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European Solid Mechanics Confrtrnce
7th ESMC Lisbon 2009

Welcome

On behalf of European Mechanics Society (EUROMECH) we are pleased to welcome you to Lisbon for the 7th EUROMECH Solid Mechanics Conference held at Instituto Superior Tecnico of the Technical University of Lisbon.

EUROMECH has the objective to engage in all activities intended to promote in Europe the development of mechanics as a branch of science and engineering. Activities within the field of mechanics range from fundamental research to applied research in engineering. The approaches used comprise theoretical, analytical, computational and experimental methods. Within these objectives, the society organizes the EUROMECH Solid Mechanics Conference every three years. Following the very successful Conferences in Munich, Genoa, Stockholm, Metz, Thessaloniki and Budapest, the 7th EUROMECH Solid Mechanics Conference (ESMC2009) is now held in Lisbon. The purpose of this Conference is to provide opportunities for scientists and engineers to meet and to discuss current research, new concepts and ideas and establish opportunities for future collaborations in all aspects of Solid Mechanics.

We invite you to be an active participant in this Conference and to contribute to any topic, or mini-symposium, of your scientific interest. By promoting a relaxed atmosphere for discussion and exchange of ideas we expect that new paths for research are stimulated and promoted and that new collaborations can be fostered. We hope that the 7th EUROMECH Solid Mechanics Conference will have an important impact on the research in all topics included in its programme.

We want to express our appreciation to all members of the committees involved in the preparation of this Conference, to all mini-symposia organizers who identified and promoted some of the most active topics of research in Solid Mechanics, to all the staff who are managing the different aspects of the Conference and to all the contributing authors and participants who will create the real Conference. We hope that all of you feel rewarded for your participation and contribution.

Yours Sincerely,

Jorge A.C. Ambrosio
ESMC2009 Chairman

Ray W. Ogden
ESMC Committee Chairman

A Session at ESMC Budapest 2006 – 6th European Solid Mechanics Conference

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ESMC Lisbon 2009

Minisymposium MS-24

Kinetics, Control and Vibrorheology

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The 7th EUROMECH Solid Mechanics Conference (ESMC2009), to be held at Instituto Superior Técnico, Lisbon, Portugal,

ESMC2009 Lisbon
Minisymposia title:
MS – 24 "KINETICS, CONTROL and VIBRORHELOGY – KINCONVIB 2009"
(Kinetics and Control on the basis of the vibrorheology)
Organizer: Katica (Stevanović) Hedrih

Topics:
Kinetics and vibrorhelogy
Control and Vibrorheology
Theory of vibrorhelogical processes
Hybrid system kinetics and integrity of kinetics components
Discrete hereditary system kinetics
New view in Analytical mechanics
Fractional differentiation applications in Kinetics and Control theories
Control, Biomechanics and Vibrorheology

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**The 7th EUROMECH Solid Mechanics Conference (ESMC2009),
to be held at Instituto Superior Técnico, Lisbon, Portugal**

Preliminary Program

submitted by Organizer of MS-24

MS – 24 "KINETICS, CONTROL and VIBRORHELOGY KINCONVIB 2009"

(Kinetics and control on the basis of the vibrorheology)

Organizer: Katica R. (Stevanović) Hedrih

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http://www.masfak.ni.ac.yu/sitegenius/topic.php?id=1229

I. Session

MS-24 Oppening Lecture – 40 minutes

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On Vibrational Nano-Mechanics and Nano-Vibrorheology

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MS-24 Invited Lecture 30 minute

845

Bifurcations of a parametrically excited double pendulum

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MS-24 Invited Lecture 30 minute

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Control Methods in Vibroacoustics - Active Noise and Vibration Suppression

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II. Session

MS-24 Invited Lecture 40 minute

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Relation Between Fractional Differential Equations and Ordinary Differential Equations

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MS-24 Invited Lecture 30 minute

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Further Results on Fractional Process Control Systems

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MS-24 Invited Lecture 30 minute

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Transfer of energy of oscillations through the double DNA chain helix

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III. Session

MS-24 Invited Lecture 40 minute
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On the stability of potential mechanical system with tracking and dissipation forces

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MS-24 Invited Lecture 30 minute
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Phase Trajectory of Aeroelastic Dynamic Systems in an Expanded Phase Space

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MS-24 Invited Lecture 30 minute
291

Energy Transfer through the Double Circular Plate Nonconservative System Dynamics

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IV. Session

MS-24 Invited Lecture 30 minute

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Model of actuated multilayered piezoelectric structures for antifouling process

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MS-24 Invited Lecture 30 minute

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LOCAL STRAIN ENERGY AT THE CRACK TIP VICINITY IN DISCRETE MODEL OF MATERIAL

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MS-24 Contributed Lecture 20 minute

Energy of the vibroimpact systems with Coulomb's type frictions

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MS-24 Closing Lecture 40 minutes

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Free and forced vibrations of the heavy material particle along line with frictions: Direct and inverse task of the theory of vibrorheology

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Minisymposium Poster Session

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DESIGN OF A SYSTEM FOR CONTROL, MONITORING, REGULATION AND DATA ACQUISITION ON CIVIL ENGINEERING OBJECTS, CONSTRUCTIONS AND MOBILE MODULES

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Minisymposium Opening Lecture

On Vibrational Nano-Mechanics and Nano-Vibrorheology

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ABSTRACT

Conceptions of vibrational mechanics and vibrorheology were formulated by the author of this presentation and they were adopted for solving of many applied problems where the effects of the vibration on non-linear systems were considered [1.2].
In this presentation, we consider the possibilities to extend appropriate approaches on nano-objects. The main peculiarities of nano-systems have been considered, in particular, the necessity to use high frequency vibrations and to take into account relatively large dissipative forces.
A number of problems on control of nano-system states by high frequency vibration have been considered including vibrational orientation, motion, and positioning.

References

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[2] *Selected Topics in Vibrational Mechanics*, edited by I. Blekhman, World Scientific, Singapore, 2004

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Participants of the Minisymposium: Integrity of Dynamical Systems-ECF 16 – Alexandroupolis, Greece *2006*

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Minisymposium Invited Lecture

Relation Between Fractional Differential Equations and Ordinary Differential Equations

V. Lakshmikantham and S. Leela

Abstract: Although the concept of fractional derivative goes back to the 17th century, its importance in modeling a variety of real world problems was realized only a few decades ago [1, 3]. Moreover, only very recently, there has been a great surge in the theory of fractional differential equations (FDE). A recent monograph provides a systematic study of the basic theory of FDE [2].

In this article, we shall investigate the relation between the solutions of FDE and ODE (ordinary differential equations). This will provide a framework to study the properties of solutions of FDE, knowing the corresponding properties of solutions of ODE. We hopw this study will pave the way for further work in this direction. We shall consider both Riemann-Liouville type and Caputo type FDE. These require different methods, each having certain advantage in the investigation and providing necessary tools. We shall discuss some theoretical results to demonstrate the idea behind these approaches.

References

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President, International Federation of Nonlinear Analysts IFNA
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Minisymposium Invited Lecture

Bifurcations of a parametrically excited double pendulum

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ABSTRACT

The works on the surprising phenomenon of parametric resonance form a rich body of literature, including several books and monographs. A good deal of efforts has been directed towards methods for constructing the instability regions of parametrically excited systems typically governed by linear ODEs with periodic coefficients.

Numerous works have dealt with the parametric resonance of the simple pendulum due to its paradigmatic nature and simplicity in experimental investigations which does not imply simplicity in the dynamics as was clearly pointed out by Stuart Antman in one of his well-known manuscripts titled *The simple pendulum is not so simple*. As well-known, for small-amplitude oscillations, the equation of motion reduces to Hill's equation or to Mathieu's equation if the parametric forcing term is a sinusoidal forcing [3]. Some features of the chaotic dynamics of the 'simple' pendulum were discussed in [5] and in [4] for the double pendulum; interesting features of various higher-order parametric resonances in a double pendulum were discussed in [2].

The aim of this contribution is to investigate, both experimentally and numerically, some fundamental features of the Hopf bifurcations caused by the parametric resonances. The double pendulum is driven to resonance by an oscillating external motion of its pivot point in the vertical direction. The pendulum arm attached to the oscillating pivot has a mass which is twice the mass of the second arm. The frequencies of the two modes are $f_{in} \approx 1.4$Hz (in phase) and $f_{out} \approx 2.4$Hz (out of phase). The system dynamics are described by the following two coupled ODEs:

$$\dot{\theta}_1 = \omega_1, \dot{\theta}_2 = \omega_2 \tag{1}$$

$$I\dot{\omega}_1 + \gamma\dot{\omega}_2\cos(\theta_1 - \theta_2) - \gamma\sin(\theta_1 - \theta_2)\omega_2^2 - C(\ddot{y}_p - g)\sin(\theta_1) + b\omega_1 = 0 \tag{2}$$

$$I_2\dot{\omega}_2 + \gamma\dot{\omega}_1\cos(\theta_1 - \theta_2) - \gamma\sin(\theta_1 - \theta_2)\omega_1^2 - R(\ddot{y}_p - g)\sin(\theta_2) + b\omega_2 = 0 \tag{3}$$

where $y_p = a\cos(2\pi f_p t)$, θ_1 and θ_2 are the angles that arms 1 and 2 make with the vertical direction, a is the amplitude of the pivot oscillation and f_p is its frequency. There are four fixed points $(\theta_1^*, \theta_2^*) = (0,0), (0,\pi), (\pi,0)$ and (π,π) with $(\omega_1^*, \omega_2^*) = (0,0)$ whose stability depends on a and f_p. Besides the stability regions established in the parameter plane (f_p, a), we investigated Hopf bifurcations related to the fixed point $(\theta_1^*, \theta_2^*) = (0,0)$. We observed a subcritical Hopf at low excitation frequencies f_p and a supercritical bifurcation at higher frequencies. Between these two bifurcations we observed two more bifurcations that evolve, as

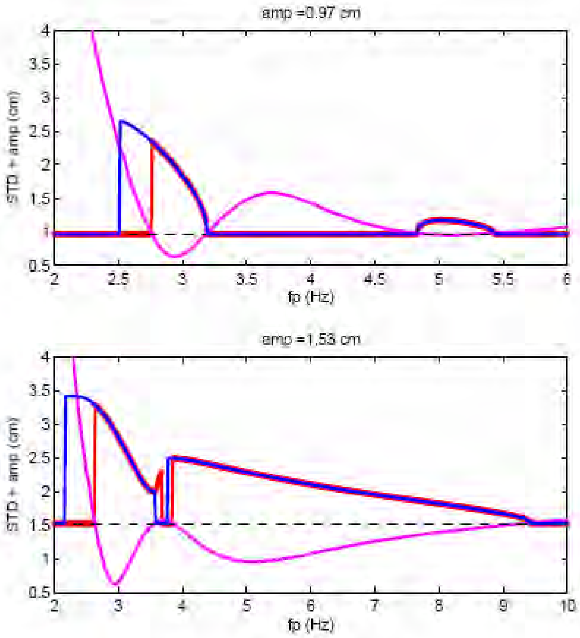

Figure 1: The Hopf bifurcation diagrams before and after the transitions from subcritical to supercritical.

the excitation amplitude a is increased, from supercritical to subcritical until they collide and disappear. The figure shows the Hopf bifurcations before and after the transitions from subcritical to supercritical. The radius of the limit cycle was calculated as the standard deviation (STD) of stationary states. The stability boundary of the fixed point point $(0,0)$ is in magenta; STD, in red(blue), is shown when increasing (decreasing) the frequency.

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Minisymposium Invited Lecture

Control Methods in Vibroacoustics – Active Noise and Vibration Suppression

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ABSTRACT

Design and implementation of active vibration and noise control in smart structures gains more and more interest in recent years [1]. For this purposes different approaches have been developed. Depending on special requirements, different controller design approaches or their combinations can be applied in order to solve complex problems of active vibration and noise attenuation. Control methods considered in this contribution regard in the first place applications to smart mechanical structures, i.e. their active control in the sense of vibration and/or noise reduction using piezoelectric materials for actuation and sensing.

For design and implementation of controllers as an integral part of smart structures a model based approach is supposed, which considers two different ways to obtain models of smart structures. The first, numerical approach, results in appropriate models of smart structures under consideration of integrated active materials and their electro-mechanical behavior [2]. In relevant cases an appropriate model-description of environmental influence, like acoustic fluid, can be included. A suitable basis for such considerations is offered by finite element (FE) models of structural behavior (electro-mechanical), which can be augmented by acoustic fluid behavior [3]. The second approach to modeling of smart structures for active vibration and noise control is experimental system identification. This approach is possible only if prototypes or real structures are available. For linear time-invariant models, the subspace identification approach [4] has been proven to result in suitable state space models, comparable with FE-models, which can serve as a good starting point for controller design purposes [5].

In this contribution controller design methods for vibration and/or noise reduction of smart piezoelectric structures are proposed and their feasibility is demonstrated by examples.

Proposed controllers are discrete-time controllers, which enables their real-time implementation. Two approaches, optimal LQ tracking system with additional dynamics and model reference adaptive control, as well as their combination, are considered. The novelty in the proposed approach represents implementation of the optimal LQ controller for the reference model design of an adaptive controller. In this way the desired behavior of the

controlled model reference adaptive system is prescribed by the optimal tracking system, which guaranties realization of the control task, providing at the same time stability and robustness of the closed-loop system with respect to the controller gains convergence. Special excitation cases of interest involve periodic excitations with frequencies equal to the structural eigenfrequencies, due to the possibility of resonance. Proposed controllers can successfully cope with such excitations.

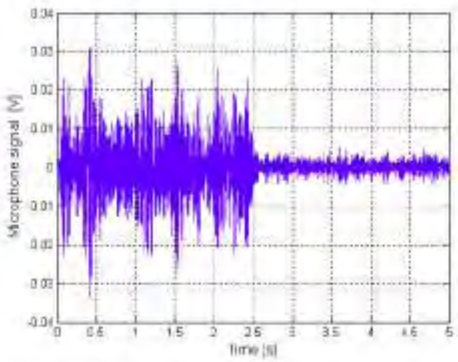

Figure 1 *Uncontrolled and controlled air-pressure signal in the presence of a random excitation*

As an example of the controller implementation results, the noise attenuation of a piezoelectric structure due to optimal LQ control with additional dynamics is represented in Fig. 1 in terms of the air-pressure magnitude reduction in the presence of a random excitation.

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Minisymposium Invited Lecture

On The Stability Of Potential Mechanical System With Tracking And Dissipation Forces

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The stability of trivial solution $q=\dot{q}=0$ of mechanical system with two degrees of freedom are investigated. For ideal constraints, the equations of motion take the form:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}}-\frac{\partial T}{\partial q}=-\frac{\partial U}{\partial q}+Q(q)+\varepsilon\frac{\partial \Phi}{\partial \dot{q}},\ q=(q_1,q_2) \tag{1}$$

Here U is a potential energy, Q is a generalized tracking force, $\Phi=\langle \bar{B}\dot{q},\dot{q}\rangle$ is a Rayleigh's dissipative function, ε is an arbitrary parameter.

The linear equations reduced to normal coordinates can be written as follows

$$\ddot{x}+\varepsilon B\dot{x}+Cx+Px=0 \tag{2}$$

Here C is a symmetric matrix, P is a skew-symmetric matrix, B is a positive definite matrix of dissipation.

When $\varepsilon=0$, necessary and sufficient conditions for stability of trivial equilibrium of equations (2) are well known [1]:

$$\operatorname{tr} C>0,\ \det(C+P)>0,\ (\operatorname{tr} C)^2-4\det(C+P)>0 \tag{3}$$

Let $h=\operatorname{tr}B\operatorname{tr}C-\operatorname{tr}(CB)$, $\omega_{1,2}$ be positive roots of equation $\det\left(-\omega^2 I+C+P\right)=0$, ε_* be the positive parameter which defined by

$$\varepsilon_*^2=\frac{1}{h\det B}\left\{\frac{h^2}{\operatorname{tr}B}-h\operatorname{tr}C+\operatorname{tr}B\det(C+P)\right\},$$

where $\operatorname{tr}B>0$, $\det B>0$ by virtue of positive definiteness of matrix B.

The following theorem gets the criterion of stability for trivial equilibrium when dissipative forces are arbitrary.

Theorem. I. *Suppose conditions* (3) *are satisfied; then, if a parameter h satisfies inequalities*

$$0<h<\omega_1^2\operatorname{tr}B \quad or \quad h>\omega_2^2\operatorname{tr}B,$$

the trivial solution $q=\dot{q}=0$ *is unstable for* $\varepsilon\in(0,\varepsilon_*)$ *and asymptotical stable for* $\varepsilon\in(\varepsilon_*,\infty)$. *If*

$$\omega_1^2 \operatorname{tr} B \le h \le \omega_2^2 \operatorname{tr} B,$$

then the asymptotical stability takes place for any $\varepsilon \in (0, \infty)$.

For $h < 0$, *the equilibrium* $q = \dot{q} = 0$ *is unstable for arbitrary* ε.

II. *If conditions* (3) *are broken, then we have the following. Suppose that inequalities*

$$(\operatorname{tr} C)^2 - 4\det(C+P) < 0, \ h > 0$$

or

$$(\operatorname{tr} C) < 0, \ (\operatorname{tr} C)^2 - 4\det(C+P) > 0, \ \det(C+P) > 0, \ h > 0$$

are satisfies, then the trivial solution $q = \dot{q} = 0$ *is unstable for* $\varepsilon \in (0, \varepsilon_*)$ *and asymptotical stable for* $\varepsilon \in (\varepsilon_*, \infty)$. *Otherwise the equilibrium is instable for arbitrary* ε.

These results are applied to investigate the stability of equilibriums of double-hinged mechanism [2].

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Minisymposium Invited Lecture

Further Results on Fractional Process Control Systems

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ABSTRACT

The paper presents a new algorithms of fractional process control - $PI^{\alpha}D^{\beta}$ type which may include iterative learning feedback control (ILC). When the structure of object is not known or when many parameters cannot be determined, iterative learning control may be considered, [1]. Motivated by human learning, the basic idea of iterative learning control is to use information from previous execution of a trial in order to improve performance from trial to trial. This is an advantage, when accurate model of the system is not available as friction and actuator dynamics, though present in the system, are not modeled to reduce the computational complexity. Therefore, iterative learning control requires less *a priori* knowledge about the controlled system in the controller design phase and also less computational effort than many other kinds of control. ILC is a technique to control systems operating in a repetitive mode with the additional requirement that a specified output trajectory $y_d(t)$ in an interval $[0,T]$ be followed to a high precision and in order to improve performance from trial to trial in the sense that the tracking error is sequentially reduced. Also, the fractional integro-differential operators-(fractional calculus) is a generalization of integration and derivation to non-integer order (fractional) operators (α,β) and they provide an excellent instrument for the description of memory and hereditary properties of various materials and processes and, also obtaining more degrees of freedom in the model, [2]. Fractional order controllers can significantly improve static and dynamic control system properties and they are less sensitive to controlled systems and controllers parameters variations and can be used as robust controllers. In this paper different aspects of $PI^{\alpha}D^{\beta}$ including the design schemes and control algorithms are covered. The control scheme comprises two types of control laws: a $PI^{\alpha}D^{\beta}$ feedback law and a feed-forward control law where the $PI^{\alpha}D^{\beta}$ controller provides stability of the system and keeps its state errors within uniform bounds and in the feed-forward path, a learning control rule/strategy is exploited to

track the entire span of a reference input over a sequence of iterations.[3]. The discretization of the continuous fractional-order operators and considered systems will be also investigated in detail and presented. Specially, for robust control law, it is also proposed the a variant of Time Delay Control [4]. Also, a finite time stability test procedure is proposed for (non)linear (non)autonomous time-invariant delay fractional order systems. Moreover, we examine the problem of sufficient conditions that enable system trajectories to stay within the *a priori* given sets for the particular class of (non)linear nonautonomous fractional order time-delay systems. Finally, to demonstrate the benefits of proposed fractional process control of $PI^{\alpha}D^{\beta}$ type an mechatronic system, (producing of technical gases), is used for the simulation study.

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Minisymposium Invited Lecture

Phase Trajectory of Aeroelastic Dynamic Systems in an Expanded Phase Space

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ABSTRACT

Research into aeroelastic oscillations in structures and buildings subjected to an air flow is of interest owing to the progressively longer bridge span frameworks and overpasses and taller mast- and tower-type structures. No matter to which type such lengthy structures under consideration belong, the aerodynamic loads, which affect the structures and cause their steady oscillations, represent the non-linear conservative functions of the displacements and the velocities of the structural components. A distinguishing feature of the oscillations of the elastic structures in a wind flow is the complex interactions between the aerodynamic loads and the parameters of oscillations of bodies in a wind flow.

In some models of the aeroelastic oscillations of structures, the values the aerodynamic force are determined not only by the aerodynamic resistance but also by the periodic load resulting from the alternate shedding of the Karman vortices from the lateral surfaces of the bluff bodies. The load of this type arises from the stalled flow-around of the structures and represents an external periodic force with its frequency depending on both the cross-sectional shape in a structural component and on the wind flow velocity.

This article describes the results of the research into peculiarities of the dynamic interactions between a surface pipeline and a horizontal uniform wind flow. The investigations have been carried out on the basis of the equations [1-4] for the interactions between a bluff body of a circular cylinder (i.e. a surface pipeline) and a uniform wind flow.

Dynamic behaviour of mechanical systems is usually presented as oscillating processes in various graphic forms such as time processes, the Lissajous patterns and hodograph. Such patterns of presentations enable to determine the type of a process and to perform numerical estimations of its characteristics, but do not disclose any properties of the governing system. Unlike them phase trajectories have the row of advantages. The image on phase plane "acceleration - displacement" is a more vivid presentation because it depicts inharmonious oscillations particularly well. Each phase trajectory represents only one definite clearly defined motion. The geometric presentation of a single phase trajectory or a set of trajectories allows coming to important conclusions about the oscillation characteristics. It is, foremost, true with the oscillations, which are described with nonlinear differential equations. As is has been shown by the investigations of author [5], the expansion of a phase space by taking into account the phase plane "acceleration - displacement" substantially promotes the efficiency in analyzing a dynamic system behaviour. Hereby, we pass on to a three-dimensional phase space confined with three co-ordinate axes, i.e. displacement, velocity and acceleration. An interest taken into accelerations in dynamic systems is conditioned by the

fact that these accelerations are more sensitive to high-frequency components in oscillating processes.
In the hybrid modeling, the analysis of the oscillations was performed for a steel pipeline having the length of $l = 35m$, the diameter of $d = 0,426\ m$, and the wall thickness of $\Delta = 0,004\ m$. The wind flow velocity values varied within the range of $V = 0 \div 30ms^{-1}$. Let us assume that the value of the initial static deflection of a pipeline is equal to $W_C = 0,05077\ m$. Time processes, spectral characteristics of distributing of energy of oscillations on their frequencies and phase trajectories in expanded phase space (Fig.1) , are received at transient and stationary behaviours of vertical vibrations in a wide frequency range.

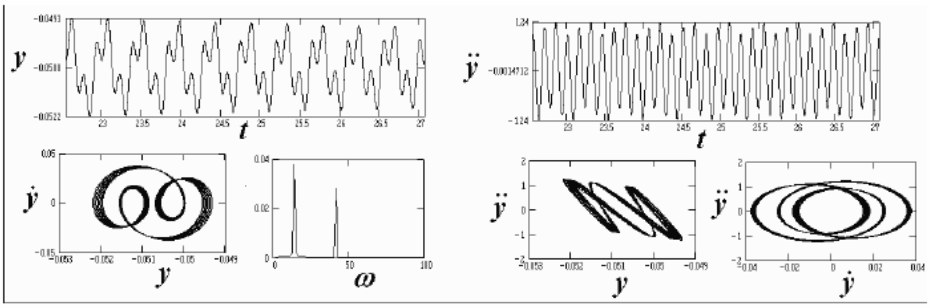

Fig. 1. Time processes, spectral characteristics and phase trajectories of vertical oscillations in a surface sagging pipeline at the velocity of $V = 14,165ms^{-1}$ of the uniform wind flow

The phase trajectories obtained for the modes of "beatings" on the planes "velocity-displacement" and "acceleration - displacement" convolve or unwind alternately. The distance between the spiral loops varies, depending on the action exerted by the dissipative forces per cycle of oscillations. The phase trajectories of the modulated oscillations on the planes "velocity-displacement" and "acceleration - displacement" consist of three closed curves, which are inversely symmetrical relative to the axes velocity and acceleration, respectively. The hysteretic effects in a resonance frequency ranges and zones with iteration of free and forced oscillations were found.

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Minisymposium Invited Lecture

Energy Transfer through the Double Circular Plate Nonconservative System Dynamics

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ABSTRACT

The study of the transfer energy between subsystems coupled in hybrid system is very important for different applications. This paper presents analytical analysis of the transfer energy between plates for free transversal vibrations of an visco-elastically connected double circular plate system. The analytical analysis showed that the visco-elastic connection between plates caused the appearance of two-frequency like regime of time function, which corresponds to one eigen amplitude function of one mode, and also that time functions of different vibration modes are uncoupled, but energy transfer between plates in one eigen mode appears. It was shown for each shape of vibrations. Series of the two Lyapunov exponents [3] corresponding to the one eigen amplitude mode are expressed by using energy of the corresponding eigen amplitude time component.

The paper (see Ref. [2]) presents analytical and numerical analysis of the free and forced transversal vibrations of an visco elastically connected double circular plate system. Analytical solutions of the system of coupled partial differential equations, which describe corresponding dynamical free and forced processes, are obtained using the method of Bernoulli's particular integral and Lagrange's method of variation constants. It was shown that one-mode vibrations correspond two-frequency regime for free vibrations induced by initial conditions and to three-frequency regime for forced vibrations induced by one-frequency external excitation and corresponding initial conditions.

This model of double plate system is suitable for considering energy transfer between plates, by using derived partial differential equations in the generalized form by introduction a visco-elastic layer between plates.

Also, by using energy analysis of the deformable bodies (see Refs. [3], [4]) for the linear case, we can conclude that transfer energy between eigen amplitude modes not appear, only there are two frequency component time processes in one mode and there are transfer energy between two one frequency time components and between plates in one eigen amplitude mode.

In the paper we will present expressions of the reduced components of kinetic energy $\tilde{\mathbf{E}}_{k,nm}^{(i)} = \dfrac{\mathbf{E}_{k,nm}^{(i)}}{M_{(1)nm}(r,\varphi)_i} = \dfrac{\rho_i h_i}{2}\left[\dot{\mathrm{T}}_{(i)nm}(t)\right]^2$, $i=1,2$, where $M_{(1)nm}(r,\varphi)_i = \iint\limits_A \left[\mathrm{W}_{(i)nm}(r,\varphi)\right]^2 r dr d\varphi$, and reduced

potential energy (see Refs. [3], [4]) $\tilde{\mathbf{E}}^{(i)}_{p,nm} = \frac{1}{2}\rho_i h_i \omega^2_{(i)nm} M_{(1)nm}\left[\mathrm{T}_{(i)nm}(t)\right]^2 = \frac{\mathbf{E}^{(i)}_{p,nm}}{M_{(1)nm}}$, as well as reduced potential energy of the light distributed visco-elastic layer $\tilde{\mathbf{E}}^{(i)}_{pnm(1,2)layer} = \frac{1}{2}c\left[T_{(2)nm}(t) - T_{(1)nm}(t)\right]^2 = \frac{\mathbf{E}_{pnm(1,2)sloja}}{M_{(1)nm}}, i = 1,2$, and reduced Raleigh function of the dissipation $\tilde{\Phi}_{nm(1,2)layer} = \frac{1}{2}b\left[\dot{T}_{(2)nm}(t) - \dot{T}_{(1)nm}(t)\right]^2 = \frac{\Phi_{nm(1,2)sloja}}{M_{(1)nm}}$ belong to corresponding mn-family mode $n,m = 1,2,3,4,\dots\infty$.

We can see that is suitable to use energy analysis in the form of the corresponding mn-family mode $n,m = 1,2,3,4,\dots\infty$ by using these reduced components of the energy depending of their times functions $T_{(i)nm}(t)$ and its first derivatives $\dot{T}_{(i)nm}(t)$ belong to corresponding mn-family mode $n,m = 1,2,3,4,\dots\infty$. For that reason we can use infinite numbers of the sets belong to corresponding mn-family mode $n,m = 1,2,3,4,\dots\infty$ by systems of the two ordinary differential equations in every of the sets corresponding to our physical model. Then we will present how we, by using phenomenological mapping, make energy analysis as in the systems with two degree of the freedom and pictured diagrams of energy forms in the time for modeled system like as pictured in the figure 1.

At the end by using this energy approach we introduce Lyapunov exponents of this type and the way for coupled hybrid systems with different type of the material properties, as it is visco elastic or creep, and to use for investigation of the stability process, or deformable forms of the deformable body motion in the hybrid systems. Than we can see that these Lyapunov exponents [4] are measures of the processes integrity or system motion integrity.

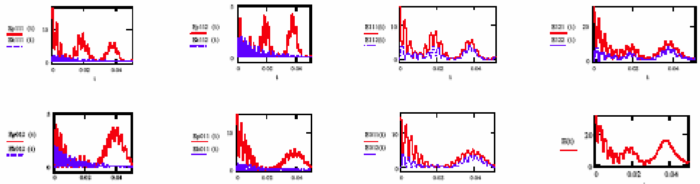

Figure 1. Reduced values of kinetic, potential and total energy corresponding to first and second mode of plates oscillation like as system oscillations

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Minisymposium Invited Lecture

Transfer of energy of oscillations through the double DNA chain helix

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ABSTRACT

Different models of two coupled homogeneous DNA chain vibrations are proposed in the literature (see Refs. [4] and [5]). As a basic approach we use DNA mathematical models published by N. Kovaleva and L. Manevich in 2005 and 2007, and investigated corresponding linearized models. We consider the linear natural and fractional order model to obtain main chain subsystems of the double DNA fractional order chain helix (see refs. [1] [2] and [3]). Analytical expressions of the eigen circular frequencies for the homogeneous linearized model of the double DNA chain helix are used to obtain corresponding eigen fractional order creep vibration modes. Two sets of eigen normal coordinates of the double DNA linearized natural and fractional order chain helix for separation of the system into two uncoupled linear, as well as fractional order main chains are identified (see Reds. [2] and [3]). The visualization of the eigen fractional order creep vibration modes of the double DNA fractional order chain helix is presented. The study of energy transfer between subsystems coupled in a hybrid system is very important for better understanding the translation process of DNA.

This paper presents an analysis of the energy transfer between partial oscillators in the main chains of the double DNA that is considered as a linearized order chain helix.

Series of two sets of Lyapunov exponents corresponding to a set of modes of partial oscillators in the corresponding main chain are expressed by using energy of the corresponding eigen time component.

The simplest model describing opening of DNA double helix is presented in Ref. [5] by Kovaleva N., L.Manevich (2005). Corresponding differential equations are solved analytically using multiple-scale expansions after transition to complex variables. Obtained solution corresponds to localized torsional nonlinear excitation – breather. Stability of breather is also investigated.

By using change of the generalized coordinates $\varphi_{k,1}$ and $\varphi_{k,2}$ for k-th bases of both chains in the DNA model into following new (for detail see Reds. [2] and [3])

$$\xi_k = \varphi_{k,1} - \varphi_{k,2} \quad \text{and} \quad \eta_k = \varphi_{k,1} + \varphi_{k,2} \tag{1}$$

system of linearized differential equations of the double DNA linear order chain helix vibrations obtain the following form:

$$\frac{1}{\omega_0^2}\ddot{\xi}_{\mathbf{k}} - \xi_{\mathbf{k+1}} + 2\xi_{\mathbf{k}}\left[1+\mu-\kappa\right] - \xi_{\mathbf{k-1}} = 0 \tag{2}$$

$$\frac{1}{\omega_0^2}\ddot{\eta}_k - \eta_{k-1} + 2\eta_k(1+\mu) - \eta_{k-1} = 0,\ k = 1,2,3,\ldots\ldots,n \tag{3}$$

where we use the following notation:

$$\kappa = \frac{K_{\alpha\beta}}{2K}\left(1 - \frac{\omega_{\alpha\beta 2}}{\omega_{\alpha\beta 1}}\right)(r_\alpha - r_\beta)^2,\ \ \mu = \frac{K_{\alpha\beta} r_\alpha (r_\alpha - r_\beta)}{K},\ \frac{1}{\omega_0^2} = \frac{2\mathbf{J}}{K} \tag{4}$$

By use trigonometric method we obtain the following sets of the eigen circular frequencies:

$$\omega_s^2 = 2\omega_0^2\left[2\sin^2\frac{\varphi_s}{2} + (\mu - \kappa)\right],\ \omega_s^2 = 2\omega_0^2\left[2\sin^2\frac{\vartheta_s}{2} + \mu\right],\ s = 1,2,3,4\ldots,n \tag{5}$$

Analytical expressions of the reduced values of kinetic and potential energies of the main chains of the double DNA linear order chain helix are:

a* for kinetic energy for first and second main chain are:

$$\widetilde{E}_{Kin,\xi} = \frac{1}{2}\sum_{k=1}^{k=n}\dot{\xi}_k^2 \text{ and } \widetilde{E}_{Kin,\eta} = \frac{1}{2}\sum_{k=1}^{k=n}\dot{\eta}_k^2 \tag{6}$$

b* for potential energy for first and second main chain are:

$$\widetilde{E}_{p,\xi} = \frac{1}{2}(\xi_k)\widetilde{\mathbf{C}}_\xi\{\xi_k\} = \frac{1}{2}\sum_{k=1}^{k=n}\sum_{j=1}^{j=n}\widetilde{c}_{\xi,kj}\xi_k\xi_j \tag{7}$$

$$\widetilde{E}_{p,\eta} = \frac{1}{2}(\eta_k)\widetilde{\mathbf{C}}_\eta\{\eta_k\} = \frac{1}{2}\sum_{k=1}^{k=n}\sum_{j=1}^{j=n}\widetilde{c}_{\eta,kj}\eta_k\eta_j \tag{8}$$

where $\widetilde{\mathbf{C}}_\xi = (c_{\xi,kj})$ and $\widetilde{\mathbf{C}}_\eta = (c_{\eta,kj})$ are reduced matrices of the main chains subsystem rigidity of the double DNA linear order chain helix visible from the system of differential equations (2) and (3).

The analysis showed that there is no transfer of energy between main chains of the double DNA considered as a linearized order chain helix, and that transfer of energy appears only between main partial oscillators in the corresponding subset of the corresponding main chain. These results may be important for future application in theoretical and experimental medical investigations.

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Minisymposium Invited Lecture

Model of actuated multilayered piezoelectric structures for antifouling process

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ABSTRACT

A smart structure is a system containing multifunctional parts that can perform sensing, control, and actuation; it is a primitive analogue of a biological body. Smart materials are used to construct these smart or adaptive structures, which can perform both sensing and actuation functions. The adaptive structures have the ability to adapt, evolve or change their properties or behaviour in response to the environment around them. The analysis and design of adaptive structures requires a highly multi-disciplinary approach which includes elements of structures, materials, dynamics, control, design and inspiration taken from biological systems [?, ?]. Development of adaptive structures has been taking place in a wide range of industrial applications, but is particularly advanced in the aerospace and space technology sector with morphing wings, deployable space structures; piezoelectric devices and vibration control of tall buildings. The concept of these active structures can be applied to oceanographic instrumentations in order to reduce the adhesion of fouling on these structures [?, ?]. The idea is to design an intelligent structural system that is able to protect itself against the fouling. In the design of actively controlled flexible structures, the determination of the actuator location is very important. In this work, the constitutive equations of multilayered piezoelectric structures are derived in a new form and the electromechanical coupling is presented. In order to obtain a solution, to system of equations is discredited using the finite element approach and thus the equations of motion are derived. The eigenvalue problem is solved. Finally the complete solution, providing the response and modal amplitude of the vibrating structure to a harmonic piezoelectrically induced excitation, is obtained. The main original results on the parameters governing the modal amplitudes and application of this study to antifouling process are presented.

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Minisymposium Invited Lecture

Energy of the vibro-impact systems with Coulomb's type frictions

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This paper was inspired by lectures and/or abstracts and papers written by the following scientists Matrosov V.M. , Chernousko F. and Blekhman I. Aim of this paper is to show that basic effects and phenomena in the systems with vibrorheological properties and vibro-impact dynamics can be investigated by using vibrations of the heavy material particle along rough curvilinear line (circle) with Coulomb type friction.

Friction is a complex physical phenomenon, which is still insufficiently investigated. Laws of friction were studied by Leonardo da Vinci who discovered that when a body moves along a horizontal surface, it experiences the force which obstructs the motion and depends on the body's weight. Coulomb introduced the concept of friction coefficient and concluded that its value is dependent on the material and on the state of surfaces interacting with friction, but does not depend on the surface of contact. The formula $F_{\mu} = -\mu F_N \, sign\, \vec{v}$ is known as the Admonton-Coulomb law.

Paper present analytical results and some visualizations of the kinetic, potential and total mechanical energy of a vibro-impact system with one degree of freedom and one side impact limiter of the system elongations. Mechanical energy of a material particle oscillations along a rough circle in vertical plane, with Coulomb's type friction and with one side impact limiter of the elongations is object of the analytical research. The corresponding ordinary nonlinear differential double equation of dynamic equilibrium states is accompanying with corresponding initial conditions and corresponding conditions of the ideally elastic or no elastic impact conditions, as well as conditions of the alternation of the friction force of the Coulomb's type for the case no large initial conditions.

In this case, that defined system at initial moment have small kinetic energy, as well as potential energy, when initial angular elongation is smaller them angle of the limiter position δ, $\varphi_0 < \delta$ and initial angular velocity is in the interval bounded by: $\sqrt{2\frac{g}{R}(1-\cos\delta)} < \dot{\varphi}_0 < \sqrt{4\frac{g}{R} - 2\frac{g}{R}(1-\cos\delta)}$.

First integrals of the governing nonlinear differential equation of the material particle motion in considered case, earlier published, with corresponding integral constants are used for analytical and graphical analysis of the kinetic, potential and total mechanical energies of the vibro-impact system, as well as power of the friction force works in characteristic intervals of the motions between impacts and between alternations of the friction force directions. Also, bifurcations of the possible different solutions of the material particle nonlinear dynamics along rough circle with two side limit impacts are considered.

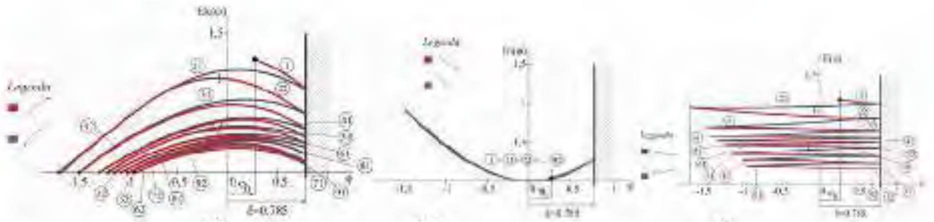
Analytical expressions of the kinetic energy branches in the intervals between two impacts, as well as between impacts and positions of the alternations of friction force directions are presented. By use these analytical expressions and by use MathCad a visualization of the vibro-impact oscillation energies were presented. Conditions for corresponding numbers of impacts and positions of the alternations of the friction force directions, before oscillations with out impacts are analyzed, as well as conditions up to heavy material particle rest state on the rough circle with friction of Coulomb's type are pointed out.

Keywords: Heavy material particle, rough circle, Coulomb's type friction, limiter, vibro-impact, initial conditions, total energy, kinetic and potential energy, analytical expression, graphical presentation, representative point.

Expressions of kinetic and potential energy. Kinetic and potential energies of the heavy material particle motion along rough circle expressed by generalized coordinate φ in the corresponding interval, or subinterval of the oscillations, are in the following forms:

$$\mathrm{E}_{k(i)}=\frac{mgR}{\left(1+4tg^{2}\alpha_{0}\right)\cos\alpha_{0}}\left[\cos(\varphi+\alpha_{0})-2tg\alpha_{0}\sin(\varphi-\alpha_{0})\right]+\frac{1}{2}mR^{2}C_{i}e^{\mp 2\varphi tg\alpha_{0}} \tag{1}$$

$$\mathrm{E}_{p(i)}=mgR(1-\cos\varphi),\quad i=1,2,3,\ldots,n \tag{2}$$



a* b* c*

Figure 1. a* Graphical presentation of the kinetic energy of the heavy material particle oscillations along rough circle with one side impact limit elongations: Kinetic energy branches dependent of the generalized coordinate φ with one side impact limit of the angular of elongation

b* Potential energy graphical presentation of the heavy material particle oscillations along rough circle with one side impact limit of elongations: Potential energy branches dependent of the generalized coordinate φ with one side impact limit of the angular elongation

c* Graphical presentation of the total mechanical energy of the heavy material particle oscillations along rough circle with one side impact limit elongations: Total mechanical energy branches dependent of the generalized coordinate φ with one side impact limit of the angular elongation

in which upper sign is for $\dot{\varphi}>0$ and lower sign for $\dot{\varphi}<0$, according alternations of the friction force alternations, correspond to the opposite direction of the material particle motion. For complete conditions of the material particle motion it is necessary to add initial conditions: $\varphi(0)=\varphi_0$ and $\dot{\varphi}(0)=\dot{\varphi}_0$, and where C_i, $i=1,2,3,...,n$, are integral constants determined by initial conditions of the motion, as well as by initial conditions of the corresponding interval after corresponding impact or corresponding alternation of the Coulomb's type friction force direction.

Concluding Remarks: Analytical expressions of the kinetic and potential energies, as well as expressions of power of the friction force work of the material particle motion along rough circle line are analytical basis for the analysis of the mechanical energy dissipation of the system impact dynamics. We considered case for the ideally elastic one side impacts, but it is easy to generalize these results to the cases no ideal elastic impacts, as well as for the case when initial conditions follow system dynamics to the case of the both side impact limits system dynamics when motion in the opposite direction can be with elongation of the angle coordinate large then $\varphi(t)<-(2\pi-\delta)$ if impact limiter is not present. Also, it is necessary to point out, that after series of the impacts it is possible to appear a no impact vibrations with decreasing total mechanical energy of the system up to the rest position of the material particle.

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Minisymposium Invited Lecture

LOCAL STRAIN ENERGY AT THE CRACK TIP VICINITY IN DISCRETE MODEL OF MATERIAL

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ABSTRACT

According to discrete model of material [2], [5], solids may be represented as systems of discrete masses (atomic lattice)**, linked by interacting forces, interatomic forces or simple bonds (Fig. 1)** . **Advantage of a discrete model is ability to explain crack healing, slow subcritical crack growth, sound generation during crack propagation, effects of chemical processes at the tip of the crack, nonlinearity of stress, strain and energy distribution in the crack tip region, influence of temperature on crack propagation. All of this give great advantage to procedures based on model of discrete masses (atomic lattice)** [5]. Different intrinsic interatomic force functions are used to represent mechanical interaction between the neighboring atoms (discrete masses) in lattice. Different two- and three-dimensional models of lattice and relations for total potential energy of selected models of lattice are analysed (Fig. 2).

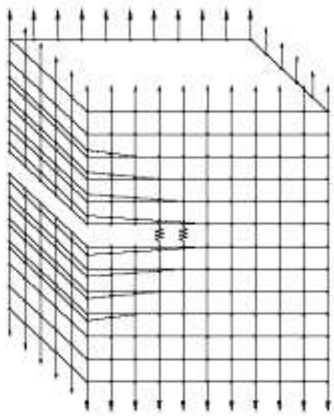
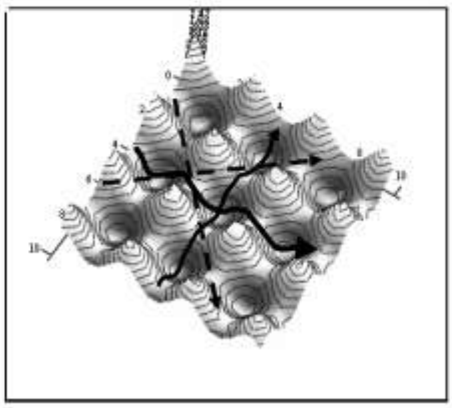


FIGURE 1. Model of three-dimensional lattice



U (x, y)

FIGURE 2. Surface of potential energy for two-dimensional x-y lattice and possible trajectories of crack propagation

In a plate with different ruling cases of global or general stress state, different local stress, and strain states will appear at vicinity of the crack, as a result of interaction between crack geometry and global stress state [3], [8], [9]. By taking in consideration of elliptically shaped crack, and by introducing different general stress states, corresponding local stress and dislocation distributions at the vicinity of such crack were obtained. Three-dimensional model of lattice with crack was applied.

Based on obtained diagrams of stress distribution, and displacements distribution, conclusions about influence of global stress state on local distribution of stresses and displacements at surrounding of the crack were drawn out.

Interactions of different physical phenomenon involved in initiation and propagation of cracks, and in the process of fracture and damage, have directed research towards analyzing processes at atomic and molecular level [1], [2],. There is consistency between conclusions based on discrete (atomic) and macroscopic models, related to strain energy distribution in vicinity of the crack tip. Assumed functions of interatomic forces are presented and their relations with the potential energy are analyzed [4], [6], and [7]. It is shown that: ***The site of fracture coincides with the location of minimum strain energy density*** (Sih G. C., see Ref. [10]), (see Fig. 2).

