

Petak, 21 mart 2014 godine

Srminar mehanke

Odeljenja za mehaniku i Odeljenja za matematiku

Mathematical Institute SANU Belgrade

**Grant OI174001**

**“Dynamics of hybrid systems with  
complex structures. Mechanics of  
Materials”**

**Report for 2013**

**Катица Р. (Стевановић) Хедрић**

Одељење за механику Математичког института САНУ у Београду  
и Машински факултет Универзитета у Нишу

Прив. адреса: 18000- Ниш, Србија, ул. Војводе Танкосића 3/22.

e-mail: [khedrih@eunet.rs](mailto:khedrih@eunet.rs)





Mathematical  
Institute  
SANU



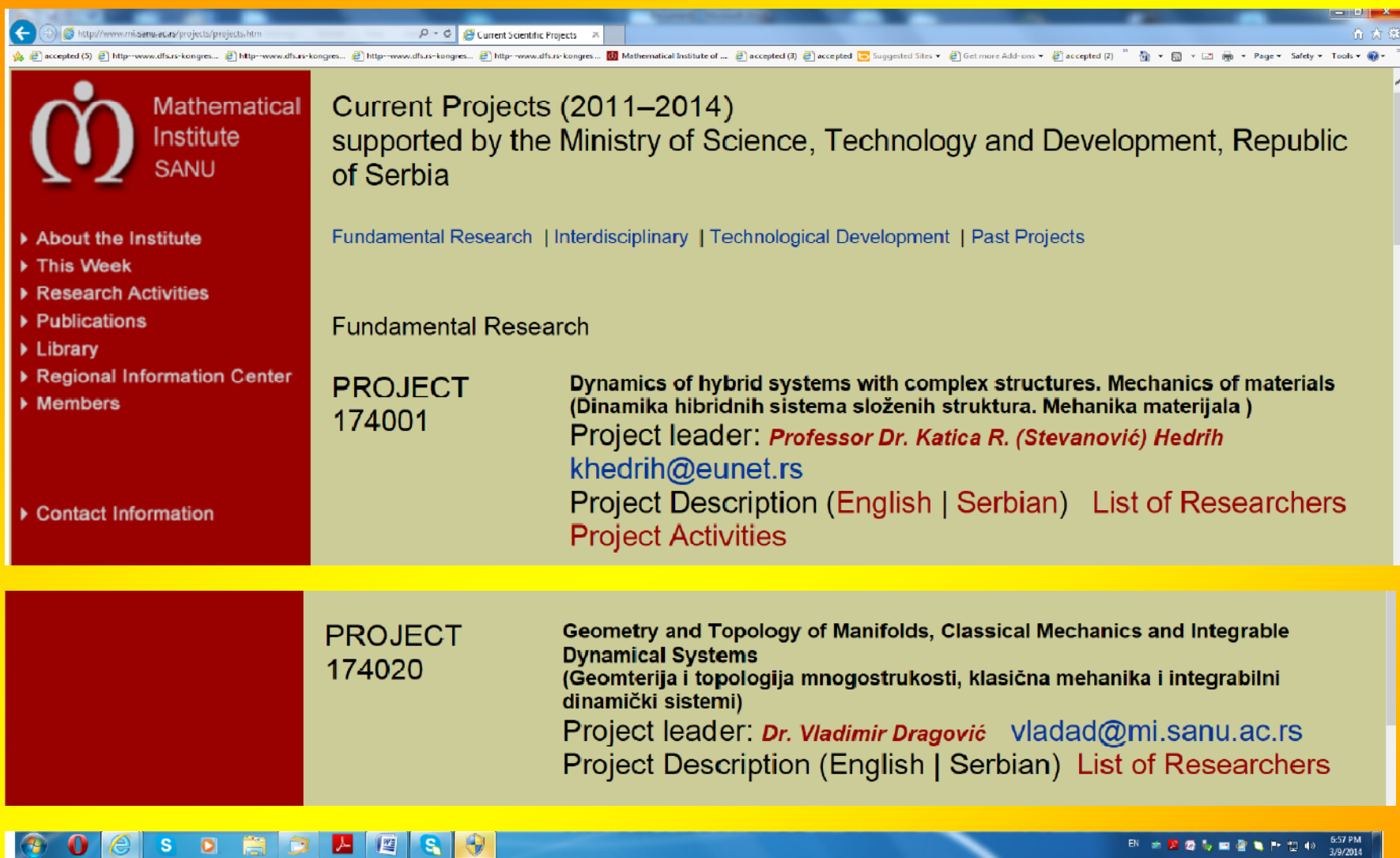
# MATHEMATICAL INSTITUTE

SERBIAN ACADEMY OF SCIENCES AND ARTS



Mathematical  
Institute  
SANU





**Mathematical Institute SANU**

- ▶ About the Institute
- ▶ This Week
- ▶ Research Activities
- ▶ Publications
- ▶ Library
- ▶ Regional Information Center
- ▶ Members
- ▶ Contact Information

## Current Projects (2011–2014)

supported by the Ministry of Science, Technology and Development, Republic of Serbia

[Fundamental Research](#) | [Interdisciplinary](#) | [Technological Development](#) | [Past Projects](#)

### Fundamental Research

|                              |                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>PROJECT 174001</b></p> | <p>Dynamics of hybrid systems with complex structures. Mechanics of materials (Dinamika hibridnih sistema slozenih struktura. Mehanika materijala)</p> <p>Project leader: <b>Professor Dr. Katica R. (Stevanović) Hedrih</b><br/> <a href="mailto:khedrih@eunet.rs">khedrih@eunet.rs</a></p> <p><a href="#">Project Description (English   Serbian)</a> <a href="#">List of Researchers</a><br/> <a href="#">Project Activities</a></p> |
| <p><b>PROJECT 174020</b></p> | <p>Geometry and Topology of Manifolds, Classical Mechanics and Integrable Dynamical Systems (Geomterija i topologija mnogostrukosti, klasična mehanika i integrabilni dinamički sistemi)</p> <p>Project leader: <b>Dr. Vladimir Dragović</b> <a href="mailto:vladad@mi.sanu.ac.rs">vladad@mi.sanu.ac.rs</a></p> <p><a href="#">Project Description (English   Serbian)</a> <a href="#">List of Researchers</a></p>                      |

**Projekat ON 174005**

**Viskoelasticnost frakcionog tipa I optimizacija oblika u teorji stapova**  
**Viscoelasticity fractional type and optimization in theory of rods**

**Project Leader Teodor Atanackovic in 2013**

**Project Leader in 2014 Dusan Zorica**

Petak, 21 mart 2014 godine

Srminar mehanke

Odeljenja za mehaniku i Odeljenja za matematiku

Mathematical Institute SANU Belgrade

**Grant OI174001**

**“Dynamics of hybrid systems with  
complex structures. Mechanics of  
Materials”**

**Report for 2013**

**Катица Р. (Стевановић) Хедрих**

Одељење за механику Математичког института САНУ у Београду  
и Машински факултет Универзитета у Нишу

Прив. адреса: 18000- Ниш, Србија, ул. Војводе Танкосића 3/22.

e-mail: [khedrih@eunet.rs](mailto:khedrih@eunet.rs)





## Teme su:

1. Analitička mehanika diskretnih sistema frakcionog reda. Dinamika reonomnih i reoloških i sistema sa neholonomnim vezama
2. Nelinearni i retki fenomeni u dinamici hibridnih sistema spregnutih krutih i deformabilnih tela. Prenos energije kroz sistem. Model hibridnog sistema, koji sadrži klatna sa oscilujućim tačkama vešanja duž krivolinijskih putanja (model prema *European patent No. EP1514026*, 28.03.2007), sa ciljem optimizacije kinetičkih parametara stabilnosti i upravljivosti dinamikom istog. Sinhronizacije podсистема hibridnih sistema sa ciljem dobijanja kriterijuma i metodologija za modeliranje prototipa hibridnog sistema. Planiranje teorijskih osnova odgovarajućeg eksperimenta.

3. Modeli bioloških oscilatora i fenomeni dinamike i prenosa signala, informacija i energije kroz njihove kompleksne strukture. Mehanika bio mehaničkih sistema sa spregnutim poljima.

4. Mehanika diskretnih modela kontinuuma - Teorija i primene. Dinamika homogenih struktura spregnutih deformabilnih tela i standardnih elemenata konstitutivnih relacija na bazi linearno elastičnih, nelinearnoelastičnih, visukoelastičnih i/ili naslednih svojstava i/ili svojstava frakcionog reda

5. Fenomeni dinamike sistema sa trenjem, diskontinuitetima svojstva kinetičkih parametara. Vibroudarni sistemi sa trenjem. Fenomeni diskontinuiteta u svojstvima kinetičkih parametara.

6. Dinamika i stabilnost hibridnih sistema u interakciji kruto, čvrsto telo i fluid

7. Dinamika loma i oštećenja materijala po metodi diskretnog kontinuuma. Kinetička stanja dinamike vrha prsline u materijalu sa spregnutim poljima.

8. Upravljanje dinamikama i hibridnim sistemima

## Ključne reči:

Hibridni,  
nelinearni,  
analitička mehanika,  
izvod necelog reda,  
diskretni kontinuum,  
stabilnost,  
vibroudar,  
upravljanje  
Fenomeni nelinearne dinamike  
Kvalitativne i matematičke analogije



**Овера извештаја****НИО реализатор**

200029-Математички институт САНУ у Београду

200104-Математички факултет у Београду

200105-Машински факултет у Београду

200109-Машински факултет у Нишу

200155-Факултет техничких наука, Косовска Митровица

200131-Технички факултет у Бору

200107-Факултет инжењерских наука Универзитета у Крагујевцу

200036-Институт Кирило Савић у Београду

200223-Иновациони центар Електротехничког факултета д.о.о., Београд

200252-Државни Универзитет у Новом Пазару

200133-Технолошки факултет у Лесковцу

200213-Иновациони центар Машинског факултета у Београду ДОО

## Истраживачи ангажовани у години за коју се подноси извештај

| Р.Б. | ЈМБГ          | Име      | С | Презиме            | Титула | Звање                  | ДАТУМ<br>стицања звања<br>(дд/мм/гггг) | Шифра<br>НИО | Тренутни<br>статус | Тренутни<br>БИМ | БИМ за<br>наредну<br>годину | Остаје<br>на<br>пројекту | Категорија<br>(стање) | Е-  |
|------|---------------|----------|---|--------------------|--------|------------------------|----------------------------------------|--------------|--------------------|-----------------|-----------------------------|--------------------------|-----------------------|-----|
| 1    | 2808944735024 | Катица   | Р | Хедрих-Стевановић  | 3-Dr   | 12-Редовни<br>професор | 15/11/1975                             | 200029       | Да                 | 0               | 0                           | Да                       | A1                    | kh  |
| 2    | 1408975177656 | Јулијана | Д | Симоновић          | 3-Dr   | 2-Асистент             | 07/12/2012                             | 200109       | Да                 | 8               | 8                           | Да                       | A1                    | bjv |
| 3    | 0203950730062 | Томислав | Б | Петровић           | 3-Dr   | 5-Редовни<br>професор  | 16/09/1981                             | 200109       | Да                 | 8               | 8                           | Да                       | A4                    | <z  |
| 4    | 0903948910004 | Владимир | М | Раичевић           | 3-Dr   | 5-Редовни<br>професор  | 16/06/1985                             | 200155       | Да                 | 8               | 8                           | Да                       | A2                    | an  |
| 5    | 2202949910015 | Златибор | С | Васић              | 3-Dr   | 5-Редовни<br>професор  | 22/05/1987                             | 200155       | Да                 | 8               | 8                           | Да                       | A7                    | srđ |
| 6    | 1409968913017 | Срђан    | В | Јовић              | 3-Dr   | 3-Доцент               | 15/02/2011                             | 200155       | Да                 | 8               | 8                           | Да                       | A2                    | srđ |
| 7    | 1609948715307 | Јулка    | Д | Кнежевић-Мијановић | 3-Dr   | 5-Редовни<br>професор  | 17/09/1979                             | 200104       | Да                 | 8               | 8                           | Да                       | A4                    | kn  |
| 8    | 3012950710178 | Драгутин | Љ | Дебељковић         | 3-Dr   | 5-Редовни<br>професор  |                                        | 200105       | Да                 | 8               | 8                           | Да                       | A4                    | dd  |
| 9    | 0802952710272 | Драгомир | Н | Зековић            | 3-Dr   | 5-Редовни<br>професор  | 22/06/1984                             | 200105       | Да                 | 8               | 8                           | Да                       | A5                    | dze |
| 10   | 1210965740080 | Сретен   | Б | Стојановић         | 3-Dr   | 4-Ванредни<br>професор | 16/03/2006                             | 200133       | Да                 | 8               | 8                           | Да                       | A3                    | srđ |
| 11   | 0108943725026 | Вера     | Б | Николић-Станојевић | 3-Dr   | 5-Редовни<br>професор  | 28/03/1988                             | 200252       | Да                 | 0               | 0                           | Да                       | A4                    | ver |
| 12   | 2110978735026 | Анђелка  | Н | Хедрих             | 2-Mr   | 2-Асистент             |                                        | 200252       | Да                 | 0               | 8                           | Да                       | A5                    | han |
| 13   | 1006971425050 | Ивана    | Д | Атанасовска        | 3-Dr   | 10-Научни<br>сарадник  | 10/06/2004                             | 200036       | Да                 | 4               | 4                           | Да                       | T2                    | ivi |
| 14   | 2712970725013 | Јелена   | М | Вељковић-Ђоковић   | 3-Dr   | 4-Ванредни<br>професор | 21/12/2001                             | 200131       | Да                 | 8               | 8                           | Да                       | A4                    | fel |
| 15   | 2402956710438 | Маринко  | Д | Угрчић             | 3-Dr   | 5-Редовни<br>професор  |                                        | 200029       | Да                 | 6               | 6                           | Да                       | A7                    | ug  |

|    |               |          |   |              |      |                       |            |        |    |    |    |    |    |    |
|----|---------------|----------|---|--------------|------|-----------------------|------------|--------|----|----|----|----|----|----|
| 16 | 0107963785010 | Наташа   | P | Тришовић     | 3-Dr | 4-Ванредни професор   | 06/11/2007 | 200105 | Да | 4  | 4  | Да | T1 | nt |
| 17 | 1208948710041 | Стеван   | M | Максимовић   | 3-Dr | 5-Спољни сарадник     | 16/07/1999 | 200029 | Да | 5  | 5  | Да | A5 | er |
| 18 | 0310973715274 | Катарина | C | Максимовић   | 3-Dr | 23-Спољни сарадник    | 27/08/2010 | 200029 | Да | 3  | 3  | Да | A5 | kr |
| 19 | 1604950725024 | Љиљана   | P | Вељовић      | 3-Dr | 3-Доцент              | 22/07/2011 | 200107 | Да | 4  | 8  | Да | A5 | ve |
| 20 | 2206965730057 | Милош    | M | Јовановић    | 3-Dr | 3-Доцент              | 27/12/2007 | 200109 | Да | 0  | 0  | Да | A6 | in |
| 21 | 0212971710095 | Горан    | C | Симеуновић   | 3-Dr | 9-Истраживач сарадник | 02/09/2010 | 200213 | Да | 12 | 12 | Да | A4 | g  |
| 22 | 2105973742012 | Небојша  | J | Димитријевић | 3-Dr | 2-Асистент            | 20/07/2012 | 200105 | Да | 8  | 8  | Да | A7 | ne |
| 23 | 3010941710030 | Душан    | J | Микичић      | 3-Dr | 5-Редовни професор    |            | 200029 | Не | 0  | 0  | Не | A4 | kl |
| 24 | 0411921710100 | Ђорђе    | Z | Мушички      | 3-Dr | 5-Редовни професор    |            | 200029 | Не | 0  | 0  | Не | A4 | kl |
| 25 | 1902940710178 | Милутин  | M | Марјанов     | 3-Dr | 5-Редовни професор    |            | 200029 | Не | 0  | 0  | Не | A4 | m  |
| 26 | 3110986730029 | Данило   | Z | Карличић     | 2-Mr | 9-Истраживач          |            | 200029 | Да | 12 | 12 | Да | A4 | da |

