## Mathematical Institute of the Serbian Academy of Sciences and Arts

# Seminar Mechanics of Machines and Mechanisms - Models and Mathematical Methods

### Program – June, 2018.

*Tuesday, June 12, 2018, at 17h,* Sala 301f, MI SANU, Kneza Mihaila 36 Katica (Stevanović) Hedrih, Mathematical Institute of SASA PHASE TRAJECTORY METHOD AND TRIGGER OF COUPLED THREE SINGULAR POINTS IN INVESTIGATION OF DIFFERENT MODEL NONLINEAR DYNAMICS OF MULTI-STEP REDUCTOR/MULTIPLIER SYSTEMS

# Lecture is dedicated to 150 years from birthday of Serbian Scientist Mihailo Petrović, Founding Father of Serbian Mathematics

**Keywords:** Phase trajectory method, trigger of coupled singular points, bifurcation, phase portrait, mechanical energy surfaces, homoclinic orbits and points, separatrice layering, couple triggers, trajectory in the form of number eight or it multiplicand, Chaotic Clock Models, coupled rotations, vector rotators, mass moment vectors, kinetic pressure, multistep reductor/multiplier systems, analogy, heavy mass particle motion along rough curvilinear trace with Amontons-Coulomb friction.

*Abstract:* For examine natural clocks of reductor, as well as source of nonlinear vibrations and noise in its nonlinear dynamics, it is necessary to investigate properties of nonlinear dynamics, and phase portraits, as well as structures of homoclinic orbits, layering and sensitivity of this layering of homoclinic orbits and bifurcation of homoclinic points.

Basic elements of the phase trajectory method, and by analyzing of the types of singular points, phase trajectory curves and total mechanical energy surfaces in phase space, will be presented.

A review of different examples with trigger of coupled three singular points in dynamics of different models of mechanical systems each with one degree of freedom will be presented and analyzed. Trigger of coupled three singular points appear in the phase portrait of dynamics of mechanical system with one degree of freedom and with coupled rotations and mass deviation with respect to the axes of rotations as it is generalized rolling pendulum along curvilinear trace with minimum and maximums in vertical plane. Phase portrait and constant total mechanical energy curves for each of the previous listed models of dynamics are mathematically described and graphically presented and analyzed. A theorem of existence of a trigger of coupled three singular points and a homoclinic phase trajectory in the form of number "eight" will be presented. Series of phase trajectory portraits with trigger of coupled singular points as results of investigation of nonlinear dynamics of one- as well as multistep geared reductor/multipliers will be presented.

In the Lecture mass moment vectors and vector rotators, introduced by author at ICTAM Haifa 92, are used to present a vector method for the analysis of kinetic parameter dynamics of coupled rigid rotors with deviational properties of mass changeable distribution and with couple rotations. A numerical experiment with the use of derived analytical expressions and of MathCAD program was used to create a visualization of phase portraits of nonlinear dynamics of coupled rotors and the layering of homoclinic orbits with respect to the system parameters change.

Kinetic pressures on bearing of rotors with simple as well as coupled rotations will be presented by deviational components of mass moment vectors and kinematical vector rotators coupled for corresponding bearing and axis of rotation.

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# *Tuesday, June 19, 2018, at 17h,* Sala 301f, MI SANU, Kneza Mihaila 36 Ivana Atanasovska, Mathematical Institute of SASA COMPLEX NONLINEARITY OF INVOLUTE GEARS DYNAMICS

Keywords: nonlinear dynamics, contact mechanics, involute gears

The basic theory of machines and mechanisms, as well as the illustrations from nature, runs gears as an unsurpassed topic for nonlinear mechanical phenomena research. Gear tooth profile could be formed by different curves. But, the most commonly used are gears with involute profile – with tooth profile in form of curve which is generated when straight line is rolling without slipping over the circle.

Developing of dynamic model of involute gear pair and calculation of main influential parameters are essential for studying the gears stability [1]. A methodology developing for analyzing the dynamic behavior of gear pairs will be presented with algorithm which includes developed procedures for calculation the main gears characteristics with special attention paid to calculations of time-varying contact deformations and mesh stiffness [2]. The new model for vibro-impact dynamics of gears will be also discussed [3, 4]. This mathematical model is applicable in the special cases when tooth profile dimensions and value of transmission ratio could cause a very small difference between pinion tooth thickness and wheel tooth spacewidth. This vibro-impact phenomenon is characterized with vibro-impact vibrations in teeth contact during a short period of time after the collision of pinion tooth and wheel tooth, when the number of teeth pair in contact has been change [5].

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