Keywords: strain energy, fracture, crack , stress state.

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Minisymposium Invited Poster

Design of system for control, monitoring, regulation and data acquisition (CMRA) on civil engineering objects, constructions and mobile modules

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ABSTRACT

Rapid development of information and communication systems as well as microprocessors controllers for monitoring, control and data acquisition has offered great opportunities for their application in many scientific disciplines [1], [2] as well as in systems for control and management. The most significant application is in solving multidisciplinary problems. The software modules used for modeling, simulation and optimization of real-time systems are also a very important part of the information and communication systems.

The CMRA system is based on usage of universal microprocessor controllers of INTEGRAF 10X series (Figure 1.) that have been developed at the Faculty of Civil Engineering and Architecture, University of Nis. Universal microprocessor controllers are applied in three different systems: System for central monitoring, management and acquisition of data on hydraulic structures, System for automatization and management of controllable conditions of plant production in protected areas and System for Central Monitoring, Control, Automatic Regulation and Data Acquisition for Advance River Water Quality Monitoring.

This paper describes an expert system developed for structures in civil engineering and architecture and for special kinds of green houses.

For achieving the set objectives of control, monitoring and regulation at the previously mentioned structures it was necessary to design and develop:

- programmable microprocessor controller with the appropriate system and application software for supervision, conversion and processing of measured parameters for described objects;
- communication software for connection between microcontroller and central system trough cable connection using 485 protocol, wireless connection using GPRS modem and connection trough low-voltage network 220V using PLC modem;
- application software of central system for appropriate sectors of application, acquisition and presentation of measuring data;

Our universal microprocessor regulator INTEGRAF 109 is based on microcontroller Philips 80C552 [3].

The final stage of development implies the creation of expert system with modified application software, developed mathematical models, the new procedures and algorithms optimized for optimal resolution of problems in the field of civil engineering and architecture primarily for the following purposes:

- Measuring thermal inertia and energy efficiency of constructions;

- Measuring dynamical characteristics of constructions: This includes measurement and acquisition of data during influence of mobile and seismic load and monitoring inelastic deformations and fractures in characteristic cross-sections and points.
- Measuring of the important parameters of facilities for tracking changes in the construction of facilities, materials aging, and fatigue for an efficient and timely maintenance of facilities. Built system monitoring will allow corrections of mathematical models for the purpose of their optimization, as well as the monitoring of construction, materials and real-time impact in order to detect damage, deformation and instability manifestations that will activate the warning system and, possibly, automatic control. This system is tested on the model of the original construction of sports halls composite supports project [4].
- Management of mobile plant modules in the protected areas: Usage of this concept creates the possibility for higher efficiency in usage of protected areas with high level of control including mobility of system units that will lead to elimination of unevenness in natural conditions and in energy saving. Technological process gains a higher level of automatization and increases productivity because of automated management of platforms. Important parameters of regulation are: temperature in characteristic points in the green houses, air humidity and lightness. Regulation of parameters in the production process, in achieving the optimization goal, is automated by the given algorithm regulated with software package and depending of plant types.

Figure1.

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Minisymposium Closing Lecture

Free and forced vibrations of the heavy material particle along line with frictions: Direct and inverse task of the theory of vibrorheology

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ABSTRACT

I. Introduction. We start with explanations of vibrorheological effects observed in the real enginering systems. Vibrorheological processes and effects appear under mutual interaction of two bodies with rough contact surfaces and in relation of relative vibrational motions one to other.

This paper was inspired by lectures and/or abstracts and papers written by the Russian scientist Matrosov V.M. [8] and [9], Chernousko F. [2] and [3] and Blekhman I. [1]. Aim of this paper is to show that basic effects and phenomena in the systems with vibrorheological properties can be investigated by using free and forced vibrations of the heavy material particle along line with Coulomb friction. Some classical problems of mechanical system motion with no ideal constraints and friction as as well as an oscillator with Coulomb friction are presented in the books by Rašković D., [10] and [11]. Series of papers and monograph by Goroško O.A. and Hedrih (Stevanović) K. founded a complete integral theory of the analytical dynamics of the hereditary systems with rheological properties using knowledge from known mechanics of the hereditary continuum. Also, some series of published papers by Hedrih (Stevanović) K. - [5], [76] and [7], presented new reserch results regardiong fractional order discrete system oscillators and their properties.

II. Free and forced vibrations of the heavy material particle along line with frictions. This part of the papaer presents theory and models of the heavy material particle motion along rough line with friction. Mathematical decription of the heavy material particle motion along rough line with friction is presented in a natural coordinate system of the line. Three cases of the material particle motion along rough line are consisdered. There special cases are when rough line are: circle, cicloid line and parabola line. We use phase trajectory lines as well as constant energy lines in the phase plane for these three cases which correspond to the classical oscillator with Coulomb friction presented in Ref. [11]. For the case of the heavy material particle motion along rough cicloid line differential equation is in the following form:

$$\ddot{\varphi} - \dot{\varphi}^2\left(\frac{1}{2}tg\frac{\varphi}{2} \mp \mu\right) + \left(tg\frac{\varphi}{2} \pm \mu\right)\frac{g}{2R} = 0 \begin{cases} for \quad v = 2R\cos\frac{\varphi}{2}\dot{\varphi} > 0 \\ for \quad v = 2R\cos\frac{\varphi}{2}\dot{\varphi} < 0 \end{cases}$$

where $s(\varphi) = 4R\sin\frac{\varphi}{2}$ is lenght of cicloid line, μ coefficient of the sliding friction and equation of the phase trajectory is in the form:

$$\dot{\varphi}^2 == -\frac{\left(\frac{g}{2R}\right)}{1+4\mu^2}\frac{1}{\cos^2\frac{\varphi}{2}}\left[(\pm 3\mu)\sin\varphi - \left(1-2\mu^2\right)\cos\varphi + \mu\frac{1+4\mu^2}{2} + Ce^{\mp 2\mu\varphi}\right], \quad \begin{cases} for \quad v = 2R\cos\frac{\varphi}{2}\dot{\varphi} > 0 \\ for \quad v = 2R\cos\frac{\varphi}{2}\dot{\varphi} < 0 \end{cases}$$

with integral constant depending on initial conditions, and changing from first branch of the phase trajectory to the next in the series with alternations of the sign depending on the material particle direction in the motion and corresponding alternation in the direction of the Coulomb friction.

For all three cases of the rough line we can identify a member in the differential equation proportional to the square of the generalized coordinate (or parameter) by which a differential equation of the motion is expressed. This corresponds to the known case of turbulent damping. Also, forced vibrations are considered, as well as vibrations of the line.

III. Direct and inverse task of the theory of vibrorheology. In this part we considered two approaches to describing and solving problems in the vibrotheological properties and how it is possible to use in the engineering practice.

IV. Smart structures are built by inclusions in the form of heavy material particle vibrations along line with frictions. By inclusion in the continuum material some elements such as heavy material particle vibration along rough line corresponding form can be used to bulid new kinds of materials for engineering systems. Nonlinear phenomena and alternations in the „position of the stability" depending on the coefficient of the sliding friction μ into continuum are sources of new construction possibilities and a new continuum model for investigations as well as a new technology method.

Acknowledgement

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(Kinetics and control on the basis of the vibrorheology)

Organizer: Katica R. (Stevanović) Hedrih

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MS-24 Invited Lecture 40 minute

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Relation Between Fractional Differential Equations and Ordinary Differential Equations

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Control Methods in Vibroacoustics - Active Noise and Vibration Suppression

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On the stability of potential mechanical system with tracking and dissipation forces

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http://www.masfak.ni.ac.yu/sitegenius/topic.php?id=1229

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Phase Trajectory of Aeroelastic Dynamic Systems in an Expanded Phase Space

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Energy Transfer through the Double Circular Plate Nonconservative System Dynamics

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Transfer of energy of oscillations through the double DNA chain helix

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MS-24 Contributed Lecture 20 minute

Energy of the vibroimpact systems with Coulomb's type frictions

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MS-24 Closing Lecture 40 minutes
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Free and forced vibrations of the heavy material particle along line with frictions: Direct and inverse task of the theory of vibrorheology

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Minisymposium Poster Session
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DESIGN OF A SYSTEM FOR CONTROL, MONITORING, REGULATION AND DATA ACQUISITION ON CIVIL ENGINEERING OBJECTS, CONSTRUCTIONS AND MOBILE MODULES

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Minisymposium Invited Lecture

RELATION BETWEEN FRACTIONAL AND ORDINARY DIFFERENTIAL EQUATIONS

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Keywords: Fractional differential equations, nonlinear variation of parameters formula, comparison theorem.

Abstract. *The relation between the solutions of fractional and ordinary differential equations is investigated. This provides a framework to study the properties of solutions of fractional differential equations by knowing the corresponding properties for solutions of ordinary differential equations and will pave the way for further work in this direction.*

1 INTRODUCTION

Although the concept of fractional derivative goes back to the 17th century, its importance in modeling a variety of real world problems was realized a few decades ago [1, 5]. Moreover, only very recently, there has been a great surge in the theory of fractional differential equations (FDE). A recent monograph provides a systematic study of the basic theory of FDE [4].

In this article, we shall investigate the relation between the solutions of FDE and ODE (ordinary differential equations). This will provide a framework to study the properties of solutions of FDE, knowing the corresponding properties of solutions of ODE. We hope this study will pave the way for further work in this direction. We shall consider both Reimann-Liouville type and Caputo type FDE. These require different methods, each having certain advantage in the investigation and providing necessary tools. We shall discuss some theoretical results to demonstrate the idea behind these approaches.

2 RIEMANN-LIOUVILLE TYPE FRACTIONAL DIFFERENTIAL EQUATIONS

Consider the IVP (initial value problem)

$$D^q x = f(t,x), \quad x(t)(t-t_0)^{1-q}|_{t=t_0} = x^0, \tag{1}$$

where $f \in C([t_0,T],\mathbb{R}^n)$, $D^q x$ is the Riemann-Liouville fractional differential operator of order q, $0 < q < 1$ and the corresponding Volterra fractional integral equation

$$x(t) = x^0(t) + \frac{1}{\Gamma(q)}\int_{t_0}^{t}(t-s)^{q-1}f(s,x(s))ds, \quad t \in [t_0,T], \tag{2}$$

where $x^0(t) = \frac{x^0(t-t_0)^{q-1}}{\Gamma(q)}$, $t_0 \geq 0$. A function $x(t)$ is called a solution of IVP (1) if $x(t) \in C_p([t_0,T],\mathbb{R}^n)$, $p = 1-q$, $D^q x(t)$ exists and is continuous on $[t_0,T]$, satisfying (1). Here,

$$C_p([t_0,T],\mathbb{R}^n) = \{u \in C((t_0,T],\mathbb{R}^n) \text{ and } (t-t_o)^p u(t) \in C([t_0,T],\mathbb{R}^n)\}.$$

We shall assume the existence and uniqueness of solutions $x(t) = x(t,t_0,x^0)$ of (1). Employing the following relation of the fractional derivative $D^q x(t)$ given by

$$D^q x(t) = \frac{1}{\Gamma(p)}\frac{d}{dt}\left(\int_{t_0}^{t}(t-s)^{p-1}x(s)ds,\right), \tag{3}$$

we shall find a link between the solutions $x(t)$ of (1) with the solutions of the resulting ODE so that one can estimate the solutions of (1) by the solutions of ODE that is generated. This would help in the investigation of the properties of solutions $x(t)$ by the solutions of corresponding ODE that are comparatively easier to obtain due to its well developed theory. For this purpose, we shall tentatively suppose the following equality

$$x(t) = x(s) + \phi(t,s), \quad t_0 \leq s \leq t \leq T, \tag{4}$$

where the function $\phi(t,s)$ needs some reasonable estimates later. Using (4) in (3), we have

$$D^q x(t) = \frac{1}{\Gamma(p)}\frac{d}{dt}\left(\int_{t_0}^{t}(t-s)^{p-1}[x(t)-\phi(t,s)]ds\right),$$

which yields

$$D^q x(t) = \frac{1}{\Gamma(1+p)}\frac{d}{dt}\left(x(t)(t-t_0)^p - \eta(t,p,\phi)\right),$$

where

$$\eta(t,p,\phi) = \frac{1}{\Gamma(p)}\frac{d}{dt}\left(\int_{t_0}^{t}(t-s)^{p-1}\phi(t,s)ds\right), \quad t_0 \leq t \leq T.$$

Setting

$$y(t) = \frac{1}{\Gamma(1+p)}x(t)(t-t_0)^p \tag{5}$$

results in the following ODE

$$y'(t) = \frac{d}{dt}y(t) = F(t,y(t)) + \eta(t,p,\phi), \quad y(t_0) = x^0, \tag{6}$$

where $F(t,y) = f(t,\Gamma(1+p)y(t)(t-t_0)^{-p})$. If $x(t)$ is any solution of IVP (1), we arrive at the perturbed equation (6), treating

$$y'(t) = F(t, y(t)), \quad y(t_0) = x^0 \tag{7}$$

as the unperturbed equation and we can utilize perturbation theory when needed to obtain the estimates on $|y(t)|$, and consequently, on $|x(t)|$ in view of relation (5).

Let us first start with a simple linear estimate of F, namely,

$$|F(t, y)| \leq k(t)|y|,$$

so that we get, setting $m(t) = |y(t)|$, $m(t_0) = |x^0|$, the integral inequality

$$m(t) \leq m(t_0) + \int_{t_0}^{t} k(s)m(s)ds, \quad t \in [t_0, T].$$

If we suppose that the perturbation term in (6) has the estimate

$$|\eta(t, p, \phi)| \leq M(t - t_0)^{\lambda}, \quad 0 \leq \lambda \leq 1,$$

then, we have the Gronwall type inequality

$$m(t) \leq m(t_0) + \int_{t_0}^{t} [k(s)m(s) + M(s - t_0)^{\lambda}]ds. \tag{8}$$

Letting the right hand side of (8) equal to $v(t)$, we arrive at

$$v'(t) = k(t)m(t) + M(t - t_0)^{\lambda} \leq k(t)v(t) + M(t - t_0)^{\lambda}, \quad v(t_0) = m(t_0),$$

which yields the desired estimate on $|y(t)|$

$$|y(t)| \leq m(t_0) \exp\left(\int_{t_0}^{t} k(s)ds\right) + \int_{t_0}^{t} \left(\exp \int_{s}^{t} k(\sigma)d\sigma\right) M(s - t_0)^{\lambda}ds,$$

i.e.,

$$|y(t)| \leq x^0 \exp\left(\int_{t_0}^{t} k(s)ds\right) + \int_{t_0}^{t} \left(\exp \int_{s}^{t} k(\sigma)d\sigma\right) M(s - t_0)^{\lambda}ds, \quad t \in [t_0, T].$$

We shall next prove nonlinear variation of parameters formula. For this purpose, suppose that $F_y(t, y)$ exists and is continuous on $[t_0, T] \times \mathbb{R}^n$. It is known [3] that the solution $y(t, t_0, x^0)$ of IVP (7) satisfies the following identity (Theorem 2.1.2 in [3])

$$\frac{\partial}{\partial t_0} y(t, t_0, x^0) + \frac{\partial}{\partial x^0} y(t, t_0, x^0) F(t_0, x^0) \equiv 0, \tag{9}$$

where $\frac{\partial}{\partial t_0} y(t, t_0, x^0)$, $\frac{\partial}{\partial x^0} y(t, t_0, x^0)$ are the solutions of the IVP of linear system

$$z' = F_y(t, y(t, t_0, x^0))z,$$

with the initial conditions

$$z(t_0) = -F(t_0, x^0), \quad z(t_0) = I, \quad \text{identity matrix},$$

respectively such that the identity (9) holds. Using this information, we can find the nonlinear variation of parameters formula for the solutions of the IVP (6) as follows. Setting

$$p(s) = y(t, s, z(s)),$$

where $z(t, t_0, x^0)$ is the solution of the IVP (6), i.e. the perturbed equation, we see that

$$\frac{d}{ds}p(s) = \frac{\partial}{\partial t_0}y(t, s, z(s)) + \frac{\partial}{\partial x^0}y(t, s, z(s))[F(s, z(s)) + \eta(s, p, \phi)]$$

which, in view of (9), reduces to

$$p'(s) = \frac{\partial}{\partial x^0}y(t, s, z(s))\eta(s, p, \phi).$$

We therefore obtain, integrating from t_0 to t,

$$p(t) = p(t_0) + \int_{t_0}^{t} \frac{\partial}{\partial x^0}y(t, s, z(s))\eta(s, p, \phi)ds$$

which implies the following nonlinear variation of parameters formula

$$z(t, t_0, x^0) = y(t, t_0, x^0) + \int_{t_0}^{t} \frac{\partial}{\partial x^0}y(t, s, z(s))\eta(s, p, \phi)ds, \tag{10}$$

on $[t_0, T]$.

If $f(t, x)$ in (1) is linear, that is, $f(t, x) = A(t)x$, where $A(t)$ is a $n \times n$ continuous matrix, then (7) reduces to

$$y' = \tilde{A}(t)y \tag{11}$$

where $\tilde{A}(t) = \Gamma(1 + p)A(t)(t - t_0)^{-p}$. Then $U(t)$ will be the matrix solution of (11) and (10) reduces to

$$z(t, t_0, x^0) = U(t)x^0 + \int_{t_0}^{t} U(t)U^{-1}(s)\eta(s, p, \phi)ds. \tag{12}$$

We therefore see that one can find several qualitative properties of FDE (1) from the corresponding properties of ODE (6), by employing the relation (5).

3 CAPUTO TYPE FRACTIONAL DIFFERENTIAL EQUATIONS

In this section, we shall discuss the IVP for frctional differential equations of Caputo type, namely,

$$^cD^q x = f(t, x), \quad x(t_0) = x_0, \tag{13}$$

for $0 < q < 1$. If $x \in C^q([t_0, T], \mathbb{R}^n)$ satisfies (13), it also satisfies the Volterra fractional integral equation given by

$$x(t) = x_0 + \frac{1}{\Gamma(q)}\int_{t_0}^{t}(t - s)^{q-1}f(s, x(s))ds \tag{14}$$

and vice versa [4]. If we suppose that the solutions $x(t, t_0, x_0)$ of (13) exist and are unique for $t_0 \le t \le T$, then one can utilize the following relation [4]

$$^cD^q x(t) = \frac{1}{\Gamma(1 - q)}\int_{t_0}^{t}\left[(t - s)^{-q}\frac{d}{ds}x(s)\right]ds$$

to connect the IVP (13) to a certain ODE to be obtained. Let us proceed and suppose that

$$x'(t) = x'(s) + \phi(t, s, q), \tag{15}$$

where the function $\phi(t, s, q)$ will be chosen later in an appropriate manner depending on our requirements. Then we get

$$^cD^q x(t) = \frac{1}{\Gamma(1-q)} \int_{t_0}^{t} (t-s)^{-q}[x'(t) - \phi(t, s, q)]ds$$

which reduces to

$$^cD^q x(t) = \frac{1}{\Gamma(2-q)} \left(x'(t)(t-t_0)^{1-q} \right) - \eta(t, t_0, q), \tag{16}$$

where

$$\eta(t, t_0, q) = \frac{1}{\Gamma(1-q)} \int_{t_0}^{t} (t-s)^{-q} \phi(t, s, q) ds.$$

Using the IVP (13), we arrive at

$$x'(t) = F(t, x) + \tilde{\eta}(t, t_0, q), \quad x(t_0) = x_0, \tag{17}$$

where

$$\left. \begin{aligned} F(t, x) &= \Gamma(2-q) f(t, x)(t-t_0)^{q-1} \\ \tilde{\eta}(t, t_0, q) &= \Gamma(2-q)\eta(t, t_0, q)(t-t_0)^{q-1}. \end{aligned} \right\} \tag{18}$$

By imposing various suitable estimates on $\tilde{\eta}(t, t_0, q)$ and choosing $f(t, x)$ appropriately, we can get bounds on $x(t, t_0, x_0)$ by means of solutions of the corresponding ODE (17). As one possible simple choice, let us suppose that

$$f(t, x) = -\lambda(t, t_0)x, \quad \phi(t, t_0, q) \le k(t-t_0)^q, \tag{19}$$

where $\lambda(t, t_0) \ge 0$ is continuous and $k > 0$ is a constant. Then, using (16)-(18), we have a differential inequality, component-wise,

$$x'(t) \le \Gamma(2-q) \left[-\lambda(t, t_0)x(t)(t-t_0)^{q-1} + \frac{k(t-t_0)}{\Gamma(1-q)} \right], \quad x(t_0) = x_0.$$

Now we choose $\lambda(t, t_0) = \frac{1}{\Gamma(2-q)}\lambda_0(t-t_0)^{1-q}$, $\lambda_0 > 0$, so that the corresponding comparison system becomes

$$v'(t) = -\lambda_0 v + k_0(t-t_0), \quad v(t_0) = x_0, \quad k_0 = \frac{k\Gamma(2-q)}{\Gamma(1-q)},$$

whose solution is

$$v(t) = x_0 \exp(-\lambda_0(t-t_0)) + \int_{t_0}^{t} \exp(-\lambda_0(t-s))k_0(s-t_0)ds, \quad t \ge t_0.$$

Hence, by comparison theorem [2, 3], we have

$$x(t) \le v(t), \quad t_0 \le t \le T.$$

As in Section 2, if we suppose that $F_x(t, x)$ exists and is continuous, and proceed in the same way, we arrive at the nonlinear variation of parameters formula, corresponding to (10), namely

$$x(t, t_0, x_0) = y(t, t_0, x_0) + \int_{t_0}^{t} \frac{\partial}{\partial x_0} y(t, s, x(s, t_0, x_0))\tilde{\eta}(s, t_0, q)ds$$

where $y(t, t_0, x_0)$ is th solution of

$$y'(t) = F(t, y(t)), \quad y(t_0) = x_0,$$

and $x(t, t_0, x_0)$ is the solution of (13). In a similar way, one can obtain the linear variation of parameter formula corresponding to (12), given by the relation

$$x(t, t_0, x_0) = U(t)x_0 + \int_{t_0}^{t} U(t)U^{-1}(s)\tilde{\eta}(s, t_0, q)ds.$$

Also, when one can find explicitly the solution of ODE (17), then the same function will be the solution, in explicit form, of FDE (13). Several qualitative properties can be obtained from the equation (13) and (17).

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Minisymposium Invited Lecture

CONTROL METHODS IN VIBROACOUSTICS – ACTIVE NOISE AND VIBRATION SUPPRESSION

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Keywords: Control methods, Vibration and noise suppression, Optimal LQ tracking system, Adaptive control.

Abstract. *Design and implementation of active vibration and noise control play an important role in overall design of smart structures. In this paper controller design methods for vibration and/or noise reduction of smart piezoelectric structures are proposed and their feasibility is demonstrated by examples. Proposed controllers are discrete-time controllers. Two approaches, optimal LQ tracking system with additional dynamics and model reference adaptive control, as well as their combination, are considered. The novelty in the proposed approach represents implementation of the optimal LQ controller for the reference model design of an adaptive controller. In this way the desired behavior of the controlled model reference adaptive system is prescribed by the optimal tracking system, which guaranties realization of the control task, providing at the same time stability and robustness of the closed-loop system with respect to the controller gains convergence. Special excitation cases of interest involve periodic excitations with frequencies equal to the structural eigenfrequencies, due to the possibility of resonance. Proposed controllers can successfully cope with such excitations.*

1 INTRODUCTION

Design and implementation of active vibration and noise control in smart structures gains more and more interest in recent years [1]. For this purposes different approaches have been developed. Depending on special requirements, different controller design approaches or their combinations can be applied in order to solve complex problems of active vibration and noise attenuation. Control methods considered in this contribution regard in the first place applications to smart mechanical structures, i.e. their active control in the sense of vibration and/or noise reduction using piezoelectric materials for actuation and sensing.

For design and implementation of controllers as an integral part of smart structures a model based approach is supposed in this paper. Different approaches can be applied to modeling of smart structures. Numerical approach results in appropriate models of smart structures under consideration of integrated active materials and their electro-mechanical behavior [2]. In relevant cases an appropriate model-description of environmental influence, like acoustic fluid, can be included. A suitable basis for such considerations is offered by finite element (FE) models of structural behavior (electro-mechanical), which can be augmented by acoustic fluid behavior [3]. Another approach to modeling of smart structures for active vibration and noise control is experimental system identification. This approach is possible only if prototypes or real structures are available. For linear time-invariant models, the subspace identification approach [4] has been proven to result in suitable state space models, comparable with FE-models [5,6].

Starting point for the controller design in this paper is a general model in the state-space form, represented by the state equation (1) and the output/measurement equation (2), which can be obtained using some of the above mentioned approaches,

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}\mathbf{f}(t) \tag{1}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) + \mathbf{F}\mathbf{f}(t) \tag{2}$$

with the state matrix $\mathbf{A}$, control input matrix $\mathbf{B}$, disturbance coupling matrix $\mathbf{E}$, output matrix $\mathbf{C}$, input-to-output coupling matrix $\mathbf{D}$ and disturbance-to-output coupling matrix $\mathbf{F}$. Vector $\mathbf{f}(t)$ represents the vector of external disturbances, $\mathbf{u}(t)$ is the vector of the controller influence and $\mathbf{y}(t)$ represents the output measurement vector. Vector $\mathbf{x}$ is the state vector, which is in case of numerical FE modeling a vector of modal coordinates obtained by ortho-normalization. In a general vibroacoustic problem the state vector contains mechanical, electrical and acoustic degrees of freedom [3].

For real-time applications discrete-time controllers should be designed. A starting point for the controller design in that case is a discrete-time realization of the state space model:

$$\mathbf{x}[k+1] = \mathbf{\Phi}\mathbf{x}[k] + \mathbf{\Gamma}\mathbf{u}[k] + \boldsymbol{\varepsilon}\mathbf{f}[k] \tag{3}$$

$$\mathbf{y}[k] = \mathbf{C}\mathbf{x}[k] + \mathbf{D}\mathbf{u}[k] + \mathbf{F}\mathbf{f}[k] \tag{4}$$

obtained by discretization of the model (1)–(2) with the sampling time T:

$$\mathbf{\Phi} = e^{\mathbf{A}T}, \quad \mathbf{\Gamma} = \int_0^T e^{\mathbf{A}\tau}\mathbf{B}d\tau, \quad \boldsymbol{\varepsilon} = \int_0^T e^{\mathbf{A}\tau}\mathbf{E}d\tau. \tag{5}$$

In the following sections two approaches to the controller design will be presented: optimal LQ controller with additional dynamics and Model Reference Adaptive Control (MRAC).

2 OPTIMAL LQ CONTROLLER DESIGN

The control technique suggested here is the optimal LQ controller with additional dynamics. The controller design includes available a priori knowledge about occurring disturbance type contained in the additional dynamics [7]. Such an a priori knowledge is available in terms of type of the disturbance function which has to be rejected or whose influence should be suppressed by the controller. Periodic disturbances with frequencies corresponding to the eigenfrequencies of a smart structure can cause resonance states and their suppression is therefore important.

A discrete-time state-space equivalent (3)–(4) of the continuous state-space model (1)–(2) is used for the controller design.

Using the a priori knowledge about the disturbance type, which has to be suppressed, the model of the disturbance is represented in an appropriate state-space form, where the disturbance is assumed to be the output of the state-space representation. The poles λ_i of the disturbance transfer function are used to define the additional dynamics using the coefficients of the polynomial:

$$\delta(z) = \prod_i (z - e^{\lambda_i T})^{m_i} = z^r + \delta_1 z^{r-1} + \ldots + \delta_r \tag{6}$$

where m_i represents the multiplicity of the pole λ_i. Additional dynamics is expressed in a state-space form:

$$\mathbf{x}_a[k+1] = \mathbf{\Phi}_a \mathbf{x}_a[k] + \mathbf{\Gamma}_a \mathbf{e}[k], \tag{7}$$

where $\mathbf{x}_a$ is the vector of the state variables for the additional dynamics, $\mathbf{e}$ is the error signal and the state-space matrices of the additional dynamics are:

$$\mathbf{\Phi}_a = \begin{bmatrix} -\delta_1 & 1 & 0 & \cdots & 0 \\ -\delta_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\delta_{r-1} & 0 & 0 & \cdots & 1 \\ -\delta_r & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \mathbf{\Gamma}_a = \begin{bmatrix} -\delta_1 \\ -\delta_2 \\ \vdots \\ -\delta_{r-1} \\ -\delta_r \end{bmatrix}. \tag{8}$$

For multiple-input multiple-output (MIMO) systems additional dynamics is replicated q times (once per each output). In this case the replicated additional dynamics is defined as:

$$\overline{\mathbf{\Phi}} \overset{def}{=} diag(\underbrace{\mathbf{\Phi}_a, \ldots, \mathbf{\Phi}_a}_{q \text{ times}}), \quad \overline{\mathbf{\Gamma}} \overset{def}{=} diag(\underbrace{\mathbf{\Gamma}_a, \ldots, \mathbf{\Gamma}_a}_{q \text{ times}}) \tag{9}$$

The discrete-time design model (Φ_d, Γ_d) is formed as a cascade combination of the additional dynamics (Φ_a, Γ_a) or ($\overline{\mathbf{\Phi}}, \overline{\mathbf{\Gamma}}$) and the discrete-time plant model (Φ, Γ):

$$\mathbf{x}_d[k+1] = \mathbf{\Phi}_d \mathbf{x}_d[k] + \mathbf{\Gamma}_d \mathbf{u}[k]; \tag{10}$$

$$\mathbf{\Phi}_d = \begin{bmatrix} \mathbf{\Phi} & \mathbf{0} \\ \mathbf{\Gamma}^* \mathbf{C} & \mathbf{\Phi}^* \end{bmatrix}, \quad \mathbf{\Gamma}_d = \begin{bmatrix} \mathbf{\Gamma} \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{x}_d = \begin{bmatrix} \mathbf{x}[k] \\ \mathbf{x}_a[k] \end{bmatrix} \tag{11}$$

where $\mathbf{\Phi}^*$ and $\mathbf{\Gamma}^*$ denote respectively $\mathbf{\Phi}_a$ and $\mathbf{\Gamma}_a$ in the case of single-input single-output systems or $\overline{\mathbf{\Phi}}$ and $\overline{\mathbf{\Gamma}}$ for MIMO systems. For the design model (10) the feedback gain matrix $\mathbf{L}$ of the optimal LQ controller is calculated in such a way that the feedback law $\mathbf{u}[k] = -\mathbf{L}\mathbf{x}_d[k]$ minimizes the performance index (12) subject to the constraint (10), where $\mathbf{Q}$ and $\mathbf{R}$ are symmetric, positive-definite matrices. For the solution of the optimal LQ control problem the Matlab functions can be used.

$$J = \frac{1}{2} \sum_{k=0}^{\infty} (\mathbf{x}_d[k]^T \mathbf{Q} \mathbf{x}_d[k] + \mathbf{u}[k]^T \mathbf{R} \mathbf{u}[k]) \tag{12}$$

The feedback gain matrix $\mathbf{L}$ is partitioned into

$$\mathbf{L} = [\mathbf{L}_1 \quad \mathbf{L}_2] \tag{13}$$

so that $\mathbf{L}_1$ corresponds to the state-space model of the controlled structure, and $\mathbf{L}_2$ to the modeled additional dynamics. Block diagram of the optimal LQ control system with additional dynamics is represented in Fig. 1.

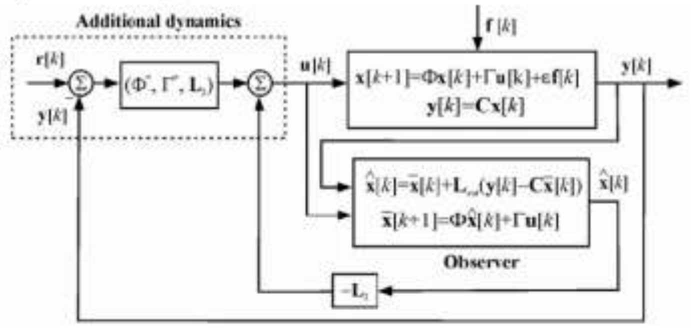

Figure 1: Optimal LQ control system.

The role of the observer is to estimate the model state variables, which cannot be directly measured. For the state estimation the Kalman filter can be used. Equations for the Kalman filter design based on the current estimator assume the state-space equation of the plant in the form (3) and the measurements depending on the state variables and influenced by the measurement noise $\mathbf{y}[k]=\mathbf{C}\mathbf{x}[k]+\mathbf{v}[k]$. The covariances of the process and measurement noise are denoted as $E(\boldsymbol{w}\boldsymbol{w}^{\mathrm{T}})=\mathbf{Q}_w$ and $E(\boldsymbol{v}\boldsymbol{v}^{T})=\mathbf{R}_v$, respectively. Then the Kalman estimator is defined by the following equations:

$$\begin{aligned}\hat{\mathbf{x}}[k]&=\bar{\mathbf{x}}[k]+\mathbf{L}_{est}[k](\mathbf{y}[k]-\mathbf{C}\bar{\mathbf{x}}[k])\\ \bar{\mathbf{x}}[k]&=\boldsymbol{\Phi}\hat{x}[k-1]+\boldsymbol{\Gamma}\mathbf{u}[k-1]\end{aligned} \tag{14}$$

with the Kalman gain matrix:

$$\mathbf{L}_{est}[k]=\mathbf{P}[k]\mathbf{C}^{\mathrm{T}}\mathbf{R}_v^{-1} \tag{15}$$

and:

$$\mathbf{P}[k]=\mathbf{M}_k[k]-\mathbf{M}_k[k]\mathbf{C}^{\mathrm{T}}(\mathbf{C}\mathbf{M}_k[k]\mathbf{C}^{\mathrm{T}}+\mathbf{R}_v)^{-1}\mathbf{C}\mathbf{M}_k[k] \tag{16}$$

$$\mathbf{M}_k[k+1]=\boldsymbol{\Phi}\mathbf{P}[k]\boldsymbol{\Phi}^{\mathrm{T}}+\boldsymbol{\varepsilon}\mathbf{Q}_w\boldsymbol{\varepsilon}^{\mathrm{T}} \tag{17}$$

Matrices $\mathbf{P}$ and $\mathbf{M}_k$ are determined by solving equations (16)–(17).

3 MODEL REFERENCE ADAPTIVE CONTROLLER

Another approach to the controller design suggested in this paper is model reference adaptive control. Controller design is based on the prescribed reference model, which defines the desired behavior of the controlled structure. In this case the available structural model is used for the investigation on the reference model prescription. Applied control technique is a direct model reference adaptive controller [8,9], which includes the innovative integral term in the adaptation law of the adaptive gains [10,11] to achieve robustness with respect to the boundness of the system states and adaptive gains, with small tracking errors. The model reference adaptive controller is designed as a discrete-time controller.

With general bounded, unknown and unmeasurable plant and output disturbances $f_x[k]$ and $f_y[k]$ respectively, discrete-time model (3)–(4) can be represented in the following form:

$$\mathbf{x}[k+1] = \mathbf{\Phi}\mathbf{x}[k] + \mathbf{\Gamma}\mathbf{u}[k] + \mathbf{f}_x[k] \tag{18}$$

$$\mathbf{y}[k] = \mathbf{C}\mathbf{x}[k] + \mathbf{D}\mathbf{u}[k] + \mathbf{f}_y[k] \tag{19}$$

For a general discrete-time state space plant model (18)–(19) the control objective of the MRAC system is to find without an explicit knowledge of the state matrices $\mathbf{\Phi}$ and $\mathbf{\Gamma}$, which contain the system parameters, such control law $\mathbf{u}[k]$ that the plant output $\mathbf{y}[k]$ follows the output $\mathbf{y}_m[k]$ of a specified reference model with the least possible error. Direct robust MRAC algorithm is derived from the general model reference adaptive tracking problem [8]. MRAC system is based on the reference model, specified by the designer, which reflects the desired behavior of the controlled structure (Fig. 2). The reference model is prescribed in a discrete-time state space form:

$$\mathbf{x}_m[k+1] = \mathbf{\Phi}_m\mathbf{x}_m[k] + \mathbf{\Gamma}_m\mathbf{u}_m[k] \tag{20}$$

$$\mathbf{y}_m[k] = \mathbf{C}_m\mathbf{x}_m[k] \tag{21}$$

where $\mathbf{\Phi}_m$ and $\mathbf{\Gamma}_m$ represent the discrete-time state and control matrices, respectively, $\mathbf{C}_m$ is the output matrix, $\mathbf{x}_m \in R^{n_m \times 1}$ is the state vector, $\mathbf{u}_m \in R^{m_m \times 1}$ the command vector and $\mathbf{y}_m \in R^{p_m \times 1}$ the output of the reference model.

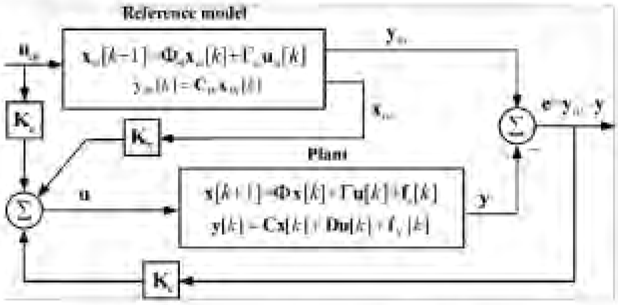

Figure 2: General form of a discrete-time MRAC system

The output tracking error is defined as

$$\mathbf{e}_y[k] = \mathbf{y}_m[k] - \mathbf{y}[k]. \tag{22}$$

The reference model is designed to meet some desired performance properties. Since its output prescribes the behavior of the plant output, the number of reference model outputs has to be equal to the number of the plant outputs ($p_m = p$). Otherwise it is independent of the controlled plant. Further it is required that the reference model is asymptotically stable. The model is assumed to be bounded-input/bounded-state stable. Since the reference model only represents desired behavior of the controlled structure, the dimension n_m of the reference model state vector may be much less than the dimension n of the plant state, which is practically the case with large flexible smart structures. Regarding the stated requirements, the reference model can be designed by selecting the parameters which provide asymptotic stability. Desired responses of the reference model can be obtained by an appropriate parameter selection and confirmed through an iterative simulation and through tuning procedures. Generally it is required to achieve the desired properties of the reference model and therefore of the controlled system output, maintaining at the same time the simplicity of the control system. The lower reference model orders are preferred on one hand due to reduced computational effort. On the other hand, the simulated prescribed behavior of the reference model must comply with the real behavior of the controlled plant, i.e. of its model in the simulation. Too low orders of the reference model sometimes do not fulfill this requirement and therefore cannot be used to prescribe the controlled behavior which complies with the realistic behavior of the controlled plant. The task of the reference model selection becomes therefore a trade-off between the requirement for the lower reference model order and control algorithm simplicity on one hand, and the requirement that the reference model represents the realistic possible behavior of the controlled plant.

With this regard in this paper the selection of the reference model is proposed, based on the equivalent representation of the closed-loop feedback control system (Fig. 3) with an optimal LQ controller, designed using the procedure explained in section 2, which provides the desired behavior in the sense of the suppressed output magnitudes subjected to control in the presence of periodic excitations. Realistic prescription of the desired behavior is possible if the influence of the excitations is taken into account in the design of the reference model. In such a case the excitations represent the input of the reference model and the reference model in turn outputs the optimally controlled behavior.

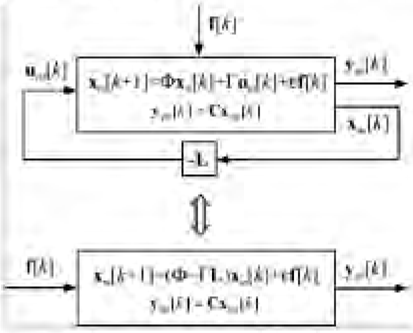

Figure 3: Equivalent representations of the reference model.

In Fig. 3 the first block diagram represents the feedback system with the plant model used for the reference model design. Discrete-time plant model is defined based on the equations (3)–(4) and has the form given in the upper block of the first block-diagram in Fig. 3.

$\mathbf{f}$ represents periodic excitations with the frequencies corresponding to selected eigenfrequencies of the plate or acoustic fluid. The feedback loop is closed by

$$\mathbf{u}_m[k]=-\mathbf{L}\mathbf{x}_m[k] \tag{23}$$

where $\mathbf{L}$ represents the feedback gain matrix of the optimal LQ controller, designed using the procedure explained in section 2 in such a way that the controller minimizes the performance index:

$$J=\frac{1}{2}\sum_{k=0}^{\infty}\left(\mathbf{x}_m[k]^T\mathbf{Q}_m\mathbf{x}_m[k]+\mathbf{u}_m[k]^T\mathbf{R}_m\mathbf{u}_m[k]\right) \tag{24}$$

with symmetric, positive-definite weighting matrices $\mathbf{Q}_m$ and $\mathbf{R}_m$. An equivalent representation of the closed-loop reference system in the upper block-diagram is represented in the lower block-diagram, which corresponds to the reference model designed to meet the requirements of the desired plant behavior with the reduced output.

With regard to the control objective the realization (Φ, Γ) is supposed to be controllable and output stabilizable, the realization (Φ, $\mathbf{C}$) is supposed to be observable and the matrix Γ is assumed to have a maximum rank. Then, a discrete-time direct model reference adaptive law is expressed in the following form:

$$\mathbf{u}[k] = \mathbf{K}_r[k]\mathbf{r}[k] = \mathbf{K}_e[k]\mathbf{e}_y[k] + \mathbf{K}_x[k]\mathbf{x}_m[k] + \mathbf{K}_u[k]\mathbf{u}_m[k] \tag{25}$$

where the adaptive gains as well as the vectors $\mathbf{e}_y$, $\mathbf{x}_m$ and $\mathbf{u}_m$ are concatenated within appropriate matrices of dimensions $m \times n_r$ and $n_r \times 1$, respectively:

$$\mathbf{K}_r[k] = \begin{bmatrix} \mathbf{K}_e[k] & \mathbf{K}_x[k] & \mathbf{K}_u[k] \end{bmatrix}, \quad \mathbf{r}[k] = \begin{bmatrix} \mathbf{e}_y[k] \\ \mathbf{x}_m[k] \\ \mathbf{u}_m[k] \end{bmatrix}. \tag{26}$$

The control task involves several objectives. The output tracking error $\mathbf{e}_y$ (22) should be minimized by the adaptive system. In a general case the output can be affected by an external disturbance or a measurement disturbance, which are represented by the term $\mathbf{f}_y[k]$ in the general plant output equation (19). Disturbance or excitation affecting the states of the plant $\mathbf{f}_x[k]$ is included in the state equation (18). The robust stability and performance of the controlled system in the presence of a wide class of input signals and input or output disturbances/excitations is the aim of the control. In [8] it was shown that the adaptive controller is able to maintain small tracking errors in nonideal environment. This property reflects the robustness of the adaptive controller with respect to boundness of the states, errors and adaptive gains. The adaptive gain $\mathbf{K}_r[k]$ in (26) is determined as a sum of proportional and integral parts $\mathbf{K}_p$ and $\mathbf{K}_I$ respectively:

$$\mathbf{K}_r[k] = \mathbf{K}_p[k] + \mathbf{K}_I[k] \tag{27}$$

According to the basic model reference adaptive algorithm the proportional and integral gains are adapted in the following way:

$$\begin{aligned} &\mathbf{K}_p[k] = \mathbf{e}_y \mathbf{r}^T(t)\overline{\mathbf{T}}, \\ &\mathbf{K}_I[k+1] = \mathbf{e}_y \mathbf{r}^T[k]\mathbf{T}, \quad \mathbf{K}_I(0) = \mathbf{K}_{I0} \end{aligned} \tag{28}$$

where $\mathbf{T}$ and $\overline{\mathbf{T}}$ are $n_r \times n_r$ time-invariant weighting matrices and $\mathbf{K}_{I0}$ is the initial integral gain.

Another aspect of the control requirements regards the convergence of the adaptive gains. In the robust model reference adaptive control approach the integral gain differs from the basic adaptive algorithm in (28). The robust model reference control system should successfully face disturbances (or the parameter variation viewed in terms of unmodeled or unknown dynamics). In ideal conditions without disturbances the integral gain increases as long as the error exists. When the integral gain reaches a certain stabilizing value the error begins to decrease and it decreases further till it reaches the zero value. Than the integral gain stops increasing and maintains some stabilizing constant value. In realistic environment due to disturbances the error does not reach the zero value and thus the integral gain never stops increasing. Although almost strictly positive real structures are theoretically proven to be stable in the presence of high gains, the infinite increase of the integral gains can lead to divergence of the adaptive control system or to numeric instability in the presence of disturbances. A modification of the integral gain in (27) by adding a σ-term is therefore introduced [10,12], in order to guarantee the convergence. Discrete-time form of the robust adaptation with respect to the integral gain convergence is:

$$\mathbf{K}_I[k+1] = \mathbf{e}_y[k]\mathbf{r}^{\mathrm{T}}[k]\mathbf{T} - \sigma\mathbf{K}_I[k]. \tag{29}$$

A condition, which the plant (18)–(19) including disturbances or excitations should fulfill in order to be globally stable with respect to boundness, is that it is almost strictly positive real [8,12] and that the disturbances are bounded. In that case the states, gains and errors involved in the adaptive control are bounded. In order to guarantee robust stability, perfect tracking is not obtained in general, but the adaptive controller maintains a small tracking error over large ranges of nonideal conditions and uncertainties.