| Р.Б. | ЈМБГ           | Име            | С | Презиме            | Титула | Звање                      | ДАТУМ<br>стицања звања<br>(дд/мм/гггг) | Шифра<br>ННО | Тренутни<br>статус | Тренутни<br>БИМ | БИМ за<br>наредну<br>годину | Остаје<br>на<br>пројекту | Категорија<br>(стање) | Е  |
|------|----------------|----------------|---|--------------------|--------|----------------------------|----------------------------------------|--------------|--------------------|-----------------|-----------------------------|--------------------------|-----------------------|----|
|      |                |                |   |                    |        | сарадник                   |                                        |              |                    |                 |                             |                          |                       |    |
| 27   | 2903984730079  | Милан          | С | Цајић              | 2-Мг   | 9-Истраживач<br>сарадник   |                                        | 200029       | Да                 | 12              | 12                          | Да                       | A4                    | са |
| 28   | 0305970727227  | Тамара         | Н | Несторовић-Трајков | 3-Дг   | 20-Странац                 |                                        | 200029       | Да                 | 0               | 0                           | Да                       | A1                    | та |
| 29   | 00000000GiuReg | Rega           |   | Giuseppe           | 3-Дг   | 20-Странац                 |                                        | 200029       | Да                 | 0               | 0                           | Да                       | A1                    | G  |
| 30   | 00000000MacJ.  | J. A. Tenreiro |   | Machado            | 3-Дг   | 20-Странац                 |                                        | 200029       | Да                 | 0               | 0                           | Да                       | A1                    | it |
| 31   | 00000000AwrJan | Jan            |   | Awrejcewicz        | 3-Дг   | 20-Странац                 |                                        | 200029       | Да                 | 0               | 0                           | Да                       | A1                    | av |
| 32   | 00000000BalJos | Jose Manuel    |   | Balthazar          | 3-Дг   | 20-Странац                 |                                        | 200029       | Да                 | 0               | 0                           | Да                       | A1                    | jo |
| 33   | 00000000SinSub | Subhash        |   | Sinha              | 3-Дг   | 20-Странац                 |                                        | 200029       | Да                 | 0               | 0                           | Да                       | A1                    | si |
| 34   | 00000000WarJer | Jerzy          |   | Warminski          | 3-Дг   | 20-Странац                 |                                        | 200029       | Да                 | 0               | 0                           | Да                       | A1                    | je |
| 35   | 00000000BalDum | Dumitru        |   | Baleanu            | 3-Дг   | 20-Странац                 |                                        | 200029       | Да                 | 0               | 0                           | Да                       | A1                    | du |
| 36   | 00000000YabHir | Hiroshi        |   | Yabuno             | 3-Дг   | 20-Странац                 |                                        | 200029       | Да                 | 0               | 0                           | Да                       | A1                    | ya |
| 37   | 00000000NayAli | Ali Hasan      |   | Nayfeh             | 3-Дг   | 20-Странац                 |                                        | 200029       | Да                 | 0               | 0                           | Да                       | A1                    | an |
| 38   | 00000000CarMat | Matthew        |   | Cartmell           | 3-Дг   | 20-Странац                 |                                        | 200029       | Да                 | 0               | 0                           | Да                       | A1                    | ca |
| 39   | 00000000MikYur | Yuri           |   | Mikhlin            | 3-Дг   | 20-Странац                 |                                        | 200029       | Да                 | 0               | 0                           | Да                       | A1                    | mi |
| 40   | 1906949720023  | Илија          | Ж | Николић            | 3-Дг   | 5-Редовни<br>професор      |                                        | 200107       | Да                 | 4               | 4                           | Да                       | A7                    | ih |
| 41   | 0309986735063  | Марија         | Б | Стаменковић        | 2-Мг   | 9-Истраживач<br>сарадник   |                                        | 200029       | Да                 | 12              | 12                          | Да                       | A4                    | se |
| 42   | 1702986730018  | Никола         | Д | Нешић              | 2-Мг   | 9-Истраживач<br>сарадник   |                                        | 200029       | Да                 | 0               | 0                           | Да                       | A4                    | ni |
| 43   | 1205987766039  | Марија         | А | Микић              | 2-Мг   | 2-Асистент                 |                                        | 200104       | Да                 | 8               | 8                           | Да                       | A4                    | mi |
| 44   | 2605987102388  | Љубинко        | Б | Кевац              | 2-Мг   | 9-Истраживач<br>сарадник   |                                        | 200223       | Да                 | 12              | 12                          | Да                       | A4                    | lj |
| 45   | 3012987782624  | Владимир       | Р | Вељић              | 2-Мг   | 8-Истраживач<br>приправник |                                        | 200213       | Не                 | 0               | 0                           | Не                       | A4                    | ve |
| 46   | 2405986930008  | Радослав       | Д | Радуловић          | 2-Мг   | 2-Асистент                 |                                        | 200105       | Да                 | 8               | 8                           | Да                       | A4                    | ra |

Докторанти ангажовани на пројекту

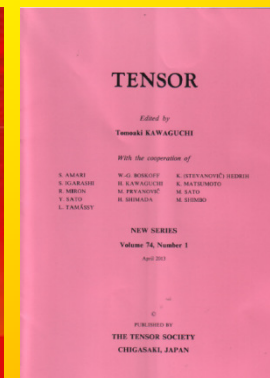
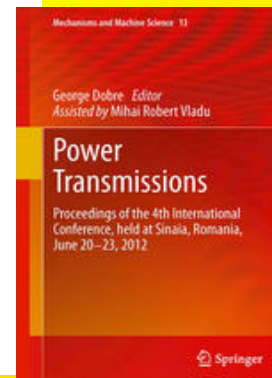
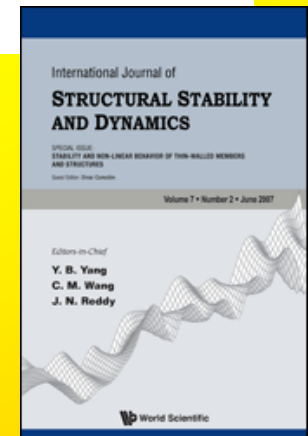
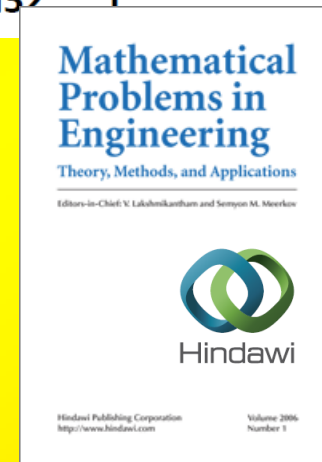
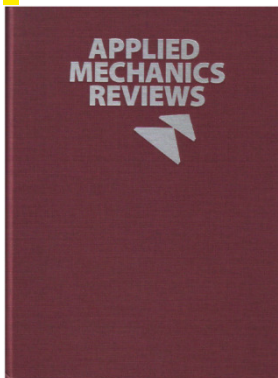
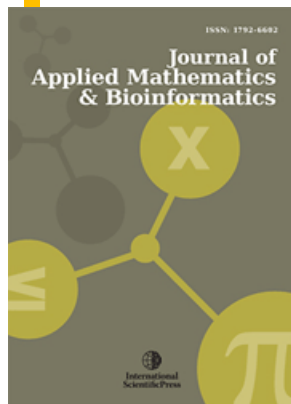
**44 истраживача, 12 из иностранства, 8 истраживача је има мање од 28 година, сви остали 25 др наука, 6 је са NULA IM, 19 ЈЕ PLACENIH, 5 је докторирао од 2011 године до 2013,**

## Број референци по категоријама

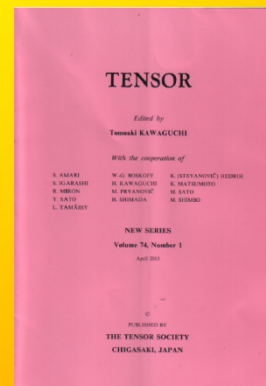
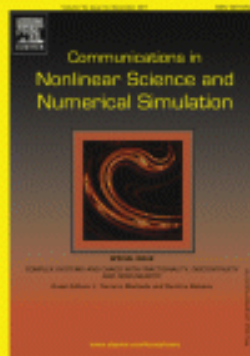
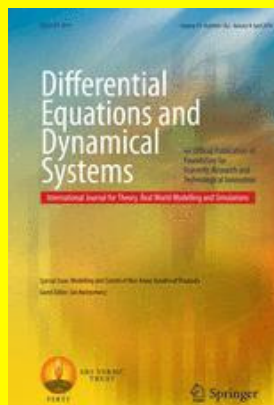
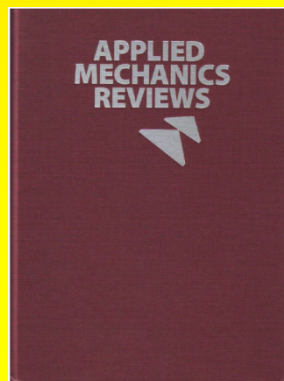
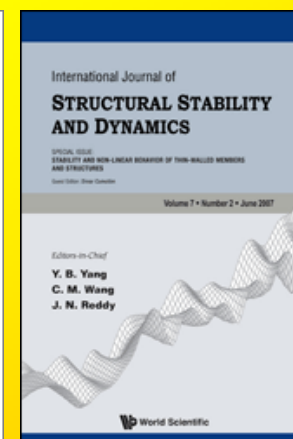
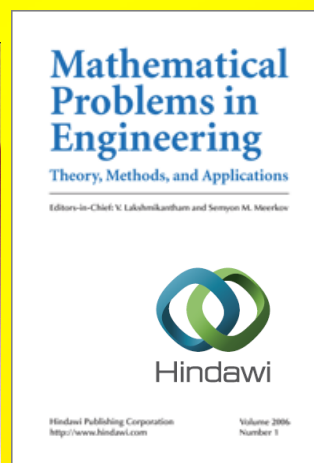
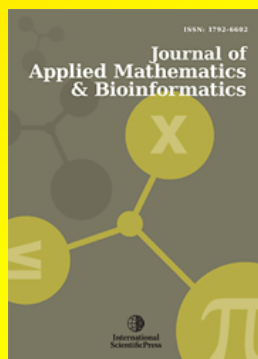
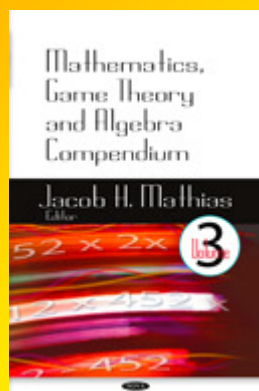
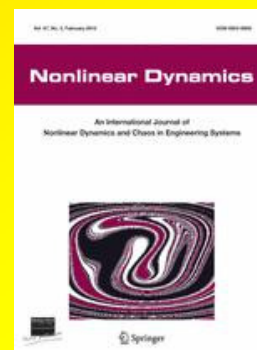
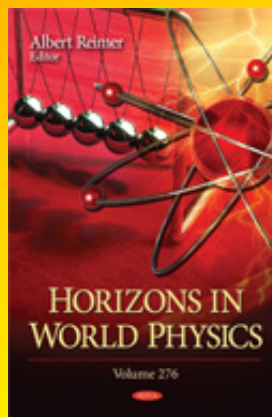
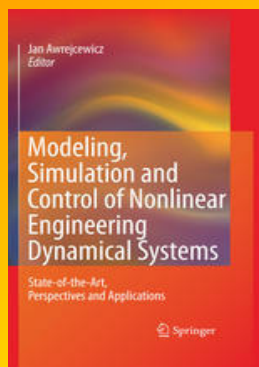
|     |    |
|-----|----|
| M14 | 11 |
| M18 | 1  |
| M21 | 4  |
| M22 | 3  |
| M23 | 13 |
| M24 | 13 |
| M31 | 2  |
| M32 | 1  |
| M34 | 9  |
| M37 | 1  |
| M32 | 3  |
| M33 | 40 |
| M42 | 5  |
| M51 | 5  |
| M52 | 3  |
| M63 | 3  |
| M66 | 3  |

## 2\* Број пленарних предавања и предавања по по

|     |   |
|-----|---|
| M31 | 2 |
| M32 | 1 |









I\* Уређивање и садржај специјалног броја часописа

НАУЧНО ДРУШТВО СРБИЈЕ  
**SCIENTIFIC REVIEW**

New Series: Series: Scientific and Engineering

Special Issue **Nonlinear Dynamics S2 (2013)**

Dedicated to **Milutin Milanković (1879- 1958)**

**Guest Editors: Katica R. (Stevanović) Hedrih and Žarko Mijajlović**  
Belgrade, 2013.

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>UDK 001<br/>SCIENTIFIC REVIEW<br/>SERBIAN SCIENTIFIC SOCIETY<br/><a href="http://www.ssrn.org/SCIENTIFIC-REVIEW">http://www.ssrn.org/SCIENTIFIC-REVIEW</a><br/>НАУЧНО ДРУШТВО СРБИЈЕ<br/>SCIENTIFIC REVIEW<br/>New Series<br/>Series: Scientific and Engineering<br/>Special Issue <b>Nonlinear Dynamics S2 (2013)</b><br/>Dedicated to <b>Milutin Milanković (1879- 1958)</b><br/>Guest Editors:<br/><b>Katica R. (Stevanović) Hedrih and Žarko Mijajlović</b><br/>CONTENT<br/>Milutin Milanković (1879-1958) and Astrodynamics ..... 1-2<br/>ADDRESS by Guest Editors ..... 3-6<br/>ОРАВИТЕЛНАЈАВНО АННОТИРАНОСКИ МЕТОДИ ИНТЕРПРЕТАЦИЈЕ<br/>НА ПУТАНИЈАМИ БЕЛОРУСИЈЕ,<br/>ПЛАНИ КРАСНОЈАРСКОЈ<br/>A generalization of Lagrange method of variation constants,<br/>Katica R. (Stevanović) Hedrih ..... 37-66<br/>Linear and nonlinear reaction systems,<br/>Ljiljana Kolar-Arč, Zeljko Čupić ..... 67-88<br/>Short review on the models of Bray-Liebhafsky oscillatory reaction,<br/>B. Stepanović, S. Anić ..... 89-112<br/>Bright solitons from defocusing nonlinearities,<br/>Olga V. Borovikova, Yaroslav V. Kartashov, Lilia Torner, and Boris A. Malomed ..... 113-122<br/>Modern concepts of actively controlled smart structures - Anewroll design approach,<br/>Tomara Nestorović ..... 123-148<br/>Further results on applications of fractional calculus in nonlinear dynamics<br/>- Stability and control issues,<br/>Mihailo Lazarević ..... 149-178<br/>Differential equations of motion for mechanical systems with nonlinear nonholonomic<br/>constraints - various forms and their equivalence,<br/>Dragomir N. Zaković ..... 179-196<br/>The new form of force function of two finite bodies in terms of modified Delaunay's<br/>and Andoyer's angle variables,<br/>Alexander Zienko ..... 197-206<br/>Some aspects of bird impact theory,<br/>Marinka Ugrčić ..... 207-222</p> | <p>SCIENTIFIC REVIEW - SPECIAL ISSUE NONLINEAR DYNAMICS S2 (2013)<br/>Dedicated to Milutin Milanković (1879- 1958)</p> | <p>UDK 001<br/>SERBIAN SCIENTIFIC SOCIETY<br/><a href="http://www.ssrn.org/SCIENTIFIC-REVIEW">http://www.ssrn.org/SCIENTIFIC-REVIEW</a><br/>НАУЧНО ДРУШТВО СРБИЈЕ<br/>SCIENTIFIC REVIEW<br/>New Series<br/>Series: Scientific and Engineering<br/>Special Issue <b>Nonlinear Dynamics S2 (2013)</b><br/>Dedicated to<br/><b>Milutin Milanković (1879- 1958)</b><br/>Guest Editors:<br/><b>Katica R. (Stevanović) Hedrih<br/>and Žarko Mijajlović</b><br/>Belgrade<br/>2013</p> |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

**THEORETICAL AND APPLIED MECHANICS  
TEORIJSKA I PRIMENJENA MEHANIKA**

Series: Special Issue - Address to Mechanics, Vol. 40 (S1), 2012.

