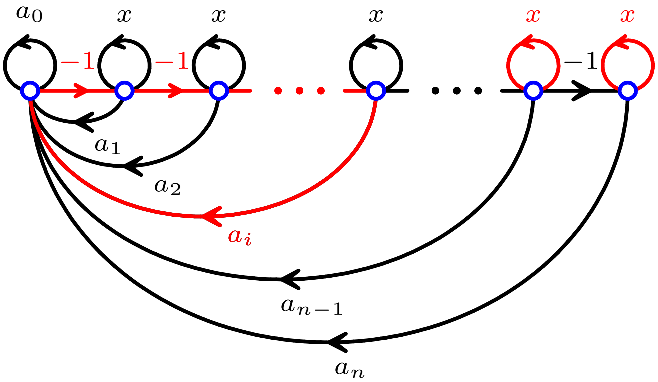
$$\det A = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_ix^{n-i} + \ldots + a_{n-1}x + a_n = \sum_{i=0}^{n} a_ix^{n-i}$$



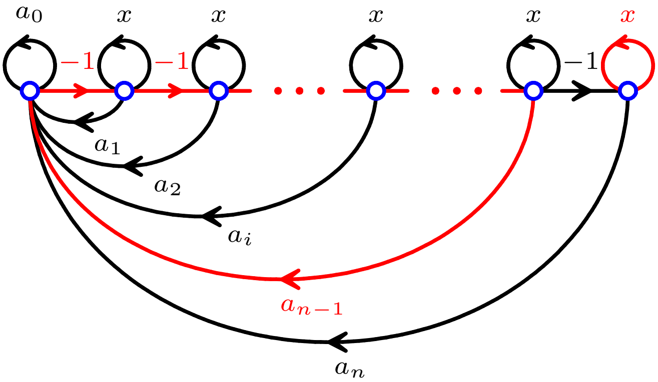
$$\det A \quad = \quad a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_ix^{n-i}$$
$$+ \ldots + a_{n-1}x + a_n = \sum_{i=0}^{n} a_ix^{n-i}$$



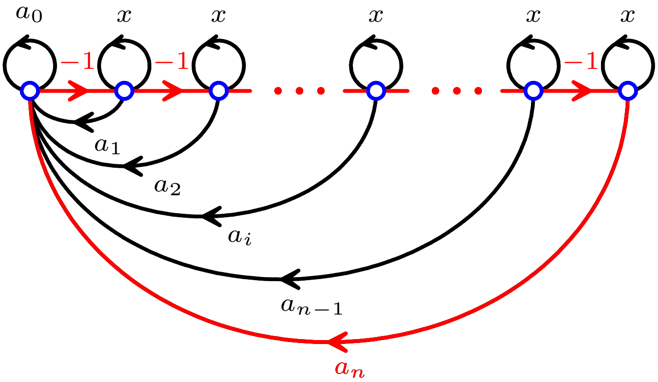
$$\begin{aligned}\det A \quad &= \quad a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_ix^{n-i} \\ &\qquad + \ldots + a_{n-1}x + a_n = \sum_{i=0}^{n} a_ix^{n-i}\end{aligned}$$



$$\det A = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_ix^{n-i} + \ldots + a_{n-1}x + a_n = \sum_{i=0}^{n} a_ix^{n-i}$$



$$\det A = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_ix^{n-i} + \ldots + a_{n-1}x + a_n = \sum_{i=0}^{n} a_ix^{n-i}$$



$$
\begin{aligned}
\det A \quad &= \quad a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_ix^{n-i} \\
&\qquad + \ldots + a_{n-1}x + a_n = \sum_{i=0}^{n} a_ix^{n-i}
\end{aligned}
$$