4 CONTROLLER IMPLEMENTATION IN VIBRATION AND ACOUSTIC PROBLEMS – EXAMPLES

4.1 Active vibration suppression of a car roof

Vibration suppression of a car roof with attached piezoelectric patches using optimal LQ controller with additional dynamics is demonstrated through a numerical simulation for a test structure. Piezoelectric patches attached to the surface of the car roof are used as actuators and sensors. Excitation by shakers at prescribed points is intended for the experimental investigations (Fig. 4).

Figure 4: Passenger compartment and inner surface of the car roof with attached piezo-patches and shakers

Modeling of the structure including the piezoelectric effect of the actuator/sensor groups was performed using the FEM approach. Based on the generated FEM mesh, an optimization of the actuator/sensor placement was performed under consideration of the eigenmodes of interest and the controllability index. The actuator/sensor placement in Fig. 5 describes one of the test cases, which was calculated based on the controllability index. Comparison of the calculated an experimentally determined eigenfrequencies shows a good agreement in the considered frequency range.

For the controller design a modally reduced state space model was used, which takes into account five selected eigenfrequencies of interest: f_1=48.45Hz, f_2=51.12Hz, f_3=63.23Hz, f_4=64.67Hz and f_5=68.00Hz.

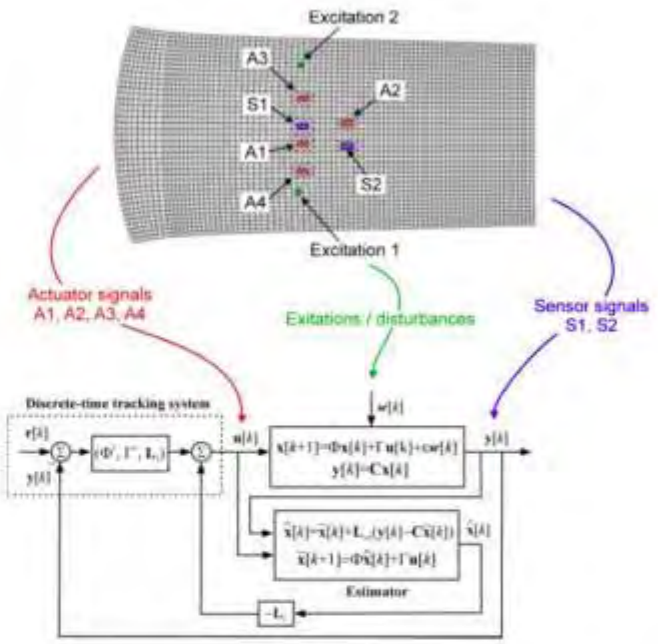

Figure 5: FEM mesh of the car roof with actuator/sensor placement.

Using the control concept with optimal LQ controller, additional dynamics and Kalman estimator (section 2) the simulation of the control action was performed in order to show the potentials of the control strategy for the vibration suppression of the car roof. The results are represented in Fig. 6.

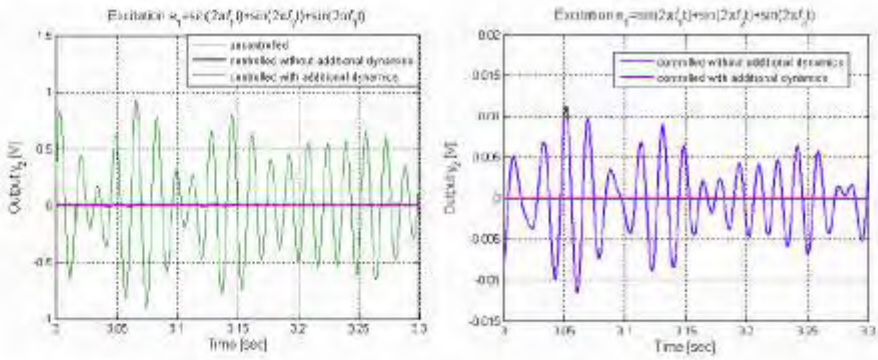

Figure 6: Controlled and uncontrolled responses of the sensor patches.

The comparison of the uncontrolled and controlled cases shows significant reduction of the vibration magnitudes in the presence of the controller. The controller was also compared with the standard optimal LQ controller without additional dynamics which compensates for the presence of the periodic sinusoidal excitations with critical frequencies. The comparison shows much better vibration suppression in the presence of the controller with additional dynamics.

4.2 Noise control of a smart acoustic box

An actively controlled smart acoustic box consisting of the clamped plate with attached piezoelectirc patches used as actuators and of the wooden box surrounding the clamped plate is designed and investigated in order to reduce the plate vibrations and the air pressure at selected points within the box (Fig. 7).

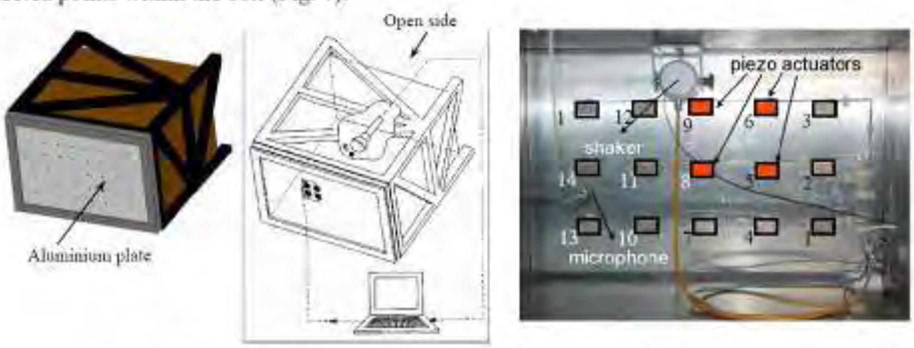

Figure 7: Scheme of the acoustic box and photo of the clamped plate with piezo-actuators.

The inner side of the aluminium plate is attached with fifteen piezoelectric patches, which can be used as actuators and sensors. Multifunctional piezoelectric material integrated with the plate enables actuation and sensing as well as the active control of the structure, when the control algorithm is implemented. For the state-space model development using the FEM approach, four piezo-patches (5, 6, 8, 9 in Fig. 7) are designated as actuator-patches. Using an appropriate modeling procedure it is possible to obtain models, which correspond to different actuator-sensor constellations. Modal reduction can also result in state-space models of different orders, determining in that way the number of inputs and outputs considered for the controller design.

The plate is excited by a shaker. Plate vibrations are measured by the laser scanning vibrometer (for the velocity and displacement measurements at selected points on the plate surface). Excitation of the plate can cause its vibration and the consequent acoustic effects within the box. Especially undesirable are the periodic excitations with frequencies corresponding to some of the structural or acoustic eigenmodes, since they can lead to resonant states. The behavior of the acoustic field under excitation is expressed in terms of the air pressure change, which can be sensed by a microphone with accompanying supply located within the acoustic box.

The control aim is the active noise reduction in a specified field of the acoustic box. It should be achieved using the supplied pizeo-patches, with the goal to cause the plate vibrations which will influence in turn the acoustic field inside the box in such a way that the air pressure at the selected point (where the microphone is placed) is reduced when the controller is active. For this purpose the optimal LQ controller was designed as suggested in section 2, based on the available information on the acoustic structure contained in the state-space model. The experimental setup for the control implementation is represented in Fig. 8.

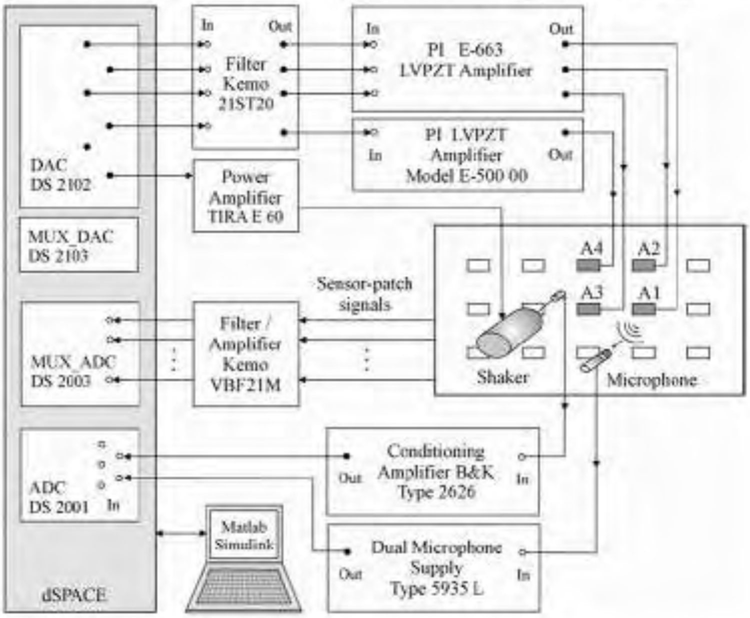

Figure 8: Experimental setup for the controller implementation.

The optimal LQ controller was tested with different excitation signals. Some of the results are represented in with Fig. 9. The results for the excitation obtained as a sum of three periodic sinusoidal signals with the frequencies corresponding to the eigenfrequencies of the plate (f_{w1}=66.7Hz, f_{w2}=106.2Hz, f_{w2}=163.8Hz) are shown in Fig. 9(*a*). Fig. 9(*b*) shows the results with the random excitation signal. The pressure amplitude reduction can be observed in both cases.

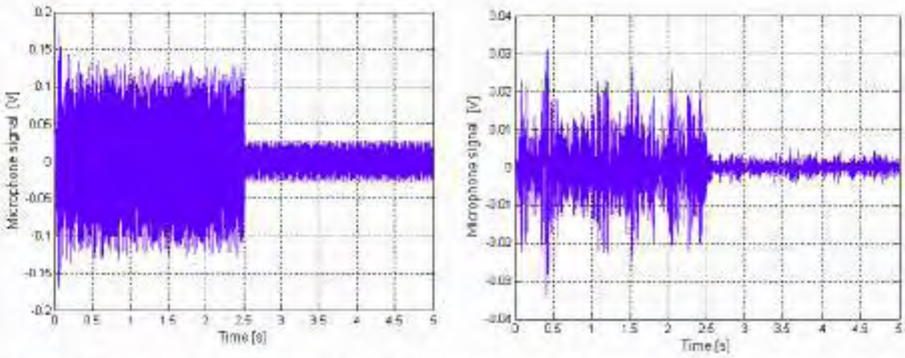

Figure 9: Optimal LQ controller with additional dynamics: uncontrolled and controlled microphone output signal in the presence of (*a*) excitation $\sum_{i=1}^{3}\sin(f_{wi})$, (*b*) random excitation.

The results of the adaptive MRAC controller testing are shown in Fig. 10. The adaptive controller is compared with the optimal LQ controller in the presence of the periodic excitation with the frequency f_{e2}. Uncontrolled and controlled signals are represented in Fig. 10(a), and a zoomed portion of the signals in Fig. 10(b). Both controllers perform the air pressure reduction at the microphone point. In this case the optimal controller performs a slightly higher reduction degree.

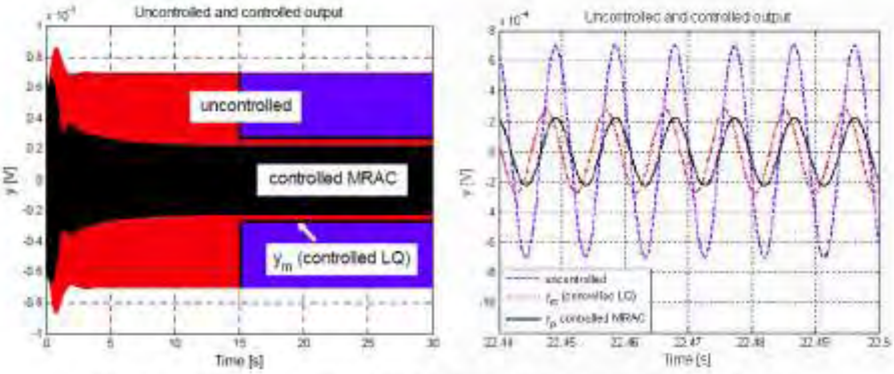

Figure 10: (a) Comparison of the MRAC and optimal LQ controller; (b) zoomed portion.

5 CONCLUSION

Two controller design approaches for vibration and/or noise reduction of smart piezoelectric structures are proposed in this paper. Control techniques are model-based and therefore it is assumed that the plant model is available. For concrete smart structures reliable models can be obtained using the FEM approach or the model identification.

Proposed controllers are discrete-time controllers. An optimal LQ tracking system with additional dynamics and model reference adaptive control, as well as their combination, are considered. The novelty in the proposed approach represents implementation of the optimal LQ controller for the reference model design of an adaptive controller. In this way the desired behavior of the controlled model reference adaptive system is prescribed by the optimal tracking system, which guaranties realization of the control task, providing at the same time stability and robustness of the closed-loop system with respect to the controller gains convergence.

The feasibility of the controllers was shown on examples. Worst-scenario excitation cases are considered, which involve periodic excitations with frequencies equal to the structural eigenfrequencies. Such excitations can lead to dangerous undesirable resonant states. Due to controller action even in the presence of such excitations, the vibration suppression or the noise attenuation were achieved.

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Minisymposium Invited Lecture

ОБ УСТОЙЧИВОСТИ ПОТЕНЦИАЛЬНОЙ МЕХАНИЧЕСКОЙ СИСТЕМЫ СО СЛЕДЯЩЕЙ И ДИССИПАТИВНОЙ СИЛАМИ

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Ключевые слова: следящая сила, сила вязкого трения, эффект Циглера, стабилизация равновесия

Аннотация. *Рассматривается эффект дестабилизации устойчивого равновесия неконсервативной системы под действием сколь угодно малой линейной силы вязкого трения. Получены необходимые и достаточные условия устойчивости системы с несколькими степенями свободы для случая малого трения и, как следствие, условия существования эффекта дестабилизации (эффекта Циглера). Рассмотрен также вопрос о стабилизации равновесия с помощью больших сил трения. Построены критерии устойчивости равновесия системы с двумя степенями свободы, когда силы трения принимают произвольные значения. Результаты исследования применены к задаче устойчивости двухзвенного механизма на плоскости, построены области устойчивости и области Циглера в пространстве параметров задачи.*

1. ВВЕДЕНИЕ.

Рассмотрим голономную механическую систему со стационарными идеальными связями под действием потенциальных, следящих сил, а также диссипативных сил вязкого трения, линейно зависящих от обобщенных скоростей. Уравнения Лагранжа для такой системы имеют вид

$$\frac{d}{dt}\frac{\partial T}{\partial \dot q}-\frac{\partial T}{\partial q}=-\frac{\partial \Pi}{\partial q}+Q(q)-\frac{\partial \Phi}{\partial \dot q};\quad \Phi=\frac{\varepsilon}{2}\ \tilde{B}\dot q,\dot q \qquad (1.1)$$

Здесь $q=(q_1,\dots,q_n)^T$ – вектор обобщенных координат, кинетическая энергия T – квадратичная форма по обобщенным скоростям, $Q(q)$ – вектор обобщенных сил, соответствующий следящей силе, Φ – диссипативная функция Релея, квадратичная по скоростям, $\varepsilon>0$ – коэффициент вязкого трения. Диссипация предполагается полной, поэтому матрица $\tilde{B}$ положительно определена. Считаем, без ограничения общности, что $q=\dot q=0$ – изолированная точка покоя уравнений (1.1).

Рассмотрим задачу о влиянии сил трения на устойчивость тривиального равновесия $q=\dot q=0$. Как известно, в отсутствие сил трения и следящих сил устойчивость равновесия обеспечивается условием минимума потенциальной энергии в положении равновесия. Добавление диссипативных сил с полной диссипацией ведет в этом случае к асимптотической устойчивости точки покоя (теорема Томсона–Тета–Четаева). Однако появление неконсервативной силы $Q(q)$ может изменить характер поведения системы в окрестности тривиального равновесия кардинально. Циглер [1], исследуя устойчивость двухзвенного маятника, нагруженного следящей силой, пришел к неожиданному выводу, что критическая сила F_* (при которой происходит потеря устойчивости равновесной конфигурации системы) в отсутствие диссипации превосходит критическую силу $F_{**}(\varepsilon b)$ системы, содержащей диссипативные силы со сколь угодно малым коэффициентом трения ε, включая предельный случай, когда $\varepsilon=0$. Это значит, что существует область $F_*>F>F_{**}(0)$ изменения модуля следящей силы, в которой система устойчива в первом приближении в отсутствие сил трения, и эта устойчивость разрушается силами трения, сколь угодно малыми по величине. Наличие таких областей в пространстве параметров будем называть эффектом Циглера, а сами области – зонами Циглера.

Исследовалось влияние сил трения на устойчивость равновесия голономной системы под действием потенциальных, гироскопических и следящих сил [2–5]. Было показано [2], что при добавлении диссипативных сил с равными коэффициентами диссипации устойчивое в первом приближении равновесие становится асимптотически устойчивым (простейшие результаты см. [6]). Однако одна из задач, которой посвящена статья, – построение необходимых и достаточных условий существования эффекта Циглера, до сих пор не решена.

Рассмотрим уравнение линейных колебаний

$$A\ddot q+\varepsilon\tilde{B}\dot q+\tilde{C}q=0 \qquad (1.2)$$

где A – положительно определенная матрица коэффициентов кинетической энергии, $\tilde{B}$ – положительно определенная матрица сил трения (диссипация полная), $\tilde{C}=\left[\Pi_{q_j q_k}\right]_0-Q_q\big|_0$ – матрица позиционных сил, состоящая из симметричной $\tilde{C}_1$ и кососимметричной $\tilde{C}_2$ компонент. Предполагается, что $\Pi \in C^2(\Omega)$, $Q \in C^1(\Omega; \mathbb{R}^n)$, Ω – область в $\mathbb{R}^n$, содержащая начало координат.

Введем нормальные координаты x по формуле $q = Sx$, где S – неособенная матрица, удовлетворяющая условиям $S^T AS = I$ и $S^T \tilde{C}_1 S = C$, причем C – диагональная матрица [7]. Тогда уравнение (1.2) примет вид

$$\ddot{x} + \varepsilon B\dot{x} + Cx + Px = 0 \qquad (1.3)$$

Положительно определенная матрица $\tilde{B}$ вычисляется по формуле $B = S^T \tilde{B} S$, а кососимметричная матрица P – по формуле $P = S^T \tilde{C}_2 S$.

Характеристическое уравнение системы (1.3) имеет вид

$$\Delta(\lambda; \varepsilon) \equiv \det(\lambda^2 I + \varepsilon\lambda B + C + P) = 0$$

В отсутствие сил трения ($\varepsilon = 0$) линейные уравнения движения

$$\ddot{x} + Cx + Px = 0 \qquad (1.5)$$

обратимы, поэтому асимптотическая устойчивость точки покоя невозможна; устойчивость имеет место лишь в случае, когда ее собственные значения чисто мнимые и полупростые.

Определение. Пусть стационарное решение $q = \dot{q} = 0$ уравнений (1.1) устойчиво в первом приближении при $\varepsilon = 0$. Будем говорить, что в системе (1.1) имеем место эффект Циглера (эффект дестабилизации силами трения), если решение $q = \dot{q} = 0$ неустойчиво по Ляпунову при сколь угодно малом $\varepsilon > 0$.

2. УСТОЙЧИВОСТЬ СИСТЕМЫ В СЛУЧАЕ МАЛЫХ СИЛ ТРЕНИЯ.

Начнем с простого случая системы с двумя степенями свободы ($n = 2$). Характеристический полином имеет вид

$$\Delta(\lambda; \varepsilon) \equiv \lambda^4 + \varepsilon \operatorname{tr} B \lambda^3 + (\operatorname{tr} C + \varepsilon^2 \det B)\lambda^2 + \varepsilon h\lambda + \det(C + P)$$

где

$$h = \operatorname{tr} B \operatorname{tr} C - \operatorname{tr}(BC) = c_1 b_2 + c_2 b_1$$

Очевидно, что $\operatorname{tr} B > 0$ в силу положительной определенности симметричной матрицы B.

Рассмотрим вспомогательный полином

$$\Delta_0(\omega) \equiv \Delta(i\omega; 0) = \omega^4 - \operatorname{tr} C\omega^2 + \det(C + P)$$

Он имеет корни $\omega_2 > \omega_1 > 0$ тогда и только тогда, когда выполнены неравенства

$$\operatorname{tr} C > 0, \quad \det(C + P) > 0, \quad (\operatorname{tr} C)^2 - 4\det(C + P) > 0 \qquad (2.1)$$

Очевидно, если выполнены неравенства (2.1), то решение $x = \dot{x} = 0$ уравнений (1.4) будет устойчивым; нарушение этих неравенств ведет к неустойчивости в силу появления корней характеристического уравнения с положительной действительной частью. В вырожденных случаях

$$\operatorname{tr} C > 0, \quad (\operatorname{tr} C)^2 - 4\det(C + P) = 0$$

$$\operatorname{tr} C \geq 0, \quad \det(C+P) = 0 \tag{2.2}$$

имеем чисто мнимые кратные корни характеристического уравнения системы (1.4) (в том числе и кратные нулевые корни). Эти случаи оставляем без рассмотрения.

Формула для определения корней имеет вид

$$\omega_{1,2}^2 = \frac{1}{2} \operatorname{tr} C \mp \sqrt{(\operatorname{tr} C)^2 - 4\det(C+P)}$$

Теорема 1. Пусть выполнены неравенства (2.1), гарантирующие устойчивость точки покоя уравнений (1.1) в первом приближении. Если параметр h удовлетворяет неравенствам

$$\omega_1^2 \operatorname{tr} B \leq h \leq \omega_2^2 \operatorname{tr} B \tag{2.3}$$

то существует $\varepsilon_1 > 0$, такое, что равновесие $q = \dot{q} = 0$ уравнений (1.1) асимптотически устойчиво при $\varepsilon \in (0, \varepsilon_1]$, тогда как при

$$h < \omega_1^2 \operatorname{tr} B \text{ или } h > \omega_2^2 \operatorname{tr} B \tag{2.4}$$

имеет место неустойчивость, когда $\varepsilon \in (0, \varepsilon_1]$.

Предположим, что условия (2.1) нарушены. Тогда, с точностью до вырожденных случаев (2.2), равновесие $q = \dot{q} = 0$ уравнений (1.1) неустойчиво, если $\varepsilon \in (0, \varepsilon_1]$.

Доказательство. Пусть λ_* – корень уравнения $\Delta(\lambda;0) = 0$. Возможны варианты: $\lambda_* = \pm i\omega_2$ или $\lambda_* = \pm i\omega_1$. Согласно теореме о неявной функции, если производная Δ отлична от нуля в точке $(\lambda_*, 0)$ (т.е. корень λ_* простой), то существует аналитическая функция $\lambda = \lambda(\varepsilon)$, такая, что $\lambda(0) = \lambda_*$ и $\Delta(\lambda(\varepsilon); \varepsilon) = 0$ в окрестности $\varepsilon = 0$. Из третьего неравенства условий (2.1) следует, что корень λ_* простой.

В разложении $\lambda(\varepsilon) = \lambda_* + \lambda^{(1)}\varepsilon + O(\varepsilon^2)$ коэффициент $\lambda^{(1)}$ оказывается действительным

$$\lambda^{(1)} = -\frac{\Delta'(\lambda_*;0)}{\Delta'_\lambda(\lambda_*;0)} = \frac{h - \omega_*^2 \operatorname{tr} B}{4\omega_*^2 - 2\operatorname{tr} C}$$

Величина в знаменателе последнего выражения может быть как положительной (если $\omega_* = \omega_2$), так и отрицательной (если $\omega_* = \omega_1$). Коэффициент $\lambda^{(1)} < 0$ для обоих корней $\omega_* = \omega_1$ и $\omega_* = \omega_2$ тогда и только тогда, когда

$$\omega_1^2 \operatorname{tr} B < h < \omega_2^2 \operatorname{tr} B$$

В этом случае $\operatorname{Re} \lambda(\varepsilon) < 0$ для обоих корней при малом ε, и точка покоя уравнений (1.1) асимптотически устойчива. Наоборот, если выполнено условие (2.4), то для некоторого корня $\lambda^{(1)} > 0$, $\operatorname{Re} \lambda(\varepsilon) > 0$ и развивается неустойчивость.

Остается исследовать случай $\lambda^{(1)} = 0$. Он имеет место, когда $h = \omega_*^2 \operatorname{tr} B$. Подставляя разложение $\lambda(\varepsilon) = \lambda_* + \lambda^{(1)}\varepsilon + \lambda^{(2)}\varepsilon^2 + \lambda^{(3)}\varepsilon^3 + \ldots$ в уравнение $\Delta(\lambda; \varepsilon) = 0$, можно записать

$$(4\lambda_*^3\lambda^{(2)} + 2\lambda_*\lambda^{(2)}\operatorname{tr}C + \lambda_*^2\det B)\varepsilon^2 +$$
$$+\ (4\lambda_*^3\lambda^{(3)} + 2\lambda_*^2\lambda^{(2)}\operatorname{tr}B + 2\lambda_*\lambda^{(3)}\operatorname{tr}C + h\lambda^{(3)})\ \varepsilon^3 + \ldots = 0$$

Приравнивая коэффициенты при ε^2 и ε^3 к нулю, последовательно вычисляем $\lambda^{(2)}$ и $\lambda^{(3)}$. Получаем

$$\lambda^{(2)} = -\frac{\lambda_*\det B}{2\lambda_*^2 + 2\operatorname{tr}C} = \frac{\mp i\omega_*\det B}{-4\omega_*^2 + 2\operatorname{tr}C}$$

$$\lambda^{(3)} = \frac{-\lambda_*^2(2\lambda_*^2\operatorname{tr}B + h)}{4\lambda_*^3 + 2\lambda_*\operatorname{tr}C} = -\frac{\omega_*^2\det B\operatorname{tr}B}{(-4\omega_*^2 + 2\operatorname{tr}C)^2}$$

Видно, что величина $\lambda^{(2)}$ – чисто мнимая, а $\lambda^{(3)} < 0$, поэтому $\operatorname{Re}\lambda(\varepsilon) < 0$. Итак, исследован случай равенства в соотношениях (2.3).

Предположим, что условие (2.1) нарушено. Это значит, что характеристический полином имеет корни с положительной вещественной частью. Малые силы трения не влияют на знак вещественной части, поэтому равновесие неустойчиво. Теорема доказана.

Элементарные вычисления показывают, что неравенства (2.1) и (2.4) несовместны, когда $P = 0$. Это значит, что эффект дестабилизации равновесия с помощью малых сил трения невозможен в отсутствие следящих сил. Теорема 1 позволяет сформулировать необходимые и достаточные условия дестабилизации устойчивого равновесия системы (1.1) малыми силами трения при наличии следящих сил.

Следствие 1. Силы вязкого трения, сколь угодно малые по величине, дестабилизируют устойчивое в первом приближении равновесие системы (1.1) с двумя степенями свободы тогда и только тогда, когда одновременно выполняются следующие условия:

1) система подвержена действию следящих сил, т.е. $P \neq 0$;
2) параметры системы удовлетворяют неравенствам (2.1), (2.4).

Покажем, что при условии (2.1) и равных коэффициентах диссипации, условие (2.3) асимптотической устойчивости выполнено. Действительно, из уравнения $\Delta_0(\omega) = 0$ следует, что $\omega_1^2 + \omega_2^2 = c_1 + c_2$, и поэтому неравенство (2.3) можно переписать как

$$2b_1\omega_1^2 \le b_1(\omega_1^2 + \omega_2^2) \le 2b_1\omega_2^2$$

что выполнено, так как $\omega_1 < \omega_2$. Таким образом, результаты статьи [2] об асимптотической устойчивости положения равновесия при равных коэффициентах диссипации являются частным случаем теоремы 1.

Рассмотрим производные параметры

$$\beta = \frac{b_1}{\operatorname{tr}B}, \quad \theta = \frac{2\det P}{(c_1 - c_2)^2}$$

Тогда неравенство (2.4) можно переписать как $\theta > -4\beta^2 + 4\beta$. На фиг. 1 зона Циглера выделена штриховкой.

Переходим к анализу устойчивости точки покоя для неконсервативных систем с n степенями свободы. Характеристический полином имеет общий вид

$$\Delta(\lambda;\varepsilon)=\lambda^{2n}+\sum_{k=0}^{2n}\alpha_k\lambda^{2n-k} \tag{2.5}$$

Коэффициенты α_k, определяемые с помощью алгоритма Леверье [8], представляют собой полиномы от ε, причем

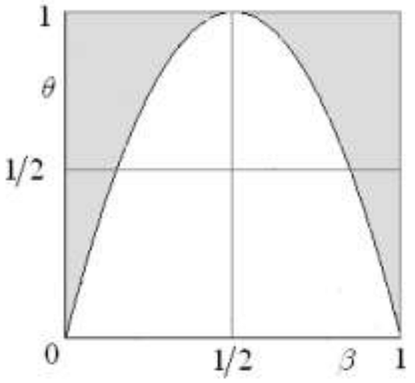
$$\alpha_{2j}=\alpha_{2j}^{(0)}+\alpha_{2j}^{(2)}\varepsilon^2+\ldots,\ \ \alpha_{2j-1}=\alpha_{2j-1}^{(1)}\varepsilon+\ldots,\ \ j=1,\ldots,n$$

Рассмотрим вспомогательные полиномы

$$\Delta_0(\omega)=\Delta(i\omega;0)=\det(-\omega^2I+C+P),\ \ \Delta_1(\omega)=-\sum_{j=1}^{n}\alpha_{2j-1}^{(1)}(-\omega^2)^{n-j}$$

$$\Delta_2(\omega)=\sum_{j=0}^{n-1}2(n-j)\alpha_{2j}^{(0)}(-\omega^2)^{n-j-1},\ \ \alpha_0^{(0)}=1$$

Точка покоя уравнений (1.5) устойчива в том и только в том случае, когда уравнение



Фиг. 1. Зона Циглера в параметрах β, θ

частот $\Delta_0(\omega)=0$ имеет только действительные корни. Пусть $\omega_n>\ldots>\omega_1>0$ – простые действительные корни этого уравнения.

Следующая теорема обобщает теорему 1 и определяет критерии устойчивости системы с n степенями свободы с малыми силами трения.

Теорема 2. Пусть полином Δ_0 имеет n действительных положительных корней ω_l ($l=1,\ldots,n$). Если коэффициент $\alpha_{2n-1}^{(0)}$ удовлетворяет неравенству

$$\min_s\Delta_1(\omega_{2s-1})<\alpha_{2n-1}^{(0)}<\max_s\Delta_1(\omega_{2s}),\ s=1,\ldots,[n/2] \tag{2.6}$$

то существует $\varepsilon_1>0$, такое, что равновесие $q=\dot q=0$ уравнений (1.1) асимптотически устойчиво при $\varepsilon\in(0,\varepsilon_1]$, тогда как при

$$\alpha_{2n-1}^{(0)} < \min_s \Delta_1(\omega_{2s-1}) \text{ или } \alpha_{2n-1}^{(0)} > \max_s \Delta_1(\omega_{2s}) \tag{2.7}$$

имеет место неустойчивость, когда $\varepsilon \in (0, \varepsilon_1]$.

Предположим, что полином Δ_0 имеет корни, не принадлежащие действительной оси. Тогда, с точностью до вырожденных случаев, когда уравнение частот имеет кратные или нулевые корни, равновесие $q = \dot{q} = 0$ уравнений (1.1) неустойчиво, если $\varepsilon \in (0, \varepsilon_1]$.

Доказательство. Рассмотрим, как ведет себя корень $\lambda_l^* = \pm i\omega_l$ при возмущении в системе (1.3). Согласно теореме о неявной функции, если корень λ_l^* простой, то существует однозначная аналитическая в окрестности $\varepsilon = 0$ функция $\lambda(\varepsilon)$, являющаяся решением задачи $\Delta(\lambda; \varepsilon) = 0$, $\lambda(0) = \lambda_l^*$. В разложении

$$\lambda(\varepsilon) = \lambda_l^* + \lambda_l^{(1)}\varepsilon + \lambda_l^{(2)}\varepsilon^2 + \ldots$$

определим коэффициент $\lambda_l^{(1)}$. Для этого вычислим производную $\Delta'_\varepsilon(\lambda_l^*; 0)$. Имеем

$$\Delta'_\varepsilon(\lambda_l^*; 0) = \sum_{j=1}^{n} \alpha_{2j-1}^{1}(\pm i\omega_l)^{2(n-j)+1} = \pm i\omega_l \Delta_1(\omega_l)$$

Вычисления показывают, что

$$\Delta'_\lambda(\lambda_l^*; 0) = 2n(\pm i\omega_l)^{2n-1} + \sum_{j=1}^{n-1} 2(n-j)\alpha_{2j}^{(0)}(\pm i\omega_l)^{2n-2j-1} = \mp i\omega_l \Delta_2(\omega_l)$$

Наконец, запишем выражение

$$\lambda_l^{(1)} = -\frac{\Delta'_\varepsilon(\lambda_l^*; 0)}{\Delta'_\lambda(\lambda_l^*; 0)} = \frac{\Delta_1(\omega_l)}{\Delta_2(\omega_l)} \tag{2.8}$$

Коэффициент $\lambda_l^{(1)}$ оказывается действительным.

Дестабилизация в системе (1.1) связана с появлением некоторого коэффициента $\lambda_l^{(1)} > 0$. В этом случае $\operatorname{Re}\lambda_l(\varepsilon) > 0$ в окрестности $\varepsilon = 0$. Наоборот, если все $\lambda_l^{(1)} < 0$, то возмущенная система остается устойчивой при малых ε.

Определим знак коэффициента $\lambda_l^{(1)}$. С этой целью подсчитаем знак знаменателя выражения (2.8) через знак производной

$$\frac{d\Delta_0}{d\omega} = \frac{1}{\omega}\Delta_2(\omega_l)$$

Знаки $\dfrac{d\Delta_0}{d\omega}(\omega_l)$ чередуются, причем, $\dfrac{d\Delta_0}{d\omega}(\omega_1) < 0$. Это объясняется тем, что старший коэффициент полинома Δ_0 равен $(-1)^n$.

Производная $\dfrac{d\Delta_0}{d\omega}(\omega_l)$ отличается от выражения в знаменателе правой части равенства (2.8) коэффициентом ω_l^{-1}. С учетом всего вышесказанного заключаем, что условие $\lambda_l^{(1)} < 0$ ($l = 1, \ldots, n$) эквивалентно условию

$$(-1)^{l-1} \Delta_1(\omega_l) > 0, \; l = 1, \ldots, n \tag{2.9}$$

Наоборот, дестабилизация в системе происходит, когда существует значение $l \in \{1, \ldots, n\}$, такое, что выполнено неравенство, противоположное (2.9). Легко видеть, что условие (2.9) эквивалентно условию (2.6), а неравенство, противоположное (2.9), эквивалентно условию (2.7).

Если полином Δ_0 имеет корни, не принадлежащие действительной оси, то один из характеристических корней будет иметь положительную действительную часть, поэтому тривиальное равновесие системы (1.1) будет неустойчивым. Малые силы трения этот вывод не меняют.

На основании алгоритма Леверье, запишем алгоритм вычисления коэффициентов $\alpha_k^{(0)}$ и $\alpha_k^{(1)}$:

$$\alpha_1^{(0)} = 0, \; \alpha_1^{(1)} = \operatorname{tr} B, \; C_0^{(1)} = 0, \; C_1^{(1)} = 0, \; C_1^{(0)} = -B + I \operatorname{tr} B$$

$$\alpha_k^{(0)} = \frac{2}{k} \operatorname{tr}\{(C+P)C_{k-2}^{(0)}\}, \; \alpha_k^{(1)} = \frac{1}{k} \operatorname{tr}\{BC_{k-1}^{(0)}\} + \frac{2}{k} \operatorname{tr}\{(C+P)C_{k-2}^{(1)}\}$$

$$C_k^{(0)} = -(C+P)C_{k-2}^{(0)} + \alpha_k^{(0)} I, \; C_k^{(1)} = -BC_{k-1}^{(0)} - (C+P)C_{k-2}^{(1)} + \alpha_k^{(1)}; \quad k = 2, \ldots, 2n-2$$

$$\alpha_{2n-1}^{(1)} = BC_{2n-2}^{(0)} + (C+P)C_{2n-3}^{(1)}, \; \alpha_{2n}^{(0)} I = (C+P)C_{2n-2}^{(0)}$$

3. ВЛИЯНИЕ БОЛЬШИХ СИЛ ТРЕНИЯ.

До сих пор рассматривались лишь малые силы вязкого трения в неконсервативной системе (1.1). Из известных результатов [4] можно сделать вывод, что силы вязкого трения с достаточно большими коэффициентами могут оказывать стабилизирующее влияние на равновесие системы. Покажем, что при определенных условиях в системе с двумя степенями свободы существует такое большое значение $\varepsilon_* > 0$ коэффициента вязкого трения, что точка покоя уравнений (1.1) асимптотически устойчива при $\varepsilon > \varepsilon_*$ и неустойчива при $\varepsilon < \varepsilon_*$. Ясно, что такая постановка задачи корректна, если точка покоя возмущенной системы неустойчива, либо в системе имеет место эффект дестабилизации малыми силами трения. Величину ε_* назовем критической.

Очевидно, при $\varepsilon = \varepsilon_*$ характеристическое уравнение обладает чисто мнимым корнем $i\omega_*$. Отсюда легко определить ε_* и ω_*. Действительно, разделяя в комплексном уравнении

$$\Delta(i\omega_*; \varepsilon) \equiv \omega_*^4 - (\operatorname{tr} C + \varepsilon^2 \det B)\omega_*^2 + \det(C+P) + \varepsilon i\omega_*(-\omega_*^2 \operatorname{tr} B + h) = 0$$

вещественную и мнимую части, получим два равенства

$$-\omega_*^2 \operatorname{tr} B + h = 0, \quad \omega_*^4 - (\operatorname{tr} C + \varepsilon^2 \det B)\omega_*^2 + \det(C+P) = 0$$

Если исключить вырожденный случай $\varepsilon = \varepsilon_*$ (считаем $\det(C+P) \neq 0$), находим

$$\omega_*^2 = \frac{h}{\operatorname{tr} B}, \quad \varepsilon_*^2 = \frac{1}{h \det B}\{(\operatorname{tr} B)^{-1} h^2 - h \operatorname{tr} C + \operatorname{tr} B \det(C+P)\}. \tag{3.1}$$

Отсюда следует, что для решения поставленной задачи необходимо потребовать выполнения неравенств

$$h > 0, \quad \varphi(h) > 0$$

где

$$\varphi(u) = \frac{1}{\operatorname{tr} B} u^2 - u \operatorname{tr} C + \operatorname{tr} B \det(C+P)$$

Однако эти условия не обязательно достаточные, так как наличие двух мнимых корней $\pm i\omega_*$ характеристического уравнения означает, что оставшиеся два корня сохраняют знак вещественной части при $\varepsilon = \varepsilon_*$, их влияние существенно.

Лемма. С точностью до соотношений типа равенства, необходимым и достаточным условием существования критической величины ε_* является выполнение одного из следующих условий:

$$(\operatorname{tr} C)^2 - 4\det(C+P) < 0, \quad h > 0 \tag{3.2}$$

$$\operatorname{tr} C < 0, \quad (\operatorname{tr} C)^2 - 4\det(C+P) > 0, \quad \det(C+P) > 0, \quad h > 0 \tag{3.3}$$

$$\operatorname{tr} C > 0, \quad (\operatorname{tr} C)^2 - 4\det(C+P) > 0, \quad \det(C+P) > 0,$$
$$h \in (0, \omega_1^2 \operatorname{tr} B) \cup (\omega_2^2 \operatorname{tr} B, \infty) \tag{3.4}$$

Величина ε_* определяется формулой (3.1).

Доказательство. Заметим, что если дискриминант уравнения $\varphi(u) = 0$

$$D = (\operatorname{tr} C)^2 - 4\det(C+P)$$

меньше нуля (первое условие (3.2)), то $\varphi(u) > 0$ для любого u, и в частности, $\varphi(h) > 0$ при $h > 0$.

Рассмотрим случай $D > 0$. Если выполнены первые три неравенства (3.3), то уравнение $\varphi(u) = 0$ имеет два действительных отрицательных корня, поэтому $\varphi(h) > 0$ при любом $h > 0$. Если выполняются неравенства (3.4), то $\varphi(h) > 0$ тогда и только тогда, когда $h < u_1$ или $h > u_2$, где $u_{1,2}$ – корни уравнения $\varphi(u) = 0$, $u_1 < u_2$. Так как $u_{1,2} = \omega_{1,2}^2 \operatorname{tr} B$, приходим к условию (2.4).

Теперь выясним, при каких условиях параметр ε_* будет критическим. Рассмотрим матрицу Гурвица для характеристического уравнения

$$\begin{Vmatrix} \varepsilon \operatorname{tr} B & \varepsilon h & 0 & 0 \\ 1 & \operatorname{tr} C + \varepsilon^2 \det B & \det(C+P) & 0 \\ 0 & \varepsilon \operatorname{tr} B & \varepsilon h & 0 \\ 0 & 1 & \operatorname{tr} C + \varepsilon^2 \det B & \det(C+P) \end{Vmatrix} \tag{3.5}$$

Критерием Рауса–Гурвица отсутствия у характеристического уравнения корней с неотрицательной вещественной частью является система неравенств $\delta_j > 0$, где δ_j – j -

й угловой минор матрицы (3.5). Так как старший коэффициент полинома Δ положителен, эти неравенства эквивалентны следующим: $\alpha_j > 0$, $\delta_3 > 0$, где α_j – коэффициент характеристического полинома перед λ^{4-j}. Так как B – положительно определенная матрица, то $\alpha_1 > 0$. Получим

$$\delta_3 = -\varepsilon^2 (\operatorname{tr} B)^2 \det(C+P) + \varepsilon^2 h \operatorname{tr} B(\operatorname{tr} C + \varepsilon^2 \det B) - \varepsilon^2 h^2$$

Сокращая неравенство $\delta_3 > 0$ на ε^2 и разрешая относительно ε, приходим к неравенству $\varepsilon > \varepsilon_*$. Система неравенств $\alpha_j > 0$ эквивалентна двум: $h > 0$ и $\det(C+P) > 0$. Последнее неравенство также следует из неравенств (3.2). Коэффициент $\operatorname{tr} C + \varepsilon^2 \det B > 0$ при $\varepsilon > \varepsilon_*$, что устанавливается непосредственным вычислением.

Если же $0 < \varepsilon < \varepsilon_*$, то $\delta_3 > 0$ и точка покоя уравнений (1.1) неустойчива. Лемма доказана.

В остальных случаях, когда параметры системы не удовлетворяют условиям леммы, имеем следующее. Либо критическое значение ε_* не существует из-за того, что нарушаются условия теоремы Гурвица ($\det(C+P) < 0$) при любых значениях параметра ε и, следовательно, равновесие неустойчиво независимо от величины силы трения, либо $h < 0$ (равенства (3.1) теряют смысл); возможен также случай асимптотической устойчивости при малых силах трения (выполняются первые три неравенства (3.4), но h удовлетворяет условиям (2.3)), тогда задача поиска ε_* теряет смысл.

Итак, объединяя утверждения теоремы 1 и леммы, получим следующие критерии устойчивости равновесия по первому приближению для случая системы с двумя степенями свободы, когда силы трения принимают произвольные значения.

Теорема 3. I. Предположим, что выполнены условия (2.1) устойчивости равновесия уравнений (1.1) в первом приближении в отсутствие сил трения. Тогда, если параметр h удовлетворяет неравенствам

$$0 < h < \omega_1^2 \operatorname{tr} B \text{ или } h > \omega_2^2 \operatorname{tr} B$$

то тривиальное равновесие системы неустойчиво при $\varepsilon \in (0, \varepsilon_*)$ и асимптотически устойчиво при $\varepsilon \in (\varepsilon_*, \infty)$; если параметр h удовлетворяет неравенствам

$$\omega_1^2 \operatorname{tr} B \le h \le \omega_2^2 \operatorname{tr} B,$$

то равновесие асимптотически устойчиво независимо от значений параметра ε.

Если $h < 0$, равновесие неустойчиво при любых значениях параметра ε.

II. Предположим, что нарушены условия (2.1), т.е. равновесие системы неустойчиво по первому приближению в отсутствие сил трения. Тогда имеет место следующее: если выполняются неравенства

$$(\operatorname{tr} C)^2 - 4\det(C+P) < 0,\ h > 0$$

либо

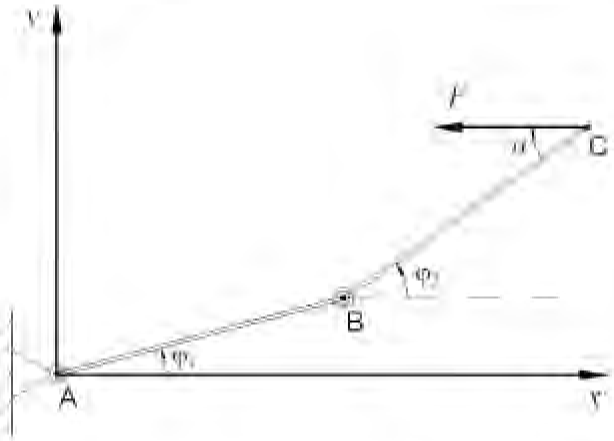
$$\operatorname{tr} C < 0,\ (\operatorname{tr} C)^2 - 4\det(C+P) > 0,\ \det(C+P) > 0,\ h > 0$$

то равновесие неустойчиво при $\varepsilon \in (0, \varepsilon_*)$ и асимптотически устойчиво при $\varepsilon \in (\varepsilon_*, \infty)$; в противном случае равновесие неустойчиво при любых значениях параметра ε.