ADDRESS TO MECHANICS:  
SCIENCE, TEACHING AND APPLICATIONS  
GUEST EDITOR: KATICA R. (STEVANOVIĆ) HEDRIH

|                                                                                                                                    |     |
|------------------------------------------------------------------------------------------------------------------------------------|-----|
| Ramislav M. Bulatović:                                                                                                             |     |
| ON THE RESIDUAL MOTION IN DAMPED VIBRATING SYSTEMS                                                                                 | 5   |
| Zoran Drašković:                                                                                                                   |     |
| ON THE DETERMINATION OF SHIFTING OPERATORS ALONG<br>GEODESICS ON A SURFACE                                                         | 17  |
| Irena Čonić and Radu Miron:                                                                                                        |     |
| DIFFERENT STRUCTURES ON SUBSPACES OF $Osc^k M$                                                                                     | 27  |
| Livija Cvetkanić:                                                                                                                  |     |
| NINETY YEARS OF DUFFING'S EQUATION                                                                                                 | 49  |
| Julka Knežević-Miljanović:                                                                                                         |     |
| ON A SHOCKLEY-READ-HALL MODEL FOR SEMICONDUCTORS                                                                                   | 65  |
| Miloš M. Jovanović:                                                                                                                |     |
| VORTEXICITY EVOLUTION IN PERTURBED POISEUILLE FLOW                                                                                 | 71  |
| Slavica Ristić:                                                                                                                    |     |
| A - A VIEW IN THE INVISIBLE                                                                                                        | 87  |
| Veľjko A. Vujčić:                                                                                                                  |     |
| MODIFICATION OF EARTH'S GRAVITY SPHERE                                                                                             | 121 |
| Milutin Marjanov:                                                                                                                  |     |
| LOOPS IN THE SUN'S ORBIT                                                                                                           | 127 |
| Aleksandar S. Tomić:                                                                                                               |     |
| THE LUNAR ORBIT PARADOX                                                                                                            | 135 |
| Miloš Kojić and Nenad Filipović:                                                                                                   |     |
| COMPUTATIONAL MECHANICS IN SCIENCE,<br>APPLICATIONS AND TEACHING                                                                   | 147 |
| Mihailo P. Lazarević:                                                                                                              |     |
| BIOLOGICALLY INSPIRED CONTROL AND MODELING<br>OF (BIO)ROBOTIC SYSTEMS AND SOME APPLICATIONS<br>OF FRACTIONAL CALCULUS IN MECHANICS | 163 |
| Anđelka Hedrih and Ugrčić Marinko:                                                                                                 |     |
| VIBRATIONAL PROPERTIES CHARACTERIZATION OF<br>MOUSE EMBRYO DURING MICROINJECTION                                                   | 189 |
| Tamara N. Nestorović:                                                                                                              |     |
| ACTIVE CONTROL OF MECHANICAL<br>STRUCTURES IN RESEARCH AND EDUCATION                                                               | 203 |
| Sreten B. Stojanović and Dragutin Lj. Debeljković:                                                                                 |     |
| DELAY DEPENDENT STABILITY OF LINEAR<br>TIME-DELAY SYSTEMS                                                                          | 223 |
| Stevan M. Maksimović and Katarina S. Maksimović:                                                                                   |     |
| IMPROVED COMPUTATION METHOD IN<br>RESIDUAL LIFE ESTIMATION OF<br>STRUCTURAL COMPONENTS                                             | 247 |
| Nataša R. Trifković:                                                                                                               |     |
| ABOUT EIGENSENSITIVITY ANALYSIS<br>OF MECHANICAL STRUCTURES                                                                        | 263 |
| Predrag Janković, Tošoslav Igrić and Dragiša Nikodijević:                                                                          |     |
| PROCESS PARAMETERS EFFECT ON MATERIAL<br>REMOVAL MECHANISM AND CUT QUALITY<br>OF ABRASIVE WATER JET MACHINING                      | 277 |
| Katica R. (Stevanović) Hedrih:                                                                                                     |     |
| ADVANCES IN CLASSICAL AND ANALYTICAL MECHANICS:<br>A REVIEWS OF AUTHOR'S RESULTS                                                   | 293 |

**SERBIAN SOCIETY OF MECHANICS  
SRPSKO DRUŠTVO ZA MEHANIKU**

ISSN 1450-5584

THEORETICAL AND APPLIED MECHANICS  
TEORIJSKA I PRIMENJENA MEHANIKA

# THEORETICAL AND TEORIJSKA I PRIMENJENA MEHANIKA APPLIED MECHANICS

Series: Special Issue - Address to Mechanics, Vol. 40 (S1), 2012.

**VOL 1-2 , 2013**

**Serbian Society of Mechanics**

**BELGRADE 2012.**

[http://www.ssm.org.rs/WebTAM/private/VOL40\\_4/TAMM\\_AddressToMechanics.pdf](http://www.ssm.org.rs/WebTAM/private/VOL40_4/TAMM_AddressToMechanics.pdf)

<http://www.ssm.org.rs/WebTAM/journal.html>

http://www.ssm.org.rs/WebTAM/journal.html

accepted (5) http://www.dfs.rs-kongres... http://www.dfs.rs-kongres... http://www.dfs.rs-kongres... Mathematical Institute of ... accepted (3) accepted Suggested Sites Get more Ad



# Theoretical and Applied Mechanics

Teorijska i primenjena mehanika

*An International Journal*

[ [Home](#) ] [ [About](#) ] [ [Editorial Board](#) ] [ [Previous editors-in-chief: \(№ 1-21\) \(№ 22-26\)](#) ] [ [Contact](#) ]

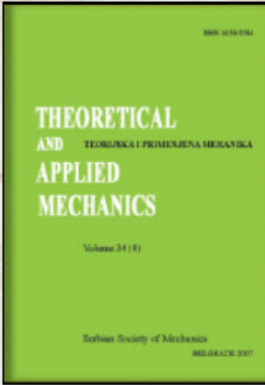
[About the Journal](#)

[Information for Authors](#)

[Paper review sheet](#)

[Archives](#)

LaTeX template  
PDF TEX



**Volume 40 / 2013**  
[Number 1](#) / [Number 2](#) / [Number 3](#) / [Number 4](#)

**Volume 39 / 2012**  
[Number 1](#) / [Number 2](#) / [Number 3](#) / [Number 4](#)

**Volume 38 / 2011**  
[Number 1](#) / [Number 2](#) / [Number 3](#) / [Number 4](#)

**Volume 37 / 2010**  
[Number 1](#) / [Number 2](#) / [Number 3](#) / [Number 4](#)

**Volume 36 / 2009**  
[Number 1](#) / [Number 2](#) / [Number 3](#) / [Number 4](#)

**Volume 35 / 2008**  
[Number 1](#) / [Number 2](#) / [Number 3](#) / [Number 4](#)

Windows taskbar icons: Internet Explorer, Firefox, S, VLC, File Explorer, Mail, Word, PowerPoint, Skype, etc.



CONGRESS 2013

SERBIAN SOCIETY OF MECHANICS.



FOURTH SERBIAN (29TH YU ) CONGRESS ON THEORETICAL AND  
APPLIED MECHANICS

4TH -7TH OF JUNE 2013, HOTEL BREZA - VRNJAČKA BANJA, SERBIA

**M2:** Nonlinear Dynamics – Milutin Milankovic

Corresponding Organizer: Katica Stevanović-Hedrih

Mathematical Institute of the Serbian Academy of Sciences and Arts, Serbia,

Email: [khedrih@eunet.rs](mailto:khedrih@eunet.rs)

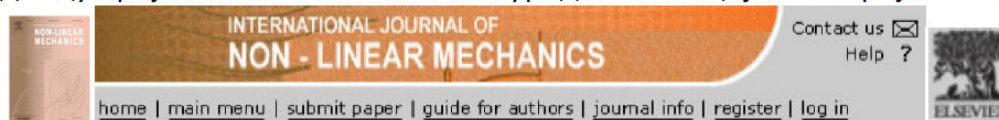








II\* Позив упућен руководиоцу пројекта **ON174001** за госта уредника специјалног броја часописа



<http://ees.elsevier.com/nlm/default.asp>

## **International Journal of Non-Linear Mechanics (M21)**

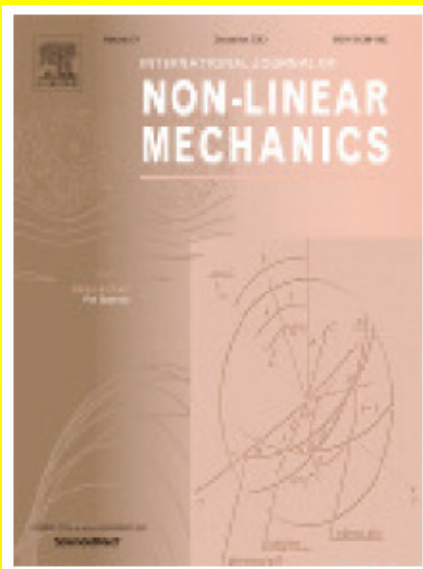
и прихватање садржаја специјалног броја

**Special issue: “*Elements of mathematical phenomenology and phenomenological mapping in non-linear dynamics*”**

**of International Journal Non-Linear Mechanics (IJNM)**

по идејама Најзначајнијег дела Михаила Петровића, једног од три студента докторанта  
ИМПОЗАНТНОГ Poincaré-a (Jules Henri Poincaré (1854 –1912)).

У току је рецензирање радова подентих за овај број и пријем нових радова.



**III\*** Именовање руководиоца Пројекта **ON174001** у тринаесточлану редакцију интернационалног часописа "TENSOR" јапанског друштва **Tensor Society**, који се успешно публикује већ 74 године и који је јединствен по садржају публикованих радова из области диференцијалне геометрије.

**IV\*** Успешност 8 младих истраживача у полагању испита на докторским студијама са просечном оценом 10 (десет) који су отпочели докторске студије са почетком пројектног циклуса.

**V\*** Пленарна предавања и предавања по позиву руководиоца пројекта у којима је приказо и **увео нову функцију дисипације енергије система фракционих својстава**. Резултати су штампани у изводима, а и предати за штампу у интегралном облику.







Petak, 21 mart 2014 godine

Srminar mehanke

Odeljenja za mehaniku i Odeljenja za matematiku

Mathematical Institute SANU Belgrade

Grant OI174001

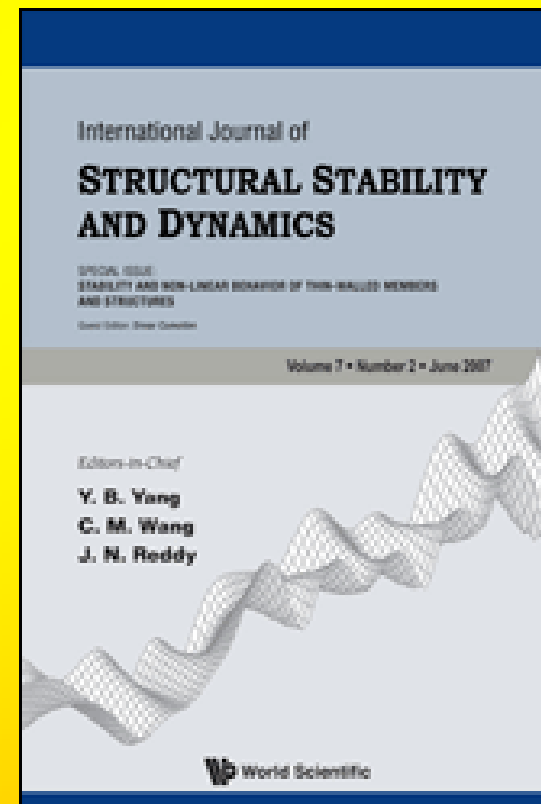
**“Dynamics of hybrid systems with  
complex structures. Mechanics of  
Materials”**

**Катица Р. (Стевановић) Хедрих**

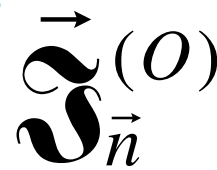
Одељење за механику Математичког института САНУ у Београду  
и Машински факултет Универзитета у Нишу

Прив. адреса: 18000- Ниш, Србија, ул. Војводе Танкосића 3/22.

e-mail: [khedrih@eunet.rs](mailto:khedrih@eunet.rs)



International Journal of Structural Stability and Dynamics  
Vol. 13, No. 7 (2013) 1340007 (20 pages)  
© World Scientific Publishing Company  
DOI: 10.1142/S0219455413400075



 **World Scientific**  
www.worldscientific.com

**Hedrih (Stevanović) R. Katica, (2013), Vector method based on mass moment vectors and vector rotators applied to rigid-body multi-coupled rotations around no intersecting axes, International Journal of Structural Stability and Dynamics, Vol. 13, No. 7 (2013) 1340007 (20 pages), #.c World Scientific Publishing Company, ISSN: 0219-4554, DOI: 10.1142/S0219455413400075**  
**<http://dx.doi.org/10.1142/S0219455413400075>**

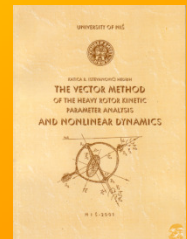
**Hendrix (Stevanović) K., Veljović Lj., New Vector Description of Kinetic Pressures on Shaft Bearings of a Rigid Body Nonlinear Dynamics with Coupled Rotations around No Intersecting Axes, Acta Polytechnica Hungarica - Journal of Applied Sciences, Special Issue on Applied Mathematics, Guest Editors: Aurél Galántai and Péter T. Nagy, Volume 10, Issue Number 7, 2013, pp. 151-170. ISSN 1785-8860**  
**<http://uni-obuda.hu/journal/Issue45.htm>**  
**<http://www.uni-obuda.hu/journal/Issue45.htm>**

**Hedrih (Stevanović) K., Milosavljević D., Veljović Lj.,**  
**Multi-parameter Analysis of a Rigid Body Nonlinear**  
**Coupled Rotations around No Intersecting Axes Based**  
**on the Vector Method, Adv. Theor. Appl. Mech., Vol. 6,**  
**2013, no. 2, 49 - 70, HIKARI Ltd, [www.m-hikari.com](http://www.m-hikari.com),**  
**<http://dx.doi.org/10.12988/ataam.2013.378>**  
**<http://www.m-hikari.com/ataam/ataam2013/ataam1-4-2013/hedrihATAM1-4-2013.pdf>**

**Katina R. (Stevanović) Hedrih and Marija B. Stamenković,**  
**(201`3), Mass moment vector applications to the multi**  
**body dynamics with coupled rotations about no intersecting**  
**axes, PAMM \_ Proc. Appl. Math. Mech. 13, 35 – 36 (2013) /**  
**DOI 10.1002/pamm.201310013**  
**<http://onlinelibrary.wiley.com/doi/10.1002/pamm.v13.1/issuetoc>**



**<https://www.lap-publishing.com/>**





*This monograph defines three kinetic vectors fixed to a certain point and axis passing through the given rigid body point. These are:*

*1\* Vector  $\vec{M}_{\vec{n}}^{(O)}$  of the body mass at the point  $O$  for the axis oriented by the unit vector  $\vec{n}$ :*

