

**Seminar**

**Mechanics of Machines and Mechanisms - Models and Mathematical Methods**

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**Program – October, 2018.**

*Tuesday, October 09, 2018, at 17h, room 301f*

Dušan Zorica, Mathematical Institute of SASA, Belgrade, Serbia; Faculty of Sciences, University of Novi Sad, Novi Sad, Serbia

**NON-LOCAL AND MEMORY EFFECTS IN THE DYNAMIC STABILITY ANALYSIS OF RODS**

Lateral vibrations of a simply rod loaded by an axial force of constant intensity and positioned on a foundation are the subject of analysis. The main goal in dynamic stability analysis is to determine the conditions that guarantee stability, i.e., conditions under which the rod will vibrate with constant or decreasing amplitude.

First, Bernoulli-Euler moment-curvature constitutive equation, describing elastic material the rod is made of, is assumed. The rod-foundation interaction is modeled by the complex-order fractional Kelvin-Voigt model of the viscoelastic body, with the restrictions on model parameters following from the Second law of thermodynamics.

Second, Eringen's type moment-curvature constitutive equation, describing the material of the rod that shows non-local effects, is assumed. This type of non-locality is usually associated with nano-rods. The foundation shows Pasternak and viscoelastic type properties. Thus, rod-foundation interaction is modeled through rotational elastic springs, describing the foundation ability to influence the curvature of the rod and through general rheological model of the viscoelastic body corresponding to the distributed-order constitutive equation.

The solutions to problems are obtained by the separation of variables method. The critical value of axial force, guaranteeing stability, is determined. The influence of various model parameters on the value of critical axial load is examined.

This talk aims to review and summarize the dynamic stability problems analyzed in the cooperation with T. Atanacković, M. Janev, S. Konjik, B. Novaković, S. Pilipović and Z. Vrcelj.

**REFERENCES**

[1] D. Zorica, T. M. Atanackovic, Z. Vrcelj, B. N. Novakovic, Dynamic stability of an axially loaded non-local rod on a generalized Pasternak foundation, *Journal of Engineering Mechanics*. ASCE, 143 (2017) D4016003–1–10.

[2] T. M. Atanackovic, M. Janev, S. Konjik, S. Pilipovic, D. Zorica, Vibrations of an elastic rod on a viscoelastic foundation of complex fractional Kelvin-Voigt type, *Meccanica*, 50 (2015) 1679–1692.

*Tuesday, October 16, 2018, at 17h, room 301f*

Andjelka N. Hedrih, Mathematical Institute of SASA, Belgrade, Serbia

**MODELING COUPLED OSCILLATORY MECHANISM WITH PHASE-DELAY**

Coupling oscillators are present in industry, engineer and biological systems. Way of their coupling, strength of coupling will affect their dynamics and stability. We model a complex structure that consists of four coupled oscillators with viscoelastic properties and phase-delay. Visco-elastic properties of coupled oscillators were modeled using differential equations with fractional derivatives. We investigated how different phase-delay of this coupled mechanical system is related with phenomenon of dynamical absorption and resonance and stability of the system.

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## REFERENCES

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- [2] Hedrih (Stevanović) K., Machado T. *Discrete fractional order system vibrations*. International Journal Non-Linear Mechanics, 73(2015)2–11, 2013. DOI: 10.1016/j.ijnonlinmec.
- [3] Hedrih, A (Stevanovic) Hedrih, K. (2018) Kinetic energy of homologue chromosome pairs in biomechanical oscillatory model of mitotic spindle. in Book of Abstracts [Elektronski izvor] / Sixth International Conference on Radiation and Applications in Various Fields of Research, RAD 2018, 18.06 - 22.06. 2018, Ohrid, Macedonia ; [editor Goran Ristić]. - Niš: RAD Association, 2018 (Niš : RAD Association). - 1 elektronski optički disk (CDROM); 12 cm.pp. 431. ISBN 978-86-80300-03-0. COBISS.SR-ID 266468620. <http://www.rad-conference.org/books.php>
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*Tuesday, October 30, 2018, at 17h, room 301f*

Aleksandar Obradović, Faculty of Mechanical Engineering in Belgrade, Serbia

FREE VIBRATION ANALYSIS OF AXIALLY FUNCTIONALLY GRADED TAPERED, STEPPED, AND CONTINUOUSLY SEGMENTED RODS AND BEAMS

This article [1] is a continuation of research in papers [2] and [3]. In this paper a new non-iterative computational technique referred to as the symbolic-numeric method of initial parameters (SNMIP) is proposed. The SNMIP represents a modification of the iterative numeric method of initial parameters in differential form known in the literature. The SNMIP is applied to study free vibrations of Euler-Bernoulli axially functionally graded tapered, stepped, and continuously segmented rods and beams with elastically restrained end with attached masses. Both the longitudinal vibration of rods and transverse vibration of beams are considered. The influence of the attached masses and springs on the natural frequencies of vibration of axially functionally graded rods and beams is examined. The validity and accuracy of the method are proven through the comparison with the known results in the available literature. The subject of this research in the following articles will be applied to Timoshenko's beams as well as to cases of longitudinal and transverse oscillations coupled across boundary conditions.

## REFERENCES

- [1] Šalinić S., Obradović A., Tomović A., Free vibration analysis of axially functionally graded tapered, stepped, and continuously segmented rods and beams, Composites Part B, ISSN:1359-8368, Vol. 150, pp. 135 - 143, 2018.
- [2] Obradović A., Šalinić S., Trifković D., Zorić N., Stokić Z, Free vibration of structures composed of rigid bodies and elastic beam segments, Journal of Sound and Vibration, ISSN: 0022-460X, Vol. 347, pp. 126–138, 2015.
- [3] Tomović A., A Novel Approach to the Free Axial-Bending Vibration Problem of Inhomogeneous Elastic Beams With Variable Cross-Sectional Profiles, S3c, Proceedings of the 6th International Congress of Serbian Society of Mechanics, Tara, 2017.

September 17, 2018

*Натка Р. (Стевановић) Хедрић*

The chair  
Prof. dr Katica R. (Stevanović) Hedrih

*Ivana Atanasovska*

The co-chair  
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