4. ДВУХЗВЕННАЯ СТЕРЖНЕВАЯ СИСТЕМА.

В качестве прикладной задачи, иллюстрирующей эффект Циглера, рассмотрим задачу исследования устойчивости равновесия двухзвенного механизма, находящегося в горизонтальной плоскости. Неконсервативный характер данной системы связан с воздействием на свободный конец второго звена следящей силы. Следящие силы составляют постоянный угол с осями тел, к которым они приложены. Эта задача рассматривалась ранее [9-10], она обобщает задачу Циглера.

Итак, рассматривается двухзвенная система, находящаяся в горизонтальной плоскости и состоящая из весомых однородных стержней AB, BC, массы которых m (фиг. 2). Стержни соединены спиральными пружинами с коэффициентом упругости c. Следящая сила F приложена к свободному концу стержня BC и составляет с BC постоянный угол α. В шарнирах A и B имеет место вязкое трение с коэффициентом b.



Фиг. 2. Двухзвенная стержневая система со следящей силой F

Введем обобщенные координаты φ_1 и φ_2. Исследуемая система является голономной системой с идеальными стационарными связями и активными силами: силой упругости в шарнирах A и B, силой трения в этих шарнирах и следящей силой F. Составим уравнения Лагранжа

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\varphi}_j} - \frac{\partial T}{\partial \varphi_j} = -\frac{\partial \Pi}{\partial \varphi_j} - \frac{\partial \Phi}{\partial \dot{\varphi}_j} + Q_j, \quad j = 1, 2$$

Здесь T – кинетическая энергия системы, Π – потенциальная энергия, Φ – диссипативная функция Релея, Q_j – обобщенная сила, соответствующая следящей силе (неконсервативная позиционная сила). Запишем выражения для функций T, Π, Φ и Q

$$T = \frac{m}{6}\left[4l^2\dot\varphi_1^2 + 3l^2\dot\varphi_1\dot\varphi_2\cos(\varphi_2-\varphi_1) + l^2\dot\varphi_2^2\right], \quad \Pi = \frac{c}{2}[\varphi_1^2 + (\varphi_2-\varphi_1)^2]$$

$$\Phi = \frac{b}{2}[\dot\varphi_1^2 + (\dot\varphi_2-\dot\varphi_1)^2], \quad Q_1 = -Fl\sin(\varphi_2-\varphi_1-\alpha), \quad Q_2 = Fl\sin\alpha$$

Уравнения Лагранжа записываются так:

$$\begin{aligned}
&\frac{4}{3}ml^2\ddot\varphi_1 + \frac{1}{2}ml^2\cos(\varphi_2-\varphi_1)\ddot\varphi_2 - \frac{1}{2}ml^2\sin(\varphi_2-\varphi_1)\dot\varphi_2^2 = \\
&= c(\varphi_2-2\varphi_1) + b(\dot\varphi_2-2\dot\varphi_1) - Fl\sin(\varphi_2-\varphi_1-\alpha) \\
&\frac{1}{2}ml^2\cos(\varphi_2-\varphi_1)\ddot\varphi_1 + \frac{1}{3}ml^2\ddot\varphi_2 + \frac{1}{2}ml^2\sin(\varphi_2-\varphi_1)\dot\varphi_1^2 = \\
&= c(\varphi_1-\varphi_2) + b(\dot\varphi_1-\dot\varphi_2) + Fl\sin\alpha
\end{aligned} \tag{4.1}$$

Положения равновесия φ_1^* и φ_2^* рассматриваемой системы определяются системой уравнений

$$\begin{aligned}
&\varphi_2 - 2\varphi_1 = \gamma\sin(\gamma\sin\alpha - \alpha) \\
&\varphi_1 - \varphi_2 = -\gamma\sin\alpha
\end{aligned} \tag{4.2}$$

где $\gamma = Flc^{-1}$ – безразмерный параметр. Решая систему (4.2), находим

$$\varphi_1^* = \gamma\sin\alpha - \gamma\sin(\gamma\sin\alpha - \alpha)$$

$$\varphi_2^* = 2\gamma\sin\alpha - \gamma\sin(\gamma\sin\alpha - \alpha)$$

При $\alpha = 0$ имеем $\varphi_1^* = \varphi_2^* = 0$ (случай, рассмотренный Циглером). Исследуемая механическая система имеет единственное положение равновесия. Исследуем это равновесие на устойчивость.

Составим уравнения возмущенного движения в окрестности положения равновесия. Введем возмущения

$$\varphi_1 = \varphi_1^* + \beta_1, \quad \varphi_2 = \varphi_2^* + \beta_2$$

и обезразмерим уравнения движения, рассматривая в качестве единицы измерения угловых переменных один радиан, а в качестве единицы измерения времени – характерное значение $T_* = \sqrt{ml^2c^{-1}}$. Уравнения возмущенного движения примут вид

$$\begin{aligned}
&\frac{4}{3}\ddot\beta_1 + \frac{1}{2}\cos(\beta_2-\beta_1+\gamma\sin\alpha)\ddot\beta_2 - \frac{1}{2}\sin(\beta_2-\beta_1+\gamma\sin\alpha)\dot\beta_2^2 = \\
&= (\beta_2 - 2\beta_1 + \gamma\sin(\gamma\sin\alpha-\alpha)) + \varepsilon(\dot\beta_2 - 2\dot\beta_1) - \gamma\sin(\beta_2-\beta_1-\alpha+\gamma\sin\alpha) \\
&\frac{1}{2}\cos(\beta_2-\beta_1+\gamma\sin\alpha)\ddot\beta_1 + \frac{1}{3}\ddot\beta_2 + \frac{1}{2}\sin(\beta_2-\beta_1+\gamma\sin\alpha)\dot\beta_1^2 = \\
&= (\beta_1 - \beta_2) + \varepsilon(\dot\beta_1 - \dot\beta_2)
\end{aligned} \tag{4.3}$$

Здесь $\varepsilon = b/(l\sqrt{mc})$, точкой теперь обозначена производная по безразмерному времени.

Получим систему первого приближения для уравнений (4.3) В векторно-матричной форме ее можно записать так:

$$A\ddot\beta + \varepsilon B\dot\beta + C\beta = 0, \quad \beta = (\beta_1, \beta_2)^T \tag{4.4}$$

где

$$A=\left\|\begin{array}{cc}\frac{4}{3} & \frac{1}{2}\cos(\gamma\sin\alpha)\\ \frac{1}{2}\cos(\gamma\sin\alpha) & \frac{1}{3}\end{array}\right\|,\ B=\left\|\begin{array}{cc}2 & -1\\ 1 & 1\end{array}\right\|$$

$$C=\left\|\begin{array}{cc}2-\gamma\cos(\gamma\sin\alpha-\alpha) & \gamma\cos(\gamma\sin\alpha-\alpha-1\\ -1 & 1\end{array}\right\|$$

Необходимо исследовать устойчивость решения $\beta=0$, $\dot{\beta}=0$ уравнения (4.4) при $\varepsilon=0$ и при малых ε, отличных от нуля. Приведем соответствующее характеристическое уравнение

$$\lambda^4\det A+\varepsilon(2+\cos(\gamma\sin\alpha))\lambda^3+\kappa\lambda^2+2\varepsilon\lambda+1=0 \tag{4.5}$$

Здесь

$$\det A=\frac{4}{9}-\frac{1}{4}\cos^2(\gamma\sin\alpha)$$

$$\kappa=-\gamma\cos(\gamma\sin\alpha-\alpha)(3\cos(\gamma\sin\alpha)+2)/6+\cos(\gamma\sin\alpha)+2+\varepsilon^2$$

Запишем условия Рауса–Гурвица отсутствия у уравнения (4.5) корней с неотрицательной вещественной частью

$$\begin{aligned}&\delta_1=\varepsilon(2+\cos(\gamma\sin\alpha))>0\\ &\delta_2=\delta_1\kappa-2\varepsilon\det A>0\\ &\delta_3=2\varepsilon\delta_2-\delta_1^2>0\end{aligned} \tag{4.6}$$

Величины δ_k представляют собой угловые миноры матрицы

$$\left\|\begin{array}{cccc}\varepsilon(2+\cos(\gamma\sin\alpha)) & 2\varepsilon & 0 & 0\\ \det A & \kappa & 1 & 0\\ 0 & \varepsilon(2+\cos(\gamma\sin\alpha)) & 2\varepsilon & 0\\ 0 & 1 & \kappa & 1\end{array}\right\|$$

Минор $\delta_4=\delta_3$, поэтому условие $\delta_4>0$ в список условий (4.6) не включено. Также, очевидно, $\delta_1>0$ при любых α, $\gamma>0$. Неравенства (4.6) эквивалентны неравенству $\delta_3>0$, которое можно записать в виде

$$\begin{gathered}\varepsilon^2+1+\frac{1}{2}\cos(\gamma\sin\alpha)-\gamma\cos(\gamma\sin\alpha-\alpha)\left(\frac{1}{2}\cos(\gamma\sin\alpha)+\frac{1}{3}\right)>\\ \left[\frac{8}{9}-\frac{1}{2}\cos^2(\gamma\sin\alpha)\right][2+\cos(\gamma\sin\alpha)]^{-1}\end{gathered} \tag{4.7}$$

Итак, неравенство (4.7) определяет область устойчивости точки покоя уравнений (4.4). Построим ее при $\varepsilon\downarrow 0$. Для этого введем новые параметры u и v, связанные с γ и α соотношениями

$$u = \gamma \sin\alpha, \quad v = \gamma\cos\alpha$$

и перепишем неравенство (4.7) в виде

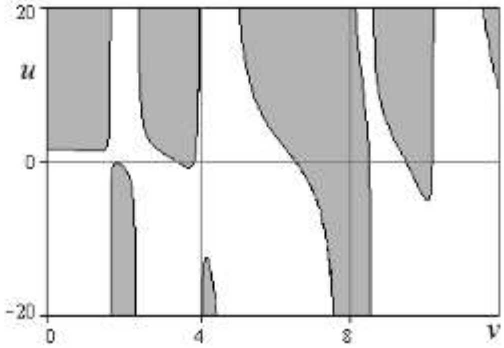
$$v(\cos u + 2/3)\cos u < 2 + \cos u - u(\cos u + 2/3)\sin u + \frac{\cos^2 u - 16/9}{2 + \cos u} \qquad (4.8)$$

Рассмотрим функцию $f(u) = (\cos u + 2/3)\cos u$ и найдем множества, на которых она принимает значения одного знака. Пусть $0 \le u \le 2\pi$. Тогда

$$f(u) > 0 \text{ при } u \in [0, \pi/2) \cup (\pi - \arccos(2/3), \pi + \arccos(2/3)) \cup (3\pi/2, 2\pi];$$

$$f(u) < 0 \text{ при } u \in (\pi/2, \pi - \arccos(2/3)) \cup (\pi + \arccos(2/3), 3\pi/2) \qquad (4.9)$$

Обозначим правую часть неравенства (4.8) через $g(u)$; тогда для тех u, когда $f(u) > 0$ ($f(u) < 0$), неравенство (4.8) запишется как $v < g(u)/f(u)$ ($v > g(u)/f(u)$). Область (4.8) изображена на фиг. 3.



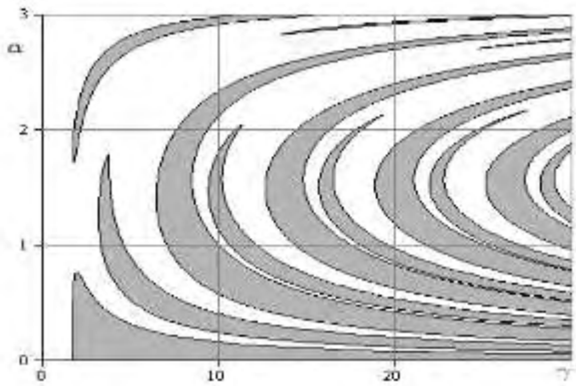
Фиг. 3. Области устойчивости в переменных u, v

Заштрихованные участки соответствуют точкам неустойчивости, незаштрихованные – устойчивости, т.е. решениям неравенства (4.8). Видно, что область устойчивости состоит из счетного числа связных компонент.

Перенесем область устойчивости в полосу (γ, α). Для этого строим кривые

$$\gamma(u) = \sqrt{u^2 + \frac{g(u)^2}{f(u)^2}}, \quad \alpha(u) = -\operatorname{arctg}\frac{g(u)}{uf(u)} + \frac{\pi}{2}$$

Параметр u пробегает один из интервалов, фигурирующих в формуле (4.9), со сдвигом на $2\pi k$ $(k = 0, 1, 2\ldots)$. Объединение счетного числа этих кривых является границей области устойчивости в параметрах (γ, α) (фиг. 4).



Фиг. 4. Области устойчивости в переменных α,γ

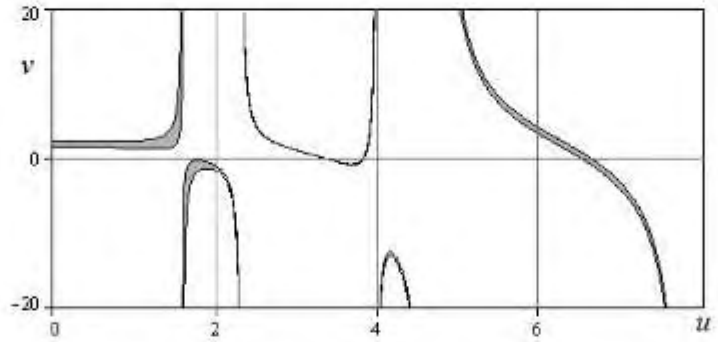
Сравним область устойчивости (4.8) и область устойчивости уравнения (4.4), когда $\varepsilon = 0$. Было показано [10], что область устойчивости (в первом приближении) системы без диссипации задается неравенством

$$\frac{1}{6}(2+3\cos(\gamma\sin\alpha))(2-\gamma\cos(\gamma\sin\alpha-\alpha)) > -\frac{4}{3}+\sqrt{\frac{16}{9}-\cos^2(\gamma\sin\alpha)} \qquad (4.10)$$

Используя параметры u и v, это неравенство можно переписать в виде

$$v\cos u\,(\cos u + 2/3) < 4 - 2\sqrt{\frac{16}{9}-\cos^2 u} - \frac{2}{3}u\sin u + 2\cos u - u\sin u\cos u \quad (4.11)$$

Элементарные вычисления показывают, что неравенство (4.10) − следствие неравенства (4.8). Т.е. область устойчивости без диссипации включает область устойчивости системы с диссипацией и оказывается шире последней. Точка (u,v) принадлежит зоне Циглера тогда и только когда, когда выполнено неравенство (4.10), но не выполнено неравенство (4.8) (фиг. 5)



Фиг. 5. Зоны Циглера

Построим область стабилизации точки покоя системы (4.11) большими силами трения. Обратимся к лемме. Условия (3.4) отвечают зонам Циглера. Можно показать, что неравенства (3.2) и (3.3) несовместны для рассматриваемой задачи. Таким образом, область стабилизации совпадает с зонами Циглера.

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Minisymposium Invited Lecture

PHASE TRAJECTORY OF AEROELASTIC DYNAMIC SYSTEMS IN AN EXPANDED PHASE SPACE

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Keywords: aeroelastic oscillations .surface pipeline. phase trajectories.

Abstract. *Research into aeroelastic oscillations in structures and buildings subjected to an air flow is of interest owing to the progressively longer bridge span frameworks and overpasses and taller mast- and tower-type structures. No matter to which type such lengthy structures under consideration belong, the aerodynamic loads, which affect the structures and cause their steady oscillations, represent the non-linear conservative functions of the displacements and the velocities of the structural components. A distinguishing feature of the oscillations of the elastic structures in a wind flow is the complex interactions between the aerodynamic loads and the parameters of oscillations of bodies in a wind flow.*

This article is devoted to research dynamic behaviour of surface pipeline in a horizontal uniform wind flow. The investigations have been carried out on the basis of the conventional equations for the interactions between a bluff body of a circular cylinder and a uniform wind flow. The analysis was performed by a hybrid computer complex on the base of analogue devices and digital computer treatment. Time processes, spectral characteristics of distributing of energy of oscillations on their frequencies and phase trajectories, are received at transient and stationary behaviours of vertical vibrations in a wide frequency range. The hysteretic effects in a resonance frequency ranges and zones where interaction of free and forced oscillations appear.

1 INTRODUCTION

Research into aeroelastic oscillations in structures and buildings subjected to an air flow is of interest owing to the progressively longer bridge span frameworks and overpasses and taller mast- and tower-type structures. No matter to which type such lengthy structures under consideration belong, the aerodynamic loads, which affect the structures and cause their steady oscillations, represent the non-linear conservative functions of the displacements and the velocities of the structural components. A distinguishing feature of the oscillations of the elastic structures in a wind flow is the complex interactions between the aerodynamic loads and the parameters of oscillations of bodies in a wind flow. Dynamic behaviour of mechanical systems is usually presented as oscillating processes in various graphic forms such as time processes, the Lissajous patterns and hodograph. Such patterns of presentations enable to determine the type of a process and to perform numerical estimations of its characteristics, but do not disclose any properties of the governing system. Unlike them phase trajectories have the row of advantages. The image on phase plane "acceleration - displacement" is a more vivid presentation because it depicts inharmonious oscillations particularly well. Each phase trajectory represents only one definite clearly defined motion. The geometric presentation of a single phase trajectory or a set of trajectories allows coming to important conclusions about the oscillation characteristics. It is, foremost, true with the oscillations, which are described with nonlinear differential equations. As is has been shown by the investigations of author [5], the expansion of a phase space by taking into account the phase plane "acceleration - displacement" substantially promotes the efficiency in analyzing a dynamic system behaviour. Hereby, we pass on to a three-dimensional phase space confined with three co-ordinate axes, i.e. displacement, velocity and acceleration. An interest taken into accelerations in dynamic systems is conditioned by the fact that these accelerations are more sensitive to high-frequency components in oscillating processes.

2 THE MODEL OF AEROELSTIC OSCILLATIONS OF SURFACE PIPELINE

In some models of the aeroelastic oscillations of structures, the values the aerodynamic force are determined not only by the aerodynamic resistance but also by the periodic load resulting from the alternate shedding of the Karman vortices from the lateral surfaces of the bluff bodies.

The load of this type arises from the stalled flow-around of the structures and represents an external periodic force with its frequency depending on both the cross-sectional shape in a structural component and on the wind flow velocity.

This article describes the results of the research into peculiarities of the dynamic interactions between a surface pipeline and a horizontal uniform wind flow.

The investigations have been carried out on the basis of the conventional equations [1-3] for the interactions between a bluff body of a circular cylinder (i.e. a surface pipeline) and a uniform wind flow. To adapt these conventional nonlinear differential equations to the objective of this research, the following definitions were entered:

$$m\ddot{y} + \frac{1}{\pi}\delta\omega_0\dot{y} + f(\dot{y}) + R(y) = F(t), \; Ry = \alpha y + \gamma y^2 + \beta y^3 + F_0 \qquad (1)$$

where δ, ω_0 are the logarithmic decrement and the natural frequency of a pipeline, respectively; $f(\dot{y})$ is the non-stationary aerodynamic force [1-3]; $R(y)$ is the elastic restoring force; and $F(t)$ is the aeroelastic Karman force.

$$f(\dot{y}) = \frac{1}{2}\rho V^2 d\sqrt{1+\left(\frac{\dot{y}}{V}\right)^2}\left(C_\Gamma \frac{\dot{y}}{V} - C_\Gamma^a \operatorname{arctg}\left(\frac{\dot{y}}{V}\right)\right) \qquad (2)$$

where ρ is a air density; V is a wind flow velocity; d is a pipeline diameter; C_Γ is the aerodynamic drag ratio, C_Γ^a is the ratio, taking into account the aeroelastic nature of a circular cylindrical body:

$$F(t) = \frac{2}{m\pi} C_K \rho V^2 d \cos\omega t, \qquad \omega = \frac{2\pi S_h V}{2} \qquad (3)$$

where C_K is the ratio of the Karman force; ω is the frequency of stalling the Karman vortices; and S_h is the Strouhal number.

The form of equation (2) suggests that the described system should be the one with self-induced periodic oscillations in its right-hand side. In this instance, the self-induced oscillating properties are conditioned by the nonlinear aerodynamic resistance.

3 THE METHOD OF INVESTIGATION AND RESULTS

The hybrid modelling was chosen as the basic method of investigation. At modelling on hybrid computer complexes (HCC) the real system is replaced by physical (electric) model, and the computer becomes the working model. HCC incorporate analogue and PC. HCC possess the speed of analogue computers, accuracy and the wide memory size of the PC; Unlike PC, HCC enables the user to observe visually computing process by means of oscillographs, recorders etc.; and also allows to change parameters of investigated model during computing process. Thus, HCC gives the opportunities [5] for studying influence of parameter changes on behaviour of investigated systems, and also allows to follow the modes which are not sold on PC.

In the hybrid modeling, the analysis of the oscillations was performed for a steel pipeline having the length of $l = 35m$, the diameter of $d = 0{,}426\,m$, and the wall thickness of $\Delta = 0{,}004\,m$.

The wind flow velocity values varied within the range of $V = 0 \div 30 ms^{-1}$. Let us assume that the value of the initial static deflection of a pipeline is equal to $W_c = 0.05077\,m$. Then, the parameters in equation (1) take on the following values: $\alpha = 199 s^{-1}$; $\gamma = 84 m^{-1}s^{-2}$; $\beta = 551 m^{-2}s^{-2}$; $F_0 = 9{,}97\ ms^{-2}$. The point with $y_A = -0.0508m$ corresponds to the equilibrium state of the system.

Fig.1 shows the analytical skeleton curve and the amplitude-frequency curve, which was obtained in the hybrid simulations. It is evident that these curves correlate with each other fairly well. The system under consideration is a rigid one. Within the range of the wind flow velocities from $V < 4{,}4ms^{-1}$ to $V > 6ms^{-1}$, the oscillations can be interpreted as the forced ones with small amplitudes.

As soon as the system reached the boundaries of the range of the fundamental resonance $\omega \approx \omega_0$, a beating mode established (fig. 2). On completion of the transient process, the amplitudes of the natural oscillations gradually decrease and then damp out.

With an increase of the wind flow velocities in the range of the fundamental resonance, the amplitudes of the oscillations drastically increase, too (see fig. 2). The transient time processes clearly display the "swinging" of the system (fig. 3). The oscillation amplitudes attain their peak values at the resonance on the fundamental harmonic at the wind flow velocity of $V_{cr} = 4.78ms^{-1}$. Under such conditions, the oscillation amplitudes do not exceed

$\overline{a_1} = 0.05$. Within this wind flow velocity range, the oscillations in the system under study are self-sustaining and similar to those in the systems with self-induced oscillations. In actual practice, they can hardly be observed in the systems, because the wind flow is ununiform by its nature. The gusts of the wind flow interact with the forced transversal oscillations caused by periodic stalling the vortices.

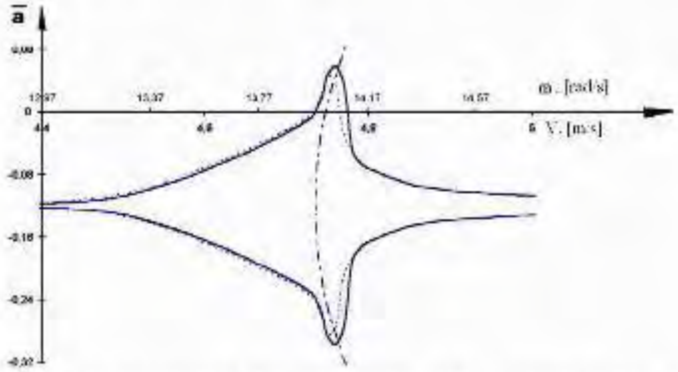

Figure 1. Amplitude-frequency curves of the vertical oscillations of a surface pipeline in a uniform wind flow

When the wind flow velocities increase still further, the oscillation amplitudes decrease, and the oscillations go onto the unresonance branch of the amplitude-frequency curve. In the transient time processes, "jumps" in the oscillation amplitudes of the system are noted (fig. 4).

The analysis of stability of the aerodynamic oscillations of the fundamental tone carried out on the basis of the constructed resonance curves allows to conclude that the oscillations of this type remain steady as long as the wind flow velocities do not exceed the value of $V = 4.8\,ms^{-1}$. This is explained by the fact that any changes in a wind flow velocity V affect not only the frequencies of the vortical excitement but also the magnitude of the non-stationary aerodynamic force.

With the gradually decreasing wind flow velocity, the amplitudes of the oscillations increased. On the graphs of the time processes in this range, "swinging" of the system is observed. The transition onto the resonance branch of the amplitude-frequency curve took place at the wind flow velocity $V_{cl} = 4.75ms^{-1}$, and it was accompanied with the mitigation of the oscillation amplitudes.

Noteworthy also is the fact that the fractional and multiple resonances, which are typical for the non-linear systems, are present at the wind flow velocities corresponding to the critical velocities for the sub- and ultraharmonic oscillations. Thus, a distinctive feature of the system under study is the availability of a regionzone of the subharmonic resonance $\omega \approx 3\omega_0$ (see fig. 5). With the wind flow velocity of $V = 14{,}165ms^{-1}$, the graph of a transient process depicts the modulated oscillations of the fundamental tone commensurable with those at the frequencies of the vortex stalling. On completion of the transient process, the amplitudes of the self-induced oscillations decrease, while those of the forced oscillations increase; hence, by this means the redistribution of oscillation energy is realized.

It should be noted that the range of the subharmonic resonance is narrower than that of the fundamental resonance, and the amplitudes of the subharmonic oscillations are less.

The analysis of the obtained results supports the assertion that both the nature of the variations in the velocities of the wind flow and its duration has a pronounced effect on the characteristics of the surface pipeline oscillations. Particularly, the wind flow velocity c $V \cong 14{,}165 ms^{-1}$ of a short duration can induce significant oscillations at the frequencies of th fundamental tone.

The system under study is defined as that with self-induced oscillations. The phas trajectories obtained on the plane $(y, \dot{y})$ represent the closed curves; and their specific featur is that they are inversely symmetric to the axis $\dot{y}$.

The "skeleton" curves of the phase trajectories on the plane $(y, \ddot{y})$ numerically estimate for in the "beating" and "swinging" modes (see fig. 2-4) looked very similar to the slopin straight lines. This is due to the fact that the non-linearity level of elastic force $R(y)$ is nc large. The angle of slope of the phase trajectories towards the axis y on the plane $(y, \ddot{y})$ i proportional to the natural frequency ω_0 in the system being studied.

Under the action of the non-linear dissipative force, the acceleration of the points of th system mitigates rapidly after this acceleration achieves its maximal value. As a consequence the ends of the phase trajectories on the plane $(y, \ddot{y})$ in the modes of the rapidly decreasin oscillations are sharp (see fig.4).

The phase trajectories obtained for the modulated oscillations on the plane $(y, \ddot{y})$ require more close consideration. They consist of three closed curves having the angle of slop proportional to the natural frequency ω_0 of the pipeline. The formation of two additiona loops on the phase trajectories $(y, \ddot{y})$ is attributed to the fact that the position of the oscillatio center in the modulation modes is changes continuously, unlike the situation existing in th previously discussed modes.

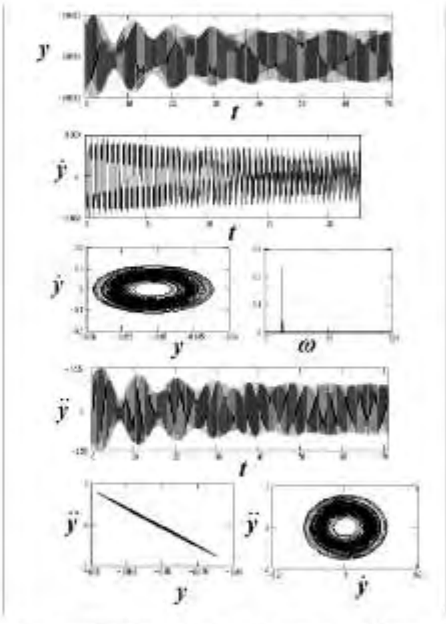

Figure 2. Time processes, spectral characteristics and phase trajectories of vertical oscillations in a surface sagging pipeline at the velocity of $V = 4{,}5 ms^{-1}$ of the uniform wind flow

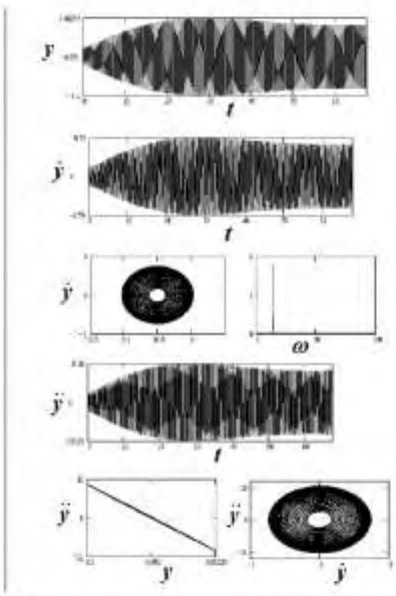

Figure 3. Time processes, spectral characteristics and phase trajectories of vertical oscillations in a surface sagging pipeline at the velocity of $V = 4,72ms^{-1}$ of the uniform wind flow

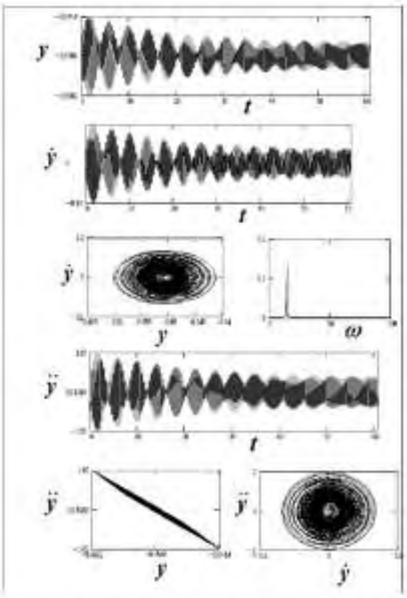

Figure 4. Time processes, spectral characteristics and phase trajectories of vertical oscillations in a surface sagging pipeline at the velocity of $V = 5,25ms^{-1}$ of the uniform wind flow

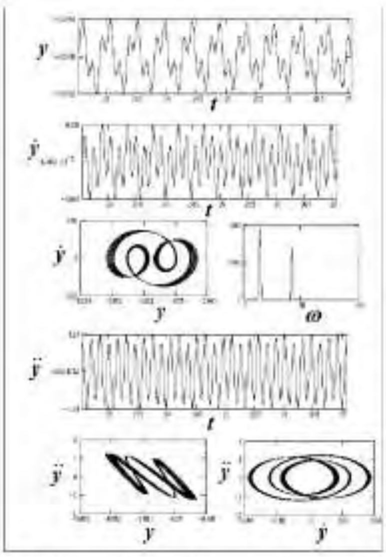

Figure 5. Time processes, spectral characteristics and phase trajectories of vertical oscillations in a surface sagging pipeline at the velocity of $V = 14{,}165ms^{-1}$ of the uniform wind flow

All the phase trajectories on both on the plane $(y, \dot{y})$ and the plane $(y, \ddot{y})$ have the shape of spirals; but these spirals are convolving at the damping oscillations and unwinding as the oscillations increase.

4 CONCLUSION

The phase trajectories obtained for the modes of "beatings" on the planes $(y, \dot{y})$ and $(y, \ddot{y})$ convolve or unwind alternately. The distance between the spiral loops varies, depending on the action exerted by the dissipative forces per cycle of oscillations. The phase trajectories of the modulated oscillations on the planes $(y, \ddot{y})$ and $(\dot{y}, \ddot{y})$ consist of three closed curves, which are inversely symmetrical relative to the axes y and $\dot{y}$, respectively.

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Seminar
Nonlinear Dynamics – Milutin Milanković

Participants of Seminar with
Academician **Anthony Kounadis** and Professor ***John T. Katsikadelis***

Participants of Seminar: **Julijana Simonović, Dragoslav Stoiljković, Katica (Stevanović) Hedrih** ***and Dragan Jovanović*** *(from left to right).*

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Minisymposium Invited Contributed Lecture

ENERGY TRANSFER THROUGHT THE DOUBLE CIRCULAR PLATE NONCONSERVATIVE SYSTEM DYNAMICS

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Keywords: Circular plate system, visco-elastic connection, energy transfer, two (three)-frequency regime, Lyapunov exponents.

Abstract. *The study of the transfer energy between subsystems coupled in hybrid system is very important for different applications. This paper presents analytical analysis of the transfer energy between plates for free transversal vibrations of a visco-elastically connected double circular plate system. The analytical analysis showed that the visco-elastic connection between plates caused the appearance of two-frequency like regime of time function, which corresponds to one eigen amplitude function of one mode, and also that time functions of different vibration modes are uncoupled, but energy transfer between plates in one eigen mode appears. It was shown for each shape of vibrations. Series of the two Lyapunov exponents corresponding to the one eigen amplitude mode are expressed by using energy of the corresponding eigen amplitude time component.*

Using the analytical solutions of the system of coupled partial differential equations, which describe corresponding dynamical free and forced processes it was shown that one-mode vibrations correspond two-frequency regime for free vibrations induced by initial conditions and to three-frequency regime for forced vibrations induced by one-frequency external excitation and corresponding initial conditions. This model of double plate system is suitable for considering energy transfer between plates, by using derived partial differential equations in the generalized form by introduction a visco-elastic layer between plates.

Also, by using energy analysis of the deformable bodies for the linear case, we can conclude that transfer energy between eigen amplitude modes not appear, only there are two frequency component time processes in one mode and there are transfer energy between two one frequency time components and between plates in one eigen amplitude mode.

1 INTRODUCTION

As it is well known, plates, beams and belts have been extensively used as structural elements in many industrial applications. The interest in the study of coupled systems (see Refs. [2], [3], [4], [5], [6], [8] and [19]) as new qualitative multi plate, multi beam as well as multi belt hybrid systems has grown exponentially over the last few years because of the theoretical challenges involved in the investigation of such systems. If these hybrid systems contain coupled subsystems with different coupled fields and different disparate natures we can generalize that words are about hybrid systems with coupled fields disparate natures and with subsystems with continuous or discrete properties.

Also, we can conclude that the impact on applications in several disciplines and industrial contexts increase. Especially in the usage of the laminate which has two facing layers that are structurally bonded to a polyurethane elastomer core which may have steel or rigid foam void sections embedded within. Such a model of coupled structure provides equivalent in plane and transverse stiffness and strength, reduces fatigue problems, minimizes stress concentrations, improves thermal and acoustical insulation, and may provides vibration control.

Recent technological innovations have caused a considerable interest in the study of dynamical processes of heterogeneous continuous and discrete nature systems, denoted as hybrid systems, characterized by the interaction of continuous system time models, governed by partial differential equations and of ordinary differential equations (see Refs. [8], [10] and [12]). Also, there is interest in the study of the transfer energy between subsystems coupled in hybrid system (see Refs. [2], [3] , [24] and [25]).

In many engineering systems with nonlinearity, high frequency excitations are sources of appearance of multi frequency resonant regimes with high frequency modes as well as low frequency modes. This is observed in many experimental research results and also theoretical results (see Refs. [24], [25] and [28]). The interaction between amplitudes and phases of different modes in nonlinear systems with many degrees of freedom as well as in the free and forced multi frequency regimes of deformable bodies with infinite number of vibration frequencies, is observed theoretically by averaging asymptotic method of Krilov-Bogoliyubov-Mitropolskiy (see Refs. [20]-[23]). This knowledge has great practical importance.

In the monograph by Nayfeh [26] a coherent and unified treatment of analytical, computational, and experimental methods and concepts of modal nonlinear interactions is presented. These methods are used to explore and unfold in a unified manner the fascinating complexities in nonlinear dynamical systems. Through the mechanisms discussed in this monograph, energy from high-frequency sources can be transferred to the low-frequency modes of supporting structures and foundations, and the result can be harmful large-amplitude oscillations that decrease their fatigue lives. On the other hand, these mechanisms can be exploited to transfer the energy from a system to a sacrificial subsystem [27] and hence decrease considerably the vibrations of the main system and increase its fatigue life.

In the numerous papers (see Refs. [2], [3], [16], [17] and [18]) Hedrih presents transfer energy between modes in nonlinear deformable body vibrations and well in the coupled linear and nonlinear oscillators by using averaging and asymptotic methods Krilov-Bogoliyubov-Mitropolyskiy for obtaining system of the differential equations of amplitudes and phases in first approximations and expression for energy of the excited modes depending of amplitudes, phases and frequencies of different nonlinear modes. By means of these obtained asymptotic approximations of the solutions, the energy analysis of the interaction of the modes in the cases of obtained multifrequency vibration regimes in the nonlinear elastic systems (beams, plates and shells) excited by initial conditions for free and forced vibrations, is realized. Also, a transfer energy between modes is identified. Also, for the case of the forced frequency of the external excitation in the resonant frequency range near to one of the natural eigen frequency of the basic linear system two or more resonant energy jumps are present and it is possible to identified by the obtained system of the differential equations of amplitudes and phases in first approximations. Trigger of coupled singularities, as well as coupled triggers of the energy values is also present in the nonlinear system multifrequency resonant stationary and nonstationary regimes during to increasing and decreasing values of the external excitation frequencies though corresponding mode resonant ranges.

In conclusion of this part we can summarize the following: Oscillatory processes in dynamical systems depend on the systems character. In such systems energy is also transformed from one form to another and has different flows inside a dynamical system. Transformation of kinetic energy into potential energy and vice versa occurs in conservative systems, but when linear systems are in question, the energy carried by a considered harmonic (mode) of adequate frequency remains constant during a dynamical process, as does the total systems energy. There is no mutual influence between harmonics and the system may be presented by partial oscillators, the number of which is equal to the number of oscillations freedom degrees, or to the number of free vibrations own circular frequencies. During that the total energy of a single partial oscillator remains constant and the transformation of kinetic energy into potential occurs. In such linear system transfer energy between modes no occurs (see Ref. [29])

When nonlinear conservative systems are in question such conclusion remains for linear systems would be incorrect (see Refs. [2-29]). The theoretical and experimental studies reveal that the interactions between widely separated modes result in various bifurcations, the coexistence of multiple attractors, and chaotic attractors. The theoretical results show also that damping may be destabilizing.

The paper (see Ref. [8] and [19]) presents analytical and numerical analysis of the free and forced transversal vibrations of an elastically and visco-elastically connected double plate rectangular like as circular system. Analytical solutions of the system of coupled partial differential equations, which describe corresponding dynamical free and forced processes, are obtained using the method of Bernoulli's particular integral and Lagrange's method of variation constants. It was shown that one-mode vibrations correspond two-frequency regime for free vibrations induced by initial conditions and to three-frequency regime for forced vibrations induced by one-frequency external excitation and corresponding initial conditions.

This model of double plate system is suitable for considering energy transfer between plates, by using derived partial differential equations in the generalized form by introduction a visco-elastic layer [1] between plates.

Also, by using previous energy analysis of the deformable bodies, we can conclude that transfer energy between eigen amplitude modes not appear, only there are two frequency component time processes in one mode and there are transfer energy between two frequency time components and between plates in one eigenamplitude mode.

A model of the double plate system with discontinuity in elastic layer is considered as a model of the interface crack between two plates connected by thin elastic layer Winkler type and obtained results are presented in [4]. The analytical analysis of free transversal vibrations of an elastically connected double plate systems with discontinuity in the elastic layer Winkler type shown that to one mode vibrations correspond infinite or finite multi-frequency regime for free and also multi-frequency plus for forced vibrations induced by initial conditions and one-frequency or corresponding number multi-frequency regime depending of external excitation. It is shown for every shape of vibrations. Also, we can conclude that discontinuity in the elastic layer is source for transfer energy between all eigen amplitude modes with infinite number frequency time component processes.

2 BASIC EQUATIONS

By using the model of double circular plate system with visco-elastic layer (similar as in [19] and [25]), we can consider the energy transfer between plates as in the Reference [3]. For that reason we use corresponding derived partial differential equations and corresponding analytical results and expressions for solutions of the transversal displacements of the both plates vibrations. This double plate system is presented in Fig.1.

The governing systems of the coupled partial differential equations for free double plates oscillations are in the following form (see Refs. [8] and [19]):

$$\frac{\partial^2 w_1(r,\varphi,t)}{\partial t^2}+c_{(1)}^4\Delta\Delta w_1(r,\varphi,t)-2\delta_{(1)}\left[\frac{\partial w_2(r,\varphi,t)}{\partial t}-\frac{\partial w_1(r,\varphi,t)}{\partial t}\right]-a_{(1)}^2\left[w_2(r,\varphi,t)-w_1(r,\varphi,t)\right]=0$$

$$\frac{\partial^2 w_2(r,\varphi,t)}{\partial t^2}+c_{(2)}^4\Delta\Delta w_2(r,\varphi,t)+2\delta_{(2)}\left[\frac{\partial w_2(r,\varphi,t)}{\partial t}-\frac{\partial w_1(r,\varphi,t)}{\partial t}\right]-a_{(2)}^2\left[w_2(r,\varphi,t)-w_1(r,\varphi,t)\right]=0 \tag{1}$$

where $w_i(r,\varphi,t)$, $i=1,2$ are small plate transverse deflections (with means, as has been discussed in books by Rašković (1965) [29] and small compared to the plates thickness, h_i, $i=1,2$,) and that plates vibrations occur only in the orthogonal direction with respect to the parallel middle surfaces of the plates passing through their parallel contours with same boundary plates conditions; $a_{(i)}^2=\frac{c}{\rho_i h_i}$, $2\delta_{(i)}=\frac{b}{\rho_i h_i}$ $i=1,2$ and $c_{(i)}^4=\frac{\mathrm{D}_i}{\rho_i h_i}$, $i=1,2$ with $\mathrm{D}_i=\frac{\mathrm{E}_i h^3}{12(1-\mu^2)}$, $i=1,2$ corresponding bending cylindrical rigidities of the plates, and $\Delta\Delta=\frac{\partial^4}{\partial x^4}+2\frac{\partial^4}{\partial x^2\partial y^2}+\frac{\partial^4}{\partial y^4}$ is differential operator; E_i modulus of elasticity, μ_i Poisson's ratio and G_i shear modulus, ρ_i plate mass distribution. The plates are interconnected by a visco-elastic layer with constant surface stiffness c and with constant surface damping force coefficient b distributed along all plates' surfaces.

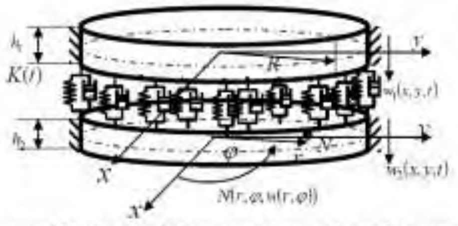


Figure 1: Double plate system with viscoelastic layer: structure and noted corresponding kinetic parameters and coorinate systems

For the solutions of the governing system of the corresponding coupled partial differential equations (1) for free double plate system oscillations, we take into account the eigen amplitude functions $\mathbf{W}_{(i)nm}(r,\varphi)$, $i=1,2$, $n,m=1,2,3,4,\ldots,\infty$ and the time expansion with the coefficients in the form of the unknown time functions $T_{(i)nm}(t)$, $i=1,2$, $n,m=1,2,3,4,\ldots,\infty$ describing their time evolution:

$$w_i(r,\varphi,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\mathrm{W}_{(i)nm}(r,\varphi)\mathrm{T}_{(i)nm}(t), \quad i=1,2 \tag{2}$$

where the eigenamplitude functions $\mathbf{W}_{(i)nm}(r,\varphi)$, $i=1,2$, $n,m=1,2,3,4,....\infty$ are the same, for both plates in the system, as in the case with decoupled plates problem (see Refs. [8] and [19]). Then after introducing the (2) into governing system of the coupled partial differential equations for free double plates oscillations in the form (1) and after multiplying first and second equation with $\mathrm{W}_{(i)sr}(r,\varphi)drd\varphi$ and after integrating along the middle plate surface and taking into account orthogonality conditions and corresponding equal boundary conditions of the plates, we obtain the mn-family of the systems containing coupled two ordinary differential equations for determination of the unknown time functions $T_{(i)nm}(t)$, $i=1,2$, $n,m=1,2,3,4,....\infty$ in the following form:

$$\ddot{\mathrm{T}}_{(1)nm}(t)+\omega^2_{(1)nm}\mathrm{T}_{(1)nm}(t)+2\delta_{(1)}\dot{\mathrm{T}}_{(1)nm}(t)-a^2_{(1)}\mathrm{T}_{(2)nm}(t)-2\delta_{(1)}\dot{\mathrm{T}}_{(2)nm}(t)=0 \qquad n,m=1,2,3,4,....\infty$$

$$\ddot{\mathrm{T}}_{(2)nm}(t)+\omega^2_{(2)nm}\mathrm{T}_{(2)nm}(t)+2\delta_{(2)}\dot{\mathrm{T}}_{(2)nm}(t)-a^2_{(2)}\mathrm{T}_{(1)nm}(t)-2\delta_{(2)}\dot{\mathrm{T}}_{(1)nm}(t)=0 \tag{3}$$

where $\omega^2_{(i)nm}=k^4_{(i)nm}c^4_{(i)}+a^2_{(i)}$ and $k^4_{(i)nm}$ characteristic numbers depending of the plates equal boundary conditions. After eliminating the time function $\mathrm{T}_{(2)nm}(t)$ from the previous mn-family of the system of coupled second order ordinary differential equations, we obtain mn-family of one four order ordinary differential equation with the corresponding mn-family characteristic equations in the form of the polynomial four order equation with respect to unknown characteristic eigen numbers $\lambda_{nm(s)}$, $n,m=1,2,3,4,....\infty$. $s=1,2$ and each of the sets with two conjugate complex roots $\lambda_{nm(s)1,2}$, $n,m=1,2,3,4,....\infty$, $s=1,2$ in the form:

$$\lambda_{nm(s)1,2}=-\tilde{\delta}_{nm(s)}\mp j\tilde{\omega}_{nm(s)}, \qquad n,m=1,2,3,4,....\infty,\ s=1,2$$

where $-\tilde{\delta}_{nm(s)}$ real and $\tilde{\omega}_{nm(s)}$ imaginary parts of the corresponding pair of the roots of the characteristic equation. We also take in consideration case when kinetic parameter satisfy condition that roots are conjugate complex numbers with negative real parts. Late we can consider other cases of the roots.