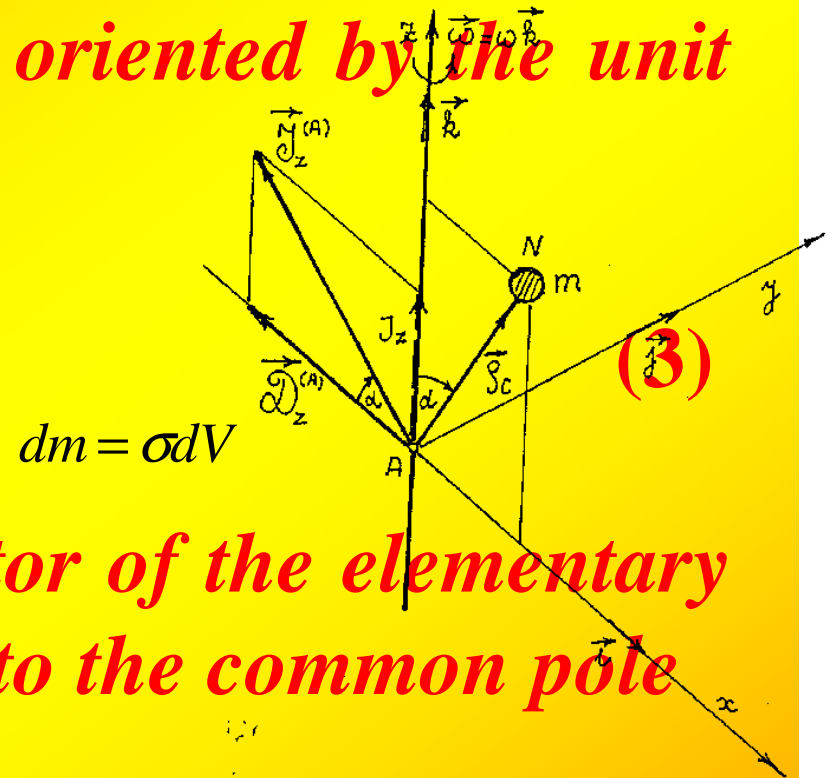
$$\vec{M}_{\vec{n}}^{(O)} \stackrel{\text{def}}{=} \iiint_V \vec{n} dm = M \vec{n} \quad dm = \sigma dV \quad (1)$$

*2\* Vector  $\vec{S}_{\vec{n}}^{(O)}$  of the body mass static (linear) moment at the point  $O$  for the axis oriented by the unit vector  $\vec{n}$  in the form:*

$$\vec{S}_{\vec{n}}^{(O)} \stackrel{\text{def}}{=} \iiint_V [\vec{n}, \vec{\rho}] dm \quad dm = \sigma dV \quad (2)$$

3\* Vector  $\vec{S}_{\vec{n}}^{(O)}$  of the body mass inertia moment at the point  $O$  for the axis oriented by the unit vector  $\vec{n}$  :

$$\vec{S}_{\vec{n}}^{(O)} \stackrel{\text{def}}{=} \iiint_V [\vec{\rho}, [\vec{n}, \vec{\rho}]] dm$$



$$dm = \sigma dV$$

where  $\vec{\rho}$  is the position vector of the elementary body mass  $dm$  with respect to the common pole  $O$ .

The spherical and the devivational parts of the inertia moment vector and of the inertia tensor are analyzed.



*The "supports" vectors of the body mass linear moments as well as of the body mass inertia moments for the pole  $O$  and axis oriented by unit vector  $\vec{n}$  are introduced by definitions and expressions. The "support" vector  $\vec{S}_{\vec{n}}^{(O)}$  of the body mass linear moment and the "support" vector  $\vec{\mathfrak{S}}_{\vec{n}}^{(O)}$  of the body mass inertia moment of the body point  $N: \vec{ON} = \vec{\rho}$ , for the pole in the point  $O$  and for the axis oriented by the unit vector  $\vec{n}$  are defined by the following expressions:*

$$\vec{S}_{\vec{n}}^{(O)} \stackrel{\text{def}}{=} \frac{\partial \vec{\mathfrak{S}}_{\vec{n}}^{(O)}}{\partial m} = [\vec{n}, \vec{\rho}]$$

$$\vec{\mathfrak{S}}_{\vec{n}}^{(O)} \stackrel{\text{def}}{=} \frac{\partial \vec{\mathfrak{S}}_{\vec{n}}^{(O)}}{\partial m} = [\vec{\rho}, [\vec{n}, \vec{\rho}]] \quad (4)$$



*Also, we can conclude that the impact on applications of the use of different possibilities of the phenomenological analogy of different model dynamics and professors, researchers and scientists, with larger area of the own scientific knowledge are also very important for optimization of the teaching processes and the application of Bologna's principle in the original form.*

*3\* Vector  $\vec{\mathfrak{S}}_{\vec{n}}^{(O)}$  of the body mass inertia moment at the point  $O$  for the axis oriented by the unit vector  $\vec{n}$  :*

$$\vec{\mathfrak{S}}_{\vec{n}}^{(O)} \stackrel{\text{def}}{=} \int \int \int_V [\vec{\rho}, [\vec{n}, \vec{\rho}]] dn$$

*where  $\vec{\rho}$  is the position vector of the elementary body mass with respect to the common pole  $O$ . For special cases see Ref. [35].*

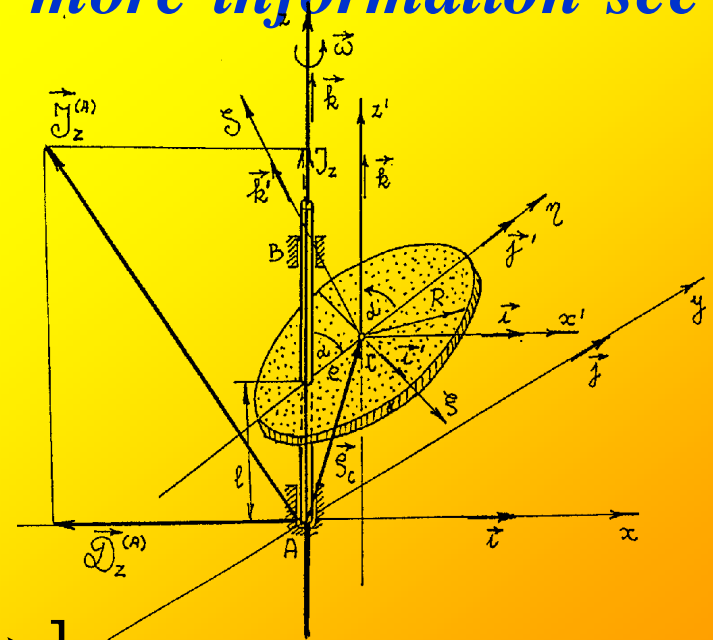
We can write two vector equations of dynamic equilibrium for rotation of the body around the stationary axis oriented by the unit vector  $\vec{n}$ , with bearing  $A$  and  $B$ , with angular velocity  $\omega$  and acceleration  $\dot{\omega}$ , and under the action of the active force system  $\vec{F}_k$ ,  $k = 1, 2, \dots, N$  (for more information see Refs. [33] in the following form :

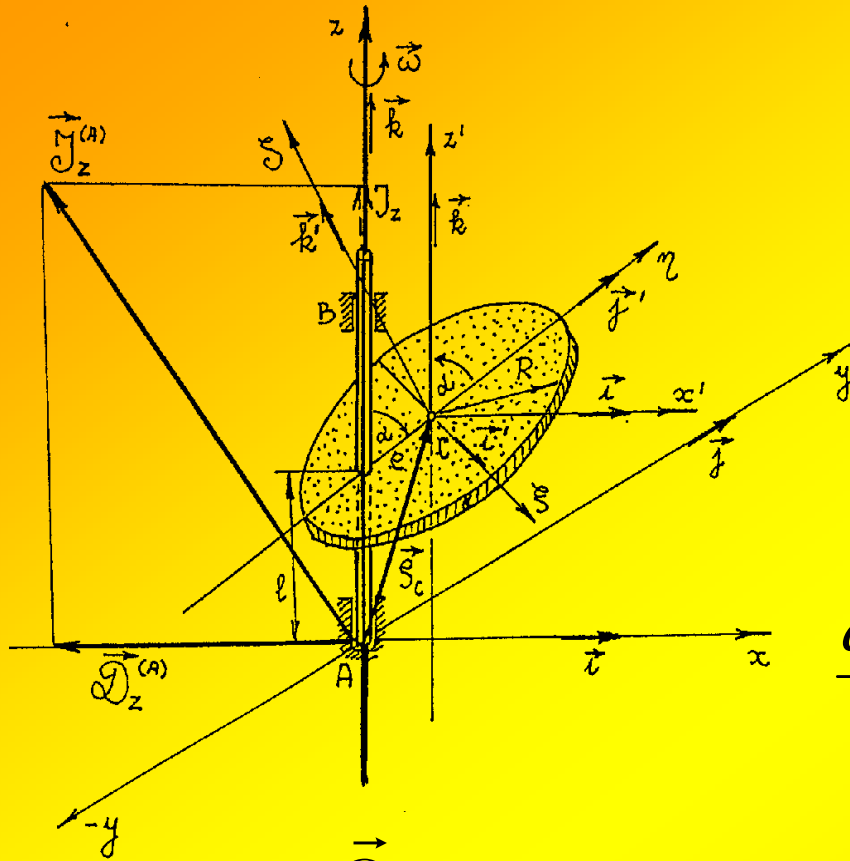
$$\mathfrak{N} = \sqrt{\dot{\omega}^2 + \omega^4}$$

$$\frac{d\vec{\mathfrak{N}}}{dt} = \mathfrak{N}_1 \left| \vec{\mathfrak{S}}_{\vec{n}}^{(A)} \right| = \sum_{k=1}^{k=N} \vec{F}_k + \vec{F}_A + \vec{F}_B$$

$$\frac{d\vec{\mathfrak{L}}_A}{dt} = \dot{\omega} J_{\vec{n}}^{(A)} + \dot{\omega} \vec{\mathfrak{S}}_{\vec{n}}^{(A)} + \omega [\vec{\omega}, \vec{\mathfrak{S}}_{\vec{n}}^{(A)}] =$$

$$= \dot{\omega} J_{\vec{n}}^{(A)} + \left| \vec{\mathfrak{S}}_{\vec{n}}^{(A)} \right| \mathfrak{N}_2 = \sum_{k=1}^{k=N} [\vec{\rho}_k, \vec{F}_k] + [\vec{\rho}_B, \vec{F}_B]$$





$$\vec{\mathfrak{S}}_{\vec{n}}^{(O)} \stackrel{\text{def}}{=} \iiint_V [\vec{\rho}, [\vec{n}, \vec{\rho}]] dm$$

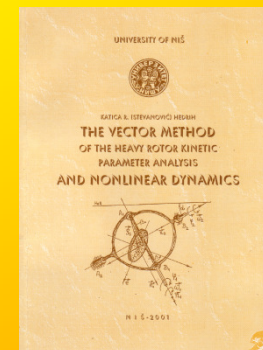
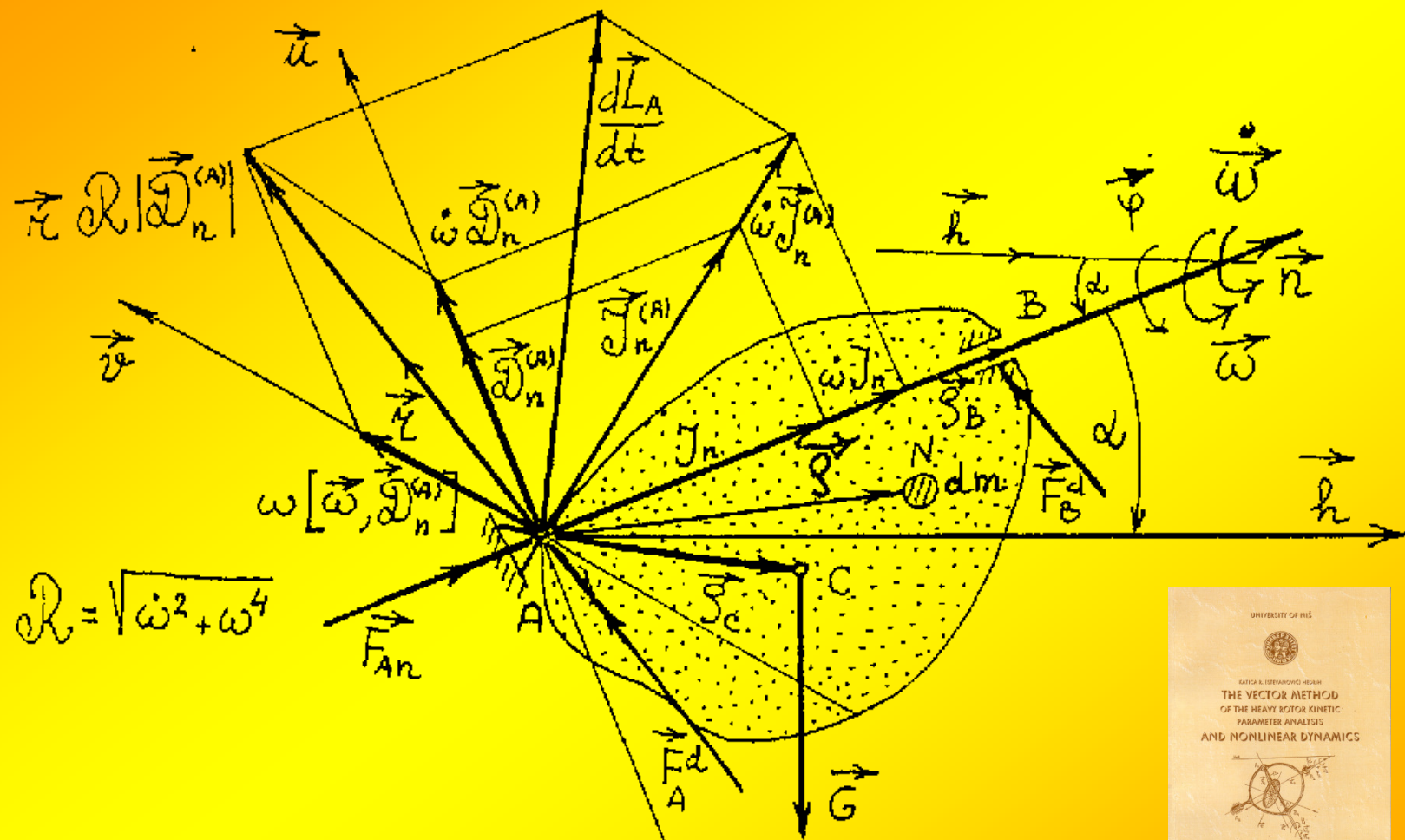
$$\vec{\mathfrak{S}}_O^{(\vec{n})} = \iiint_V [\vec{n}, \vec{r}] dm$$

$$\mathfrak{N} = \sqrt{\dot{\omega}^2 + \omega^4}$$

$$\frac{d\vec{\mathfrak{K}}}{dt} = \mathfrak{N}_1 |\vec{\mathfrak{S}}_{\vec{n}}^{(A)}| = \sum_{k=1}^{k=N} \vec{F}_k + \vec{F}_A + \vec{F}_B$$

$$\frac{d\vec{\mathfrak{Q}}_A}{dt} = \dot{\omega} J_{\vec{n}}^{(A)} + \dot{\omega} \vec{\mathfrak{S}}_{\vec{n}}^{(A)} + \omega [\vec{\omega}, \vec{\mathfrak{S}}_{\vec{n}}^{(A)}] =$$

$$= \dot{\omega} J_{\vec{n}}^{(A)} + |\vec{\mathfrak{S}}_{\vec{n}}^{(A)}| \mathfrak{N}_2 = \sum_{k=1}^{k=N} [\vec{\rho}_k, \vec{F}_k] + [\vec{\rho}_B, \vec{F}_B]$$



**Figure 12. The graphical presentation of the kinetic vectors of rotors with inclined rotation axis.**



Following the expressions (67) and (70), as well as the expression (68) and (71), we can write the following two vector equations:

$$\frac{d\vec{\mathfrak{R}}}{dt} = \mathfrak{R} \left| \vec{\mathfrak{S}}_{\vec{n}}^{(A)} \right| \vec{\mathfrak{r}}_1 = \sum_{k=1}^{k=N} \vec{F}_k + \vec{F}_A + \vec{F}_B + \vec{G} \quad (125)$$

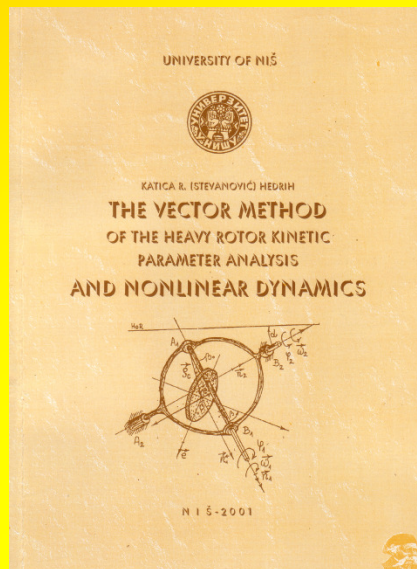
$$\frac{d\vec{\mathfrak{Q}}_A}{dt} = \dot{\omega} J_{\vec{n}}^{(A)} + \left| \vec{\mathfrak{S}}_{\vec{n}}^{(A)} \right| \mathfrak{R} = \sum_{k=1}^{k=N} [\vec{\rho}_k, \vec{F}_k] + [\vec{\rho}_C, \vec{G}] + [\vec{\rho}_B, \vec{F}_B] \quad (126)$$

These two vectorial equations are kinetic equations of dynamic equilibrium of the body rotating around the stationary axis under the action of the active force system  $\vec{F}_k$ .