We can compose formally the system equation (3) by the following matrices of the inertia $\mathbf{A}_{nm}$, of the quazielastic coefficients $\mathbf{C}_{nm}$ and of the damping force coefficients $\mathbf{B}_{nm}$ of the dynamical system corresponding to the mn-family, with two degrees of freedom:

$$\mathbf{A}_{nm}=\begin{pmatrix}\mathbf{1} & \\ & \mathbf{1}\end{pmatrix} \qquad \mathbf{C}_{nm}=\begin{pmatrix}\omega^2_{(1)nm} & -a^2_{(1)}\\ -a^2_{(2)} & \omega^2_{(2)nm}\end{pmatrix} \qquad \mathbf{B}_{nm}=\begin{pmatrix}2\delta_{(1)} & -2\delta_{(1)}\\ -2\delta_{(2)} & 2\delta_{(2)}\end{pmatrix} \tag{5}$$

and by using the solutions in the form of:

$$\mathrm{T}_{(1)nm}(t)=C_{(1)nm}e^{\lambda_{nm}t}$$

$$\mathrm{T}_{(2)nm}(t)=C_{(2)nm}e^{\lambda_{nm}t} \tag{6}$$

where λ_{nm}, $n,m=1,2,3,4,....\infty$ are unknown own characteristic numbers, $C_{(i)nm}$ unknown amplitude. Then characteristic equation of the mn-family is in the form of:

$$f_{nm}(\lambda_{nm})=\left|\lambda^2_{nm}\mathbf{A}_{nm}+\lambda_{nm}\mathbf{B}_{nm}+\mathbf{C}_{nm}\right|=\begin{vmatrix}\lambda^2_{nm}+2\delta_{(1)}\lambda_{nm}+\omega^2_{(1)nm} & -2\delta_{(1)}\lambda_{nm}-a^2_{(1)}\\ -2\delta_{(2)}\lambda_{nm}-a^2_{(2)} & \lambda^2_{nm}+2\delta_{(2)}\lambda_{nm}+\omega^2_{(2)nm}\end{vmatrix}=0 \tag{7}$$

with the sets of the two pair of roots $\lambda_{nm(s)1,2}$, $n,m=1,2,3,4,\dots,\infty$, $s=1,2$ in the form (4).

We can write:

$$\frac{C^{(s)}_{(1)nm}}{C^{(s)}_{(2)nm}}=\frac{2\delta_{(1)}\lambda_{nm(s)}+a^2_{(1)}}{\lambda^2_{nm(s)}+2\delta_{(1)}\lambda_{nm(s)}+\omega^2_{(1)nm}}=\frac{\lambda^2_{nm(s)}+2\delta_{(2)}\lambda_{nm(s)}+\omega^2_{(2)nm}}{2\delta_{(2)}\lambda_{nm(s)}-a^2_{(2)}}=\frac{1}{C_{nm(s)}} \tag{8}$$

The solutions of the nm-family mode time functions $T_{(i)nm}(t)$, $i=1,2$, $n,m=1,2,3,4,\dots,\infty$ from system (3), for the case different plate thicknesses and materials, are in the form of:

$$\begin{gathered}
T_{(1)nm}(t)=e^{-\bar{\lambda}_{nm(1)}t}\left(A_{nm}\cos\tilde{\omega}_{nm(1)}t+B_{nm}\sin\tilde{\omega}_{nm(1)}t\right)+e^{-\bar{\lambda}_{nm(2)}t}\left(C_{nm}\cos\tilde{\omega}_{nm(2)}t+D_{nm}\sin\tilde{\omega}_{nm(2)}t\right)\\
T_{(2)nm}(t)=C_{nm(1)}e^{-\bar{\lambda}_{nm(1)}t}\left[A_{nm}\cos\tilde{\omega}_{nm(1)}t+B_{nm}\sin\tilde{\omega}_{nm(1)}t\right]-C_{nm(2)}e^{-\bar{\lambda}_{nm(2)}t}\left[C_{nm}\cos\tilde{\omega}_{nm(2)}t+D_{nm}\sin\tilde{\omega}_{nm(2)}t\right]
\end{gathered} \tag{9}$$

where $C_{nm(s)}$, $n,m=1,2,3,4,\dots,\infty$, $s=1,2$ are known constants as two eigen characteristic sets of the nm-family characteristic numbers as ratio between cofactors of the corresponding characteristic determinant.

The solutions of the mn-family mode time functions $T_{(i)nm}(t)$, $i=1,2$, $n,m=1,2,3,4,\dots,\infty$ from system (3), for the case same plate dimensions and materials, are in the form of:

$$\begin{gathered}
T_{(2)nm}(t)=e^{-\bar{\lambda}_{nm}t}\left(A_{nm}\cos\tilde{\omega}_{nm(1)}t+B_{nm}\sin\tilde{\omega}_{nm(1)}t+C_{nm}\cos\tilde{\omega}_{nm(2)}t+D_{nm}\sin\tilde{\omega}_{nm(2)}t\right)\\
T_{(1)nm}(t)=e^{-\bar{\lambda}_{nm}t}\left\{\left[A_{nm}\cos\tilde{\omega}_{nm(1)}t+B_{nm}\sin\tilde{\omega}_{nm(1)}t\right]-\left[C_{nm}\cos\tilde{\omega}_{nm(2)}t+D_{nm}\sin\tilde{\omega}_{nm(2)}t\right]\right\}
\end{gathered} \tag{10}$$

where mn-family mode $n,m=1,2,3,4,\dots,\infty$ contains the following set of the unknown constants $A_{nm}, B_{nm}, C_{nm}, D_{nm}$ defined by initial plates conditions.

Then, the particular solutions of governing system of the coupled partial differential equations for free system oscillations corresponding to plate displacements

$$\begin{gathered}
w_1(r,\varphi,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}W_{(1)nm}(r,\varphi)\left\{e^{-\bar{\lambda}_{nm(1)}t}\left(A_{nm}\cos\tilde{\omega}_{nm(1)}t+B_{nm}\sin\tilde{\omega}_{nm(1)}t\right)+e^{-\bar{\lambda}_{nm(2)}t}\left(C_{nm}\cos\tilde{\omega}_{nm(2)}t+D_{nm}\sin\tilde{\omega}_{nm(2)}t\right)\right\}\\
w_2(r,\varphi,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}W_{(2)nm}(r,\varphi)\left\{C_{nm(1)}e^{-\bar{\lambda}_{nm(1)}t}\left[A_{nm}\cos\tilde{\omega}_{nm(1)}t+B_{nm}\sin\tilde{\omega}_{nm(1)}t\right]-C_{nm(2)}e^{-\bar{\lambda}_{nm(2)}t}\left[C_{nm}\cos\tilde{\omega}_{nm(2)}t+D_{nm}\sin\tilde{\omega}_{nm(2)}t\right]\right\}
\end{gathered} \tag{11}$$

where the unknown constants $A_{nm}, B_{nm}, C_{nm}, D_{nm}$ for mn-family mode $n,m=1,2,3,4,\dots,\infty$ are defined by initial plates conditions functions, for middle plate point displacements $g_i(x,y)$ and for middle plate points velocities $\dot{g}_i(x,y)$, $i=1,2$ (which satisfied boundary conditions).

3 ENERGY EXPRESSIONS FOR THE PLATES

3.1 Kinetic energy expressions for the plates

The kinetic energies of the plates may be expressed in the following forms (see Ref. [3]):

$$E_k^{(i)}=\frac{\rho_i h_i}{2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\dot{T}_{(i)nm}(t)\right]^2 M_{(i)nm}(r,\varphi),\ i=1,2 \tag{12}$$

where

$$M_{(i)nmsr}=\iint_{(A)}W_{(i)nm}(r,\varphi)W_{(i)sr}(r,\varphi)r\,d\varphi\,dr=\begin{cases}0 & sr\neq nm\\ M_{(i)nm} & sr=nm\end{cases} \tag{13}$$

The kinetic energy of the one plate we can express in the form of the sum by components $\mathbf{E}^{(i)}_{k,nm}$ belong to corresponding mn-family mode $n,m=1,2,3,4,\dots,\infty$ in the following form:

$$\mathbf{E}_k^{(i)} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \mathbf{E}_{k,nm}^{(i)}, \; i=1,2 \tag{14}$$

where the kinetic energy components $\mathbf{E}_{k,nm}^{(i)}, i=1,2$ belong to corresponding nm -family mode $n,m=1,2,3,4,....\infty$ was expressed by derivatives of the component time functions belong to same corresponding nm -family mode

$$\mathbf{E}_{k,nm}^{(i)} = \frac{1}{2}\rho_i h_i M_{(i)nm}(r,\varphi)\left[\dot{T}_{(i)nm}(t)\right]^2 = M_{(i)nm}\tilde{\mathbf{E}}_{k,nm}^{(i)}, \; i=1,2 \tag{15}$$

Also, we can introduce reduced component kinetic energy $\tilde{\mathbf{E}}_{k,nm}^{(i)}, i=1,2$ belong to corresponding nm -family mode $n,m=1,2,3,4,...\infty$

$$\tilde{\mathbf{E}}_{k,nm}^{(i)} = \frac{\mathbf{E}_{k,nm}^{(i)}}{M_{(i)nm}(r,\varphi)} = \frac{\rho_i h_i}{2}\left[\dot{T}_{(i)nm}(t)\right]^2, \; i=1,2 \tag{16}$$

3.2 Potential energy expressions for the plates

The potential energy of the plate is equal to energy of the deformation of elastic plate in the vibration state and expression we can write in the following form:

$$E_p = \mathbf{A}_d = \frac{1}{2}\iiint_V \left[\varepsilon_x\sigma_x + \varepsilon_y\sigma_y + \varepsilon_z\sigma_z + \gamma_{xy}\tau_{xy} + \gamma_{xz}\tau_{xz} + \gamma_{yz}\tau_{yz}\right] dV \tag{17}$$

where $\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}$ are tensor strain components, $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz}$ are tensor stress components of the plate strain and stress vibration state. Tensor stress components τ_{zx} and τ_{zy} are small, as tensor strain components γ_{zx} and γ_{zy} are also small and can be neglected in the comparison with other members in expression for work of elastic deformation of the thin plate.

Also, we can take into account that plate are thin an the stress state is plain and that we can calculate with middle plate surface, and make averaging with respect to the middle plate surface (see Ref. [29]) and for the work of elastic plate deformation we can write the following approximate expression:

$$E_p = \frac{\mathbf{D}}{2}\iint_{(A)}\left\{\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2}\right)^2 - 2(1-\mu)\left[\frac{\partial^2 w}{\partial r^2}\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2}\right) - \left(\frac{1}{r}\frac{\partial^2 w}{\partial r\partial \varphi} - \frac{1}{r^2}\frac{\partial w}{\partial \varphi}\right)^2\right]\right\} r\,dr\,d\varphi \tag{18}$$

After introducing solutions (2) in previous expression (20) for expression of the potential energy of the plates we obtain the following:

$$\mathbf{E}_p^{(i)} = \frac{\mathbf{D}_i}{2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty}\sum_{r=1}^{\infty} C_{nmsr(i)} T_{(i)nm}(t) T_{(i)sr}(t), \; i=1,2 \tag{19}$$

where $C_{nmsr(i)}$ is known depending of eigen amplitude functions (for detail see Ref. [30]). By use partial integration with respect to coordinate r or φ, and taking into account possible boundary condition we obtain:

$$C_{nmsr(i)} = \iint_{(A)} W_{(i)sr}(r,\varphi)\Delta\Delta W_{(i)nm}(r,\varphi)\,dA = k_{nm}^4 \iint_{(A)} W_{(i)sr}(r,\varphi) W_{(i)nm}(r,\varphi)\,dA \tag{20}$$

and after taking into account ortogonallity conditions (13) we obtain:

$$C_{nm,rs(i)} = k_{nm}^4 \begin{cases} 0 & rs \neq nm \\ M_{(i)nm} = \iint\limits_{(A)} \left[W_{(i)n}(r,\varphi)\right]^2 dA & rs = nm \end{cases} \tag{21}$$

The potential energies of the separate plates are in the following forms:

$$\mathbf{E}_p^{(i)} = \frac{\mathsf{D}_i}{2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} k_{nm}^4 M_{(i)nm} \left[\mathrm{T}_{(i)nm}(t)\right]^2 \tag{22}$$

or in the forms:

$$\mathbf{E}_p^{(i)} = \frac{\rho_i h_i}{2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \omega_{(i)nm}^2 M_{(i)nm} \left[\mathrm{T}_{(i)nm}(t)\right]^2 \tag{23}$$

where $\omega_{(i)nm}^2 = \frac{\mathsf{D}_i}{\rho_i h_i} k_{nm}^4$.

The potential energy of the one plate, we can express in the form of the sum by components $\mathbf{E}_{p,nm}^{(i)}$ belong to corresponding mn-family mode $n,m = 1,2,3,4,\ldots,\infty$ in the following form:

$$\mathbf{E}_p^{(i)} = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \mathbf{E}_{p,nm}^{(i)}, \quad i = 1,2 \tag{24}$$

where the energy components $\mathbf{E}_{p,nm}^{(i)}$, $i = 1,2$ belong to corresponding mn-family mode $n,m = 1,2,3,4,\ldots,\infty$ was expressed by the component time functions belong to same corresponding mn-family mode

$$\mathbf{E}_{p,nm}^{(i)} = \frac{\rho_i h_i}{2} \omega_{(i)nm}^2 M_{nm} \left[\mathrm{T}_{(i)nm}(t)\right]^2 = M_{(i)nm} \widetilde{\mathbf{E}}_{p,nm}^{(i)} \tag{25}$$

Also, we can introduce reduced component potential energy $\widetilde{\mathbf{E}}_{p,nm}^{(i)}$, $i = 1,2$ belong to corresponding mn-family mode $n,m = 1,2,3,4,\ldots,\infty$

$$\widetilde{\mathbf{E}}_{p,nm}^{(i)} = \frac{1}{2} \rho_i h_i \omega_{(i)nm}^2 M_{(i)nm} \left[\mathrm{T}_{(i)nm}(t)\right]^2 = \frac{\mathbf{E}_{p,nm}^{(i)}}{M_{(i)nm}}. \tag{26}$$

3.3 Potential energy expression for the visco-elastic layer

For analysis of the double plate system with visco-elastic layer we can write expression for the potential energy of the constraints between coupled plates in the form of the energy of deformation of the distributed elastic layer neglected mass and properties of inertia and neglected kinetic energy. Then expression for the potential energy of the coupling of the plates is in the form:

$$\mathbf{E}_{p(a,b)layer} = \iint\limits_{A} \frac{1}{2} c (w_2 - w_1)^2 dA \tag{27}$$

After introducing solutions (2) in previous expression (27), and taking into account ortogonallity conditions, for expression of the potential energy of the plate coupling we obtain the following:

$$\mathbf{E}_{p(1,2)layer} = \frac{1}{2} c \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} M_{(1)nm} \left[\mathrm{T}_{(1)nm}^2(t) + \mathrm{T}_{(2)nm}^2(t) - 2\mathrm{T}_{(1)nm}(t)\mathrm{T}_{(2)nm}(t)\right] \tag{28}$$

The potential energy of the plate coupling we can express in the form of the sum by components $\mathbf{E}^{(i)}_{p,nm(1,2)layer}$ belong to corresponding mn-family mode $n,m=1,2,3,4,\ldots\infty$ in the following form:

$$\mathbf{E}_{p(1,2)layer}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\mathbf{E}_{p,nm(1,2)layer}$$

where the energy components $\mathbf{E}^{(i)}_{p,nm(1,2)layer}$, $i=1,2$ belong to corresponding mn-family mode $n,m=1,2,3,4,\ldots\infty$ was expressed by the component time functions belong to same corresponding mn-family mode

$$\mathbf{E}_{pnm(1,2)layer}=\frac{1}{2}cM_{(1)nm}\left[\mathrm{T}^2_{(1)nm}(t)+\mathrm{T}^2_{(2)nm}(t)-2\,\mathrm{T}_{(1)nm}(t)\mathrm{T}_{(2)nm}(t)\right] \tag{29}$$

Also, we can introduce reduced component potential energy of the light distributed elastic layer $\tilde{\mathbf{E}}^{(i)}_{p,nm(1,2)layer}$, $i=1,2$ belong to corresponding mn-family mode $n,m=1,2,3,4,\ldots\infty$

$$\tilde{\mathbf{E}}^{(i)}_{p,nm(1,2)layer}=\frac{1}{2}c\left[T_{(2)nm}(t)-T_{(1)nm}(t)\right]^2=\frac{\mathbf{E}_{pnm(1,2)layer}}{M_{(i)nm}} \tag{30}$$

then it is:

$$\mathbf{E}_{p(1,2)layer}=\frac{1}{2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}M_{(1)nm}\tilde{\mathbf{E}}^{(i)}_{p,nm(1,2)layer} \tag{31}$$

3.4 Rayleigh function of dissipation for the visco-elastic layer

For analysis of the double plate system with visco-elastic layer, we can write expression for the Rayleigh function of the dissipation of the constraint between coupled plates in the form of the power of the damping force depending of velocity of the deformation of the distributed visco-elastic layer neglected mass and properties of inertia and neglected kinetic energy. Then expression for the Rayleigh function of the dissipation in the visco-elastic layer of the plates is in the form:

$$\Phi_{(1,2)layer}=\frac{1}{2}b\iint_A\left(\frac{\partial w_2(r,\varphi,t)}{\partial t}-\frac{\partial w_1(r,\varphi,t)}{\partial t}\right)^2 dA,\; i=1,2 \tag{32}$$

or in the form:

$$\Phi_{(1,2)layer}=\frac{1}{2}b\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}M_{(1)nm}(r,\varphi)\left[\dot{\mathrm{T}}_{(2)nm}(t)-\dot{\mathrm{T}}_{(1)nm}(t)\right]^2,\; i=1,2 \tag{33}$$

$$\Phi_{(1,2)layer}=\frac{1}{2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}M_{(1)nm}\tilde{\Phi}_{nm(1,2)layer}$$

where

$$b_{i,nmsr}=b\iint_A \mathrm{W}_{(i)nm}(r,\varphi)\mathrm{W}_{(i)sr}(r,\varphi)dA=\begin{cases}0 & sr\neq nm\\ bM_{(i)nm} & sr=nm\end{cases}$$

$$\tilde{\Phi}_{nm(1,2)layer}=\frac{1}{2}b\left[\dot{T}_{(2)nm}(t)-\dot{T}_{(1)nm}(t)\right]^2=\frac{\Phi_{nm(1,2)layer}}{M_{(1)nm}} \tag{34}$$

4 ENERGY ANALYSIS OF THE DOUBLE PLATE SYSTEM

Using expressions of the reduced components of kinetic $\widetilde{\mathbf{E}}^{(i)}_{k,nm}, i=1,2$ and potential energy $\widetilde{\mathbf{E}}^{(i)}_{p,nm}, i=1,2$, as well as reduced potential energy of the light distributed visco-elastic layer $\widetilde{\mathbf{E}}^{(i)}_{p,nm(1,2)layer}, i=1,2$, and reduced Rayleigh function of the dissipation belong to corresponding mn-family mode $n,m=1,2,3,4,...,\infty$ we can see that is suitable to use energy analysis in the form of the corresponding mn-family mode $n,m=1,2,3,4,...,\infty$ by using these reduced components of the energy, as it is presented in Ref. [3]. For that reason we can use infinite numbers of the sets belong to corresponding mn-family mode $n,m=1,2,3,4,...,\infty$ by systems of the two ordinary differential equations (3) in every of the sets and energy analysis make as in the systems with two degree of the freedom.

We can compose formally the system of two equations (3) by use defined, in Ref. [3], the matrices of the inertia $\mathbf{A}_{nm}$ and $\mathbf{C}_{nm}$ quazielastic coefficients of the dynamical system corresponding to the mn-family, with two degrees of freedom in the form (5). By use these matrices of the inertia $\mathbf{A}_{nm}$ and $\mathbf{C}_{nm}$ quazielastic coefficients of the dynamical system correspondyng to the mn-family, with two degrees of freedom we can compose expression of the reduced values of the kinetic and potential energy correspond to mn-family mode in the following form:

A* Reduced kinetic energy $\widetilde{\widetilde{\mathbf{E}}}_{k,mn}, n,m=1,2,3,4,...,\infty$

$$\widetilde{\widetilde{\mathbf{E}}}_{k,nm}=\frac{1}{2}\left(\dot{T}_{(1)nm} \quad \dot{T}_{(2)nm}\right)\widetilde{\mathbf{A}}_{nm}\begin{Bmatrix}\dot{T}_{(1)nm}\\ \dot{T}_{(2)nm}\end{Bmatrix}=\frac{1}{2}\left(\dot{T}_{(1)nm} \quad \dot{T}_{(2)nm}\right)\begin{pmatrix}\rho_1 h_1 & \\ & \rho_2 h_2\end{pmatrix}\begin{Bmatrix}\dot{T}_{(1)nm}\\ \dot{T}_{(2)nm}\end{Bmatrix}$$

$$\widetilde{\widetilde{\mathbf{E}}}_{k,nm}=\frac{1}{2}\left[\rho_1 h_1\left(\dot{T}_{(1)nm}(t)\right)^2+\rho_2 h_2\left(\dot{T}_{(1)nm}(t)\right)^2\right]=\frac{\mathbf{E}_{k,nm}}{M_{(1)nm}} \tag{35}$$

where $\rho h=\frac{1}{2}\left(\rho_1 h_1+\rho_2 h_2\right)$

B* Reduced potential energy $\widetilde{\widetilde{\mathbf{E}}}_{p,nm}, n,m=1,2,3,4,...,\infty$ of the coupled plates

$$\widetilde{\widetilde{\mathbf{E}}}_{p,nm}=\frac{1}{2}\left(T_{(1)nm} \quad T_{(2)nm}\right)\widetilde{\mathbf{C}}_{nm}\begin{Bmatrix}T_{(1)nm}\\ T_{(2)nm}\end{Bmatrix}=\frac{1}{2}\left(T_{(1)nm} \quad T_{(2)nm}\right)\begin{pmatrix}\rho_1 h_1\omega^2_{(1)nm} & -\rho_1 h_1 a^2_{(1)}\\ -\rho_2 h_2 a^2_{(2)} & \rho_2 h_2\omega^2_{(2)nm}\end{pmatrix}\begin{Bmatrix}T_{(1)nm}\\ T_{(2)nm}\end{Bmatrix} \tag{36}$$

$$\widetilde{\widetilde{\mathbf{E}}}_{p,nm}=\frac{1}{2}\left[\rho_1 h_1\omega^2_{(1)nm}\left(T_{(1)nm}(t)\right)^2+\rho_2 h_2\omega^2_{(2)nm}\left(T_{(2)nm}(t)\right)^2-\left(\rho_1 h_1 a^2_{(1)}+\rho_2 h_2 a^2_{(2)}\right)T_{(1)nm}(t)T_{(2)nm}(t)\right]=\frac{\mathbf{E}_{p,nm}}{M_{(1)nm}}$$

C* Reduced Rayleigh function of the dissipation $\widetilde{\widetilde{\Phi}}_{p,nm}, n,m=1,2,3,4,...,\infty$ of the coupled plates

$$\widetilde{\widetilde{\Phi}}_{nm(1,2)layer}=\frac{1}{2}\left(\dot{T}_{(1)nm} \quad \dot{T}_{(2)nm}\right)\mathbf{B}_{nm}\begin{Bmatrix}\dot{T}_{(1)nm}\\ \dot{T}_{(2)nm}\end{Bmatrix}=\frac{1}{2}\left(\dot{T}_{(1)nm} \quad \dot{T}_{(2)nm}\right)\begin{pmatrix}2\delta_{(1)}\rho_1 h_1 & -2\delta_{(1)}\rho_1 h_1\\ -2\delta_{(2)}\rho_2 h_2 & 2\delta_{(2)}\rho_2 h_2\end{pmatrix}\begin{Bmatrix}\dot{T}_{(1)nm}\\ \dot{T}_{(2)nm}\end{Bmatrix}$$

$$\widetilde{\widetilde{\Phi}}_{nm(1,2)layer}=\left(\dot{T}_{(1)nm}-\dot{T}_{(2)nm}\right)\left(\delta_{(1)}\rho_1 h_1\dot{T}_{(1)nm}-\delta_{(2)}\rho_2 h_2\dot{T}_{(2)nm}\right)=\frac{1}{2}b\left(\dot{T}_{(1)nm}-\dot{T}_{(2)nm}\right)^2=\frac{\Phi_{nm(1,2)layer}}{M_{(i)nm}} \tag{37}$$

Now, we can separate members from expressions of the kinetic and potential energy correspond to the first and second plates:

A.1* Kinetic energy of the plates in the coupled system:

$$\tilde{\tilde{\mathbf{E}}}_{k,nm(1)} = \frac{1}{2}\rho_1 h_1 \left(\dot{T}_{(1)nm}(t)\right)^2 = \frac{\mathbf{E}_{k,nm(1)}}{M_{(1)nm}}, \quad \tilde{\tilde{\mathbf{E}}}_{k,nm(2)} = \frac{1}{2}\rho_2 h_2 \left(\dot{T}_{(2)nm}(t)\right)^2 = \frac{\mathbf{E}_{k,nm(2)}}{M_{(1)nm}} \tag{38}$$

B.1* Potential energy of the plates and reduced part of the potential energy of the visco-elastic layer to the corresponding plates:

$$\tilde{\tilde{\mathbf{E}}}_{p,nm(i)} = \frac{1}{2}\rho_i h_i \omega_{(i)nm}^2 \left(T_{(i)nm}(t)\right)^2 = \frac{\mathbf{E}_{p,nm(i)}}{M_{(1)nm}}, \quad i = 1,2$$

B.2* Pure energy interaction between plates induced by visco-elastic layer:

$$\tilde{\tilde{\mathbf{E}}}_{p,nm(1,2)} = -\left(\rho_1 h_1 a_{(1)}^2 + \rho_2 h_2 a_{(2)}^2\right) T_{(1)nm}(t) T_{(2)nm}(t) = \frac{\mathbf{E}_{p,nm(1,2)}}{M_{(1)nm}} \tag{39}$$

We can analyze system potential energy and separate members, also, in the following form:

B.3 * Potential energy of the plates without reduced part of the potential energy of the elastic layer to the corresponding plates:

$$\tilde{\tilde{\mathbf{E}}}_{p,nm(i,s)} = \frac{1}{2}\rho_i h_i \omega_{0nm}^2 \left(T_{(i)nm}(t)\right)^2 = \frac{\mathbf{E}_{p,nm(i,s)}}{M_{(1)nm}}, \quad i = 1,2 \tag{40}$$

B.4* Full potential energy of the visco-elastic layer interaction between plates:

$$\tilde{\tilde{\mathbf{E}}}_{p,nm(1,2)layer} = \frac{1}{2}c\left[T_{(2)nm}(t) - T_{(1)nm}(t)\right]^2 = \frac{\mathbf{E}_{p,nm(1,2)layer}}{M_{(1)nm}}$$

where $\quad \omega_{(i)nm}^2 = c_{(i)}^4 k_{(i)nm}^4 + a_{(i)}^2 = \omega_{0nm}^2 + a_{(i)}^2 \qquad (41)$

We can see that potential energy reduction are on the basis separate plates on the Wincler type elastic foundation, and that potential energy of the elastic layer are split (separate) to the both plates, and only one part is interaction between plates depending of elastic layer rigidity and of the both time functions of the plates.

C. 1* Rayleigh function of the dissipation - reduced part of the visco-elastic layer to the corresponding plates:

$$\tilde{\tilde{\Phi}}_{nm(1,2)layer(1)} = \delta_{(1)}\rho_1 h_1 \left(\dot{T}_{(1)nm}\right)^2 = \frac{\Phi_{nm(1,2)layer(1)}}{M_{(1)nmr}}, \quad \tilde{\tilde{\Phi}}_{nm(1,2)layer(2)} = \delta_{(2)}\rho_2 h_2 \left(\dot{T}_{(2)nm}\right)^2 = \frac{\Phi_{nm(1,2)layer(2)}}{M_{(1)nmr}} \tag{42}$$

C. 2* Part of Rayleigh function of the dissipation – pure interaction between plates induced by visco-elastic layer

$$\tilde{\tilde{\Phi}}_{nm(1,2)layer(1,2)} = -\left(\delta_{(1)}\rho_1 h_1 + \delta_{(2)}\rho_2 h_2\right) \dot{T}_{(1)nm} \dot{T}_{(2)nm} = \frac{\Phi_{nm(1,2)layer(1,2)}}{M_{(1)nmr}} \tag{43}$$

Now, by using solutions of the free vibrations in the form (10) or in the more suitable form for next energy analysis, we can write the following form:

$$T_{(1)nm}(t) = R_{nm} e^{-\hat{\delta}_{nm(1)}t}\cos\theta_{nm(1)} + D_{nm} e^{-\hat{\delta}_{nm(2)}t}\cos\theta_{nm(2)}$$

$$T_{(2)nm}(t) = C_{nm(1)} R_{nm} e^{-\hat{\delta}_{nm(1)}t}\cos\theta_{nm(1)} - C_{nm(2)} D_{nm} e^{-\hat{\delta}_{nm(2)}t}\cos\theta_{nm(2)} \tag{44}$$

where $\quad \theta_{nm(i)} = \hat{\omega}_{nm(i)}t + \alpha_{nm}, \; i = 1,2.$

By use the first derivative of the plate time functions $T_{(1)nm}(t)$ and $T_{(2)nm}(t)$ with respect to time the reduced total energy of the plates on the nm -mode we can express:

1* Reduced value of the total energy of the first plate on the nm -mode:

$$\tilde{\tilde{\mathbf{E}}}_{k,nm(1)}+\tilde{\tilde{\mathbf{E}}}_{p,nm(1)}=\frac{\rho_1 h_1}{2}\left[\left(\dot{T}_{(1)nm}(t)\right)^2+\omega^2_{(1)nm}\left(T_{(1)nm}(t)\right)^2\right]=\frac{\lfloor \mathbf{E}_{k,nm(1)}+\mathbf{E}_{p,nm(1)}\rfloor}{M_{(1)nm}} \tag{45}$$

2* Reduced values of the total energy of the second plate on the nm -mode:

$$\tilde{\tilde{\mathbf{E}}}_{k,nm(2)}+\tilde{\tilde{\mathbf{E}}}_{p,nm(2)}=\frac{\rho_2 h_2}{2}\left[\left(\dot{T}_{(2)nm}(t)\right)^2+\omega^2_{(2)nm}\left(T_{(2)nm}(t)\right)^2\right]=\frac{\lfloor \mathbf{E}_{k,nm(2)}+\mathbf{E}_{p,nm(2)}\rfloor}{M_{(1)nm}} \tag{46}$$

3* Reduced value of the total energy of the both plates:

$$\tilde{\tilde{\mathbf{E}}}_{nm(p1,p2)}=\tilde{\tilde{\mathbf{E}}}_{k,nm(1)}+\tilde{\tilde{\mathbf{E}}}_{p,nm(1)}+\tilde{\tilde{\mathbf{E}}}_{k,nm(2)}+\tilde{\tilde{\mathbf{E}}}_{p,nm(2)}$$

4* Reduced Rayleigh function of the dissipation $\tilde{\tilde{\Phi}}_{nm(1,2)layer}$, $n,m=1,2,3,4,....\infty$ of the coupled plates:

$$\tilde{\tilde{\Phi}}_{nm(1,2)layer}=\frac{1}{2}b\left(\dot{T}_{(1)nm}-\dot{T}_{(2)nm}\right)^2=\frac{\Phi_{nm(1,2)layer}}{M_{(i)nmsr}} \tag{47}$$

5* Reduced value of the total energy of the system carried by nm mode is:

$$\tilde{\tilde{\mathbf{E}}}_{nm-syst}=\tilde{\tilde{\mathbf{E}}}_{nm(p1,p2)}+\tilde{\tilde{\mathbf{E}}}_{pnm(1,2)layer}+\tilde{\tilde{\mathbf{E}}}_{p,nm(1,2)} \tag{48}$$

For the case of the free own transversal oscillations of the plates in the double plate system with pure elastic layer total energy on the corresponding mode is constant during the oscillations and equal to the system energy of the corresponding mode at initial moment. We can see that energy depend of the two amplitudes correspond to nm - mode. Relation

$$\frac{d\tilde{\tilde{\mathbf{E}}}_{nm-syst}}{dt}=-2\Phi_{nm-syst}=-2\Phi_{nm-layert} \tag{49}$$

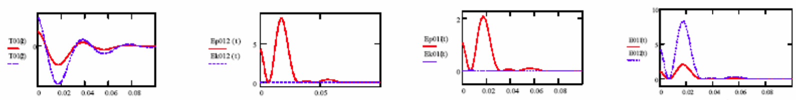
is valid for every nm - mode form infinite sets.

On the base of the backward energy analyses of the given system we are now in position to get the analyses of energy exchange in the nm mod of oscillation for special initial condition options.

Example special case 1*: For the case that initial conditions give as the following the plate time functions $T_{(1)nm}(t)$ and $T_{(2)nm}(t)$ depends of only one frequency:

$$\mathrm{T}_{(1)nm}(t)=R_{nm0}e^{-\tilde{\delta}_{nm(1)}t}\cos\tilde{\omega}_{nm(1)}t\,,\quad \mathrm{T}_{(2)nm}(t)=C_{nm(1)}R_{nm0}e^{-\tilde{\delta}_{nm(1)}t}\cos\tilde{\omega}_{nm(1)}t \tag{50}$$

The energy interaction is equal to zero, and energy transfer between plates don't appear. The figure 2. presents the times function $T_{(1)11}(t)$ and $T_{(2)11}(t)$ of the first mode of plates oscillation for referred case with corresponding reduced values of kinetic, potential and total energy for both plates.



a) b) c) d)

Figure 2: a) The times function $T_{(1)11}(t)$ and $T_{(2)11}(t)$ of the first mode of plates oscillations; b) corresponding reduced values of kinetic and potential energy of the first plate; c) corresponding reduced values of kinetic and potential energy of the second plate; and d) total energy for both plates

Example special case 2* For the case that initial conditions give as the following the plate time functions $T_{(1)nm}(t)$ and $T_{(2)nm}(t)$:

$$T_{(1)nm}(t) = R_{nm2} e^{-\delta_{nm1} t} \cos \tilde{\omega}_{nm(1)} t + D_{nm2} e^{-\delta_{nm2} t} \cos \tilde{\omega}_{nm(2)} t$$

$$T_{(2)nm}(t) = R_{nm0} e^{-\delta_{nm1} t} \cos \tilde{\omega}_{nm(1)} t - D_{nm2} e^{-\delta_{nm2} t} \cos \tilde{\omega}_{nm(2)} t$$

For that case energy interaction is different from zero, and energy transfer between plates appears.

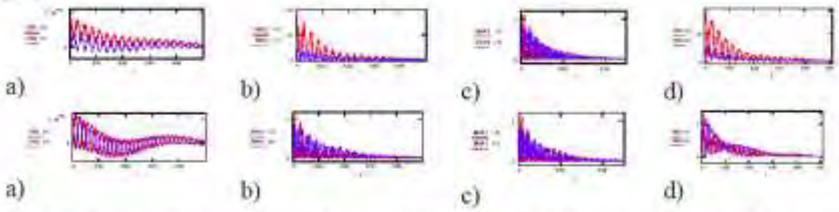


Figure 3: a) The times function $T_{(1)11}(t)$ and $T_{(2)11}(t)$ of the first mode of plates oscillation for different initial conditions; b) corresponding reduced values of kinetic and potential energy of the first plate; c) corresponding reduced values of kinetic and potential energy of the second plate; and d) total energy for both plates

The figure 3. presents the times function $T_{(1)11}(t)$ and $T_{(2)11}(t)$ of the first mode of plates oscillation for different initial conditions with corresponding reduced values of kinetic, potential and total energy for both plates.

5 THE PROPERTIES OF THE NONLINERITY IN SUCH A SYSTEMS

The analytical and numerical analysis [16, 17, 18 and 30] showed that the visco and nonlinear elastic connection of the third order between plates caused the appearance of two-frequency like regime of time function, which corresponds to one eigen amplitude function of one mode, and also that time functions of different vibration modes are coupled, as well as energy transfer between plates in one eigen mode appear.

More then two resonant jumps in the amplitude-frequency as well as in phase-frequency curves appear, what it is visible in the figure 4. and caused more then two resonant jumps of the energy and corresponding influence between nonlinear modes. As nonlinear phenomena interactions.

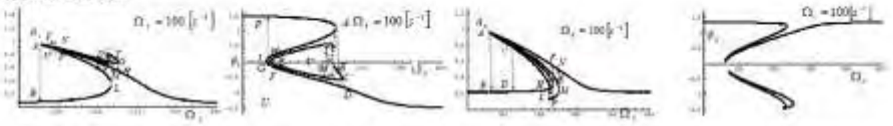


Figure 4: Resonant jumps of amplitudes and phases of two harmonics in one nm -mode of oscillations for the system of two circular plates connected with visco-elastic nonlinear layer

Using the analytical asymptotic approximation of the amplitudes and phases of multi frequency particular solutions of the nonlinear system dynamics derived by Bogoliubov-Krilov-Mitropoljsky method and presented energy analysis, it is possible to analyze transfer energy between nonlinear modes in stationary and non stationary regimes.

6 LYAPUNOV EXPONENTS AND CONCLUDING REMARKS

For every of the eigen plate time functions $T_{(1)nm}(t)$ and $T_{(2)nm}(t)$ and time processes in nm - mode we can define Lyapunov exponents in the form

$$\lambda_{(nm)i} = \lim_{t\to\infty}\frac{1}{2t}\ln\left\{\left[T_{(i)nm}(t)\right]^2 + \frac{1}{\omega^2_{(i)nm}}\left[\dot{T}_{(i)nm}(t)\right]^2\right\} = \lim_{t\to\infty}\frac{1}{2t}\ln\frac{2\tilde{\mathbf{E}}_{(nm)i}}{\omega^2_{(i)nm}} = -\tilde{\delta}_{(nm)i},\quad i=1,2,\ n,m=1,2,3,4,\ldots\infty$$

For the case of the free vibrations without damping and with pure ideally elastic layer, these Lyapunov exponents are equal to zero. But, by using this energy approach we can introduce Lyapunov exponents of this type and way for coupled hybrid systems with different type of the material properties, as it is viscoelastic or creep, and to use for investigation of the stability process, or deformable forms of the deformable body motion in the hybrid systems. Than we can see that these Lyapunov exponent are measures of the processes integrity or system motion integrity.

For the second case of a model of the double plate system with discontinuity in elastic layer considered as a model of the interface crack between two plates connected by thin elastic layer of the Winkler type and by using obtained results presented in Ref. [3], it is easy to conduct energy analysis of the transfer energy also using consideration from this paper in the part III*, IV* and V* and corresponding solutions from cited paper. In that case defined Lyapunov exponents obtain important role in analysis of the transfer energy between plates including interaction between different modes.

Future more we could add commenter about our investigation based on this energy analysis applied on similar system with nonlinearity in the coupled layer [30] what should be the backbone of our future title. Presenting the energy in the process of oscillation in the one nm-mod it is comfort to display the energy exchange between the component in the system like as the external action and also to make an analysis about inner interaction between the two harmonics in the one nm-mod what is the property of the nonlinearity. Probably it is possible to discover targered energy exchange in such a system with strong nonlinearity and that results will be presented in some coming titles.

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Minisymposium Invited Contributed Lecture

TRANSFER OF ENERGY OF OSCILLATIONS THROUGH THE DOUBLE DNA CHAIN HELIX

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Keywords: Double DNA chain helix, Eigen main chains, Kinetic and potential energies, transfer energy.

Abstract. *Expressions for the kinetic and potential energy as well as energy interaction between chains in the double DNA chain helix are obtained and analyzed for a linearized model. Corresponding expressions of the kinetic and potential energies of these uncoupled main chains are also defined for the eigen main chains of the double DNA chain helix. By obtained expressions we concluded that there is no energy interaction between eigen main chains of the double DNA chain helix. Time expressions of the main coordinates of the two eigen main chains are expressed by time, and eigen circular frequencies. Also generalized coordinates of the double DNA chain helix are expressed by time and eigen circular frequencies. These data contribute to better understanding of biomechanical events of DNA transcription that occur parallel with biochemical processes. Considered as a linear mechanical system, DNA molecule as a double helix has its eigen circular frequencies and that is its characteristic. Mathematically it is possible to decouple it into two chains with their eigen circular frequencies which are different. This may correspond to different chemical structure (the order of base pairs) of the complementary chains of DNA. We are free to propose that every specific set of base pair order has its eigen circular frequencies and its corresponding oscillatory energy and it changes when DNA chains are coupled in the system of double helix. Oscillations of base pairs and corresponding oscillatory energy for specific set of base pairs may contribute to conformational chances of DNA double helix, and its unzipping and folding.*

1 INTRODUCTION

DNA is a biological polymer that chemically consists of two long polymers of simple units called nucleotides, with backbone made of sugars and phosphate groups joined by ester bonds. One of four types of molecules called bases is attached to each sugar. Two bases on opposite strands are linked via hydrogen bonds holding the two strands of DNA together. It is the sequence of these four bases along the backbone that encodes information. The basic function of DNA in the cell is to encode the genetic material. For using that information to make proteins, DNA molecule has to interact with other molecules in the cell nucleus. DNA molecule is moving, changing its position and shape during the interactions. DNA molecules can be considered to be a mechanical structure on the nanolevel. There are different approaches to studding the mechanical properties of the DNA molecule (experimental, theoretical modeling).

The mechanical properties of DNA are closely related to its molecular structure and sequence, particularly the weakness of hydrogen bonds and electronic interactions that hold strands of DNA together compared to the strength of bonds within each strand. Every process that binds or reads DNA is able to use or modify the mechanical properties of DNA for purposes of recognition, packaging and modification. Binding of proteins and other ligands induces a strong deformation of the DNA structure.

During transcription process double DNA molecule is partially unfolded. The transcription occurrs only on the unfolded segment. Transcription process considers transfer of information from DNA to iRNA, process that preceded the translation process. The final results of translation process is newly synthesized protein molecules. Proteins are vital for cell functioning, cell reproduction, growth - for all biochemical reactions, for normal functions of tissues and whole organism.

"Transcription activation seems to be governed by underlying dynamical mechanisms related to several distortions of the double chain structure: a dynamical approach on a mesoscopic description level could then allow a deeper understanding of this complex process." [1]

1.1. DNA mechanical models

A number of mechanical models of the DNA double helix have been proposed so far. Different models (see Fig. 1) are focusing on different aspects of the DNA molecule (biological, physical and chemical processes in which DNA is involved). A number of models have been constructed to describe different kinds of movements in a DNA molecule: asymmetric and symmetric motion; movements of long and short segments. Some models have, for example, been made for circular double-stranded DNA molecules in viral capsids. There are also polymer models [2], elastic rod models [3], network models [4], torsional springs models [5], soliton -existence supporting models [6,7,8].

There are some models that deal with energy transfers through DNA molecule (sequence dependence energy transfer, chare transfer through DNA, tosional energy of DNA, fluorescence resonance energy transfer). Some of these models have practical application: "Fluorescence resonance energy transfer (FRET) has been especially used to study protein structures [9,10].