If we now multiply scalarly and vectorially these equations (125) and (126) with the unit vector  $\vec{n}$ , and, having in mind that the  $\vec{\rho}_B = \rho_B \vec{n}$ , we obtain:

1\* the rotation equation around the axes oriented by the unit vector  $\vec{n}$  in the form:

$$\left( \vec{\mathfrak{S}}_{\vec{n}}^{(A)}, \dot{\vec{\omega}} \right) = \left( \left[ \vec{\rho}_C, \vec{G} \right], \vec{n} \right) + \sum_{k=1}^{k=N} \left( \left[ \vec{\rho}_k, \vec{F}_k \right], \vec{n} \right) \quad (127)$$



**2\* the equations for determining the bearings' kinetic pressures, that is, pressures upon the bearings,  $\vec{F}_A$  and  $\vec{F}_B$ , that is, their components in the axis direction  $\vec{n}$  and normal to the rotation axis:**

$$\vec{F}_{A\vec{n}} = (\vec{F}_A, \vec{n})\vec{n} = -\vec{n} \sum_{k=1}^{k=N} (\vec{F}_k, \vec{n}) - \vec{n}(\vec{G}, \vec{n}) \quad (128)$$

$$\vec{F}_{AT} = -\vec{F}_B + \vec{\mathfrak{R}}_1 \left| \vec{\mathfrak{S}}_{\vec{n}}^{(A)} \right| - [\vec{n}, [\vec{G}, \vec{n}]] - \sum_{k=1}^{k=N} [\vec{n}, [\vec{F}_k, \vec{n}]] \quad (129)$$

$$\vec{F}_B = \frac{1}{\rho_B} \vec{\mathfrak{R}} \left| \vec{\mathfrak{S}}_{\vec{n}}^{(A)} \right| - \frac{1}{\rho_B} [\vec{n}, [\vec{\rho}_C, \vec{G}], \vec{n}] - \frac{1}{\rho_B} \sum_{k=1}^{k=N} [\vec{n}, [\vec{\rho}_k, \vec{F}_k], \vec{n}] \quad (130)$$

**where is:  $\vec{\mathfrak{R}} = \mathfrak{R} \vec{r}$ ,  $\mathfrak{R} = \sqrt{\dot{\omega}^2 + \omega^4}$ . (131)**

**The rotator  $\vec{\mathfrak{R}} = \mathfrak{R} \vec{r}$  is rotating and increasing by the angular velocity and by the angular acceleration.**

***around fixed axis – Kinetic pressure on bearings***

$$\frac{d\vec{p}(t)}{dt} = \dot{\omega}\vec{S}_O^{(\bar{n})} + \omega[\vec{\omega}, \vec{S}_O^{(\bar{n})}] = \dot{\omega}\vec{S}_O^{(\bar{n})} + \omega^2[\vec{n}, \vec{S}_O^{(\bar{n})}] = \vec{F}_{AMN} + \vec{F}_{An} + \vec{F}_B + \vec{F} + \vec{G}$$

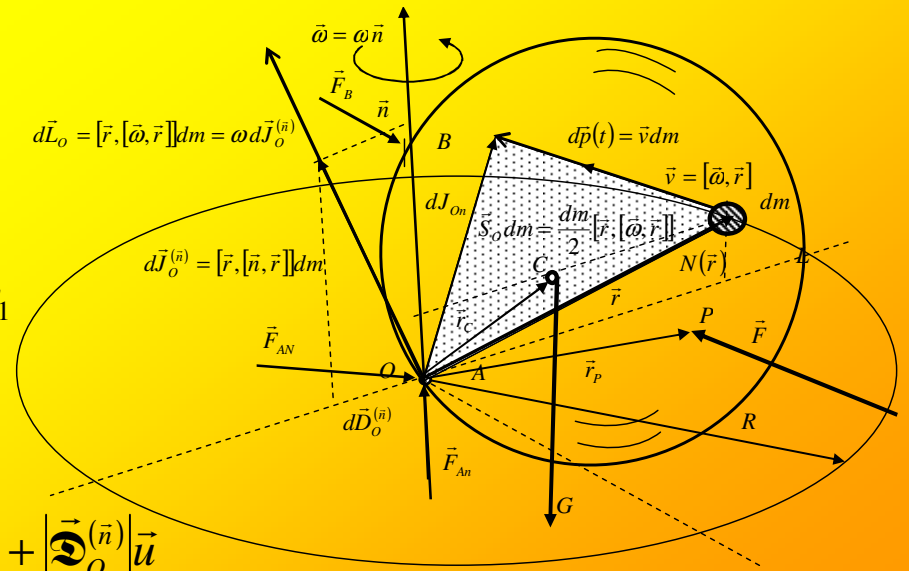
$$\frac{d\vec{L}_O}{dt} = \dot{\omega}\vec{J}_O^{(\vec{n})} + \omega[\vec{\omega}, \vec{J}_O^{(\vec{n})}] = \dot{\omega}\vec{J}_O^{(\vec{n})} + \omega^2[\vec{n}, \vec{J}_O^{(\vec{n})}] = [\vec{r}_P, \vec{F}] + [\vec{r}_C, \vec{G}] + [\vec{r}_B, \vec{F}_B]$$

$$\vec{S}_O^{(\bar{n})} = \iiint_V [\vec{n}, \vec{r}] dm = [\vec{n}, \vec{r}_C] M = \left| \vec{S}_O^{(\bar{n})} \right| \vec{u}_1$$

$$[\vec{n}, \vec{S}_O^{(\bar{n})}] = M[\vec{n}, [\vec{n}, \vec{r}_C]] = M\langle (\vec{n}, \vec{r}_C) \vec{n} - \vec{r}_C \rangle = |\vec{S}_O^{(\bar{n})}| \vec{w}_1$$

$$[\vec{n}, \vec{J}_O^{(\vec{n})}] = [\vec{n}, J_{On} \vec{n} + \vec{\mathfrak{D}}_O^{(\vec{n})}] = [\vec{n}, \vec{\mathfrak{D}}_O^{(\vec{n})}] = |\vec{\mathfrak{D}}_O^{(\vec{n})}| \vec{w}$$

$$\vec{J}_O^{(\bar{n})} = (\vec{n}, \vec{J}_O^{(\bar{n})})\vec{n} + [\vec{n}, [\vec{J}_O^{(\bar{n})}, \vec{n}]] = J_{On}\vec{n} + \vec{\mathfrak{S}}_O^{(\bar{n})} = J_{On}\vec{n} + \left| \vec{\mathfrak{S}}_O^{(\bar{n})} \right| \vec{u}$$



For this case the expressions of the linear momentum  $\vec{\mathfrak{K}}$  and of the angular momentum  $\vec{\mathfrak{Q}}_{O_1}$  of the gyro-rotor system are:

$$\begin{aligned}\vec{\mathfrak{K}} &= [\vec{\omega}_2, \vec{d}_1]M + \omega_1 \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_1)} + \omega_2 \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_1)} \\ \vec{\mathfrak{Q}}_{O_2} &= \omega_1 \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_1)} + \omega_2 \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_1)} + \omega_1 [\vec{d}_1, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_1)}] + \omega_2 [\vec{d}_1, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_1)}] + \\ &\quad + \omega_2 \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2 \rightarrow O_1)} + \omega_2 [\vec{\rho}_C, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2 \rightarrow O_1)}]\end{aligned}\quad (62)$$

For special case of the gyro-rotor with many shaft rotor axes with one section  $O$ , these expression of the linear momentum and of the angular momentum are very simple:

$$\vec{\mathfrak{K}} = \sum_{i=1}^{i=p} |\vec{\omega}_i| \vec{\mathfrak{S}}_{\vec{n}_i}^{(O)} \quad \vec{\mathfrak{Q}}_O = \sum_{i=1}^{i=p} |\vec{\omega}_i| \vec{\mathfrak{S}}_{\vec{n}_i}^{(O)} \quad (63)$$



$$\begin{aligned} \frac{d\vec{\mathfrak{Q}}_{O_2}}{dt} = & \dot{\omega}_1 \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_1)} + \omega_1^2 [\vec{n}_1, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_1)}] + \dot{\omega}_2 \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_1)} + \omega_2^2 [\vec{n}_2, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_1)}] + \\ & + \omega_1 \omega_2 [\vec{n}_2, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_1)}] + \omega_1 \omega_2 [\vec{n}_1, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_1)}] + \omega_1 \omega_2 \vec{\mathfrak{S}}_{[\vec{n}_2, \vec{n}_1]}^{(O_1)} \end{aligned}$$

$$\begin{aligned} \frac{d\vec{\mathfrak{Q}}_{O_2}}{dt} = & \dot{\omega}_1 J_{\vec{n}_1}^{(O_1)} \vec{n}_1 + \dot{\omega}_2 J_{\vec{n}_2}^{(O_1)} \vec{n}_2 + \vec{\mathfrak{N}}_1 \left| \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_1)} \right| + \vec{\mathfrak{N}}_2 \left| \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_1)} \right| + \\ & + \vec{\mathfrak{N}}_{21} \left| \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_1)} \right| + \vec{\mathfrak{N}}_{12} \left| \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_1)} \right| + \vec{\mathfrak{N}}_3 \left| \vec{\mathfrak{S}}_{[\vec{n}_2, \vec{n}_1]}^{(O_1)} \right| \end{aligned}$$

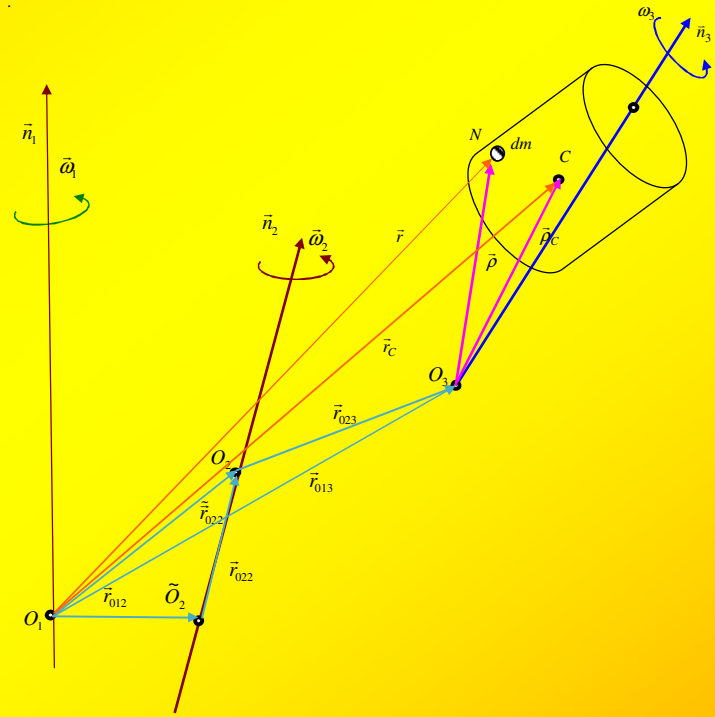
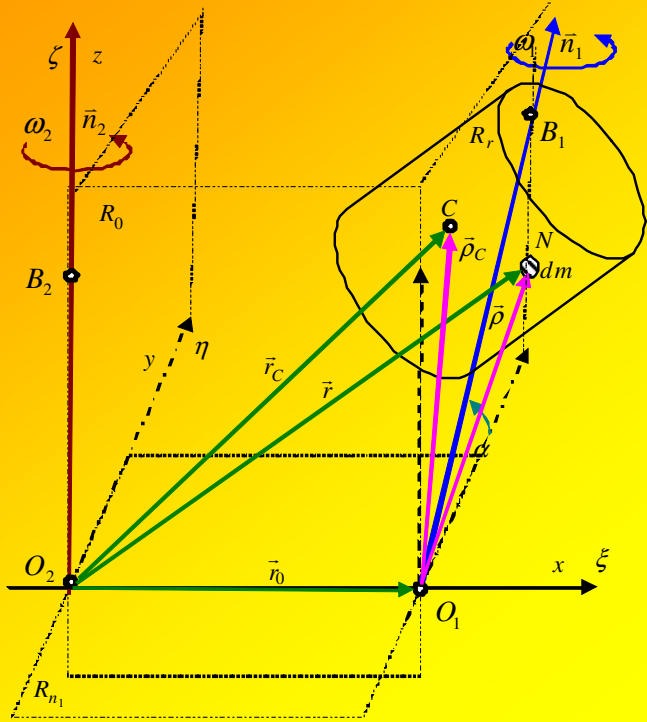
**where the kinematic vectors rotator are introduced in following form:**  $\vec{\mathfrak{N}}_2 = \dot{\omega}_2 \vec{u}_2 + \omega_2^2 \vec{v}_2$ ;  $\mathfrak{N}_2 = \sqrt{\dot{\omega}_2^2 + \omega_2^4}$

$$\vec{\mathfrak{N}}_3 = \omega_1 \omega_2 \vec{u}_3; \quad \mathfrak{N}_3 = \omega_1 \omega_2 \quad \vec{\mathfrak{N}}_1 = \dot{\omega}_1 \vec{u}_1 + \omega_1^2 \vec{v}_1; \quad \mathfrak{N}_1 = \sqrt{\dot{\omega}_1^2 + \omega_1^4}$$

$$\vec{\mathfrak{N}}_{12} = \omega_1 \omega_2 [\vec{n}_1, \vec{u}_2]; \quad \mathfrak{N}_{12} = \omega_1 \omega_2 \sin \gamma(=)0 \quad (65)$$

$$\vec{\mathfrak{N}}_{21} = \omega_1 \omega_2 [\vec{n}_2, \vec{u}_1]; \quad \mathfrak{N}_{21} = \omega_1 \omega_2 \sin \varphi_1$$

# Linear momentum of a rigid body coupled multi-rotations around no intersecting axes



$$\vec{\mathfrak{R}} = \vec{\mathfrak{R}}_{\vec{n}_1}^{(O_{1-2})} + \vec{\mathfrak{R}}_{\vec{n}_1}^{(O_2)} + \vec{\mathfrak{R}}_{\vec{n}_2}^{(O_2)} = [\vec{\omega}_1, \vec{r}_{012}]M + \omega_1 \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)} + \omega_2 \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)}$$