Experiments of Shore & Baldwin (Shore & Baldwin, 1983) provide a direct measurement of the torsional free energy of DNA, and they show that the DNA twisting potential is symmetric. Their experiments indicate that the DNA helix is continuous, or nearly so, in a nicked circle. These data give the free energy of DNA twisting as a function of twist. The curve of j versus DNA length can be fitted to a harmonic twisting potential with a torsional constant of C = 2.4

X 10(-19) erg cm [11]. Xu and Nordlund (Xu and Nordlund, 2000) investigated sequence, temperature, concentration, and solvent dependence of single energy transfer from normal DNA bases to the 2-aminopurine base in synthesized DNA oligomers by optical spectroscopy. Transfer was shown directly by a variable fluorescence excitation band at 260-280nm. Adenine (A) is the most efficient energy donor by an order of magnitude. Stacks of A adjacent to 2AP act as an antenna for 2AP excitation. An interposed G, C, or T base between A and 2AP effectively blocks transfer from A to 2AP. Base stacking facilitates transfer, while base pairing reduces energy transfer slightly [12]. Charge transfers through DNA depending on the sequence of the stretch of DNA propagating the electron transfer, DNA can either act as a conductor, a semiconductor, or an insulator [13].

Figure 1. a* Figure 1. b* Figure 1. c*

Figure 1. a* "Toy mechanical" model of DNA. a, DNA is modeled as an elastic rod (grey) wrapped helically by a stiff wire (red), see Ref. [9] by Jeff Gore, Zev Bryant, Marcelo (2006)

Figure 1. b* The model scheme of a double helix on six coarse-grained particles [10].

Figure 1.c* Fragment of the DNA double chain consisting of three AT base pairs. Longitudinal pitch of the helix $a=3.4\mathring{A}$; transverse pitch $b=16.15\mathring{A}$ [11]

"The DNA is an ordered, well-structured geometrical chain of atoms describing a charged polymer. Biomolecules are *fashion*-moving objects: biological function is a consequence of movement… In the mode of vibration *DNA bubbles* the base pairs are opened and closed in an orderly way. The bubbles are nucleated by the effect of temperature T in a localized manner. Some bubbles can grow at the expense of some others. The time between pulses is long enough in order that excitations do not become exhausted and form biphotonic excitations, which favour the oxidation of guanine (G). In the presence of bubbles, for sites near to the perturbation the probability of bases being closed slightly increases as T decreases. The effect is much greater for distant sites. There is energy transfer, confirmed by differential scanning nanocalorimetry (DSNC) in two promoter regions of bacteria."[14].

None of this models deal with energy of oscillations. Interactions between DNA molecule and transcription factors may give initial force for specific set of oscillatory frequencies. Every set of oscillatory frequencies has its corresponding set of oscillatory energies.

Our model of DNA was based on a model of DNA models by N. Kovaleva and L. Manevich [6,7].

2 DNA MODELS BY N. K. KOVALEVA AND L. MANEVICH

They show that a localized excitation (breather) can exist in a double DNA helix, which corresponds to predominant rotation of one chain and small perturbation of second chain using coarse-grained model of DNA double helix. In this model, three beads represent each nucleotide with interaction sites corresponding to phosphate group, group of sugar ring, and the base [6]. N. Kovaleva et al. [6] point out that solitons and breathers play a functional role in DNA chains.

Different models of two coupled homogeneous DNA chain vibrations are proposed in the literature (see Refs. [6] and [7]). As a basic approach we use DNA mathematical models published by N. Kovaleva and L. Manevich in 2005 and 2007, and investigated corresponding linearized models. We consider the linear natural and fractional order model to obtain main chain subsystems of the double DNA fractional order chain helix (see refs. [15] [26] and [16]). Analytical expressions of the eigen circular frequencies for the homogeneous linearized model of the double DNA chain helix are used to obtain corresponding eigen fractional order creep vibration modes (see Ref. [26]). Two sets of eigen normal coordinates of the double DNA linearized natural and fractional order chain helix for separation of the system into two uncoupled linear, as well as fractional order main chains are identified (see Reds. [15] and [26]). The visualization of the eigen fractional order creep vibration modes of the double DNA fractional order chain helix is presented. The study of energy transfer between subsystems coupled in a hybrid system is very important for better understanding the translation process of DNA.

This paper presents an analysis of the energy transfer between partial oscillators in the main chains of the double DNA that is considered as a linearized order chain helix.

Series of two sets of Lyapunov exponents corresponding to a set of modes of partial oscillators in the corresponding main chains are expressed by using energy of the corresponding eigen time component.

The simplest model describing opening of DNA double helix is presented in Ref. [6] by Kovaleva N., L.Manevich (2005). Corresponding differential equations are solved analytically using multiple-scale expansions after transition to complex variables. Obtained solution corresponds to localized torsional nonlinear excitation – breather. Stability of breather is also investigated.

Authors deal with the planar DNA model in which the chains of the macromolecule form two parallel straight lines placed at a distance h from each other, and the bases can make only rotation motions around their own chain, being all the time perpendicular to it. Authors accepted as generalized (independent) coordinates $\varphi_{k,1}$ that is the angular displacement of the k-th base of the first chain, and as generalized (independent) coordinates $\varphi_{k,2}$ is the angular displacement of the k-th base of the second chain. Then, by use accepted generalized coordinates $\varphi_{k,1}$ and $\varphi_{k,2}$ for k-th bases of the both chains in the DNA model, authors derived the system of differential equations describing DNA model vibrations in the following forms(see refs. [6] and [7]):

$$\mathbf{J}_{k,1}\ddot{\varphi}_{k,1}-\frac{K_{k,1}}{2}\left[\sin(\varphi_{k+1,1}-\varphi_{k,1})-\sin(\varphi_{k,1}-\varphi_{k-1,1})\right]+K_{\alpha\beta}r_{\alpha}(r_{\alpha}-r_{\beta})\sin\varphi_{k,1}-$$
$$-K_{\alpha\beta}\frac{1}{4}\left(1-\frac{\omega_{\alpha\beta 2}}{\omega_{\alpha\beta 1}}\right)(r_{\alpha}-r_{\beta})^{2}\sin(\varphi_{k,1}-\varphi_{k,2})=0$$
$$\mathbf{J}_{k,2}\ddot{\varphi}_{k,2}-\frac{K_{k,2}}{2}\left[\sin(\varphi_{k+1,2}-\varphi_{k,2})-\sin(\varphi_{k,2}-\varphi_{k-1,2})\right]+K_{\alpha\beta}r_{\alpha}(r_{\alpha}-r_{\beta})\sin\varphi_{k,2}+$$
$$+K_{\alpha\beta}\frac{1}{4}\left(1-\frac{\omega_{\alpha\beta 2}}{\omega_{\alpha\beta 1}}\right)(r_{\alpha}-r_{\beta})^{2}\sin(\varphi_{k,1}-\varphi_{k,2})=0 \tag{1}$$

Here $\mathbf{J}_{k,1}$ is the axial moment of mass inertia of the k-th base of the first chain; $\mathbf{J}_{k,2}$ is the axial moment of mass inertia of the k-th base of the second chain, and the point denotes differentiation in time t. For the base pair the axial moments of mass inertia are equal to $\mathbf{J}_{k,1} = m_\alpha r_\alpha^2$, $\mathbf{J}_{k,12} = m_\beta r_\beta^2$. The value of the base mass m_α, the length r_α, and the corresponding axial moment of mass inertia $\mathbf{J}_{k,1} = m_\alpha r_\alpha^2$ for all possible base pair authors accepted as in the References [16] an [26]. The fourth terms in previous system equations describe interaction of the neighboring bases along each of the macromolecule chains. Parameter $K_{k,i}$, $i = 1,2$ characterizes the energy of interaction of the k-th base with the $(k+1)$-th one along the i-th chain $i = 1,2$. There are different estimations of rigidity. For the calculation that the most appropriate value is close $K_{k,i} = K = 6\times 10^3 [kJ/mol]/$

For beginning it is necessary to consider a corresponding linearized system of the previous system of differential equations in the following form:

$$\begin{aligned}
&\mathbf{J}_{k,1}\ddot{\varphi}_{k,1} - \frac{K_{k,1}}{2}\left[(\varphi_{k+1,1} - \varphi_{k,1}) - (\varphi_{k,1} - \varphi_{k-1,1})\right] + K_{\alpha\beta} r_\alpha (r_\alpha - r_\beta)\varphi_{k,1} - \\
&\qquad - K_{\alpha\beta}\frac{1}{4}\left(1 - \frac{\omega_{\alpha\beta 2}}{\omega_{\alpha\beta 1}}\right)(r_\alpha - r_\beta)^2(\varphi_{k,1} - \varphi_{k,2}) = 0 \\
&\mathbf{J}_{k,2}\ddot{\varphi}_{k,2} - \frac{K_{k,2}}{2}\left[(\varphi_{k+1,2} - \varphi_{k,2}) - (\varphi_{k,2} - \varphi_{k-1,2})\right] + K_{\alpha\beta} r_\alpha (r_\alpha - r_\beta)\varphi_{k,2} + \\
&\qquad + K_{\alpha\beta}\frac{1}{4}\left(1 - \frac{\omega_{\alpha\beta 2}}{\omega_{\alpha\beta 1}}\right)(r_\alpha - r_\beta)^2(\varphi_{k,1} - \varphi_{k,2}) = 0
\end{aligned} \tag{2}$$

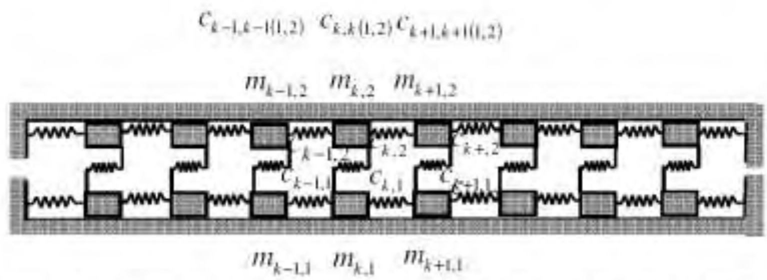

Figure 2. a* Model of the double DNA chain helix in the form of multichain system with fixed ends

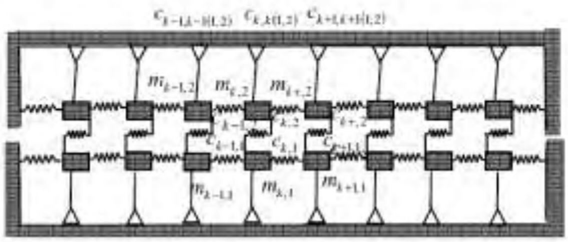

Figure 2. b* Model of the double DNA chain helix model in the form of multipendulum system with fixed ends

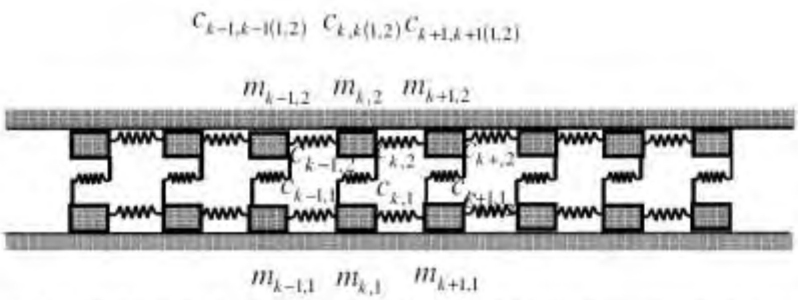

Figure 3. a* Model of the double DNA Chain helix model in the form of multichain system with free ends

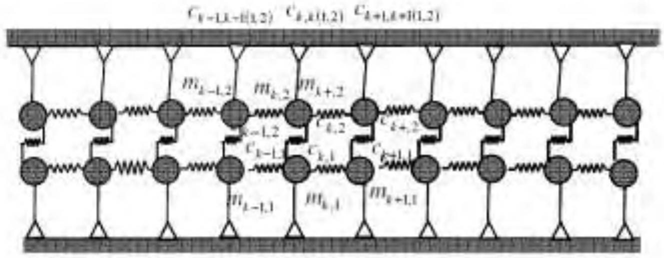

Figure 3. b* Model of the Double DNA Chain helix in the form of multipendulum model with free ends

3 EIGEN MAIN CHAINS AND CORRESPONDING SUBSYSTEM MAIN COORDINATES

For the case of homogeneous systems we can take into consideration that are $\mathbf{J}_{k,1} = \mathbf{J}_{k,2} = \mathbf{J}$ and $K_{k,1} = K_{k,2} = K$.

By using change of the generalized coordinates $\varphi_{k,1}$ and $\varphi_{k,2}$ for k-th bases of both chains in the DNA model into following new (for detail see Reds. [15], [18], [17] and [26])

$$\xi_k = \varphi_{k,1} - \varphi_{k,2} \quad \text{and} \quad \eta_k = \varphi_{k,1} + \varphi_{k,2} \tag{3}$$

system of linearized differential equations of the double DNA linear order chain helix vibrations (2) obtain the following form:

$$\frac{1}{\omega_0^2}\ddot{\xi}_k - \xi_{k+1} + 2\xi_k\left[1 + \mu - \kappa\right] - \xi_{k-1} = 0 \tag{4}$$

$$\frac{1}{\omega_0^2}\ddot{\eta}_k - \eta_{k+1} + 2\eta_k(1+\mu) - \eta_{k-1} = 0\,,\; k = 1,2,3,\dots,n \tag{5}$$

where we use the following notation:

$$\kappa = \frac{K_{\alpha\beta}}{2K}\left(1 - \frac{\omega_{\alpha\beta 2}}{\omega_{\alpha\beta 1}}\right)(r_\alpha - r_\beta)^2\,,\quad \mu = \frac{K_{\alpha\beta} r_\alpha (r_\alpha - r_\beta)}{K}\,,\quad \frac{1}{\omega_0^2} = \frac{2\mathbf{J}}{K} \tag{6}$$

By using trigonometric method we obtain the following two subsets of eigen circular frequencies (see refs. [27], [15] , [18], [17] and [26]):

$$\omega_s^2 = 2\omega_0^2\left[2\sin^2\frac{\varphi_s}{2} + (\mu - \kappa)\right],$$

$$\omega_r^2 = 2\omega_0^2\left[2\sin^2\frac{\vartheta_r}{2} + \mu\right],\qquad s,r = 1,2,3,4\dots,n \tag{7}$$

where φ_s and ϑ_r are characteristic numbers depending of boundary conditions of the model of the double DNA linear order chain helix and corresponding boundary conditions of the obtained eigen main chains of the considered double DNA linear order chain helix.

4 BOUNDARY CONDITIONS OF THE DOUBLE DNA CHAIN HELIX

Now, it is necessary to consider some boundary conditions of the double DNA chain helix in accordance with possible real situations. For that reason we take into account two cases of the double DNA chain helix, when ends of chains are free (see Fig. 3) and when ends of chains are fixed (see Fig. 2) . Then, we can write following boundary conditions of the double DNA chain helix (for detail see Ref. [27] and [16], as well as [15-27]):

a* case: both ends of the of the double DNA chain helix are free (see Fig. 3.a* and 3,b*):

$$\varphi_s = \frac{s\pi}{n}\qquad \text{and}\qquad \vartheta_r = \frac{r\pi}{n}\qquad s,r = 1,2,3,4,\dots,n \tag{9}$$

b* case: both ends of the of the double DNA chain helix are fixed (see Fig. 2.a* and 2,b*):

$$\varphi_s = \frac{s\pi}{(n+1)}\qquad \vartheta_r = \frac{r\pi}{(n+1)}\,,\quad s,r = 1,2,3,4\dots,n \tag{10}$$

Then analytical expressions of the quadrate of ω_s - *eigen circular frequencies of the vibration modes of the separate chains in the* the of the double DNA chain helix are:

a* case: both ends of the of the double DNA chain helix are free (see Fig. 3.a* and 3,b*):

$$\omega_s^2 = 2\omega_0^2\left[2\sin^2\frac{s\pi}{2n} + (\mu - \kappa)\right],\qquad s = 1,2,3,4\dots,n \tag{11}$$

$$\omega_r^2 = 2\omega_0^2\left[2\sin^2\frac{r\pi}{2n} + \mu\right],\qquad r = 1,2,3,4\dots,n \tag{12}$$

b* case: both ends of the of the double DNA chain helix are fixed (see Fig. 2.a* and 2,b*):

$$\omega_s^2 = 2\omega_0^2\left[2\sin^2\frac{s\pi}{2(n+1)} + (\mu - \kappa)\right],\quad s = 1,2,3,4\dots,n \tag{13}$$

$$\omega_r^2 = 2\omega_0^2\left[2\sin^2\frac{r\pi}{2(n+1)} + \mu\right], \qquad r = 1,2,3,4,...,n \tag{14}$$

5 EIGEN MAIN COORDINATES OF THE EINEG MAIN CHAINS OF THE DNA DOUBLE HELIX

Then, it is possible to apply trigonometric method (see References [27] by Rašković and [15-26] by Hedrih (Stevanović)) to both subsystems – eigen main chains to obtain eigen main coordinates of the eigen main chains in the form:

$$\xi_k = \sum_{s=1}^{n} C_s \sin k\varphi_s \cos(\omega_s t + \alpha_s), \; k = 1,2,3,......,n \tag{15}$$

$$\eta_k = \sum_{r=1}^{n} D_r \sin k\vartheta_r \cos(\omega_r t + \beta_r), \; k = 1,2,3,......,n \tag{16}$$

where C_s, α_s and D_r, β_r are unknown integral constants depending of initial conditions of the double DNA chain helix, and ω_s and ω_r, $s,r = 1,2,3,4,...,n$ eigen circular frequencies of the vibration modes expressed by the formulas (11)-(12) for the case that both ends of the double DNA chain helix are free and (13)-(14) for the case that both ends of the double DNA chain helix are fixed. Also, corresponding φ_s and ϑ_r are expressed by formulas (9) or (10) depending of the free, or fixed ends of the double DNA chain helix.

6 SOLUTIONS FOR THE GENERALIZED COORDINATES

Accepted generalized coordinates $\varphi_{k,1}$ and $\varphi_{k,2}$ for k-th bases of the both coupled chains in the double DNA chain helix are possible to express in the following form:

$$\varphi_{k,1} = \frac{1}{2}(\xi_k + \eta_k) = \frac{1}{2}\left[\sum_{s=1}^{n} C_s \sin k\varphi_s \cos(\omega_s t + \alpha_s) + \sum_{r=1}^{n} D_r \sin k\vartheta_r \cos(\omega_r t + \beta_r)\right] \tag{17}$$

$$\varphi_{k,2} = \frac{1}{2}(\eta_k - \xi_k) = \frac{1}{2}\left[\sum_{r=1}^{n} D_r \sin k\vartheta_r \cos(\omega_r t + \beta_r) - \sum_{s=1}^{n} C_s \sin k\varphi_s \cos(\omega_s t + \alpha_s)\right] \tag{18}$$

where C_s, α_s and D_r, β_r are unknown constant depending of the initial conditions of the double DNA chain helix in the following form:

$$\varphi_{k,1}(0) = \varphi_{k,1(0)}, \; \dot{\varphi}_{k,1}(0) = \dot{\varphi}_{k,1(0)}, \; \varphi_{k,2}(0) = \varphi_{k,2(0)}, \; \dot{\varphi}_{k,2}(0) = \dot{\varphi}_{k,2(0)}, \; k = 1,2,3,......,n \tag{19}$$

7 KINETIC AND POTENTIAL ENERGY OF THE DOUBLE DNA CHAIN HELIX

Then, kinetic energies of the coupled chains in the double DNA chain helix system is possible to express by generalized coordinates $\{\varphi_{k,1}\}$ and $\{\varphi_{k,2}\}$ in separate form by the following matrix forms:

$$2\mathbf{E}_{kin,i} = (\dot{\varphi}_{k,i})\mathbf{A}_i\{\dot{\varphi}_{k,i}\}\ ,\ i=1,2 \tag{20}$$

because in the linearized double DNA chain helix system there is no dynamical constraints, only statical constraints. In the previous expressions for kinetic energies $\mathbf{A}_i$, $i=1,2$ are matrices of the inertia coefficients in the following forms:

$$\mathbf{A}_i = \mathbf{J}\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \dots & & \\ & & & \dots & \\ & & & & 1 \end{pmatrix},\ i=1,2 \tag{21}$$

Taking into account that for case of the homogeneous linearized double DNA chain helix system differential equations of the oscillations expressed by generalized coordinates are in the following form:

$$\mathbf{J}\ddot{\varphi}_{k,1} - \frac{K}{2}\left[(\varphi_{k+1,1}-\varphi_{k,1})-(\varphi_{k,1}-\varphi_{k-1,1})\right] + \mu K\varphi_{k,1} - \frac{1}{2}\kappa K(\varphi_{k,1}-\varphi_{k,2}) = 0 \tag{22}$$

$$\mathbf{J}\ddot{\varphi}_{k,2} - \frac{K}{2}\left[(\varphi_{k+1,2}-\varphi_{k,2})-(\varphi_{k,2}-\varphi_{k-1,2})\right] + \mu K\varphi_{k,2} + \frac{1}{2}\kappa K(\varphi_{k,1}-\varphi_{k,2}) = 0 \tag{23}$$

It is now possible to extract two matrices containing quasielastic coefficients of the system in the following form:

$$\mathbf{C}_i = \frac{K}{2}\begin{pmatrix} 1+2\mu & -1 & & & \\ -1 & 2+2\mu & -1 & & \\ & -1 & \dots & & \\ & & & 2+2\mu & -1 \\ & & & -1 & 1+2\mu \end{pmatrix},\ i=1,2 \tag{24}$$

for the case of the free ends (see Fig. 3.a* and 3,b*)of the both double DNA chain helix , and

$$\mathbf{C}_i = \frac{K}{2}\begin{pmatrix} 2+2\mu & -1 & & & \\ -1 & 2+2\mu & -1 & & \\ & -1 & \dots & & \\ & & & 2+2\mu & -1 \\ & & & -1 & 2+2\mu \end{pmatrix},\ i=1,2 \tag{24*}$$

for the case of the fixed both ends (see Fig. 2.a* and 2,b*) of the double DNA chain helix.

By using previous matrices (24) or (24*) for different DNA Chain helix boundary conditions, potential energies of the chains in the DNA chain helix can be expressed by generalized coordinates $\{\varphi_{k,1}\}$ and $\{\varphi_{k,2}\}$ in separate forms by the following matrix forms:

$$2\mathbf{E}_{pot,i} = (\varphi_{k,i})\mathbf{C}_i\{\varphi_{k,i}\}\ ,\ i=1,2 \tag{25}$$

Also from system of the differential equations (22)-(23) is possible to extract a matrix of the quasielastic coefficients of the energy interaction between coupled chains in the Double DNA helix in the following form:

$$\mathbf{C}_{1,2} = -\frac{\kappa K}{2}\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \dots & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} \tag{26}$$

and by using previous matrix (26) we can express a part of the system potential energy as a potential energy of coupled chains in the double chain helix system interaction:

$$2\mathbf{E}_{pot.inter.} = \left(\varphi_{k,1} - \varphi_{k,2}\right)\mathbf{C}_{1,2}\left\{\varphi_{k,1} - \varphi_{k,2}\right\} = -\sum_{k=1}^{n} \frac{\kappa K}{2}\left(\varphi_{k,1} - \varphi_{k,2}\right)^2 \tag{27}$$

Last expression (27) for potential energy interactions is sum of the energies interactions between two coupled pairs in the double DNA chain helix. No dynamical interactions between these pars, because no kinetic energy interactions.

8 KINETIC AND POTENTIAL ENERGY OF THE DOUBLE DNA CHAIN HELIX MAIN CHAINS

Then, kinetic energies of the eigen main chains of the double DNA chain helix system is possible to express by normal coordinates of the main chains $\{\xi_k\}$ and $\{\eta_k\}$ in the in separate form by the following matrix forms:

$$2\mathbf{E}_{kin,\xi} = \left(\dot{\xi}_k\right)\mathbf{A}_{\xi}\left\{\dot{\xi}_k\right\} \quad , \qquad 2\mathbf{E}_{kin,\eta} = \left(\dot{\eta}_k\right)\mathbf{A}_{\eta}\left\{\dot{\eta}_k\right\} \tag{28}$$

because in the linearized double DNA chain helix system, between eigen main chains there is no dynamical interactions, In the previous expressions for kinetic energies $\mathbf{A}_{\xi}$, and $\mathbf{A}_{\eta}$ are matrices of the inertia coefficients in the following forms:

$$\mathbf{A}_{\xi} = \mathbf{J}\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ldots & & \\ & & & \ldots & \\ & & & & 1 \end{pmatrix}, \qquad \mathbf{A}_{\eta} = \mathbf{J}\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ldots & & \\ & & & \ldots & \\ & & & & 1 \end{pmatrix} \tag{29}$$

Taking into account that for case of the homogeneous linearized double DNA chain helix system differential equations of the oscillations expressed by normal coordinates of the main chains $\{\xi_k\}$ and $\{\eta_k\}$, it is no possible to extract two matrices containing quasielastic coefficients of the system in the following form:

$$\mathbf{C}_{\xi} = \frac{K}{2}\begin{pmatrix} 1+2\mu-2\kappa & -1 & & & \\ -1 & 2+2\mu-2\kappa & -1 & & \\ & -1 & \ldots & & \\ & & & 2+2\mu-2\kappa & -1 \\ & & & -1 & 1+2\mu-2\kappa \end{pmatrix} = \frac{K}{2}\tilde{\mathbf{C}}_{\xi} \tag{30}$$

for the first eigen main chain, and

$$\mathbf{C}_{\eta} = \frac{K}{2}\begin{pmatrix} 1+2\mu & -1 & & & \\ -1 & 2+2\mu & -1 & & \\ & -1 & \ldots & & \\ & & & 2+2\mu & -1 \\ & & & -1 & 1+2\mu \end{pmatrix} = \frac{K}{2}\tilde{\mathbf{C}}_{\eta} \tag{31*}$$

for the second eigen main chain, and both for the case of the free ends (see Fig. 3.a* and 3.b*) of the double DNA chain helix , and

$$\mathbf{C}_{\xi}=\frac{K}{2}\begin{pmatrix} 2+2\mu-2\kappa & -1 & & & \\ -1 & 2+2\mu-2\kappa & -1 & & \\ & -1 & \dots & & \\ & & & 2+2\mu-2\kappa & -1 \\ & & & -1 & 2+2\mu-2\kappa \end{pmatrix}\frac{K}{2}\tilde{\mathbf{C}}_{\xi} \tag{32}$$

for the first eigen main chain, and

$$\mathbf{C}_{\eta}=\frac{K}{2}\begin{pmatrix} 2+2\mu & -1 & & & \\ -1 & 2+2\mu & -1 & & \\ & -1 & \dots & & \\ & & & 2+2\mu & -1 \\ & & & -1 & 2+2\mu \end{pmatrix}=\frac{K}{2}\tilde{\mathbf{C}}_{\eta} \tag{33}$$

for the second eigen main chain, and both for the case of the fixed both ends (see Fig. 2.a* and 2.b*) of the double DNA chain helix.

By using previous matrices (30)-(31 or (32)-(33) for different DNA chain helix boundary conditions, potential energies of the eigen main chains of double DNA chain helix, it is possible to express by normal coordinates of the main chains $\{\xi_k\}$ and $\{\eta_k\}$ in the separate form by the following matrix forms:

$$2\mathbf{E}_{pot,\xi}=(\xi_k)\mathbf{C}_{\xi}\{\xi_k\}\quad , \qquad 2\mathbf{E}_{pot,\eta}=(\eta_k)\mathbf{C}_{\eta}\{\eta_k\} \tag{34}$$

Between eigen main chains of the double DNA chain helix no energy interaction, neither kinetic, nor potential energies. Eigen main chains are independent subsystems and no energy interaction, but it is possible to analyze total energies of each of the two eigen main chains and compare kinetic, potential ad total mechanical energies of these eigen main chains.

Analytical expressions of the reduced values of kinetic and potential energies of the main chains of the double DNA linear order chain helix are:

a* for kinetic energy for first and second main chain are:

$$\tilde{E}_{Kin,\xi}=\frac{E_{Kin,\xi}}{\mathbf{J}}=\frac{1}{2}\sum_{k=1}^{k=n}\dot{\xi}_k^2 \text{ and } \tilde{E}_{Kin,\eta}=\frac{E_{Kin,\eta}}{\mathbf{J}}=\frac{1}{2}\sum_{k=1}^{k=n}\dot{\eta}_k^2 \tag{35}$$

b* for potential energy for first and second main chain are:

$$\tilde{E}_{p,\xi}=\frac{E_{pot,\xi}}{\mathbf{J}}=\frac{1}{2}\omega_0^2(\xi_k)\tilde{\mathbf{C}}_{\xi}\{\xi_k\}=\frac{1}{2}\omega_0^2\sum_{k=1}^{k=n}\sum_{j=1}^{j=n}\tilde{c}_{\xi,kj}\xi_k\xi_j \tag{36}$$

$$\tilde{E}_{p,\eta}=\frac{E_{pot,\eta}}{\mathbf{J}}=\frac{1}{2}\omega_0^2(\eta_k)\tilde{\mathbf{C}}_{\eta}\{\eta_k\}=\frac{1}{2}\omega_0^2\sum_{k=1}^{k=n}\sum_{j=1}^{j=n}\tilde{c}_{\eta,kj}\eta_k\eta_j \tag{37}$$

where $\tilde{\mathbf{C}}_{\xi}=(\tilde{c}_{\xi,kj})$ and $\tilde{\mathbf{C}}_{\eta}=(\tilde{c}_{\eta,kj})$ are reduced matrices of the main chains subsystem rigidity of the double DNA linear order chain helix visible from expressions of the previous matrices (30)-(31) or (32)-(33) depending of the boundary chains conditions.

9 CONCLUSIONS

The analysis showed that there is no transfer of energy between main chains of the double DNA chain helix considered as a linearized order chain helix, and that transfer of energy appears only between material particles in the corresponding subset of the corresponding main chain. These results may be important for future application in theoretical and experimental medical investigations. As we take into account a linearized conservative model of the double DNA chain helix, then it is possible to conclude that main chains oscilate with constant total mechanical energy but with different total energy values, as well as with different set of the eigen frequencies. Under the external one frequency excitation, in pnly in one main chains is possible that resonance regime appear, but also there are possibilities for dynamical absorption existence.

Transcription process of DNA is well described at biochemical level. During transcription part of double DNA is unzipped, and only one chain helix is used as a matrice for transcription. For better understanding DNA and its function it is necessary to consider its behavior through bioelectrical and mechanical point of view. If we know what is happened to DNA at biomechanical, bioelectrical and biochemical level during transcription our understanding of its function will be more complete. This may open a wide array of possibilities of using DNA as an essential structure in technical devices.

Also, by use modification of the linearized model by introducing standard light elements with constitutive relation on the coupled fields with thermo modifications is possible to introduce a hybrid model of the double DNA chain helix with analysis of the more complex process of the transfer energy and corresponding analogy this model with real DNA system.

10 ACKNOWLEDGEMENT

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APPENDIX

NOMENCLATURE

DNA – Deoxyribonucleic acid (DNA)

$\varphi_{k,1}[\,rad\,]$ - generalized coordinate – angles of the k-th base of the first chain of the double DNA chain helix;

$\varphi_{k,2}[\,rad\,]$ - generalized coordinate – angles of the k-th base of the second chain of the double DNA chain helix;

$\mathbf{J}_{\mathbf{k},1}$ $[kgm^2]$- is the axial moment of mass inertia of the k-th base of the first chain of the double DNA chain helix;

$\mathbf{J}_{\mathbf{k},2}[kgm^2]$- is the axial moment of mass inertia of the k-th base of the second chain of the double DNA chain helix;

$\dot{\varphi}_{k,1}$[rads^{-1}] - angular velocity of the k-th base of the first chain of the he double DNA chain helix;

$\mathbf{J}_{\mathbf{k},1} = m_\alpha r_\alpha^2$, $\mathbf{J}_{\mathbf{k},12} = m_\beta r_\beta^2$ $[kgm^2]$ - the base pair the axial moments of mass inertia ;

m_α $[kg]$- the value of the base mass

$r_\alpha[m]$ - the length

$\mathbf{J}_{\mathbf{k},1} = m_\alpha r_\alpha^2$ $[kgm^2]$ - the corresponding axial moment of mass inertia for all possible base pair authors accepted as in the Reference [17].

$K_{k,i}$, $i = 1,2$ [KJmol^{-1}]- parameters characterize the energy of interaction of the k-th base with the $(k+1)$-th one along the i-th chain $i = 1,2$.

$K_{k,i} = K = 6\times 10^3$[KJmol-1]- for the calculation that the most appropriate value is close /

ξ_k , η_k [rad], $k = 1,2,3,......,n$ - main orthogonal coordinates of the eigen main chains of the double DNA chain helix;

$\xi_k = \varphi_{k,1} - \varphi_{k,2}$ and $\eta_k = \varphi_{k,1} + \varphi_{k,2}$, $k = 1,2,3,......,n$ - functional dependence between main orthogonal coordinates ξ_k and η_k of the eigen main chains and generalized coordinates $\varphi_{k,1}$ and $\varphi_{k,2}$ [rad] of the double DNA chain helix;

$\omega_{\alpha\beta 2}$ [$\sec^{-1}$] - are frequencies of rotational motions of the bases, in similar and opposite directions accordingly, of the k-th base of the first chain of the
double DNA chain helix;

$\omega_{\alpha\beta 1}$ [$\sec^{-1}$] - are frequencies of rotational motions of the bases, in similar and opposite directions accordingly, of the k-th base of the first chain of the
double DNA chain helix;

$K_{k,1} = K_{k,2} = K$ - for the case of homogeneous double DNA chain helix;

$\mathbf{J}_{k,1} = \mathbf{J}_{k,2} = \mathbf{J}$ [kgm^2] - for the case of homogeneous double DNA chain helix;

A_k - amplitude

$u = JK^{-1}\omega^2$ - eigen characteristic number of the homogeneous double DNA chain helix;

$k = K_{\alpha\beta} 2K^{-1}(1 - \omega_{\alpha\beta 2}\, \omega_{\alpha\beta 1}^{-1})(r_\alpha - r_\beta)^2$ - parameter of the homogeneous double DNA chain helix;

$\mu = K_{\alpha\beta} r_\alpha (r_\alpha - r_\beta) K^{-1}$ - parameter of the homogeneous double DNA chain helix;

$\omega_{s\xi}^2$ [$\sec^{-2}$], $s = 1,2,3,4....n$ - set of the n eigen circular frequencies of the first eigen main chain of the homogeneous double DNA chain helix;

$\omega_{s\eta}^2$ [$\sec^{-2}$], $s = 1,2,3,4....n$ - set of the n eigen circular frequencies of the first eigen main chain of the homogeneous double DNA chain helix;

$\omega_{s\xi}^2$ and $\omega_{s\eta}^2$, $s = 1,2,3,4....n$ -two subsets of the set of the homogeneous double DNA chain helix;


Prof. CHIEN Wei-zang

Minisymposium Poster Presentation

DESIGN OF A SYSTEM FOR CONTROL, MONITORING, REGULATION AND DATA ACQUISITION ON CIVIL ENGINEERING OBJECTS, CONSTRUCTIONS AND MOBILE MODULES

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Keywords: Monitoring, Acquisition, Control, Sensors, Civil Engineering Objects, Mobile Modules

Abstract. *Rapid development of informational and communicational systems as well as microprocessors controllers for monitoring, control and data acquisition offers great opportunities for their appliance in many scientific disciplines [1,2] as well as in systems for control and management. The most significant appliance is in solving multidisciplinary problems. As a very important part of the informational and communicational systems are also the software modules used for modeling, simulation and optimization of real-time systems.*

The system for control, monitoring, regulation and data acquisition (CMRA system) on civil engineering objects, constructions and mobile modules is based on usage of universal microprocessor controllers of series INTEGRAF 10X. CMRA system is an expert system developed at the Faculty of Civil Engineering and Architecture, University of Nish. Universal microprocessor controller INTEGRAF 10X has been applied in three different systems: System for central monitoring, management and acquisition of data on hydraulic objects, System for automatization and management of controllable conditions of plant production in protected areas and System for Central Monitoring, Control, Automatic Regulation and Data Acquisition for Advance River Water Quality Monitoring.

This paper describes architecture of the CMRA expert system developed for objects in civil engineering and architecture and its appliances for measuring different parameters on different types of buildings.

1 INTRODUCTION

With the production of low cost sensors a new way is opened for many applications that will monitor parameters of objects in many areas of science. Object monitoring has been a very popular field of research in recent years.

2 CMRA SYSTEM ARCHITECTURE

Architecture of the system for control, monitoring, regulation and data acquisition on civil engineering objects, constructions and mobile modules (CMRA) is shown in the Figure 1.

The system consists of two parts: server of the central system and a client of the central system.

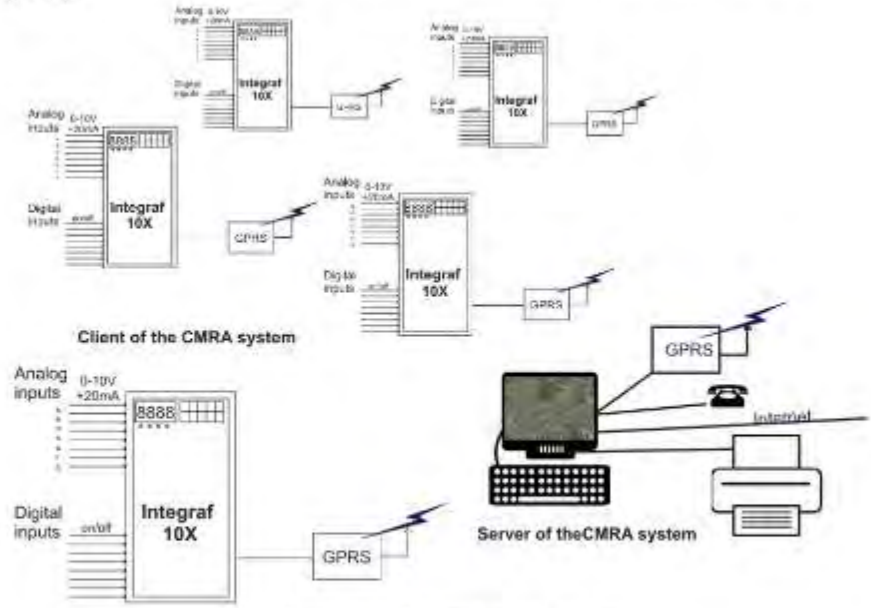

Figure 1: CMRA system architecture

2.1 Server Application of the central system

Application software for PC written in C programming language for Windows environment /surroundings/ is used for gathering and processing data received from microprocessor controller. The software can be also used for governing objects in systems, with certain algorithm, on the basis of processed result of measuring parameters, given by microprocessor controller.

Application software along with microcontroller software, Client of the Central system, INTEGRAF 10X, and communication software, for communication between microcontroller through GPRS modem and PC, Server of the Central System, makes a unique software package of Central System for gathering, processing and data supervision on purpose of automatization and increasement of system reliability.

The software package allows connecting several systems for tracking and governing, into unique system for Central control, automatic regulation and data acquisition.

2.2 Client of the Central System

Client of the central system is installed at the measuring object location or on the measuring object itself.

As a base in construction of typical measuring station, for measuring and data acquisition, we propose universal regulator INTEGRAF 10X based on microcontroller Philips 80C552 [3]. It is possible to use microcontroller with similar characteristics from other manufacturers (Intel, Atmel…).

INTEGRAF 10X (Figure 2.) as a compact universal regulator, provides acquisition, regulation, governing, automatization of processes and production systems, supervision and control, simply in any field where is necessary to use microcontroller governing. It is a high performance microcontroller suitable for instrumentation, industrial governing, automatization of industrial, waterworks or agricultural equipment.

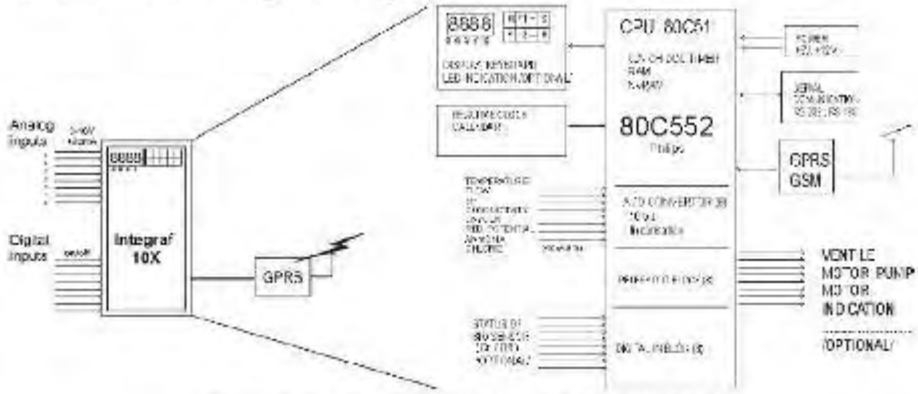

Figure 2: Structural Block Schema of INTEGRAF 10X

On the basis of collected data (input analog and digital signals) and data processing (scaling, calculating technical values with built in algorithms), controller is able to administer outputs completely independently.

Monitoring of measuring values, state of regulation circuits, distant assigning and accessing to regulation parameters is available thanks to real-time connection of controller and PC.

2.3 Communication system

Communication infrastructure of CMRA system based on GPRS modem completely solves the problem of connection between the central computer (server) and microprocessor regulators INTEGRAF 10X (client) within any area covered with the local mobile network. The use of GPRS modem within CMRA system assures great flexibility of system for data acquisition, central supervision and management, in sense of eased expansion of network or excluding of some parts from the existing network.

The main lack of using GPRS modem within CMRA system (real-time system), in comparison with cable connection trough 485 communication interface, is the functional dependence of CMRA system from the quality of mobile service that Telecom Serbia as the local operator of digital GSM network. Unreliability of the service and the often traffic blockage would give negative reflection on the work reliability of CMRA system, mostly in the system with possible managing and maintaining given water quality level. Time of response in CMRA system with GPRS modems is longer then with using wire connections, and is also dependant from the load of the mobile network.

On account of all that's said above, in the CMRA system, real-time system is provided possibility of using both GPRS modems and RS232 and RS 485 communication interfaces. (Figure 3.)

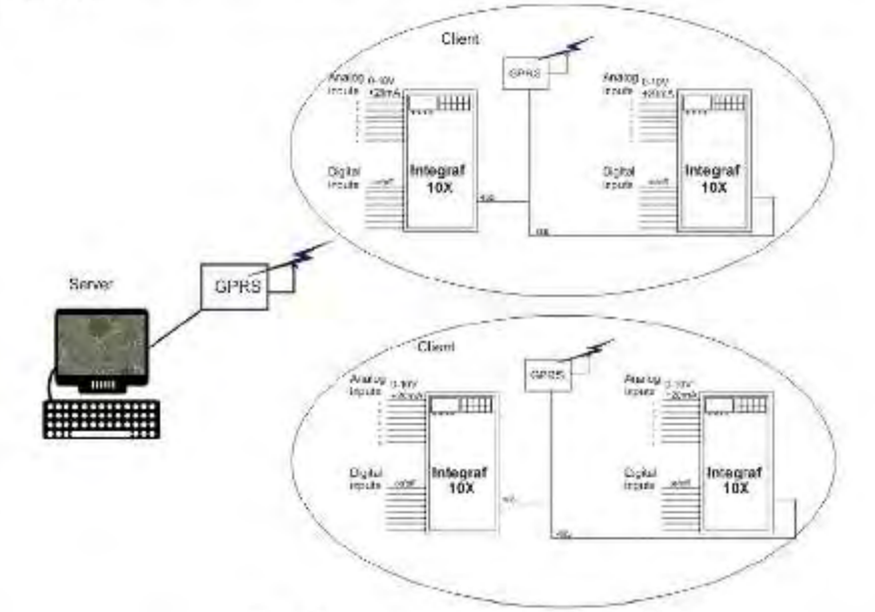

Figure 3: Communication in CMRA system

It is also necessary to enable data buffering within INTEGRAF 10X in systems where the only connection is trough GPRS, as a mean of prevention of data lost in cases when there is a traffic blockage or connection break.

3 CMRA SYSTEM IMPLEMENTATION

The implementation of CMRA system will be described in the fallowing three chapters. We will describe separately three systems in which we implemented CMRA system. Those systems are used in different areas of work. First system is used on the hydraulic object, second system for plant production object and the third one for the river water protection.

3.1 System for central monitoring, management and acquisition of data on hydraulic objects

The CMRA system for hydraulic objects for water distribution allows continuous measurement, monitoring and acquisition of measuring data obtained through microprocessor controller Integraf 10X and remote control systems for water supply distribution in real-time.

Measuring parameters of vital importance in water distribution systems are level of water, flow, speed, etc.

The system allows connection to several measuring points on one or more objects into a single information system for monitoring, data acquisition, centralized control and management of water supply and optimal water exploitation.

CMRA system enables defining of the optimal strategy for system management with minimization of the exploitation costs. This approach allows that any problem or shortcoming in the system can be discovered and minimized in the short time.

The system enables development of mathematical models of the real system and it's testing in various conditions of exploitation and the definition of optimal control algorithms whose effect will be judged on the model before making the real system.

The system was tested in the laboratory for hydraulics and sanitary equipment at the Faculty of civil Engineering and Architecture, University of Niš. (Figure 4.)