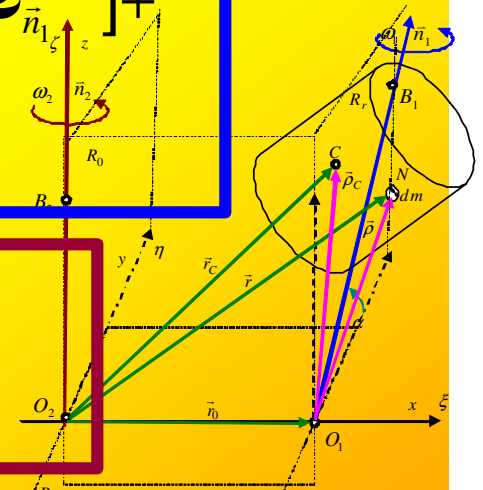
$$\begin{aligned}\vec{\mathfrak{K}} &= \vec{\mathfrak{K}}_{\vec{n}_1}^{(O_{1-3})} + \vec{\mathfrak{K}}_{\vec{n}_2}^{(O_{2-3})} + \vec{\mathfrak{K}}_{\vec{n}_1}^{(O_3)} + \vec{\mathfrak{K}}_{\vec{n}_2}^{(O_3)} + \vec{\mathfrak{K}}_{\vec{n}_3}^{(O_3)} = \\ &= \omega_1 [\vec{n}_1, \vec{r}_{012} + \vec{r}_{022} + \vec{r}_{023}]M + \omega_2 [\vec{n}_2, \vec{r}_{023}]M + \omega_1 \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)} + \omega_2 \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)} + \omega_3 \vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)}\end{aligned}$$

$$\vec{\mathfrak{R}} = \vec{\mathfrak{R}}_{\vec{n}_1}^{(O_{1-2})} + \vec{\mathfrak{R}}_{\vec{n}_1}^{(O_2)} + \vec{\mathfrak{R}}_{\vec{n}_2}^{(O_2)} = [\vec{\omega}_1, \vec{r}_{012}]M + \omega_1 \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)} + \omega_2 \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)}$$

$$\vec{\mathfrak{Q}}_{O_1} = \omega_1 \vec{n}_1 r_{012}^2 M + \omega_1 [\vec{\rho}_C, [\vec{n}_1, \vec{r}_{012}]]M + \omega_1 [\vec{r}_{012}, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)}] + \omega_2 [\vec{r}_{012}, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)}] + \omega_1 \vec{\mathfrak{J}}_{\vec{n}_1}^{(O_2)} + \omega_2 \vec{\mathfrak{J}}_{\vec{n}_2}^{(O_2)}$$

$$\begin{aligned} \frac{d\vec{\mathfrak{R}}}{dt} &= \dot{\omega}_1 [\vec{n}_1, \vec{r}_0]M + \omega_1^2 [\vec{n}_1, [\vec{n}_1, \vec{r}_0]]M + \dot{\omega}_1 \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)} + \omega_1^2 [\vec{n}_1, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)}] + \\ &+ \dot{\omega}_2 \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)} + \omega_2^2 [\vec{n}_2, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)}] + 2\omega_1 \omega_2 [\vec{n}_2, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)}] \end{aligned}$$

$$\frac{d\vec{\mathfrak{M}}}{dt} = \vec{\mathfrak{M}}_{01} [\vec{n}_1, \vec{r}_0]M + \vec{\mathfrak{M}}_{11} |\vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)}| + \vec{\mathfrak{M}}_{022} |\vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)}| + 2\omega_1 \omega_2 [\vec{n}_2, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)}]$$



$$\vec{\mathfrak{M}}_{01} = \dot{\omega}_1 \vec{u}_{01} + \omega_1^2 \vec{v}_{01} = \dot{\omega}_1 \left[ \vec{n}_1, \frac{\vec{r}_0}{r_0} \right] + \omega_1^2 \left[ \vec{n}_1, \left[ \vec{n}_1, \frac{\vec{r}_0}{r_0} \right] \right]; \vec{\mathfrak{M}}_{011} = \dot{\omega}_1 \vec{u}_{01} + \omega_1^2 \vec{v}_{02} = \dot{\omega}_1 \frac{\vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)}}{\mathfrak{S}_{\vec{n}_1}^{(O_2)}} + \omega_1^2 \left[ \vec{n}_1, \frac{\vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)}}{\mathfrak{S}_{\vec{n}_1}^{(O_2)}} \right]$$

$$\vec{\mathfrak{M}}_{022} = \dot{\omega}_2 \vec{u}_{02} + \omega_2^2 \vec{v}_{02} = \dot{\omega}_2 \frac{\vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)}}{\mathfrak{S}_{\vec{n}_2}^{(O_2)}} + \omega_2^2 \left[ \vec{n}_2, \frac{\vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)}}{\mathfrak{S}_{\vec{n}_2}^{(O_2)}} \right]$$

$$\vec{\mathfrak{Q}}_{O_1} = \omega_1 \bar{n}_1 r_{012}^2 M + \omega_1 [\vec{\rho}_C, [\bar{n}_1, \vec{r}_{012}]] M + \omega_1 [\vec{r}_{012}, \vec{\mathfrak{S}}_{\bar{n}_1}^{(O_2)}] + \omega_2 [\vec{r}_{012}, \vec{\mathfrak{S}}_{\bar{n}_2}^{(O_2)}] + \omega_1 \vec{\mathfrak{S}}_{\bar{n}_1}^{(O_2)} + \omega_2 \vec{\mathfrak{S}}_{\bar{n}_2}^{(O_2)}$$

$$\begin{aligned} \frac{d\vec{\mathfrak{Q}}_{O_1}}{dt} = & \bar{\chi}_{12}(\bar{r}_0, \bar{\rho}_C, M, \dot{\omega}_1, \dot{\omega}_2, \omega_1, \omega_2, \bar{n}_1, \bar{n}_2) + \dot{\omega}_1 \bar{n}_1 r_0^2 M + 2\omega_1 \omega_2 [\bar{n}_1, \vec{\mathfrak{S}}_{\bar{n}_2}^{(O_2)}] \\ & + \dot{\omega}_1 (\bar{n}_1, \vec{\mathfrak{S}}_{\bar{n}_1}^{(O_2)}) \bar{n}_1 + \dot{\omega}_2 (\bar{n}_2, \vec{\mathfrak{S}}_{\bar{n}_2}^{(O_2)}) \bar{n}_2 + \mathfrak{N}_1 |\vec{\mathfrak{S}}_{\bar{n}_1}^{(O_2)}| + \mathfrak{N}_2 |\vec{\mathfrak{S}}_{\bar{n}_2}^{(O_2)}| \end{aligned}$$

$$\vec{\mathfrak{N}}_1 = \dot{\omega}_1 \frac{\vec{\mathfrak{S}}_{\bar{n}_1}^{(O_2)}}{\mathfrak{S}_{\bar{n}_1}^{(O_2)}} + \omega_1^2 \left[ \bar{n}_1, \frac{\vec{\mathfrak{S}}_{\bar{n}_1}^{(O_2)}}{\mathfrak{S}_{\bar{n}_1}^{(O_2)}} \right], \quad \vec{\mathfrak{N}}_2 = \dot{\omega}_2 \frac{\vec{\mathfrak{S}}_{\bar{n}_2}^{(O_2)}}{\mathfrak{S}_{\bar{n}_2}^{(O_2)}} + \omega_2^2 \left[ \bar{n}_2, \frac{\vec{\mathfrak{S}}_{\bar{n}_2}^{(O_2)}}{\mathfrak{S}_{\bar{n}_2}^{(O_2)}} \right]$$

$$\vec{\mathfrak{K}} = \omega_1 [\vec{n}_1, \vec{r}_{012} + \vec{r}_{022} + \vec{r}_{023}]M + \omega_2 [\vec{n}_2, \vec{r}_{023}]M + \omega_1 \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)} + \omega_2 \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)} + \omega_3 \vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)} \omega_3$$

$$\begin{aligned} \frac{d\vec{\mathfrak{K}}}{dt} = & \dot{\omega}_1 [\vec{n}_1, \vec{r}_{012} + \vec{r}_{022} + \vec{r}_{023}]M + \omega_1^2 [\vec{n}_1, [\vec{n}_1, \vec{r}_{012} + \vec{r}_{022} + \vec{r}_{023}]] + \dot{\omega}_2 [\vec{n}_2, \vec{r}_{023}]M + \omega_2^2 [\vec{n}_2, [\vec{n}_2, \vec{r}_{023}]]M + \\ & + 2\omega_1 \omega_2 [\vec{n}_1, [\vec{n}_2, \vec{r}_{023}]]M + \dot{\omega}_1 \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)} + \omega_1^2 [\vec{n}_1, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)}] + \dot{\omega}_2 \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)} + \omega_2^2 [\vec{n}_2, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)}] + \dot{\omega}_3 \vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)} + \omega_3^2 [\vec{n}_3, \vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)}] \\ & + 2\omega_1 \omega_2 [\vec{n}_1, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)}] + 2\omega_1 \omega_3 [\vec{n}_1, \vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)}] + 2\omega_2 \omega_3 [\vec{n}_2, \vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)}] \vec{r} \end{aligned}$$

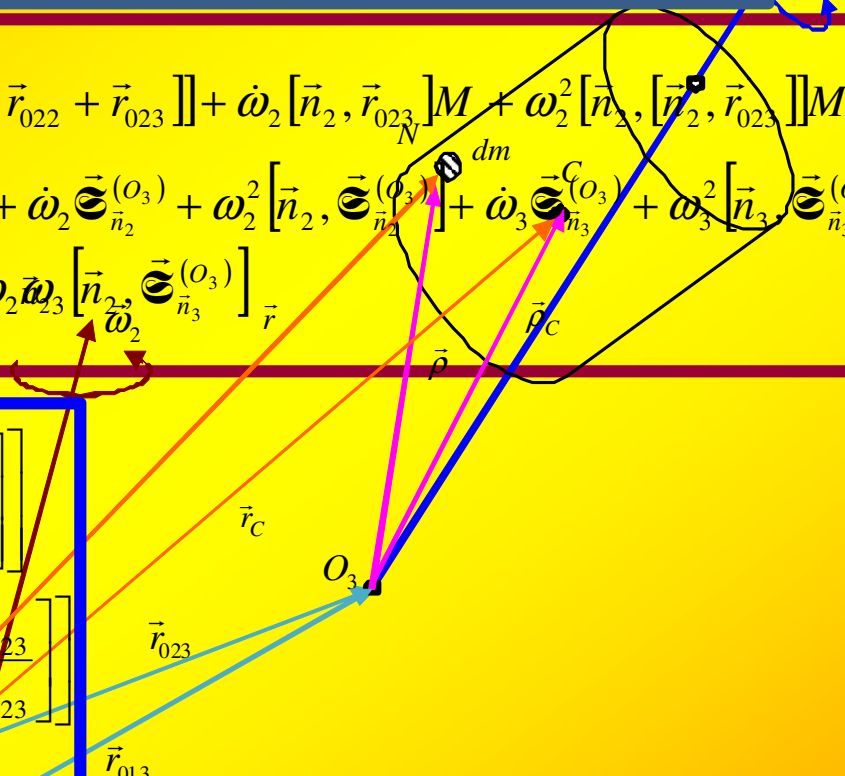
$$\vec{\mathfrak{N}}_{01} = \dot{\omega}_1 \vec{u}_{01} + \omega_1^2 \vec{v}_{01} = \dot{\omega}_1 \left[ \vec{n}_1, \frac{\vec{r}_{O_3}}{r_{O_3}} \right] + \omega_1^2 \left[ \vec{n}_1, \left[ \vec{n}_1, \frac{\vec{r}_{O_3}}{r_{O_3}} \right] \right]$$

$$\vec{\mathfrak{N}}_{02} = \dot{\omega}_2 \vec{u}_{02} + \omega_2^2 \vec{v}_{02} = \dot{\omega}_2 \left[ \vec{n}_2, \frac{\vec{r}_{023}}{r_{023}} \right] + \omega_2^2 \left[ \vec{n}_2, \left[ \vec{n}_2, \frac{\vec{r}_{023}}{r_{023}} \right] \right]$$

$$\vec{\mathfrak{N}}_{022} = \dot{\omega}_1 \vec{u}_{02} + \omega_1^2 \vec{v}_{02} = \dot{\omega}_1 \frac{\vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)}}{\mathfrak{S}_{\vec{n}_1}^{(O_3)}} + \omega_2^2 \left[ \vec{n}_1, \frac{\vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)}}{\mathfrak{S}_{\vec{n}_1}^{(O_3)}} \right]$$

$$\vec{\mathfrak{N}}_{022} = \dot{\omega}_2 \vec{u}_{02} + \omega_2^2 \vec{v}_{02} = \dot{\omega}_2 \frac{\vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)}}{\mathfrak{S}_{\vec{n}_2}^{(O_3)}} + \omega_2^2 \left[ \vec{n}_2, \frac{\vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)}}{\mathfrak{S}_{\vec{n}_2}^{(O_3)}} \right]$$

$$\vec{\mathfrak{N}}_{033} = \dot{\omega}_3 \vec{u}_{02} + \omega_3^2 \vec{v}_{02} = \dot{\omega}_3 \frac{\vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)}}{\mathfrak{S}_{\vec{n}_3}^{(O_3)}} + \omega_3^2 \left[ \vec{n}_3, \frac{\vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)}}{\mathfrak{S}_{\vec{n}_3}^{(O_3)}} \right]$$





# THEOREM 1.

Vector expression of linear momentum derivatives of the rigid N bodies, multi coupled rotations, around no intersecting axis in all cases, placed bodies on the each axis, between other terms, contain sum of products by intensity of rigid N bodies mass linear moment vectors

$$\left| \mathfrak{S}_{\mathbf{i}\vec{n}_j}^{(o_K)} \right| = \left| \iiint_{V_i} [\vec{n}_j, \vec{\rho}_i] dm_i \right|, \quad i = 1, 2, 3 \dots N, \quad j = 1, 2, 3 \dots K$$

for the axes oriented by unit vectors of component coupled rotation axes through pole on the rigid N bodies self-rotation axis and vector rotators defined by:

$$\vec{\mathfrak{N}}_{\mathbf{i}0_{jj}} = \dot{\omega}_j \frac{\mathfrak{S}_{\mathbf{i}\vec{n}_j}^{(o_K)}}{\left| \mathfrak{S}_{\mathbf{i}\vec{n}_j}^{(o_K)} \right|} + \omega_j^2 \left[ \vec{n}_j, \frac{\mathfrak{S}_{\mathbf{i}\vec{n}_j}^{(o_K)}}{\left| \mathfrak{S}_{\mathbf{i}\vec{n}_j}^{(o_K)} \right|} \right] \quad i = 1, 2, 3 \dots N, \quad j = 1, 2, 3 \dots K$$