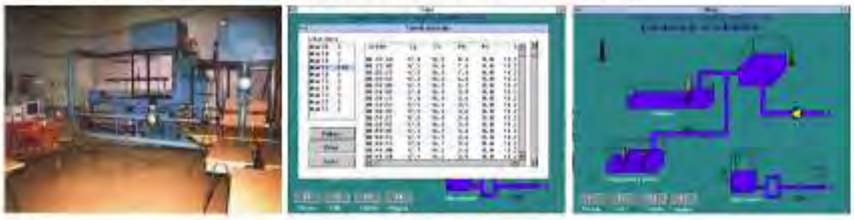

Figure 4: CMRA system for water distribution management

3.2 System for automatization and management of controllable conditions of plant production in protected areas

The CMRA system for automatization and management of controllable conditions of plant production in protected areas solves many problems in plant production. It enables creating a spatial type of microclimate totally adoptable to growing couture and controllable trough software modules. Creating microclimates in power plant production to optimize light or reduce container temperatures, disease pressure and crop stress can improve water and nutrient efficacy.

CMRA system for plant production enables acquisition of data as well as supervision and management of important conditions for development of the technological process of production plant in the protected areas (plant containers). (Figure 5.)

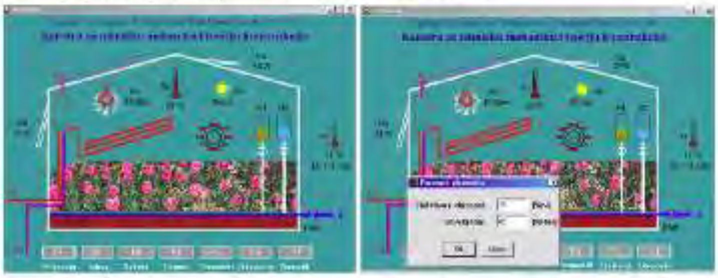

Figure 5: Regulation of parameters trough software module on CMRA server

Measuring and regulatory parameters in CMRA system are: temperature in the soil or surface, air temperature inside and outside the protected space, air humidity, soil pH value, the flow of water in the pipes, the brightness of a particular place, the angle windows, wind speed, wind direction and propagation. Measuring of parameters is realized trough sensors connected to microcontroller Integraf 10X. (Figure 6.)

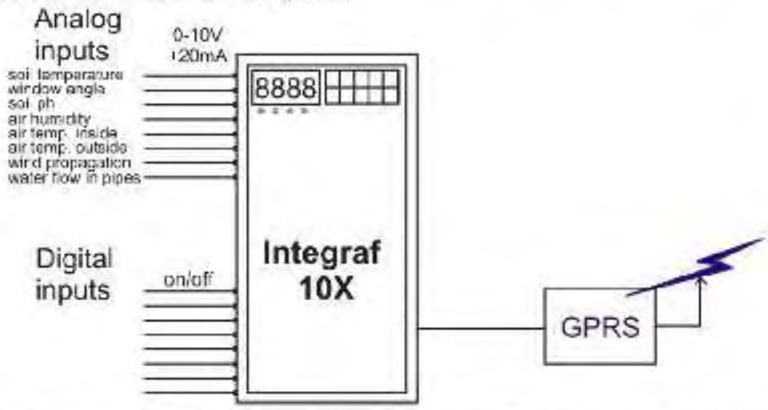

Figure 6: Integraf 10X for measuring and regulation of parameters in plant production

Regulation of parameters in the process of plant production is carried out by automatic algorithms given trough our software package. Regulation of parameters varies depending on the growing culture. For the purpose of optimization of plant growing culture, and on the basis of insight into the real state of culture, regulatory parameters can be modified for a certain period of time.

A special review is given by the management and control mechanisms in centralized or decentralized operation to open roof or side windows, as well as the horizontal and vertical transport.

Shown in the simulation process with the appropriate visualization of the computer and foresees the demonstration of certain elements in the real circumstances and with the appropriate sensor.

3.3 System for Central Monitoring, Control, Automatic Regulation and Data Acquisition for Advance River Water Quality Monitoring (ARWQM)

Environment polluted with herbicides and pesticides used in agricultural production, industrialization, increased waste waters in settlements etc., bring us to the fact that from ecological and environmental standpoint monitoring and keeping water quality in the river basin is of great importance.

The CMRA system for advanced river water quality management includes five clients on the basin of the river Moravica and a central CMRA server positioned at the control centre at the Faculty of Civil Engineering and Architecture. (Figure 7.)

Since the water protection is a very important subject, we created this system in the area where there are big polluters and a serious threat to population that gets water from the Moravica River. (Figure 8.)

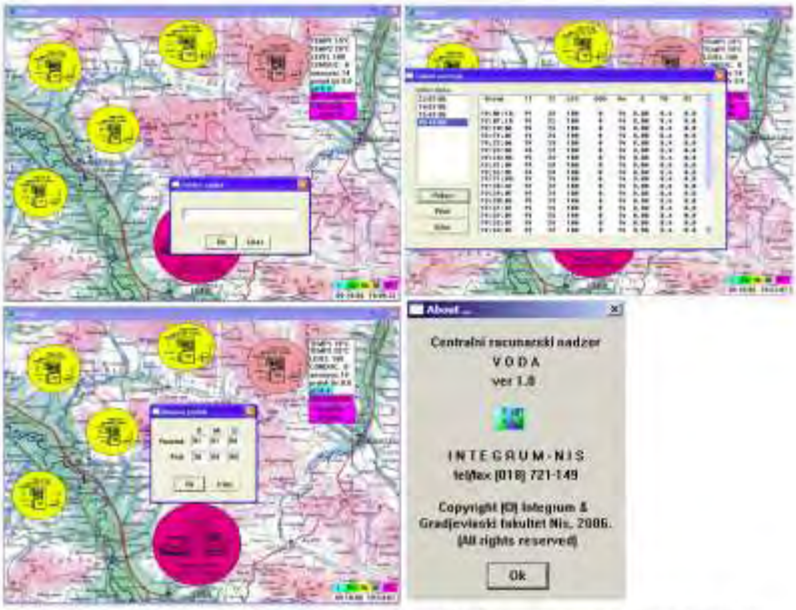

Figure 8: CMRA server side at the Faculty of Civil Engineering and Architecture

Development of CMRA system for ARWQM and it's placement on key points of Moravica river basin, enables complete monitoring of ARWQ in the basin, as well as monitoring of potential polluters. CMRA station enables studying of bio sensors and defining mathematical models of behavior for some bio sensors depending on concentration of water quality parameters.

Figure 8: CMRA client station on the river Moravica

Connecting CMRA data base, formed on the server (Figure 8.), with the data base of spread of certain diseases among people who leave near the river basin, can be useful in determination of the influence of water quality on the people's health (as well as on flora and fauna) in the region.

On the client side specific sensors are used for river water quality monitoring. (Figure 9.) Details of the whole system can be found in [3].

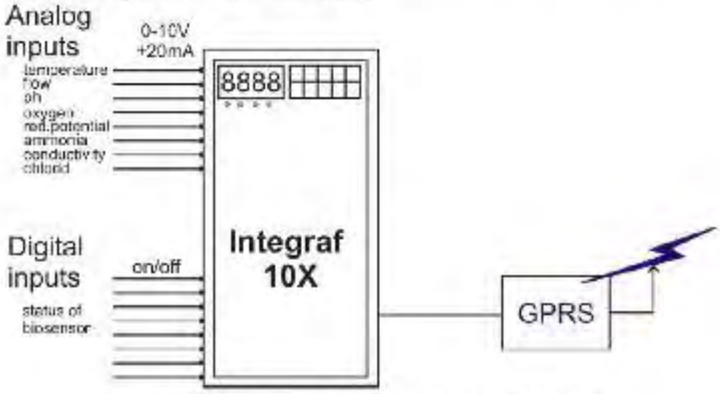

Figure 9: Integraf 10X as a client for ARWQM

The system can have built-in algorithms for water quality analyses and water categorization on the basis of well known criteria. With lake flow regulation and waste water flow regulation (water released by polluters), keeping water quality in known categories can be achieved. In order to achieve complete ARWQM system it is necessary to track level of water in accumulation, lake, define max and min quantity of water in the lake for given period of the year, use meteorological data base, install best devices on water exhaust valves. (Figure 9.)

4 CONCLUSIONS

Our future work is concentrated into creating a universal CMRA system which will connect all CMRA clients into one unique system. (Figure 10.)

This system will be used for control and regulation of CMRA clients on the level of the city or some other populated area. It represents three CMRA systems: bridge on the River Nišava, water distribution system and container for plant production connected to the server of the central system trough GPRS modems. All three systems are used for measuring specific parameters and sending their data to a server of a central system.

The final stage of development implies the creation of expert system with modified application software, developed mathematical models, the new procedures and algorithms optimized for optimal resolution of problems in the field of civil engineering and architecture primarily for the following purposes:

- Measuring thermal inertia and energy efficiency of constructions;
- Measuring dynamical characteristics of constructions; this includes measurement and acquisition of data during influence of mobile and seismic load and monitoring inelastic deformations and fractures in characteristic cross-sections and points.

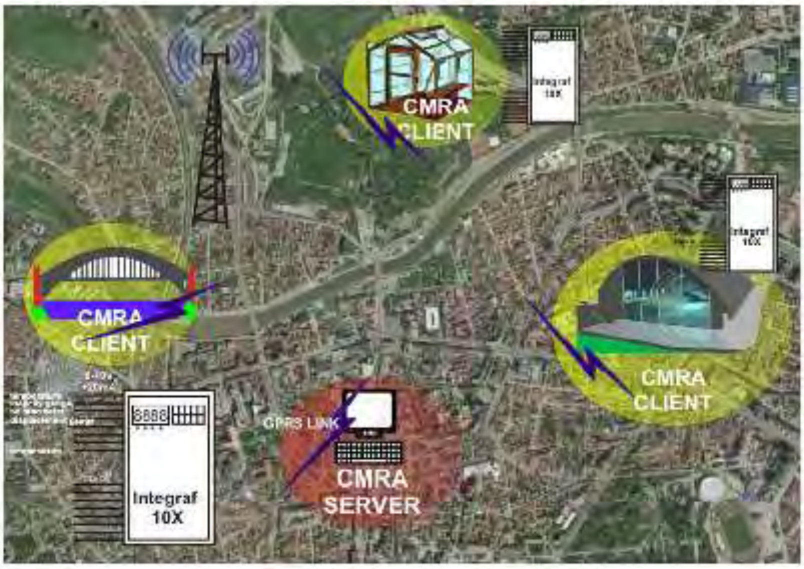

Figure 10: CMRA central system

- Measuring of the important parameters of facilities for tracking changes in the construction of facilities, materials aging, and fatigue for an efficient and timely maintenance of facilities. Built system monitoring will allow corrections of mathematical models for the purpose of their optimization, as well as the monitoring of construction, materials and real-time impact in order to detect damage, deformation and instability manifestations that will activate the warning system and, possibly, automatic control. This system is tested on the model of the original construction of sports halls composite supports project [4].
- Management of mobile plant modules in the protected areas: Usage of this concept creates the possibility for higher efficiency in usage of protected areas with high level of control including mobility of system units that will lead to elimination of unevenness in natural conditions and in energy saving. Technological process gains a higher level of automatization and increases productivity because of automated management of platforms. Important parameters of regulation are: temperature in characteristic points in the green houses, air humidity and lightness. Regulation of parameters in the production process, in achieving the optimization goal, is automated by the given algorithm regulated with software package and depending of plant types.

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Academician NANU Yuri Alekseevich Mitropolski with Colleague

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Minisymposium Closing Lecture

FREE AND FORCED VIBRATIONS OF THE HEAVY MATERIAL PARTICLE ALONG LINE WITH FRICTION: DIRECT AND INVERSE TASK OF THE THEORY OF VIBRORHELOGY

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Keywords: Nonlinear dynamics, No conservative system, Heavy material particle, Rough curvelinear line, Coulomb friction, Circle, Cycloid line, Parabola line, Phase plane method, Phase plane portrait, Analytical expressions, Vibrorheology.

Abstract. *Aim of this paper is to show that basic effects and phenomena in the systems with vibrorheological properties can be investigated by using free and forced vibrations of the heavy material particle along line with Coulomb friction. The paper presents theory and models of the heavy material particle motion along rough curvilinear line Mathematical description of the heavy material particle motion along rough curvelinear line with friction in generalized form is presented in a natural coordinate system of the corresponding line. There special cases of the roght lin with friction presented in this paper are: circle, cicloid line and parabola line. Three cases of the rough line are consisdered in the phase plane and analytical expression for the phase trajectories are obtained. We use phase trajectory lines as well as constant energy lines in the phase plane for presentation properties of the material particle motion along roght line for these three cases which correspond to the classical oscillator with Coulomb friction. On the basis of considered dynamical systems with friction an approach to the vibrorheology and to the control based on vibrorheology was presented. For all three cases of the rough line we can identify a member in the differential equation proportional to the square of the generalized coordinate (or parameter) by which a differential equation of the motion is expressed. This corresponds to the known case of turbulent damping. Also, for general case, as well as for special cases we can separate a corresponding fictive conservative system with two alternate equilibrium positions defined by an angle of friction corresponding to the coefficient of Coulomb friction. There is a complete analogy withe case of the motion of a spring oscillator with Coulomb friction. Direct and inverse task of the theory of vibrorheology was considered in the light of the heavy material particle motion along rough line. We considered two approaches to describing and solving problems in the vibrotheological properties and how it is possible to use it in the engineering practice. A corresponding fictive conservative system was introduced with two alternate equilibrium positions and series of phase trajectory was defined as energy constant line and series of energy jumps are identified.*

1 INTRODUCTION

We start with explanations of vibrorheological effects observed in the real engineering systems. Vibrorheological processes and effects appear under mutual interaction of two bodies with rough contact surfaces and in relation of relative vibrational motions one to other.
This paper was inspired by lectures and/or abstracts and papers written by the Russian scientist Matrosov V.M. [14] and [15], Chernousko F. [2] and [3] and Blekhman I. [1]. Aim of this paper is to show that basic effects and phenomena in the systems with vibrorheological properties can be investigated by using free and forced vibrations of the heavy material particle along line with Coulomb friction. Some classical problems of mechanical system motion with no ideal constraints and friction as as well as an osillator with Coulomb friction are presented in the books by Rašković D., [16] and [17]. Series of papers and monograph by Goroško O.A. and Hedrih (Stevanović) K. founded a complete integral theory of the analytical dynamics of the hereditary systems with rheological properties using knowledge from known mechanics of the hereditary continuum [4]. Also, some series of published papers by Hedrih (Stevanović) K. - [5]-[15], presented new reserch results regardiong fractional order discrete system oscillators and their properties, as well as viscoelastic properties.

2 FREE VIBRATIONS OF THE HEAVZ MATERIAL PARTICLE ALONG ROGHT CURVELINEAR LINE WITH COULOMB FRICTION

For beginning let us consider free vibrations of the heavy material particle along rough line with Coulomb Friction (see Figure 1.). For the case that curvilinear line is in the vertical plane Oxz, we can take that equation of the curve-linear line is: $z = f(x)$, or $f_1(x,z) = z - f(x) = 0$ and with the following properties $f(-x) = f(x)$ and that coordinate pole is in the zero point $f(0) = 0$ in which line have minimum. Also, it is same for the case that $f(-x) \neq f(x)$, $f(0) = 0$ and $f(x) \geq 0$, $f(-x) \geq 0$ with minimum of the $z_{\min} = f(0)$.

Heavy material particle, mass m, moving along rough curvilinear line with Coulomb sliding friction coefficient μ, is loaded by proper weight mg, as an active conservative force and by two components of non ideal constraint reactions, one F_N - normal ideal constrain reaction and F_μ tangential component of the non ideal constraint reaction induced by friction and proportional to the normal component reaction F_N, $F_\mu = -\mu F_N \, sign\, \vec{v}$.

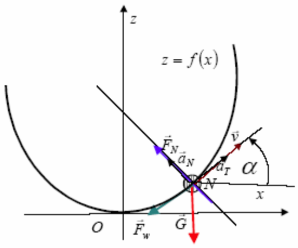


Figure 1. Heavy material particle motion along rought curvelinear line with Coulomb friction

As the material particle is loaded by active conservative force $\vec{G} = mg\vec{k}$, and for the reason that material particle constrained by non ideal constraint - rough curvilinear line as the reactions of the non ideal constraint appear two components of the reactions, one normal to the curvelinear line in the form: $\vec{F}_N = -F_N\vec{N} = \lambda\, grad\, f_1(x,z) = \lambda\left(\vec{i}\frac{\partial f(x)}{\partial x} - \vec{k}\right)$ and second tangential to the curvelinear line and colinear with material particle velocity in the form $\vec{F}_\mu = -\mu\left|\vec{F}_N\right|\frac{\vec{v}}{|\vec{v}|}$, with intensity $F_\mu = -\mu F_N\, sign\, \dot{v}$ and alternative direction with respect to material particle velocity $\vec{v}$. Force of material particle inertia has two components, one in tangential and second in normal directions, and we can write:

$$\vec{I}_F = -m\vec{a}_N - m\vec{a}_t = -m\dot{v}\vec{N} - m\frac{v^2}{R_k}\vec{T} \tag{1}$$

where R_k radius of the path line curvature at the point of the material particle termination position. $\vec{T}$ and $\vec{N}$ are unit vectors of the tangent and normal to the path line at termination position of the material particle during their motion along rough line.

We can use the principle of dynamical equilibrium and on the basis of this principle we can write vector equation of the heavy material particle motion along rought line in the following form: $\vec{I}_k + \vec{G} + \vec{F}_N + \vec{F}_\mu = 0$. By using curvilinear line natural coordinate system, it we can write the following vector equation:

$$\left(-m\ddot{s}\vec{T}\right) + \left(-m\frac{v^2}{R}\vec{N}\right) + mg\left(-\sin\alpha\vec{T} - \cos\alpha\vec{N}\right) + F_N\vec{N} - \mu\left|\vec{F}_N\right|\frac{\vec{v}}{|\vec{v}|} = 0 \tag{2}$$

where $\alpha = arctg\, z'$ is the angle of the tangent to the curvelinear line according $0x$ axis, and s is a moving material particle put along curvelinear line ($ds = dx\sqrt{1+z'^2}$).

After, scalarly multiplying previous equation by unit vectors $\vec{T}$ and $\vec{N}$, we obtain the two scalar equations. From second equation we obtain expression for the intensity of normal component of parabola constrain reaction force in the form: $F_N = m\frac{v^2}{R_k} + mg\cos\alpha$. After introducing this expression into first equation of the previous obtained system, we obtain:

$$\ddot{s} + g\sin\alpha \pm \mu\left(\frac{v^2}{R_k} + g\cos\alpha\right) = 0 \tag{3}$$

Taking into acount that for arbitrary form of rough constraint curvilinear line in plane defined by $z = f(x)$

$$\dot{v} = \frac{dv}{dt} = \frac{d^2s}{dt^2} = \ddot{s} = \frac{d}{dt}\left(\dot{x}\sqrt{1+z'^2}\right),\ R_k = \frac{\sqrt{\left(1+z'^2\right)^3}}{z''},\ \frac{v^2}{R_k} = \dot{x}^2\frac{z''}{\sqrt{1+z'^2}},\ tg\alpha = z'$$
$$\sin\alpha = \frac{tg\alpha}{\sqrt{1+tg^2\alpha}} = \frac{z'}{\sqrt{1+z'^2}} = \frac{dz}{ds},\ \cos\alpha = \frac{1}{\sqrt{1+tg^2\alpha}} = \frac{1}{\sqrt{1+z'^2}} = \frac{dx}{ds} \tag{4}$$

differential equation (3) of the heavy material particle motion along arbitrary, non ideal constraint curvilinear line, rough line in plane defined by $z = f(x)$ and with coefficient of the sliding friction μ, is in the form:

$$\frac{d}{dt}\left(\dot{x}\sqrt{1+z'^2}\right) + \frac{1}{\sqrt{1+z'^2}}\left\{gz' \pm \mu\left(\dot{x}^2 z'' - g\right)\right\} = 0 \tag{5}$$

and intensity of the normal and tangential components of the non ideal constraint rough line reaction are

$$F_N = \frac{m}{\sqrt{1+z'^2}}\left(\dot{x}^2 z'' + g\right) \tag{6}$$

$$F_\mu = -\mu \frac{m}{\sqrt{1+z'^2}}\left(\dot{x}^2 z'' + g\right) sign\vec{v} \tag{7}$$

Let us introduce new variable in the following form: $u = \dot{x}^2$, and taking into account that are $\frac{du}{dx} = 2\dot{x}\frac{d\dot{x}}{dx} = \frac{d(\dot{x})^2}{dx}$ and $\ddot{x} = \frac{1}{2}\frac{du}{dx}$, previous double differential equations of the material particle motion along rough line can be transformed in the following form:

$$\frac{du}{dx} + 2u\frac{1}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2} \pm \mu\frac{z''}{\sqrt{1+z'^2}}\right] = -\frac{2g}{\left(1+z'^2\right)}\{z' \pm \mu\} \tag{8}$$

Previous double differential equation of the material particle motion along rough curvilinear line according to new helping coordinate u is ordinary differential equation first order with changeable coefficients and type in following form: $\frac{du}{dx} \pm P(x)u = Q(x)$, with solution in the form: $u(x) = e^{-\int P(x)dx}\left[\int Q(x)e^{\int P(x)dx}dx + C\right]$, (8*)

where for considered case coefficients of the differential equation are

$$P(x) = \frac{2}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2} \pm \mu\frac{z''}{\sqrt{1+z'^2}}\right] \quad \begin{cases} for & v>0 \\ for & v<0 \end{cases} \tag{9}$$

$$Q(x) = -\frac{2g}{\left(1+z'^2\right)}\{z' \pm \mu\} \quad \begin{cases} for & v>0 \\ for & v<0 \end{cases} \tag{10}$$

Now, for considered case, we can write:

$$\left[\dot{x}(x)\right]^2 = e^{-\int\frac{2}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2} \pm \mu\frac{z''}{\sqrt{1+z'^2}}\right]dx}\left[-2g\int\frac{1}{\left(1+z'^2\right)}\{z' \pm \mu\}e^{\int\frac{2}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2} \pm \mu\frac{z''}{\sqrt{1+z'^2}}\right]dx}dx + C\right] \tag{11}$$

and also to obtain the following equation of the phase trajectories in the phase plane $(x, \dot{x})$, or afer multiplaying by $\sqrt{1+z'^2}$

$$v^2(x) = \left(1+z'^2\right)e^{-\int\frac{2}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2} \pm \mu\frac{z''}{\sqrt{1+z'^2}}\right]dx}\left[-2g\int\frac{1}{\left(1+z'^2\right)}\{z' \pm \mu\}e^{\int\frac{2}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2} \pm \mu\frac{z''}{\sqrt{1+z'^2}}\right]dx}dx + C\right] \tag{12}$$

We can see that obtained equation of the phase trajectory we can consider as a motion with corresponding conservative system motion with following differential equation:

$$\ddot{x} + \frac{g}{\left(1+z'^2\right)}\{z' \pm \mu\} = 0 \tag{13}$$

with phase trajectory equation:

$$\left[\dot{x}(x)\right]^2 - \left[\dot{x}(x_0)\right]^2 + 2g\int_{x_0}^{x}\frac{(z' \pm \mu)dx}{\left(1+z'^2\right)} = 0 \tag{14}$$

$$v^2(x) = \left(1+z'^2\right)\left[\dot{x}(x_0)\right]^2 - 2g\left(1+z'^2\right)\int_{x_0}^{x}\frac{(z' \pm \mu)dx}{\left(1+z'^2\right)} \tag{15}$$

By using previous results according to first integral (11) or (12), we can write two equations of phase trajectories for this fictive conservative system according to alternate directions of the velocity:

a* Period zero from initial moment to the first moment when velocity is equal to zero, and at initial moment initial velocity is in the same direction of the motion coordinate increase, is described by differential equation of the particle motion:

$$\frac{d}{dt}\left(\dot{x}\sqrt{1+z'^2}\right)+g\frac{z'}{\sqrt{1+z'^2}}+\mu\frac{1}{\sqrt{1+z'^2}}\left(\dot{x}^2 z''+g\right)=0 \tag{16}$$

and corresponding equation of the phase trajectory:

$$v^2(x)=\left(1+z'^2\right)e^{-\int\frac{z''}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2}+\mu\frac{z''}{\sqrt{1+z'^2}}\right]dx}\left[-2g\int\frac{1}{\left(1+z'^2\right)}\left(z'+\mu\right)e^{\int\frac{z''}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2}+\mu\frac{z''}{\sqrt{1+z'^2}}\right]dx}dx+C_1\right] \tag{17}$$

for $v>0$, when constant $C_1(x_0,\dot{x}_0)$ depends on the initial system conditions and is determined by following expression:

$$C_1(x_0,\dot{x}_0)=\dot{x}(x_0)\left\{e^{\int\frac{z''}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2}+\mu\frac{z''}{\sqrt{1+z'^2}}\right]dx}\right\}_{x=x_0}-2g\left\{\int\frac{1}{\left(1+z'^2\right)}\left(z'+\mu\right)e^{\int\frac{z''}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2}+\mu\frac{z''}{\sqrt{1+z'^2}}\right]dx}dx\right\}_{x=x_0} \tag{18}$$

Next coordinate corresponds to the the first zero velocity which we can find from the following condition:

$$v^2(x_{01})=\left\langle\left(1+z'^2\right)e^{-\int\frac{z''}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2}+\mu\frac{z''}{\sqrt{1+z'^2}}\right]dx}\left[-2g\int\frac{1}{\left(1+z'^2\right)}\left(z'+\mu\right)e^{\int\frac{z''}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2}+\mu\frac{z''}{\sqrt{1+z'^2}}\right]dx}dx+C_1(x_0,\dot{x}_0)\right]\right\rangle_{x=x_{01}}=0 \tag{19}$$

Coordinate x_{01} at which there appears an alternate change from one phase trajectory to second trajectory it is possible to obtain as solution of the previous equation.

b* Next period (*1*) of the motion is described by the following differential equation of the particle motion:

$$\frac{d}{dt}\left(\dot{x}\sqrt{1-z'^2}\right)+g\frac{z'}{\sqrt{1+z'^2}}-\mu\frac{1}{\sqrt{1+z'^2}}\left(\dot{x}^2 z''+g\right)=0 \tag{20}$$

and corresponding equation of the phase trajectory

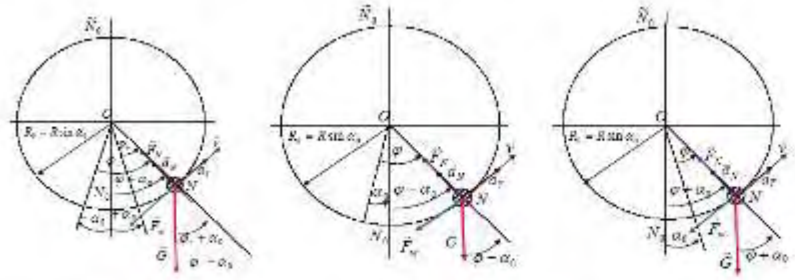
$$v^2(x)=\left(1+z'^2\right)e^{-\int\frac{z''}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2}-\mu\frac{z''}{\sqrt{1+z'^2}}\right]dx}\left[-2g\int\frac{1}{\left(1+z'^2\right)}\left(z'-\mu\right)e^{\int\frac{z''}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2}-\mu\frac{z''}{\sqrt{1+z'^2}}\right]dx}dx+C_2\right] \tag{21}$$

with initial conditions described by zero velocity and coordinate corresponding to this zero velocity $(x_{01},0)$. With corresponding integral constant $C_2(x_0,\dot{x}_0,x_{01},0)$.

Next branch of the phase trajectory is with integral constant $C_3(x_0,\dot{x}_0,x_{01},0,x_{02},0)$ and with equation alternatively with corresponding sign

3 FREE VIBRATIONS OF THE HEAVY MATERIAL PARTICLE ALONG ROGH CIRCLE LINE WITH COULOMB FRICTION

Heavy material particle, mass m, moving along rough circle (see Figure 2) with sliding friction coefficient μ, is loaded by proper weight mg, as an active conservative force and by two components of the non ideal constraint reactions, one F_N - normal ideal constrain reaction and F_{μ} tangential component of the non ideal constraint reaction induced by friction and proportional to the normal component reaction F_N, $F_{\mu} = -\mu F_N \, sign\, \vec{v}$.



a* b* c*

Figure 2. Heavy material particle, mass m, moving along rought circle with sliding friction coefficient $\mu = tg\alpha_0$; a* Generalized coordinate φ and plan of the active and reactive forces; b* and c* prezentation of the „relative" equilibrium positions with properties of the altenations $\pm\alpha_0$.

By using angle φ as a generalized coordinate considered a non conservative mechanical system with one degree of freedom, and presented content of the previous part, we can write the following system of the dynamical equilibrium

$$mR\dot{\varphi}^2 = F_N - mg\cos\varphi$$
$$mR\ddot{\varphi} = \begin{cases} -\mu F_N - mg\sin\varphi & for \quad \dot{\varphi} > 0 \\ \mu F_N - mg\sin\varphi & for \quad \dot{\varphi} < 0 \end{cases} \tag{22}$$

From the previous system we can write expression for the normal component of the non ideal constraint in the form of rogh circle

$$F_N = mR\left(\dot{\varphi}^2 + \frac{g}{R}\cos\varphi\right) \tag{23}$$

and double differential equation of the material particle along rough circle expressed by generalized coordinate in the following form:

$$\ddot{\varphi} + \mu\dot{\varphi}^2 + \frac{g}{R}(\sin\varphi + \mu\cos\varphi) = 0 \quad for \quad \dot{\varphi} \gtrless 0 \tag{23}$$

Let us introduce the following expressions for the coefficient of friction $\mu = tg\alpha_0$.

Than, the governing equations of the system is:

$$\ddot{\varphi} \pm \dot{\varphi}^2 tg\alpha_0 + \frac{g}{R\cos\alpha_0}\sin(\varphi \pm \alpha_0) = 0 \quad for \quad \dot{\varphi} \gtrless 0 \tag{24}$$

or in the form

$$\frac{d\varphi}{dt} = v$$
$$\frac{dv}{dt} = \mp v^2 tg\alpha_0 - \frac{g}{R\cos\alpha_0}\sin(\varphi \pm \alpha_0) \quad for \quad \dot{\varphi} \gtrless 0 \tag{25}$$

Coordinates of the equilibrium positions are expressed by:

$v = 0$

$$\varphi_k = k\pi \mp \alpha_0, \quad k = 0,1,2,3,\ldots,\infty \quad for \quad \dot{\varphi}\ 0 \tag{26}$$

Conditions of the stability or non stability of the equilibrium positions $v = 0$ $\varphi_k = k\pi \mp \alpha_0$, $k = 0,1,2,3,\ldots,\infty$, *for* $\dot{\varphi}\ 0$, we can obtain by corresponding system of linearized differential equations at the state around the equilibrium positions:

$$\begin{aligned} \frac{d\varphi}{dt} &= v \\ \frac{dv}{dt} &= -\frac{g}{R\cos\alpha_0}\sin(\varphi_s \pm \alpha_0)\varphi \mp 2v_s v tg\alpha \end{aligned} \tag{27}$$

Characteristic equation of the system of linearized differential equations is: $\lambda^2 + \dfrac{(-1)^k g}{R\cos\alpha_0} = 0$,

and corresponding characteristic numbers are: $\lambda_{1,2} = \mp\sqrt{\dfrac{(-1)^{k+1} g}{R\cos\alpha_0}}$.

Now, we can conclude the following:

a* For the equailibrium posittions $v = 0$, $\varphi_{2p+1} = (2p+1)\pi \mp \alpha_0$, $p = 0,1,2,3,\ldots,\infty$, *for* $\dot{\varphi}\ 0$ characteristic numbers are: $\lambda_{1,2} = \mp\sqrt{\dfrac{g}{R\cos\alpha_0}}$ real and different signs, one positive and one negative. In the phase plane these points are singular and saddle type and equilibrium position are not stable.

b* For the equailibrium posittions $v = 0$, $\varphi_{2p} = 2p\pi \mp \alpha_0$, $p = 0,1,2,3,\ldots,\infty$, *for* $\dot{\varphi}\ 0$ characteristic numbers are: $\lambda_{1,2} = \mp i\sqrt{\dfrac{g}{R\cos\alpha_0}}$ two conjugate both imaginary numbers and different signs, one positive and one negative. In the phase plane these points are singular and center type and equilibrium position are stable.

3.1 The fictive conservative system with alternate equilibrium position

By using, then, governing equation of the non conservative system dynamics (24), we can take into consideration a corresponding fictive conservative system for the case that second nonlinear member $\pm\dot{\varphi}^2 tg\alpha_0$ depending with square of the angular velocity and equivalent to "turbulent damping" was neglected. This differential equation described a fictive conservative system dynamics coressponding to the basic non conservative system with friction, we obtain in the form:

$$\ddot{\varphi} + \frac{g}{R\cos\alpha_0}\sin(\varphi \pm \alpha_0) = 0 \quad for \quad \dot{\varphi}\ 0 \tag{28}$$

Multiplying previous differential equation (28) by $2\dot{\varphi}dt = 2d\varphi$ and after integrating from initial moment to the termination moment we obtain the first integral in the following form:

$$\dot{\varphi}^2 - \dot{\varphi}_0^2 + \frac{2g}{R\cos\alpha_0}\{[1 - \cos(\varphi \pm \alpha_0)] - [1 - \cos(\varphi_0 \pm \alpha_0)]\} = 0 \quad for \quad \dot{\varphi}\ 0 \tag{29}$$

Now, we can separate same members in the previous first integral of the differential equation (28) of the fictive conservative system, for which an integral energy can be expressed in the following form:

$$\widetilde{\mathbf{E}}_k + \widetilde{\mathbf{E}}_p = \widetilde{\mathbf{E}}_{k0} + \widetilde{\mathbf{E}}_{p0} = const_{\pm} \quad for \quad \dot{\varphi} \gtrless 0 \tag{30}$$

where

$$\widetilde{\mathbf{E}}_k = \frac{1}{2} m \left(R\cos\alpha_0^{\,2} \right) \dot{\varphi}^2 \quad \text{and} \quad \widetilde{\mathbf{E}}_{k0} = \frac{1}{2} m \left(R\cos\alpha_0^{\,2} \right) \dot{\varphi}_0^2 \tag{31}$$

are analogs of kinetic energies at the arbitrary time moment and at initial moment, and

$$\widetilde{\mathbf{E}}_p = mgR\cos\alpha_0 \left[1 - \cos(\varphi \pm \alpha_0) \right] \quad for \quad \dot{\varphi} \gtrless 0$$

$$\widetilde{\mathbf{E}}_{p0} = mgR\cos\alpha_0 \left[1 - \cos(\varphi_0 \pm \alpha_0) \right] \quad for \quad \dot{\varphi} \gtrless 0 \tag{32}$$

Differential equation (28) is possible to consider as two differential equations describing two cases of the mathematical pendulum dynamics with length $R_0 = R\cos\alpha_0 = R\frac{1}{\sqrt{1+\mu^2}}$, depending on the coefficient μ of the friction, as it is presented in Figure 2. b* and in Figure 2. c*. This motion is with alternate differential equations depending on the direction of the velocity.

By using previous results according to first integral (30), we can write two equations of the phase trajectories for this ***fictive conservative system*** according to alternate directions of the material particle angular velocity:

a* Period zero from initial moment to the first moment when velocity is equal to zero, and at initial moment initial angular velocity is in the same direction of the motion coordinate increase, is described by:

$$\dot{\varphi}^2 - \dot{\varphi}_0^2 + \frac{2g}{R\cos\alpha_0} \left[\cos(\varphi_0 + \alpha_0) - \cos(\varphi + \alpha_0) \right] = 0 \quad for \quad \dot{\varphi} > 0 \tag{33}$$

Angular coordinate φ_{01} corresponding to the first zero velocity $\dot{\varphi}_1 = 0$ and this coordinate can be found from (33) in the following form:

$$\varphi_{01} = \arccos\left[\cos(\varphi_0 + \alpha_0) - \frac{R\cos\alpha_0}{2g} \dot{\varphi}_0^2 \right] - \alpha_0 + 2p\pi \quad for \quad \dot{\varphi}_1 = 0 \tag{34}$$

At angle coordinate φ_{01} an alternate change friction force from one to other direction corresponding opposite direction of the material particle angular velocity, also representative point at phase portrait pass from one branch of the phase trajectory to the second branch of the trajectory.

b* Next period (1) of the material particle motion is described by the following equation

$$\dot{\varphi}^2 + \frac{2g}{R\cos\alpha_0} \left[\cos(\varphi_{01} - \alpha_0) - \cos(\varphi - \alpha_0) \right] = 0 \quad for \quad \dot{\varphi} < 0 \tag{35}$$

with initial conditions φ_{01} described by expression (34) and zero angular velocity $\dot{\varphi}_{01} = 0$ and corresponding expression of the phase trajectory branch is in the form:

$$\dot{\varphi}^2 + \frac{2g}{R\cos\alpha_0} \left[\cos\left\langle \arccos\left[\cos(\varphi_0 + \alpha_0) - \frac{R\cos\alpha_0}{2g} \dot{\varphi}_0^2 \right] - 2\alpha_0 \right\rangle - \cos(\varphi - \alpha_0) \right] = 0 \quad for \quad \dot{\varphi} < 0 \tag{36}$$

c* Next period (2) of the material particle motion start at the moment when angular velocity is equal to zero at angular coordinate φ_{02} defined by following expression:

$$\varphi_{02} = \arccos\left[\cos(\varphi_0 + \alpha_0) - \frac{R\cos\alpha_0}{2g} \dot{\varphi}_0^2 \right] - \alpha_0 \quad for \quad \dot{\varphi}_{02} = 0 \tag{37}$$

The phase trajectory expression of the material particle motion in the next period (2) is described by the following equation

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos(\varphi_{02}+\alpha_0)-\cos(\varphi+\alpha_0)\right]=0 \quad for \quad \dot{\varphi}>0 \tag{38}$$

as well as in the following form:

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos\left\langle\arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle-\cos(\varphi+\alpha_0)\right]=0 \quad for \quad \dot{\varphi}>0 \tag{38*}$$

d* Next period (3) of the material particle motion start at the moment when angular velocity is equal to zero at angular coordinate φ_{03} defined by following expression:

$$\varphi_{03}=\cos\left\langle\arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle-\alpha_0 \quad for \quad \dot{\varphi}_{03}=0 \tag{39}$$

The phase trajectory expression of the material particle motion in the next period (3) is described by the following equation:

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos(\varphi_{03}-\alpha_0)-\cos(\varphi-\alpha_0)\right]=0 \quad for \quad \dot{\varphi}<0 \tag{40}$$

as well as in the following form:

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos\left\{\cos\left\langle\arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle-2\alpha_0\right\}-\cos(\varphi-\alpha_0)\right]=0 \tag{40*}$$

$for \quad \dot{\varphi}<0$

e* Next period (4) start at the moment when angular velocity is equal to zero at angular coordinate φ_{04} is expressed by following expression:

$$\varphi_{04}=\cos\left\langle\arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle-\alpha_0 \quad for \quad \dot{\varphi}_{04}=0 \tag{41}$$

and equation of the corresponding branch of the phase trajectory is in the form:

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos\left\{\cos\left\langle\arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle_0\right\}-\cos(\varphi+\alpha_0)\right]=0 \tag{42}$$

$for \quad \dot{\varphi}>0$

f* Next period (5) start at the moment when angular velocity is equal to zero at angular coordinate φ_{05} is expressed by following expression:

$$\varphi_{05}=\cos\left\langle\arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle-\alpha_0 \quad for \quad \dot{\varphi}_{05}=0 \tag{43}$$

and equation of the corresponding branch of the phase trajectory is in the form:
and phase trajectory is in the form:

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos\left\{\cos\left\langle\arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle-2\alpha_0\right\}-\cos(\varphi-\alpha_0)\right]=0 \tag{44}$$

$for \quad \dot{\varphi}<0$

Now, we can list a series of obtained equations of the series of a branch of trajectories. For that reason we can show the following list of expressions of series of phase trajectory branches:

$$\dot{\varphi}^2-\dot{\varphi}_0^2+\frac{2g}{R\cos\alpha_0}\left[\cos(\varphi_0+\alpha_0)-\cos(\varphi+\alpha_0)\right]=0 \quad for \quad \dot{\varphi}>0 \tag{45}$$

First pair

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos\left\langle \arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]-2\alpha_0\right\rangle-\cos(\varphi-\alpha_0)\right]=0$$

$for \quad \dot{\varphi}<0$

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos\left\langle \arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle-\cos(\varphi+\alpha_0)\right]=0 \tag{46}$$

$for \quad \dot{\varphi}>0$

Second pair

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos\left\{\cos\left\langle \arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle-2\alpha_0\right\}-\cos(\varphi-\alpha_0)\right]=0$$

$for \quad \dot{\varphi}<0$

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos\left\{\cos\left\langle \arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle_0\right\}-\cos(\varphi+\alpha_0)\right]=0 \tag{47}$$

$for \quad \dot{\varphi}>0$

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos\left\{\cos\left\langle \arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle-2\alpha_0\right\}-\cos(\varphi-\alpha_0)\right]=0$$

$for \quad \dot{\varphi}<0$

No it is easy to generalize the next third pair of the from of the equations of the branches of the trajectories

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos\left\langle\cos\left\{\cos\left\langle \arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle_0\right\}\right\rangle-\cos(\varphi+\alpha_0)\right]=0$$

$for \quad \dot{\varphi}>0$

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos\left\{\left\langle\cos\left\{\cos\left\langle \arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle_0\right\}\right\rangle-2\alpha_0\right\}-\cos(\varphi-\alpha_0)\right]=0 \tag{48}$$

$for \quad \dot{\varphi}<0$

For the next k-th pairs the operator cos multiplicities k-times

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos\left\langle\cos..\left\{\cos\left\langle \arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle_0\right\}\right\rangle-\cos(\varphi+\alpha_0)\right]=0$$

$for \quad \dot{\varphi}>0$

$$\dot{\varphi}^2+\frac{2g}{R\cos\alpha_0}\left[\cos\left\{\left\langle\cos..\left\{\cos\left\langle \arccos\left[\cos(\varphi_0+\alpha_0)-\frac{R\cos\alpha_0}{2g}\dot{\varphi}_0^2\right]\right\rangle_0\right\}\right\rangle-2\alpha_0\right\}-\cos(\varphi-\alpha_0)\right]=0 \tag{49}$$

$for \quad \dot{\varphi}<0$

Now it is easy to present graphically this analytical result for the case of a fictive conservative system corresponding to the real system of heavy material particle motion along rough circle line with Coulomb friction.

3.2 Energy jumps in the fictive conservative system corresponding to the real system

Obtained expressions of the corresponding series of phase trajectory branches for fictive conservative system corresponding to the real system with friction, are trajectories with constant total mechanical energy of the fictive conservative system, but each with different expression and different constant energy, and then there are points of trajectories continuity and energy discontinuity, the last state on the previous trajectory branch and initial conditions for the next branch of trajectory, are state of the jumps of the energy from one branch to the next branch defined by alternate of the equilibrium position of the fictive system in formal correspondence to the angle of the friction of the real system. For that case it is possible to write in the form (30), and corresponding for each branch:

$$\begin{aligned}&\tilde{\mathbf{E}}_{k}+\tilde{\mathbf{E}}_{p}=\tilde{\mathbf{E}}_{k00}+\tilde{\mathbf{E}}_{p00}=const \quad for \quad \dot{\varphi}>0\\&\tilde{\mathbf{E}}_{k}+\tilde{\mathbf{E}}_{p}=\tilde{\mathbf{E}}_{p0j}=const_{j} \quad for \quad \dot{\varphi}_{<}>0\end{aligned} \tag{50}$$

where

$$\begin{aligned}&\tilde{\mathbf{E}}_{k}=\frac{1}{2}m\left(R\cos\alpha_0{}^{2}\right)\dot{\varphi}^{2}\\&\tilde{\mathbf{E}}_{k00}=\frac{1}{2}m\left(R\cos\alpha_0{}^{2}\right)\dot{\varphi}_0^{2}\neq 0\\&\tilde{\mathbf{E}}_{k0j}=\frac{1}{2}m\left(R\cos\alpha_0{}^{2}\right)\dot{\varphi}_{0j}^{2}=0\\&\tilde{\mathbf{E}}_{p}=mgR\cos\alpha_0\left[1-\cos(\varphi\pm\alpha_0)\right] \quad for \quad \dot{\varphi}_{<}^{>}0\\&\tilde{\mathbf{E}}_{p0j}=mgR\cos\alpha_0\left[1-\cos(\varphi_{0j}\pm\alpha_0)\right] \quad for \quad \dot{\varphi}_{0j}=0\end{aligned} \tag{51}$$

Difference total mechanical energy between kinetic state at previous and the next is:

$$\Delta\tilde{\mathbf{E}}_{j}=\tilde{\mathbf{E}}_{p0,j-1}-\tilde{\mathbf{E}}_{p0,j}$$

or for each of the jumps

$$\begin{aligned}&\Delta\tilde{\mathbf{E}}_{1}=\frac{1}{2}m\left(R\cos\alpha_0{}^{2}\right)\dot{\varphi}_0^{2}+mgR\cos\alpha_0\left[\cos(\varphi_{01}-\alpha_0)-\cos(\varphi_0+\alpha_0)\right]\\&\Delta\tilde{\mathbf{E}}_{2}=\tilde{\mathbf{E}}_{p01}-\tilde{\mathbf{E}}_{p02}=mgR\cos\alpha_0\left[\cos(\varphi_{02}+\alpha_0)-\cos(\varphi_{01}-\alpha_0)\right]\\&\Delta\tilde{\mathbf{E}}_{3}=\tilde{\mathbf{E}}_{p02}-\tilde{\mathbf{E}}_{p03}=mgR\cos\alpha_0\left[\cos(\varphi_{03}-\alpha_0)-\cos(\varphi_{02}+\alpha_0)\right]\\&\Delta\tilde{\mathbf{E}}_{4}=\tilde{\mathbf{E}}_{p03}-\tilde{\mathbf{E}}_{p04}=mgR\cos\alpha_0\left[\cos(\varphi_{04}+\alpha_0)-\cos(\varphi_{03}-\alpha_0)\right]\end{aligned} \tag{52}$$

3.3 Phase trajectory equations of the no conservative system

Basic system is non conservative and heavy material particle motion along rough circle line is described by governing equation (24) and for this system total energy along system motion is non constant

$$\tilde{\mathbf{E}}_{k}+\tilde{\mathbf{E}}_{p}\neq\tilde{\mathbf{E}}_{k00}+\tilde{\mathbf{E}}_{p00}\neq const \quad for \quad \dot{\varphi}_{<}^{>}0 \tag{53}$$

And dissipation of the total energy of the system during the motion is expressed be following relation:

$$\frac{d\mathbf{E}}{dt}=-\left(\vec{F}_{\mu},\vec{v}\right)=-\mu\left|\vec{F}_{N}\right|v=-\mu mR^{2}\left(\dot{\varphi}^{2}+\frac{g}{R}\cos\varphi\right)\dot{\varphi}=-mR^{2}\left(\dot{\varphi}^{2}+\frac{g}{R}\cos\varphi\right)\dot{\varphi}tg\alpha_0<0 \tag{54}$$

Power of the friction forces work during the motion of the heavy material particle alog rough circle is:

$$P_{\mu} = \frac{d\mathbf{E}}{dt} = -\left(\vec{F}_{\mu}, \vec{v}\right) = -mR^2\left(\dot{\varphi}^2 + \frac{g}{R}\cos\varphi\right)\dot{\varphi} tg\alpha_0 < 0 \tag{54*}$$

Power rate is $P_{\mu}(\varphi, \dot{\varphi})dt = -mR^2 tg\alpha_0\left(\dot{\varphi}^2 + \frac{g}{R}\cos\varphi\right)d\varphi$.

To obtain the first integral of the heavy material particle motion along rough circle line, we start with governing differential equation (24) by introducing the following change of equation coordinates $u = \dot{\varphi}^2$, it is easy to obtain the governing equation (24) in transformed form:

$$\frac{du}{d\varphi} \pm 2utg\alpha_0 = -\frac{2g}{R\cos\alpha_0}\sin(\varphi \pm \alpha_0) \quad for \quad \dot{\varphi}_{<}^{>}0 \tag{55}$$

analogous to the (8)-(12), where

$$P(\varphi) = \pm 2tg\alpha_0 = const \quad for \quad \dot{\varphi}_{<}^{>}0$$

$$Q(\varphi) = -\frac{2g}{R\cos\alpha_0}\sin(\varphi \pm \alpha_0) \quad for \quad \dot{\varphi}_{<}^{>}0 \tag{56}$$

By using previous general solution of the differential equation (55) first order with changeable coefficients, the solution can be written in the following form:

$$u(\varphi) = -\frac{2g}{R\cos\alpha_0}e^{\mp\int 2tg\alpha_0 d\varphi}\left[\int \sin(\varphi \pm \alpha_0)e^{\pm 2tg\alpha_0\int xtg\alpha_0 d\varphi}d\varphi + C\right] \tag{57}$$

Than, the found first integral of the motion described by governing differential equation (24) is in the following form:

$$\dot{\varphi}^2 = \frac{2g}{\left(1 + 4tg^2\alpha_0\right)R\cos\alpha_0}\left\langle\left[\cos(\varphi + \alpha_0) - 2tg\alpha_0\sin(\varphi + \alpha_0)\right] + \tilde{C}e^{-2\varphi tg\alpha_0}\right\rangle \quad for \quad \dot{\varphi} > 0 \tag{58}$$

$$\dot{\varphi}^2 = \frac{2g}{\left(1 + 4tg^2\alpha_0\right)R\cos\alpha_0}\left\langle\left[\cos(\varphi - \alpha_0) + 2tg\alpha_0\sin(\varphi - \alpha_0)\right] + \tilde{C}e^{2\varphi tg\alpha_0}\right\rangle \quad for \quad \dot{\varphi} < 0 \tag{59}$$

where $\tilde{C}$ is an unknown constant depending on initial conditions of the motion, as well as conditions of the continuity in the kinetic state, when friction force alternate directions with change of the velocity directions. Change of the friction force is a discontinuity expressed in the alternation of the friction force direction, and in the double alternate equilibrium position as a consequence of the discontinuity of the friction force direction.

At initial moment initial angular velocity and angular coordinate are:

$\varphi(0) = \varphi_0$ and $\dot{\varphi}(0) = \dot{\varphi}_0$, and unknown integral constants are expressed by following expression depending

$$\tilde{C}_1 = \frac{\left(1 + 4tg^2\alpha_0\right)R\cos\alpha_0}{2g}\dot{\varphi}_0^2 e^{2\varphi_0 tg\alpha_0} - \left[\cos(\varphi_0 + \alpha_0) - 2tg\alpha_0\sin(\varphi_0 + \alpha_0)\right]e^{2\varphi_0 tg\alpha_0} \quad for \quad \dot{\varphi} > 0 \tag{60}$$

$$\tilde{C}_1 = \frac{\left(1 + 4tg^2\alpha_0\right)R\cos\alpha_0}{2g}\dot{\varphi}_0^2 e^{2\varphi_0 tg\alpha_0} - \left[\cos(\varphi_0 + \alpha_0) - 2tg\alpha_0\sin(\varphi_0 + \alpha_0)\right]e^{2\varphi_0 tg\alpha_0} \quad for \quad \dot{\varphi} > 0 \tag{61}$$

First branch of the phase trajectory passing through state $(\varphi_0, \dot{\varphi}_0)$ is in the following form

$$\begin{aligned}\dot{\varphi}^2 &= \frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left[\cos(\varphi+\alpha_0)-2tg\alpha_0\sin(\varphi+\alpha_0)\right]+\\ &+\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\frac{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}{2g}\dot{\varphi}_0^2 e^{2(\varphi_0-\varphi)tg\alpha_0}-\\ &-\frac{2ge^{-2(\varphi-\varphi_0)tg\alpha_0}}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left[\cos(\varphi_0+\alpha_0)-2tg\alpha_0\sin(\varphi_0+\alpha_0)\right]\\ &for \quad \dot{\varphi}>0\end{aligned} \tag{62}$$

Force of friction change direction passing through state $(\varphi_{02}, \dot{\varphi}_{02}=0)$ when the velocity is equal to zero, and coordinate φ_{02} is defined from the following nonlinear relation:

$$\begin{aligned}&\left[\cos(\varphi_{02}+\alpha_0)-2tg\alpha_0\sin(\varphi_{02}+\alpha_0)\right]e^{2(\varphi_{02}-\varphi_0)tg\alpha_0}=\\ &\quad=\left[\cos(\varphi_0+\alpha_0)-2tg\alpha_0\sin(\varphi_0+\alpha_0)\right]-\\ &\quad-\frac{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}{2g}\dot{\varphi}_0^2.. \quad for \quad \dot{\varphi}_{02}=0\end{aligned} \tag{63}$$

Initial conditions $\widetilde{C}_2$ for the next trajectory branch passing through state $(\varphi_{02}, \dot{\varphi}_{02}=0)$ and $\widetilde{C}_2$ is defined by following expression:

$$\widetilde{C}_2=-\left[\cos(\varphi_{02}-\alpha_0)+2tg\alpha_0\sin(\varphi_{02}-\alpha_0)\right]e^{-2\varphi_{02}tg\alpha_0} \quad for \quad \dot{\varphi}<0 \tag{64}$$

Finally, the expression of the second branch of the phase trajectory passing through state $(\varphi_{02}, \dot{\varphi}_{02}=0)$ is in the following form:

$$\begin{aligned}\dot{\varphi}^2 &= \frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left\langle\left[\cos(\varphi-\alpha_0)+2tg\alpha_0\sin(\varphi-\alpha_0)\right]\right\rangle-\\ &-\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left\langle\left[\cos(\varphi_{02}-\alpha_0)+2tg\alpha_0\sin(\varphi_{02}-\alpha_0)\right]e^{-2(\varphi_{02}-\varphi)tg\alpha_0}\right\rangle\\ &for \quad \dot{\varphi}<0\end{aligned} \tag{65}$$

Force of friction change direction passing through state $(\varphi_{03}, \dot{\varphi}_{03}=0)$ when the velocity is equal to zero and φ_{03} is defined by following expression:

$$\begin{aligned}&\left[\cos(\varphi_{03}-\alpha_0)+2tg\alpha_0\sin(\varphi_{03}-\alpha_0)\right]e^{2(\varphi_{02}-\varphi_{03})tg\alpha_0}=\\ &\quad=\left[\cos(\varphi_{02}-\alpha_0)+2tg\alpha_0\sin(\varphi_{02}-\alpha_0)\right] \quad for \quad \dot{\varphi}=0\end{aligned} \tag{66}$$

Initial conditions $\widetilde{C}_3$ for the next trajectory branch passing through state $(\varphi_{03}, \dot{\varphi}_{03}=0)$

$$\widetilde{C}_3=-\left[\cos(\varphi_{03}+\alpha_0)-2tg\alpha_0\sin(\varphi_{03}+\alpha_0)\right]e^{2\varphi_{03}tg\alpha_0}, \quad for \quad \dot{\varphi}>0 \tag{67}$$

Finally expression of the third branch of the phase trajectory passing through state $(\varphi_{03}, \dot{\varphi}_{03}=0)$ is in the following form:

$$\begin{aligned}\dot{\varphi}^2 &= \frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left[\cos(\varphi+\alpha_0)-2tg\alpha_0\sin(\varphi+\alpha_0)\right]-\\ &-\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left[\cos(\varphi_{03}+\alpha_0)-2tg\alpha_0\sin(\varphi_{03}+\alpha_0)\right]e^{-2(\varphi-\varphi_{03})tg\alpha_0}\\ &for \quad \dot{\varphi}>0\end{aligned} \tag{68}$$

Force of friction change direction passing through state $(\varphi_{04}, \dot{\varphi}_{04} = 0)$ when the velocity is equal to zero and for obtain φ_{04}, the corresponding relations take the following equating

$$\begin{aligned}&[\cos(\varphi_{04} + \alpha_0) - 2tg\alpha_0 \sin(\varphi_{04} + \alpha_0)]e^{2(\varphi_{04} - \varphi_{03})tg\alpha_0} = \\ &\qquad = [\cos(\varphi_{03} + \alpha_0) - 2tg\alpha_0 \sin(\varphi_{03} + \alpha_0)] \quad for \quad \dot{\varphi} > 0\end{aligned} \tag{69}$$

Initial conditions $\widetilde{C}_4$ for the next trajectory branch passing through state $(\varphi_{04}, \dot{\varphi}_{04} = 0)$

$$\widetilde{C}_4 = -[\cos(\varphi_{04} - \alpha_0) + 2tg\alpha_0 \sin(\varphi_{04} - \alpha_0)]e^{-2\varphi_{04}tg\alpha_0} \quad for \quad \dot{\varphi} < 0 \tag{70}$$

Finally expression of the fourth branch of the phase trajectory passing through state $(\varphi_{04}, \dot{\varphi}_{04} = 0)$ is in the following form:

$$\begin{aligned}\dot{\varphi}^2 &= \frac{2g}{(1 + 4tg^2\alpha_0)R\cos\alpha_0}[\cos(\varphi - \alpha_0) + 2tg\alpha_0 \sin(\varphi - \alpha_0)] - \\ &- \frac{2g}{(1 + 4tg^2\alpha_0)R\cos\alpha_0}[\cos(\varphi_{04} - \alpha_0) + 2tg\alpha_0 \sin(\varphi_{04} - \alpha_0)]e^{2(\varphi - \varphi_{04})tg\alpha_0} \\ &for \quad \dot{\varphi} < 0\end{aligned} \tag{71}$$

Force of friction change direction passing through state $(\varphi_{05}, \dot{\varphi}_{05} = 0)$ when the velocity is equal to zero and for obtain φ_{05}, the corresponding relation take the following equating

$$\begin{aligned}&[\cos(\varphi_{05} - \alpha_0) + 2tg\alpha_0 \sin(\varphi_{05} - \alpha_0)]e^{-2(\varphi_{05} - \varphi_{04})tg\alpha_0} = \\ &\qquad = [\cos(\varphi_{04} - \alpha_0) + 2tg\alpha_0 \sin(\varphi_{04} - \alpha_0)] \quad for \quad \dot{\varphi} < 0\end{aligned} \tag{72}$$

Finally expression of the fifth branch of the phase trajectory passing through state $(\varphi_{05}, \dot{\varphi}_{05} = 0)$ is in the following form:

$$\begin{aligned}\dot{\varphi}^2 &= \frac{2g}{(1 + 4tg^2\alpha_0)R\cos\alpha_0}\langle[\cos(\varphi + \alpha_0) - 2tg\alpha_0 \sin(\varphi + \alpha_0)]\rangle \\ &- \frac{2g}{(1 + 4tg^2\alpha_0)R\cos\alpha_0}[\cos(\varphi_{05} + \alpha_0) - 2tg\alpha_0 \sin(\varphi_{05} + \alpha_0)]e^{-2(\varphi - \varphi_{05})tg\alpha_0} \\ &for \quad \dot{\varphi} > 0\end{aligned} \tag{73}$$

It is necessary to estimate previous expression for each of the obtained branch of the phase trajectory with corresponding constants of integration $\widetilde{C}_k, k = 1,2,3,\ldots,n$; where n is number of semi periods of the material particle vibrations along rough circle line (with frictions).

Constant of integration of the next expression of the branch of phase trajectories are defined by following expressions:

$$\widetilde{C}_{2k} = -\left[\cos\left(\varphi_{0(2k)} - \alpha_0\right) + 2tg\alpha_0 \sin\left(\varphi_{0(2k)} - \alpha_0\right)\right]e^{-2\varphi_{0(2k)}tg\alpha_0} \quad for \quad \dot{\varphi} < 0 \tag{74}$$

$$\widetilde{C}_{(2k+1)} = -\left[\cos\left(\varphi_{0(2k+1)} + \alpha_0\right) - 2tg\alpha_0 \sin\left(\varphi_{0(2k+1)} + \alpha_0\right)\right]e^{2\varphi_{0(2k+1)}tg\alpha_0} \quad for \quad \dot{\varphi} > 0 \tag{75}$$

Even trajectory branch - unknown kinetic state point trajectory branch continuity $\left(\varphi_{0(2k)} = ?, \dot{\varphi}_{0(2k)} = 0\right)$

$$\begin{aligned}&\left[\cos\left(\varphi_{0(2k)} + \alpha_0\right) - 2tg\alpha_0 \sin\left(\varphi_{0(2k)} + \alpha_0\right)\right]e^{2\left(\varphi_{0(2k)} - \varphi_{0(2k-1)}\right)tg\alpha_0} = \\ &\qquad = \left[\cos\left(\varphi_{0(2k-1)} + \alpha_0\right) - 2tg\alpha_0 \sin\left(\varphi_{0(2k-1)} + \alpha_0\right)\right] \quad for \quad \dot{\varphi} > 0\end{aligned} \tag{76}$$

Add trajectory branch - - unknown kinetic state point trajectory branch continuity $\left(\varphi_{0(2k)-1} = ?, \dot{\varphi}_{0(2k-1)} = 0\right)$

$$\left[\cos\left(\varphi_{0(2k+1)}-\alpha_0\right)+2tg\alpha_0\sin\left(\varphi_{0(2k+1)}-\alpha_0\right)\right]e^{-2\left(\varphi_{0(2k+1)}-\varphi_{0(2k)}\right)tg\alpha_0}=$$
$$=\left[\cos\left(\varphi_{0(2k)}-\alpha_0\right)+2tg\alpha_0\sin\left(\varphi_{0(2k)}-\alpha_0\right)\right]\quad for\quad \dot{\varphi}<0 \tag{77}$$

2k* Even trajectory branch

$$\dot{\varphi}^2=\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left[\cos(\varphi-\alpha_0)+2tg\alpha_0\sin(\varphi-\alpha_0)\right]-$$
$$-\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left[\cos\left(\varphi_{0(2k)}-\alpha_0\right)+2tg\alpha_0\sin\left(\varphi_{(2k)0}-\alpha_0\right)\right]e^{2\left(\varphi-\varphi_{0(2k)}\right)tg\alpha_0} \tag{78}$$
$$for\quad \dot{\varphi}<0$$

2k+1* Add trajectory branch

$$\dot{\varphi}^2=\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left\langle\left[\cos(\varphi+\alpha_0)-2tg\alpha_0\sin(\varphi+\alpha_0)\right]\right\rangle$$
$$-\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left[\cos\left(\varphi_{0(2k+1)}+\alpha_0\right)-2tg\alpha_0\sin\left(\varphi_{0(2k+1)}+\alpha_0\right)\right]e^{-2\left(\varphi-\varphi_{0(2k+1)}\right)tg\alpha_0} \tag{79}$$
$$for\quad \dot{\varphi}>0$$

Angles at which velocity is equal to zero and at which force of friction change direction and analytical expression for the phase trajectory is new, can be found by graphics of the functions and zero values of the function.

Power of the work of the friction force is:

$$P_\mu(\varphi,\dot{\varphi})=$$
$$=-mR^2tg\alpha_0\left(\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left\langle\left[\cos(\varphi\pm\alpha_0)\mp 2tg\alpha_0\sin(\varphi\pm\alpha_0)\right]+\tilde{C}e^{\mp 2\varphi tg\alpha_0}\right\rangle+\frac{g}{R}\cos\varphi\right) \tag{80}$$

Work of the friction force is: $\mathbf{A}_{\mathbf{F}_\mu}=\int_0^t P_\mu(\varphi,\dot{\varphi})dt$ or in developed form:

$$\mathbf{A}_{\mathbf{F}_\mu}=-\frac{2mgRtg\alpha_0}{\left(1+4tg^2\alpha_0\right)\cos\alpha_0}\int_{\varphi_0}^{\varphi}\left\langle\left[\cos(\varphi\pm\alpha_0)\mp 2tg\alpha_0\sin(\varphi\pm\alpha_0)\right]+\tilde{C}e^{\mp 2\varphi tg\alpha_0}\right\rangle d\varphi-mgRtg\alpha_0\int_{\varphi_0}^{\varphi}\cos\varphi d\varphi$$

3.4 Condition of the heavy material particle diverb of the motion

When can the force of friction have greater values then the resultant of the active forces? Then, material particles stay in rest. Material particle is loaded by proper weight and tangential component force and by the force of inertia break motion at moment when value of the force of friction starts to be greater then this previous resultant of the active forces in the tangential direction.

Then necessary condition for material particle diverb of moving along rough circle line is that value of the friction force satisfies the following condition:

$$\left|\mu F_N\right|<mg\sin\varphi$$

or

$$\left|F_\mu(\varphi,\dot{\varphi})\right|=\left|mR^2tg\alpha_0\left(\dot{\varphi}^2+\frac{g}{R}\cos\varphi\right)\right|<\left|mg\sin\varphi\right| \tag{81}$$

or

$$\left|R^2 tg\alpha_0\left(\dot{\varphi}^2+\frac{g}{R}\cos\varphi\right)\right| < |g\sin\varphi|$$

$$\left|R^2 tg\alpha_0\left(\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left\langle\left[\cos(\varphi\pm\alpha_0)\mp 2tg\alpha_0\sin(\varphi\pm\alpha_0)\right]+\tilde{C}e^{\mp 2\varphi tg\alpha_0}\right\rangle+\frac{g}{R}\cos\varphi\right)\right| < |g\sin\varphi|$$

4 FREE VIBRATIONS OF THE HEAVZ MATERIAL PARTICLE ALONG ROGH CICLOID LINE WITH COULOMB FRICTION

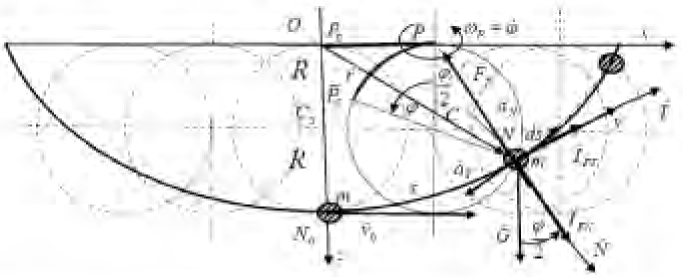


Figure 3. Heavy material point oscillations along rough cycloid with coefficient of the sliding friction μ.

Let us consider the motion of the heavy material particle, mass m along rough cichloid (see Figure 3.) non ideal constraint line, with coefficient of the sliding friction μ, as a special case of the general theory presented in the Chapter 2.

It is necessary to choose one generalized coordinate of the material particle position, and we can take the lent s, a curvilinear coordinate, starting from equilibrium position N_0 anld continuing along material particle cycloid path to the termination position at point N at cyclod (see Figure 3.). Also, by the reason for some easier calculations and relations between parametric equations of the cyclod path, $x=R(\varphi+\sin\varphi)$ and $z=R(1+\cos\varphi)$, we can and must use parameter φ (or coordinate position of the rolling circle with diameter R, describing by a point on the circle line pure geometrically cicloide path).

For the case of the heavy material particle motion along rough cicloid line differential equation is in the following form:

$$\ddot{\varphi}-\dot{\varphi}^2\left(\frac{1}{2}tg\frac{\varphi}{2}\mp\mu\right)+\left(tg\frac{\varphi}{2}\pm\mu\right)\frac{g}{2R}=0\begin{cases} for \quad v=2R\cos\frac{\varphi}{2}\dot{\varphi}>0\\ for \quad v=2R\cos\frac{\varphi}{2}\dot{\varphi}<0\end{cases} \tag{82}$$

where $s(\varphi)=4R\sin\frac{\varphi}{2}$ is lenght of cicloid line, μ coefficient of the sliding friction and equation of the phase trajectory is in the form:

$$\dot{\varphi}^2=\frac{\left(\frac{g}{2R}\right)}{1+4\mu^2}\frac{1}{\cos^2\frac{\varphi}{2}}\left[(\mp 3\mu)\sin\varphi-\left(1-2\mu^2\right)\cos\varphi+\mu\frac{1+4\mu^2}{2}+Ce^{\mp 2\mu\varphi}\right]\begin{cases} for \quad v=2R\cos\frac{\varphi}{2}\dot{\varphi}>0\\ for \quad v=2R\cos\frac{\varphi}{2}\dot{\varphi}<0\end{cases} \tag{83}$$

with integral constant depending on initial conditions, and changing from first branch of the phase trajectory to the next in the series with alternations of the sign depending on the material particle velocity direction in the motion and corresponding alternation in the direction of the Coulomb friction.

For this case of heavy material particle motion along the rough cicloid line we can identify a member in the differential equation proportional to the square of the generalized coordinate (or parameter) derivation with respect to time (generaliyed velocity) by which a differential equation of the motion is expressed. This corresponds to the known case of turbulent damping. Also, forced vibrations are considered, as well as vibrations of the line.

Bifurcation of the equilibrium position $s = 0$ appears under the influence of the force of friction during the material particle motion along the no ideal, rough cycloid, with coefficient of the sliding friction $\mu = tg\alpha_0$. One position of the stable equilibrium state disappear, and, as a result of the bifurcation caused by non ideal constraint of type cycloid, two new and alternate equilibrium positions $s(\varphi_s) = s_s = 4R\sin\frac{\varphi_s}{2} = \mp 4R\sin\alpha_0$ appear as a set of the double equilibrium positions with property of alternation. We can see that a special case of a trigger of singularities appear with force of slayding friction. A discontinuity is special case of nonlinearity and alternations of the sliding force direction with directio of the velocity. Jumps of the force friction ditrvtion is present as a soft impact in the system.

5 FREE VIBRATIONS OF THE HEAVZ MATERIAL PARTICLE ALONG ROGH PARABOLA LINE WITH COULOMB FRICTION

Let us consider motion of the material particle, mass m along rough parabola line which equation is $y = 4px^2$, where $p\left[m^{-1}\right]$ parameter of dimension compatibility, and with coefficient of the sliding friction $\mu = tg\gamma_0$.

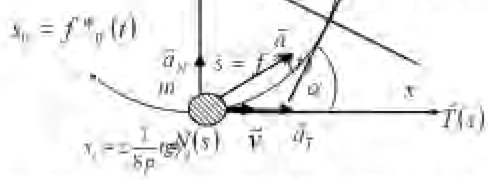


Figure 4. Heavy material point oscillations along rough parabola with coefficient of the sliding friction $\mu = tg\gamma_0$

For the case of the heavy material particle motion along rough parabola line differential equation is in the following form:

$$\frac{d}{dt}\left(\dot{x}\sqrt{1+64p^2x^2}\right)+g\frac{8px}{\sqrt{1+64p^2x^2}}\pm\mu\frac{1}{\sqrt{1+64p^2x^2}}\left(8p\dot{x}^2-g\right)=0 \qquad (84)$$

Bifurcation of the equilibrium position $s=0$, $x=0$, $y=0$ appear under the influence of the force of friction during the material particle motion along the no ideal, rough parabola, with coefficient of the sliding friction $\mu=tg\gamma_0$. One position of the stable equilibrium state disappear, and, as a result of the bifurcation is caused by no ideal constrain of type parabola, two new and alternate equilibrium positions $x_s=\pm\frac{1}{8p}tg\gamma_0$, $y_s=\frac{tg^2\gamma_0}{16p}$ or $\alpha=\pm\gamma_0$, appear as a set of the double equilibrium positions with the property of alternation. Taking into acount following coordinate transformation $u=\alpha^2$, we obtain the transformed governing equation in the form (8) or (8*) expressed by:

$$u'-2u\left(3tg\alpha\mp tg\gamma_0\right)=-2pg\left(2\sin 2\alpha+\sin 4\alpha\pm 2tg\gamma_0\left(3+4\cos 2\alpha+\cos 4\alpha\right)\right) \qquad (85)$$

where for considered case, the coefficients of the differential equation are

$$P(\alpha)=-2\left(3tg\alpha\mp tg\gamma_0\right)\quad\begin{cases} for & v>0\\ for & v<0\end{cases} \qquad (86)$$

$$Q(\varphi)=-2pg\left(2\sin 2\alpha+\sin 4\alpha\pm 2th\gamma_0\left(3+4\cos 2\alpha+\cos 4\alpha\right)\right)\quad\begin{cases} for & v>0\\ for & v<0\end{cases} \qquad (97)$$

Components of the no ideal parabola line reaction are: normal to the parabola line and friction force are:

$$F_N=\frac{m}{\sqrt{1+64p^2x^2}}\left(8p\dot{x}^2+g\right)\ \text{and}\ F_\mu=-\mu\frac{m}{\sqrt{1+64p^2x^2}}\left(8p\dot{x}^2-g\right)sign\bar{v} \qquad (87)$$

6 DIRECT AND INVERSE TASK OF THE THEORY OF VIVRORHEOLOGY

In this part we considered two approaches to describing and solving problems in the vibrorheological properties and how it is possible to use it in the engineering practice. We can take into acount two new tasks. One task is when a multifrequency external excitation is applied to the heavy material particle moving along rough curviline. Second task is when the heavy material particle is relatively moving along vibrating rough curviline line kinematically excited by multifrequency motion. In both cases we can identify qualitatively new types of vibration.

7 SMART STRUCTURES ARE BUILT BY INCLUSION IN THE FORM OF THE HEAVY MATERIAL PARTICLE VIBRATION ALONG LINE WITH FRICTION.

By inclusion in the continuum material some elements such as heavy material particle vibration along rough line corresponding form can be used to build new kinds of materials for engineering systems. Nonlinear phenomena and alternations in the „position of stability" depending on the coefficient of sliding friction μ into continuum are sources of new construction possibilities and a new continuum model for investigations as well as a new technology method.

8 CONCLUSIONS

For all three cases of rough line we can identify a member in the differential equation proportional to the square of the generalized coordinate derivation with respect to time (or parameter) by which a differential equation of the motion is expressed. This corresponds to the known case of turbulent damping. Also, forced vibrations are considered, as well as vibrations of the line.

Change of the friction force is a discontinuity expressed in the alternation of the friction force direction, and in the double alternate equilibrium position as a consequence of the discontinuity of the friction force direction.

Bifurcation of the equilibrium position at zero point, $x=0$, or $\varphi=0$, or $s=0$, appears under the influence of the force of friction during the material particle motion along the no ideal, rough curvelinear line, circle, as well as along cycloid, or Parabola line, with coefficient of the Coulomb sliding friction $\mu = tg\alpha_0$. One position of the stable equilibrium state disappear, and, as a result of the bifurcation is caused by no ideal constraint of arbitrary type of rough line, curvelinear line, circle line, cycloid, parabola line, e.g., two new and alternate equilibrium positions. For circle $\varphi=\pm\alpha_0$, for cycloid $s(\varphi_s)=s_s=\mp 4R\sin\alpha_0$, for parabola line $x_s=\pm\dfrac{1}{8p}tg\gamma_0$, e.g., appear as a set of the double equilibrium positions with property of the alternation.

We can see that a special case of the trigger of the singularities appears with force of slayding friction. A discontinuity is a special case of nonlinearity and alternations of the sliding force direction with direction of the velocity. Jumps of the force friction ditrvtion is present as a soft impact in the system.

9 ACKNOWLEDGMENT

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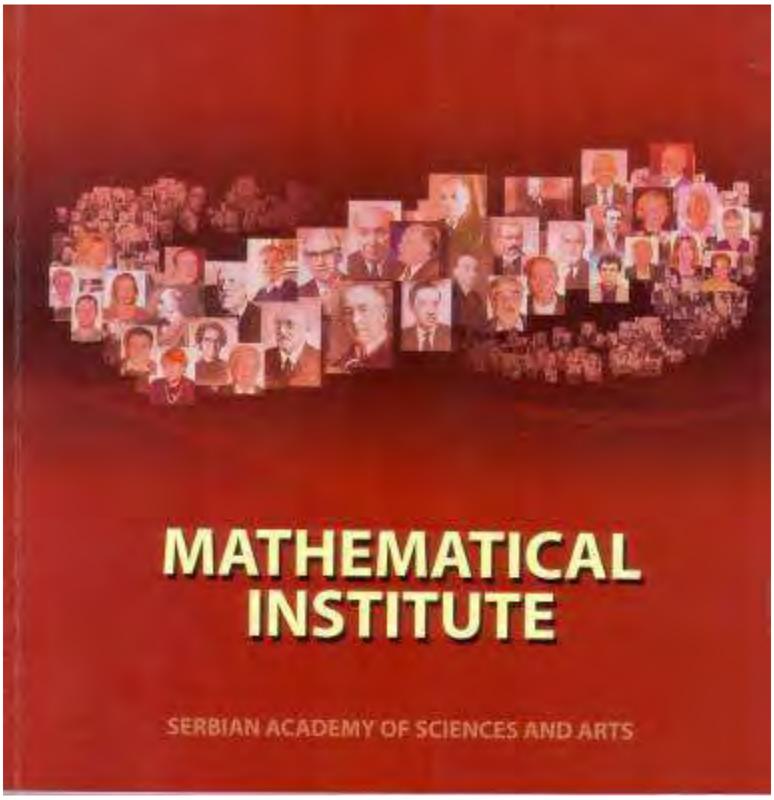
Appendix



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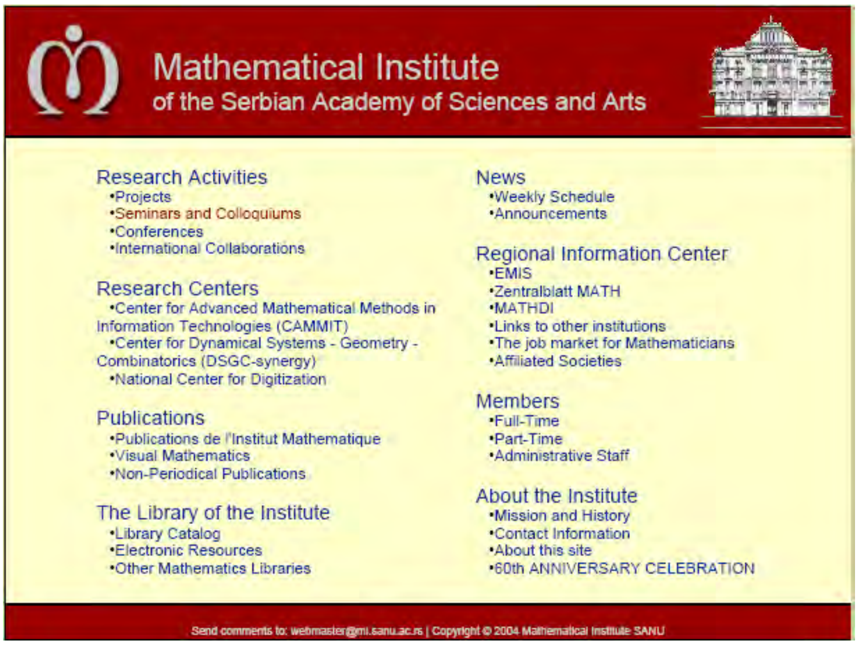
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Fundamental Research | Technological Development | Interdisciplinary | Past Projects

Fundamental Research

PROJECT 144002

Theoretical and Applied Mechanics of the Rigid and Solid Bodies. Mechanics of Materials (Problemi teorijske i tehnicke mehanike krutih i cvrstih tela. Mehanika materijala)
Project leader: ***Dr. Katica Hedrih*** katica@masfak.ni.ac.yu
Project Description (English | Serbian) List of Researchers
Recent results

Fundamental Research (2006-2010)

Theoretical and Applied Mechanics of the Rigid and Solid Bodies. Mechanics of Materials *(Project 144002)*

Today, it seems generally accepted that nonlinear dynamical problems should be cooperatively addressed through the combined use of analytical, computational, geometrical and experimental approach. Moreover, the interaction between nonlinear dynamics and control plays an important role in advanced engineering systems in order to obtain desired dynamics behavior and improved reliability during operation. The nonlinear deterministic and stochastic dynamics and control of processes in complex mechanical systems are subject of our project research.

Project Leader: Katica (Stevanović) Hedrih

New mathematical and phenomenological knowledge will be an advance to theoretical and applied mechanics of the rigid and solid bodies and complex hybrid structures and dynamical systems with applications in mechanical engineering. Mechanics of materials with coupled fields is also subject of project research. The possibilities for the development of control laws, which will enable desired behavior of the active materials and structures will be considered as well. Advances in the form of mathematical description of deterministic and stochastic dynamics of complex hybrid systems with coupled rigid and solid bodies by standard light elements and dynamic constraints are planed as research results.

Proposed research is actual in the world scientific community, and is important for both advances to the mathematical theory of mechanics and for applications in engineering. Research will be directed to the field of theoretical and applied mechanics and realized by more then 40 researchers.

Seminar for *Nonlinear Dynamics – Milutin Milanković* was spontaneously organized by the Project researchers as a forum for scientific research results evaluations and is working at Faculty of Mechanical Engineering in Niš suported by researchers from other Serbian Universities and by visiting professors from all the World.

Home page of the Project:
http://www.mi.sanu.ac.yu/projects/projects.htm
Home page of the Projet Seminar:
http://www.masfak.ni.ac.yu/sitegenius/topic.php?id=863

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Chair of Mechanics Faculty of Mechanical Engineering Niš (1963-2005)

dr Ing. Dipl. Math. Danilo P. Rašković

(1910-1985)

The First Head of Chair of Mechanics and Automatic at Faculty of Mechanical Engineering in Niš
(1963-1974)

Prof., Dr., Eng., B.Sc. Mathematician,
DANILO P. RAŠKOVIĆ,
full-professor at the Faculties of Mechanical Engineering in Belgrade, Niš, Kragujevac and Mostar, and the Faculties of Science in Belgrade and Novi Sad

Danilo Rašković, a doctor of technical sciences and mathematician with a university degree, was the founder of the first scientifically based courses of mechanics at the Faculty of Mechanical Engineering in Belgrade. He also introduced courses on the subject of resistance of material, elasticity theory, and oscillation theory all of which he taught, too. He was the author of many high-circulation textbooks of high scientific level and good mathematical foundation. He introduced vector, matrix and tensor calculus in the studies of mechanics at the Faculty of Mechanical Engineering in Belgrade and, later on, did the same at the mechanical engineering faculties in Niš, Kragujevac and Mostar. He enabled the Faculty in Belgrade, and similar schools elsewhere, to produce highly qualified and educated engineers which was one his greatest contributions. He wrote the first university textbook in Serbia on oscillation theory containing his original accomplishments in the field. He achieved considerable scientific results in the fields of elasticity theory and oscillation theory. With a good human resource base at Niš Faculty, which he had set up, he started research work into the field of nonlinear mechanics. His scientific work is important because in all of his projects he succeeded in connecting theories of elasticity and oscillation, and engineering practice. He wrote 25 university textbooks which covered the entire field of mechanics and related areas. Almost all of them had been reprinted several times, with some of them having 20 reprints. His excellent textbooks were in use on the territory of the entire former Yugoslavia, which was in tatters under the powerful influence of fascism during the Second World War.

Thanks to Professor Danilo Rašković, the faculties of mechanical engineering of Serbia, Bosnia and Herzegovina, and all the other republics of the once unified Yugoslavia, which are now separate states, produced excellent mechanical engineers. Rašković was a patriot and an honourable man. He was the recipient of the October award of the city of Niš for his contributions to the development of science at the city's university.

This distinguished scientific figure of exquisite creative energy and inspired enthusiasm, a scholar deeply attached to the Yugoslav and Serbian scientific and cultural heritage, and an exquisite pedagogue of high moral principles is in the living memory of many generations of students whom he taught how to learn and love mechanics, as a basic scientific branch of mechanical engineering either directly, through his lectures, or through his various and numerous textbooks and compilation of problems. His disciples and colleagues are glad that he had the ability to pass onto them his great enthusiasm permeated with his sincere devotion for mechanics and his exquisite scientific eagerness.

Professor Danilo P. Rašković was born in 1910, in Užice. Upon completing elementary school and six grades of high school, he graduated from the Military Academy in 1930. As an engineering military officer he enrolled in the department of mechanical and electrical engineering at the Faculty of Engineering in Belgrade, in 1933. Having graduated in 1938, he enrolled in the department of theoretical mathematics at the Faculty of Philosophy and graduated from it in 1941. As a graduate mechanical engineer he was appointed assistant section head of the Military Technical Institute in Čačak. He remained in that position during 1941. In 1942 he was appointed assistant at the Faculty of Engineering in Belgrade where he earned hid doctorate's degree in the same year, upon presenting his thesis entitled *Tangential Strains of Normally Profiled Beams*.

Professor Rašković lectured mechanics, strains of materials and oscillation theory at the faculties of mechanical engineering in Belgrade, Niš, Kragujevac, Novi Sad and Mostar, as well as at the Faculty of Science in Belgrade, Faculty of Philosophy in Novi Sad, Faculty of Electronics in Niš, and at the Military Technical College in Belgrade. More details on the research work of Professor Rašković can be found in the Belgrade University Bulletin no.75 of 1957, issued on the occasion of his appointment as a full professor at the Faculty of Mechanical Engineering in Belgrade. During his university career, he was twice elected Vice-Dean of the Faculty of Mechanical Engineering of Belgrade University. In the mechanical engineering department at the Faculty of Engineering in Niš, he lectured statistics, kinetics, kinematics, dynamics, oscillation theory, resistance of material, theory of elasticity, as well as analytical mechanics, theory of nonlinear oscillations and continuum mechanics at the postgraduate level. He was the first head of the department of mechanics and automatics at the Faculty of Mechanical Engineering in Niš. He was an extremely inspired professor, scientist and practitioner much favoured among his students and respected by his colleagues both as a professor and an engineer, because he knew how to relate engineering theory to practice.

Professor Rašković was a very fertile writer. While still in the military service he wrote five professional papers. In the period before 1957, when he was appointed full professor, he published 26 scholarly papers. As a full professor he wrote 37 pieces of scientific work that were published in scientific journals of the Serbian Academy of Sciences and Arts, Polish Academy of Science, German Society of Mechanics ZAMM and some other foreign journals. He took part in a number of scientific meetings in the country and abroad. He reviewed papers for four leading referral journals in the world: *Applied Mechanics Review* (USA), *Mathematical Review* (USA), *Zentralblatt für Mathematik* (Germany) and *Referativnii žurnal* (Moscow). Professor Rašković was a member of several professional and scientific societies/association in the country and abroad, the GAMM being one of them. He initiated the foundation of the Yugoslav Society of Mechanics during 1952.

He wrote a considerable number of university textbooks which ran through numerous editions. Some of them still hold records as for the number of editions and copies printed within the group they belong to. In addition, he wrote a series of textbooks on the subject of mechanics for secondary technical schools, as well as a number of chapters in professional technical handbooks, mimeographed course materials and textbooks for post-secondary schools of mechanical engineering. He also wrote several textbooks for postgraduate studies.

Among the publications for postgraduate studies the following should be mentioned: *Analytical Mechanics, Theory of Elasticity and Tensor Calculus.*

Most of his university textbooks and publications were at the time of their first edition the only professional literature on the subject, in the Serbian language. So, his publications played an important part in spreading of the knowledge in the field of technical mechanics among students, and mechanical and other kinds of engineers in Serbia and Yugoslavia. It is particularly worth mentioning that he has interpreted all the material by the most modern mathematical apparatus and has illustrated it by numerous examples from the engineering practice. Many of the cited university publications are being reprinted even nowadays and are still used by both students of engineering and engineers themselves.

Although it has been ten years since he left us, Professor Rašković is still present among new generations of students, and engineers, through his renowned textbooks that bear the memory of his merits and which have also left an indelible imprint on the development of mechanical engineering science and practice, and on the formation of many a generation of university professors. His life and work have set an example to future generations of students educated at the University of Niš and provided them with a creative impulse. He is an everlasting paradigm and a proof of how one's deeds can outlive one's physical existence by far.

In 1962 Professor Rašković, **as the head of mechanics department at the Institute of Mathematics of the Serbian Academy of Sciences and Arts**, organized research work in four different study groups, each one dealing with a particular subject, which were: ***Stability of motion*** - supervised by Dr Veljko Vujičić, ***Boundary layer theory*** - supervised by Dr Victor Saljnikov, ***Problems of anisotropic incompatible materials with finite strain*** - supervised by Dr Rastko Stojanović and ***Optimal problems of mechanics*** -supervised by Prof. Dr. Danilo Rašković.

According to records from the mechanical engineering faculties in Belgrade and Niš, as well as those from the Zentralblatt's data base, he traveled abroad on several occasions in order to participate in international scientific gatherings or to expend his knowledge. In 1957 he went to Berlin to do his specialization studies with a piece of work which was published in the *Proceedings of the 20th International Congress of Applied Mechanics*. In September 1956, in Brussels, he participated in the working of the said congress. He took part in international congresses of applied mathematics and mechanics of the German society GAMM a few times: 1957 - in Hamburg and 1958 - in Saarbrücken. Also, in 1959, 1961 and 1962 he was delegate of the Yugoslav Society of Mechanics. In 1963, in Karlsruhe, he represented Mathematical Institute of the Serbian Academy of Sciences. In 1966, in Darmstadt, he "*produced a scientific statement in the field of oscillation theory"* and in 1968 in Prague, Czechoslovakia, he had a paper entitled *Second order acceleration (jerk) for the relative motion of a body expressed by a matrix method*.

He also participated, several times, in the working of the International Conference of Nonlinear Oscillation (ICNO): 1962 in Warsaw, as a delegate of the Council of Science of the People's Republic of Serbia; 1969 in Kiev; 1972 in Krakow, at the '72 ICNO.

Between the 1963/64 and 1973/74 academic years he was Head of the mechanics section of the mechanical engineering department at the Technical Faculty in Niš, while giving lectures on all subjects from the mechanics group. Simultaneously, he taught mechanics at technical faculties in Kragujevac and Mostar and, for a while, also the subject of applied mathematics at Novi Sad Faculty of Mathematics. He accepted the position in Niš after being acquitted of the duty as a lecturer at the Faculty of Mechanical Engineering in Belgrade. The said acquittal was brought in by the Faculty in Belgrade, and was registered under the no. 67/8, in January 1964. Comments on the controversial decision are left to the others. For further reference readers should look into the book (*).

In 1974/75 he was arrested in Mostar, Bosnia-Herzegovina, and unjustly sentenced. Following the experience, he worked on new editions of his high-circulation textbooks, out of which the 10th edition of *Mechanics I* for university studies deserves a special mention as does the 15th edition of his handbook containing tables from the strength of materials. Last months of his life he spent preparing his textbook *Elasticity Theory* for publishing. It came out in 1985 but he did not live to see it.

He died, unexpectedly, on January 29, 1985 in Belgrade.

Seminar of Theoretical and Applied Mechanics of
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