**Where are:**

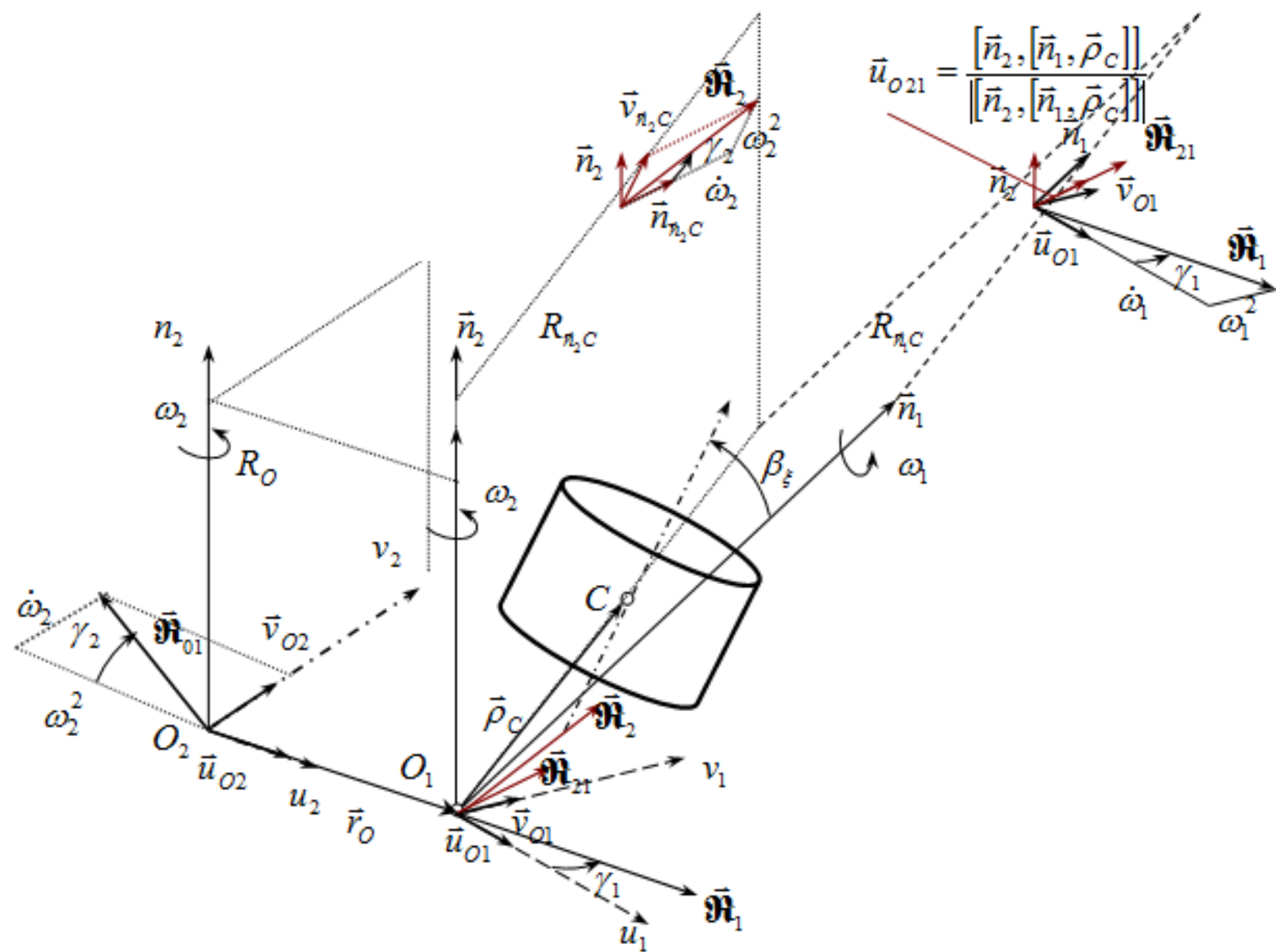
$i = 1, 2, 3 \dots N$       **number of bodies,**

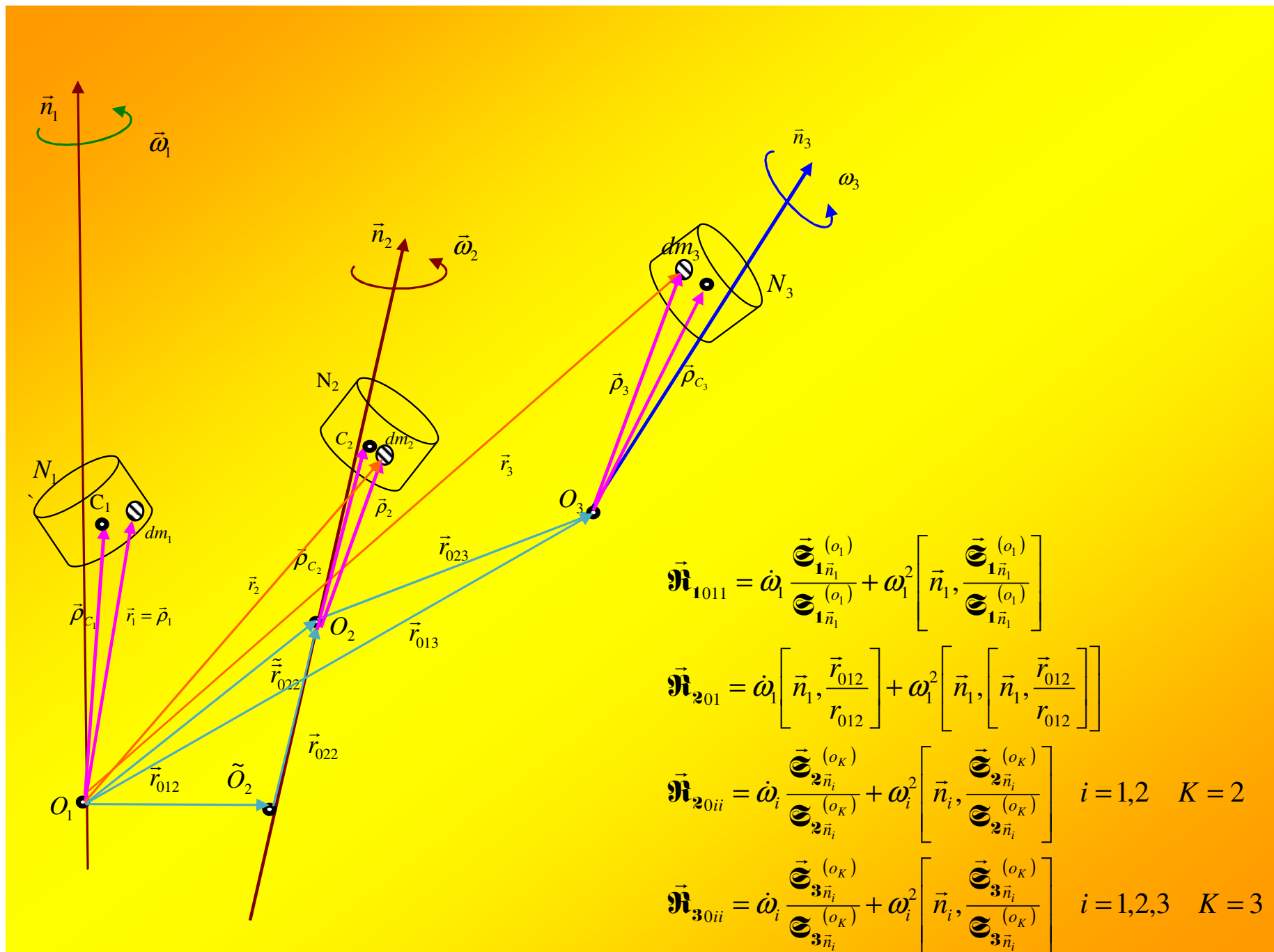
$j = 1, 2, 3 \dots K$       **number of axis**

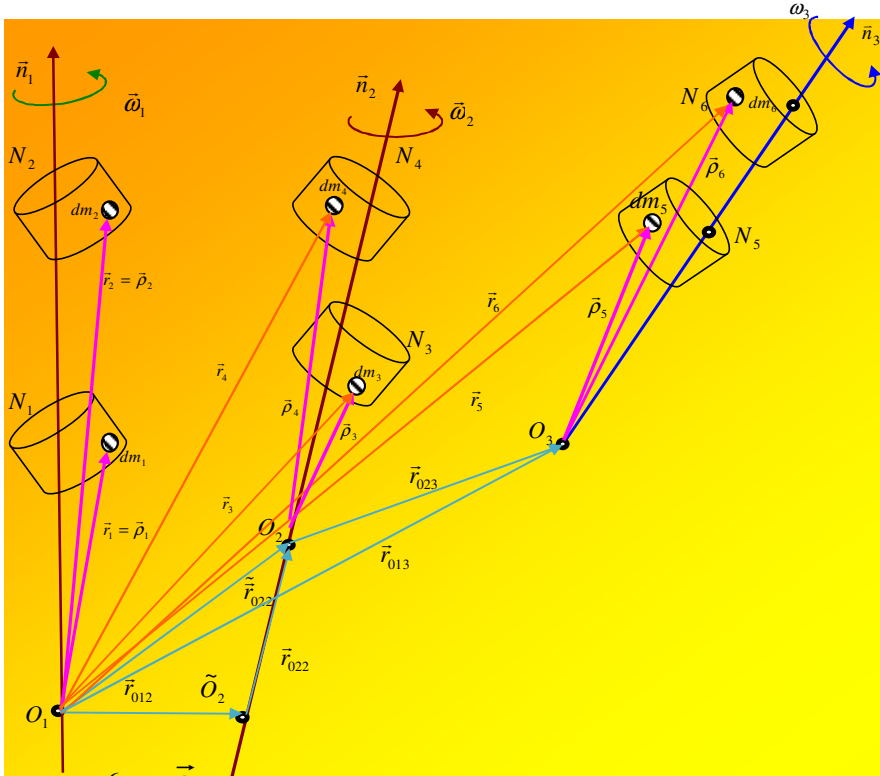
$$\begin{aligned}
\vec{\mathfrak{Q}}_{O_1} = & \omega_1 [\vec{r}_{O_3}, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_{1-2-2})}] + \omega_1 [\vec{r}_{O_3}, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_{2-3})}] + \omega_2 [\vec{r}_{O_3}, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_{2-3})}] + \omega_3 [\vec{r}_{O_3}, \vec{\mathfrak{S}}_{\vec{n}_3}^{(O_{2-3})}] \\
& + \omega_1 [\vec{\rho}_C, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_{1-2-2})}] + \omega_1 [\vec{\rho}_C, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_{2-3})}] + \omega_2 [\vec{\rho}_C, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_{2-3})}] + \omega_3 [\vec{\rho}_C, \vec{\mathfrak{S}}_{\vec{n}_3}^{(O_{2-3})}] \\
& + \omega_1 [\vec{r}_{O_3}, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)}] + \omega_2 [\vec{r}_{O_3}, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)}] + \omega_3 [\vec{r}_{O_3}, \vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)}] + \omega_1 \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)} + \omega_2 \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)} + \omega_3 \vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)}
\end{aligned}$$

$$\begin{aligned}
\frac{d\vec{\mathfrak{Q}}_{O_1}}{dt} = & \tilde{\chi}_{123}(\vec{r}_{O_3}, \vec{\rho}_C, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_{1-2-2})}, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_{2-3})}, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_{2-3})}, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)}, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)}, \vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)}, \vec{r}_C, \dot{\omega}_1, \dot{\omega}_2, \dot{\omega}_3, \omega_1, \omega_2, \omega_3, \vec{n}_1, \vec{n}_2, \vec{n}_3) + \\
& + \dot{\omega}_1(\vec{n}_1, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)})\vec{n}_1 + \dot{\omega}_2(\vec{n}_2, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)})\vec{n}_2 + \dot{\omega}_3(\vec{n}_3, \vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)})\vec{n}_3 + \mathfrak{K}_1|\vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)}| + \mathfrak{K}_2|\vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)}| + \mathfrak{K}_3|\vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)}| + \\
& + 2\omega_1\omega_2[\vec{n}_2, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)}] + 2\omega_2\omega_3[\vec{n}_3, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)}] + 2\omega_1\omega_3[\vec{n}_1, \vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)}]
\end{aligned}$$

$$\vec{\mathfrak{N}}_1 = \dot{\omega}_1 \frac{\vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)}}{\mathfrak{S}_{\vec{n}_1}^{(O_3)}} + \omega_1^2 \left[ \vec{n}_1, \frac{\vec{\mathfrak{S}}_{\vec{n}_1}^{(O_3)}}{\mathfrak{S}_{\vec{n}_1}^{(O_3)}} \right], \quad \vec{\mathfrak{N}}_2 = \dot{\omega}_2 \frac{\vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)}}{\mathfrak{S}_{\vec{n}_2}^{(O_3)}} + \omega_2^2 \left[ \vec{n}_2, \frac{\vec{\mathfrak{S}}_{\vec{n}_2}^{(O_3)}}{\mathfrak{S}_{\vec{n}_2}^{(O_3)}} \right], \quad \vec{\mathfrak{N}}_3 = \dot{\omega}_3 \frac{\vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)}}{\mathfrak{S}_{\vec{n}_3}^{(O_3)}} + \omega_3^2 \left[ \vec{n}_3, \frac{\vec{\mathfrak{S}}_{\vec{n}_3}^{(O_3)}}{\mathfrak{S}_{\vec{n}_3}^{(O_3)}} \right]$$







$$\bar{\mathfrak{M}}_{1011} = \dot{\omega}_1 \frac{\bar{\mathfrak{S}}_{1\bar{n}_1}^{(o_1)}}{\bar{\mathfrak{S}}_{1\bar{n}_1}^{(o_1)}} + \omega_1^2 \left[ \bar{n}_1, \frac{\bar{\mathfrak{S}}_{1\bar{n}_1}^{(o_1)}}{\bar{\mathfrak{S}}_{1\bar{n}_1}^{(o_1)}} \right], \quad \bar{\mathfrak{M}}_{2011} = \dot{\omega}_1 \frac{\bar{\mathfrak{S}}_{2\bar{n}_1}^{(o_1)}}{\bar{\mathfrak{S}}_{2\bar{n}_1}^{(o_1)}} + \omega_1^2 \left[ \bar{n}_1, \frac{\bar{\mathfrak{S}}_{2\bar{n}_1}^{(o_1)}}{\bar{\mathfrak{S}}_{2\bar{n}_1}^{(o_1)}} \right]$$

$$\bar{\mathfrak{M}}_{301} = \dot{\omega}_1 \left[ \bar{n}_1, \frac{\bar{r}_{012}}{r_{012}} \right] + \omega_1^2 \left[ \bar{n}_1, \left[ \bar{n}_1, \frac{\bar{r}_{012}}{r_{012}} \right] \right]$$

$$\bar{\mathfrak{M}}_{30ii} = \dot{\omega}_i \frac{\bar{\mathfrak{S}}_{3\bar{n}_i}^{(o_K)}}{\bar{\mathfrak{S}}_{3\bar{n}_i}^{(o_K)}} + \omega_i^2 \left[ \bar{n}_i, \frac{\bar{\mathfrak{S}}_{3\bar{n}_i}^{(o_K)}}{\bar{\mathfrak{S}}_{3\bar{n}_i}^{(o_K)}} \right] \quad i=1,2 \quad K=2$$

$$\bar{\mathfrak{M}}_{40ii} = \dot{\omega}_i \frac{\bar{\mathfrak{S}}_{4\bar{n}_i}^{(o_K)}}{\bar{\mathfrak{S}}_{4\bar{n}_i}^{(o_K)}} + \omega_i^2 \left[ \bar{n}_i, \frac{\bar{\mathfrak{S}}_{4\bar{n}_i}^{(o_K)}}{\bar{\mathfrak{S}}_{4\bar{n}_i}^{(o_K)}} \right] \quad i=1,2 \quad K=2$$

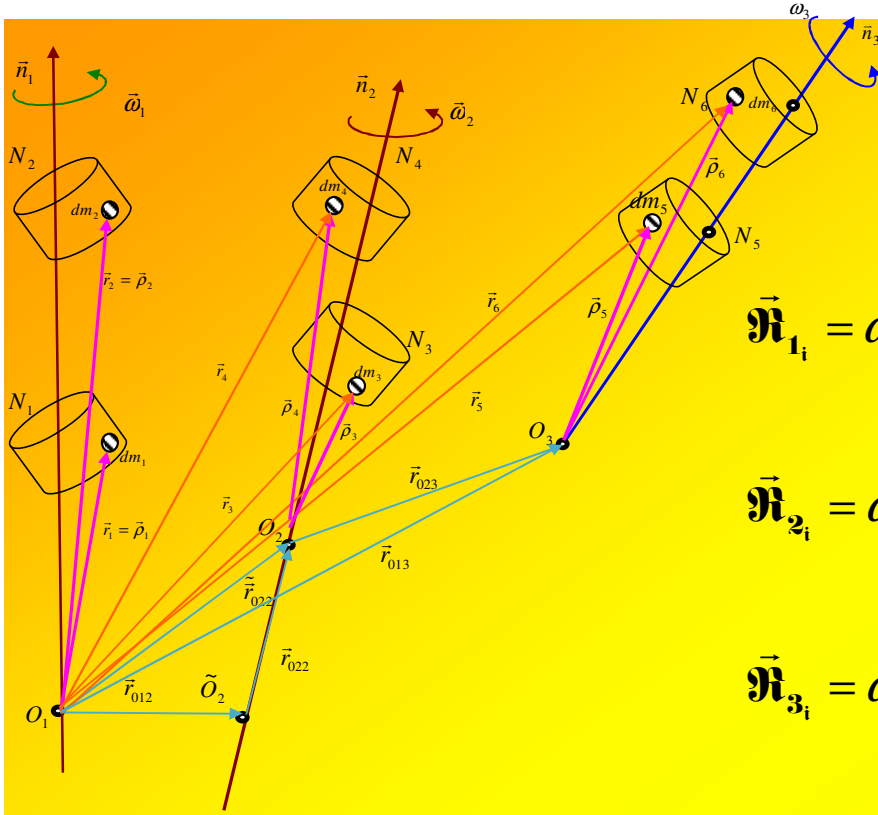
$$\bar{\mathfrak{M}}_{50ii} = \dot{\omega}_i \frac{\bar{\mathfrak{S}}_{5\bar{n}_i}^{(o_K)}}{\bar{\mathfrak{S}}_{5\bar{n}_i}^{(o_K)}} + \omega_i^2 \left[ \bar{n}_i, \frac{\bar{\mathfrak{S}}_{5\bar{n}_i}^{(o_K)}}{\bar{\mathfrak{S}}_{5\bar{n}_i}^{(o_K)}} \right] \quad i=1,2,3 \quad K=3$$

$$\bar{\mathfrak{M}}_{60ii} = \dot{\omega}_i \frac{\bar{\mathfrak{S}}_{6\bar{n}_i}^{(o_K)}}{\bar{\mathfrak{S}}_{6\bar{n}_i}^{(o_K)}} + \omega_i^2 \left[ \bar{n}_i, \frac{\bar{\mathfrak{S}}_{6\bar{n}_i}^{(o_K)}}{\bar{\mathfrak{S}}_{6\bar{n}_i}^{(o_K)}} \right] \quad i=1,2,3 \quad K=3$$

$$\begin{aligned} \sum_{i=1}^6 \frac{d\bar{\mathfrak{R}}_i}{dt} = & \bar{\mathfrak{M}}_{1011} \left| \bar{\mathfrak{S}}_{1\bar{n}_1}^{(o_1)} \right| + \bar{\mathfrak{M}}_{2011} \left| \bar{\mathfrak{S}}_{2\bar{n}_1}^{(o_1)} \right| + \bar{\mathfrak{M}}_{301} \left[ \bar{n}_1, \bar{r}_{012} \right] M_3 + \bar{\mathfrak{M}}_{3011} \left| \bar{\mathfrak{S}}_{3\bar{n}_1}^{(o_2)} \right| + \bar{\mathfrak{M}}_{3022} \left| \bar{\mathfrak{S}}_{3\bar{n}_2}^{(o_2)} \right| + 2\omega_1\omega_2 \left[ \bar{n}_2, \bar{\mathfrak{S}}_{3\bar{n}_1}^{(o_2)} \right] \\ & + \bar{\mathfrak{M}}_{401} \left[ \bar{n}_1, \bar{r}_{012} \right] M_4 + \bar{\mathfrak{M}}_{4011} \left| \bar{\mathfrak{S}}_{4\bar{n}_1}^{(o_2)} \right| + \bar{\mathfrak{M}}_{4022} \left| \bar{\mathfrak{S}}_{4\bar{n}_2}^{(o_2)} \right| + 2\omega_1\omega_2 \left[ \bar{n}_2, \bar{\mathfrak{S}}_{4\bar{n}_1}^{(o_2)} \right] + \\ & + \bar{\mathfrak{M}}_{501} \left[ \bar{n}_1, \bar{r}_{O_3} \right] M_5 + \bar{\mathfrak{M}}_{502} \left[ \bar{n}_2, \bar{r}_{023} \right] M_5 + 2\omega_1\omega_2 \left[ \bar{n}_1, \left[ \bar{n}_2, \bar{r}_{023} \right] \right] M_5 + \bar{\mathfrak{M}}_{5011} \left| \bar{\mathfrak{S}}_{5\bar{n}_1}^{(o_3)} \right| + \\ & + \bar{\mathfrak{M}}_{5022} \left| \bar{\mathfrak{S}}_{5\bar{n}_2}^{(o_3)} \right| + \bar{\mathfrak{M}}_{5033} \left| \bar{\mathfrak{S}}_{5\bar{n}_3}^{(o_3)} \right| + 2\omega_1\omega_2 \left[ \bar{n}_1, \bar{\mathfrak{S}}_{5\bar{n}_2}^{(o_3)} \right] + 2\omega_1\omega_3 \left[ \bar{n}_1, \bar{\mathfrak{S}}_{5\bar{n}_3}^{(o_3)} \right] + 2\omega_2\omega_3 \left[ \bar{n}_2, \bar{\mathfrak{S}}_{5\bar{n}_3}^{(o_3)} \right] + \\ & + \bar{\mathfrak{M}}_{601} \left[ \bar{n}_1, \bar{r}_{O_3} \right] M_6 + \bar{\mathfrak{M}}_{602} \left[ \bar{n}_2, \bar{r}_{023} \right] M_6 + 2\omega_1\omega_2 \left[ \bar{n}_1, \left[ \bar{n}_2, \bar{r}_{023} \right] \right] M_6 + \bar{\mathfrak{M}}_{6011} \left| \bar{\mathfrak{S}}_{6\bar{n}_1}^{(o_3)} \right| + \\ & + \bar{\mathfrak{M}}_{6022} \left| \bar{\mathfrak{S}}_{6\bar{n}_2}^{(o_3)} \right| + \bar{\mathfrak{M}}_{6033} \left| \bar{\mathfrak{S}}_{6\bar{n}_3}^{(o_3)} \right| + 2\omega_1\omega_2 \left[ \bar{n}_1, \bar{\mathfrak{S}}_{6\bar{n}_2}^{(o_3)} \right] + 2\omega_1\omega_3 \left[ \bar{n}_1, \bar{\mathfrak{S}}_{6\bar{n}_3}^{(o_3)} \right] + 2\omega_2\omega_3 \left[ \bar{n}_2, \bar{\mathfrak{S}}_{6\bar{n}_3}^{(o_3)} \right] \end{aligned}$$



$$\begin{aligned}
\sum_{i=1}^6 \frac{d\vec{\mathfrak{S}}_{iO_1}}{dt} = & \dot{\omega}_1 \left( \vec{n}_1, \vec{\mathfrak{S}}_{1\vec{n}_1}^{(O_1)} \right) \vec{n}_1 + \vec{\mathfrak{K}}_{1_1} \left| \vec{\mathfrak{S}}_{1\vec{n}_1}^{(O_1)} \right| + \dot{\omega}_1 \left( \vec{n}_1, \vec{\mathfrak{S}}_{2\vec{n}_1}^{(O_1)} \right) \vec{n}_1 + \vec{\mathfrak{K}}_{2_1} \left| \vec{\mathfrak{S}}_{2\vec{n}_1}^{(O_1)} \right| + \\
& + \vec{\chi}_{3_{12}} \left( \vec{r}_{012}, \vec{\rho}_{C_3}, M_3, \vec{\mathfrak{S}}_{3\vec{n}_1}^{(O_2)}, \vec{\mathfrak{S}}_{3\vec{n}_2}^{(O_2)}, \dot{\omega}_1, \dot{\omega}_2, \omega_1, \omega_2, \vec{n}_1, \vec{n}_2, \right) + 2\omega_1\omega_2 \left[ \vec{n}_1, \vec{\mathfrak{S}}_{3\vec{n}_2}^{(O_2)} \right] + \\
& + \vec{\mathfrak{K}}_{3_1} \left| \vec{\mathfrak{S}}_{3\vec{n}_1}^{(O_2)} \right| + \vec{\mathfrak{K}}_{3_2} \left| \vec{\mathfrak{S}}_{3\vec{n}_2}^{(O_2)} \right| + \dot{\omega}_1 \left( \vec{n}_1, \vec{\mathfrak{S}}_{3\vec{n}_1}^{(O_2)} \right) \vec{n}_1 + \dot{\omega}_2 \left( \vec{n}_2, \vec{\mathfrak{S}}_{3\vec{n}_2}^{(O_2)} \right) \vec{n}_2 + \\
& + \vec{\chi}_{4_{12}} \left( \vec{r}_{012}, \vec{\rho}_{C_4}, M_4, \vec{\mathfrak{S}}_{4\vec{n}_1}^{(O_2)}, \vec{\mathfrak{S}}_{4\vec{n}_2}^{(O_2)}, \dot{\omega}_1, \dot{\omega}_2, \omega_1, \omega_2, \vec{n}_1, \vec{n}_2, \right) + 2\omega_1\omega_2 \left[ \vec{n}_1, \vec{\mathfrak{S}}_{4\vec{n}_2}^{(O_2)} \right] + \\
& + \vec{\mathfrak{K}}_{4_1} \left| \vec{\mathfrak{S}}_{4\vec{n}_1}^{(O_2)} \right| + \vec{\mathfrak{K}}_{4_2} \left| \vec{\mathfrak{S}}_{4\vec{n}_2}^{(O_2)} \right| + \dot{\omega}_1 \left( \vec{n}_1, \vec{\mathfrak{S}}_{4\vec{n}_1}^{(O_2)} \right) \vec{n}_1 + \dot{\omega}_2 \left( \vec{n}_2, \vec{\mathfrak{S}}_{4\vec{n}_2}^{(O_2)} \right) \vec{n}_2 + \\
& + \vec{\chi}_{5_{123}} \left( \vec{r}_{5O_3}, \vec{\rho}_{C_5}, \vec{\mathfrak{S}}_{5\vec{n}_1}^{(O_{1-2-2})}, \vec{\mathfrak{S}}_{5\vec{n}_1}^{(O_{2-3})}, \vec{\mathfrak{S}}_{5\vec{n}_2}^{(O_{2-3})}, \vec{\mathfrak{S}}_{5\vec{n}_1}^{(O_3)}, \vec{\mathfrak{S}}_{5\vec{n}_2}^{(O_3)}, \vec{\mathfrak{S}}_{5\vec{n}_3}^{(O_3)}, \dot{\omega}_1, \dot{\omega}_2, \dot{\omega}_3, \omega_1, \omega_2, \omega_3, \vec{n}_1, \vec{n}_2, \vec{n}_3 \right) + \\
& + \vec{\mathfrak{K}}_{5_1} \left| \vec{\mathfrak{S}}_{5\vec{n}_1}^{(O_3)} \right| + \vec{\mathfrak{K}}_{5_2} \left| \vec{\mathfrak{S}}_{5\vec{n}_2}^{(O_3)} \right| + \vec{\mathfrak{K}}_{5_3} \left| \vec{\mathfrak{S}}_{5\vec{n}_3}^{(O_3)} \right| + \dot{\omega}_1 \left( \vec{n}_1, \vec{\mathfrak{S}}_{5\vec{n}_1}^{(O_3)} \right) \vec{n}_1 + \dot{\omega}_2 \left( \vec{n}_2, \vec{\mathfrak{S}}_{5\vec{n}_2}^{(O_3)} \right) \vec{n}_2 + \dot{\omega}_3 \left( \vec{n}_3, \vec{\mathfrak{S}}_{5\vec{n}_3}^{(O_3)} \right) \vec{n}_3 + \\
& + 2\omega_1\omega_2 \left[ \vec{n}_2, \vec{\mathfrak{S}}_{5\vec{n}_1}^{(O_3)} \right] + 2\omega_2\omega_3 \left[ \vec{n}_3, \vec{\mathfrak{S}}_{5\vec{n}_2}^{(O_3)} \right] + 2\omega_1\omega_3 \left[ \vec{n}_1, \vec{\mathfrak{S}}_{5\vec{n}_3}^{(O_3)} \right] + \\
& + \vec{\chi}_{6_{123}} \left( \vec{r}_{6O_3}, \vec{\rho}_{C_6}, \vec{\mathfrak{S}}_{6\vec{n}_1}^{(O_{1-2-2})}, \vec{\mathfrak{S}}_{6\vec{n}_1}^{(O_{2-3})}, \vec{\mathfrak{S}}_{6\vec{n}_2}^{(O_{2-3})}, \vec{\mathfrak{S}}_{6\vec{n}_1}^{(O_3)}, \vec{\mathfrak{S}}_{6\vec{n}_2}^{(O_3)}, \vec{\mathfrak{S}}_{6\vec{n}_3}^{(O_3)}, \dot{\omega}_1, \dot{\omega}_2, \dot{\omega}_3, \omega_1, \omega_2, \omega_3, \vec{n}_1, \vec{n}_2, \vec{n}_3 \right) + \\
& + \vec{\mathfrak{K}}_{6_1} \left| \vec{\mathfrak{S}}_{6\vec{n}_1}^{(O_3)} \right| + \vec{\mathfrak{K}}_{6_2} \left| \vec{\mathfrak{S}}_{6\vec{n}_2}^{(O_3)} \right| + \vec{\mathfrak{K}}_{6_3} \left| \vec{\mathfrak{S}}_{6\vec{n}_3}^{(O_3)} \right| + \dot{\omega}_1 \left( \vec{n}_1, \vec{\mathfrak{S}}_{6\vec{n}_1}^{(O_3)} \right) \vec{n}_1 + \dot{\omega}_2 \left( \vec{n}_2, \vec{\mathfrak{S}}_{6\vec{n}_2}^{(O_3)} \right) \vec{n}_2 + \dot{\omega}_3 \left( \vec{n}_3, \vec{\mathfrak{S}}_{6\vec{n}_3}^{(O_3)} \right) \vec{n}_3 \\
& + 2\omega_1\omega_2 \left[ \vec{n}_2, \vec{\mathfrak{S}}_{6\vec{n}_1}^{(O_3)} \right] + 2\omega_2\omega_3 \left[ \vec{n}_3, \vec{\mathfrak{S}}_{6\vec{n}_2}^{(O_3)} \right] + 2\omega_1\omega_3 \left[ \vec{n}_1, \vec{\mathfrak{S}}_{6\vec{n}_3}^{(O_3)} \right]
\end{aligned}$$



$$\vec{\mathfrak{N}}_{1_i} = \dot{\omega}_i \frac{\vec{\mathfrak{D}}_{1\vec{n}_i}^{(o_K)}}{\mathfrak{D}_{1\vec{n}_i}^{(o_K)}} + \omega_i^2 \left[ \vec{n}_i, \frac{\vec{\mathfrak{D}}_{1\vec{n}_i}^{(o_K)}}{\mathfrak{D}_{1\vec{n}_i}^{(o_K)}} \right] \quad i=1 \quad K=1$$

$$\vec{\mathfrak{N}}_{2_i} = \dot{\omega}_i \frac{\vec{\mathfrak{D}}_{2\vec{n}_i}^{(o_K)}}{\mathfrak{D}_{2\vec{n}_i}^{(o_K)}} + \omega_i^2 \left[ \vec{n}_i, \frac{\vec{\mathfrak{D}}_{2\vec{n}_i}^{(o_K)}}{\mathfrak{D}_{2\vec{n}_i}^{(o_K)}} \right] \quad i=1 \quad K=1$$

$$\vec{\mathfrak{N}}_{3_i} = \dot{\omega}_i \frac{\vec{\mathfrak{D}}_{3\vec{n}_i}^{(o_K)}}{\mathfrak{D}_{3\vec{n}_i}^{(o_K)}} + \omega_i^2 \left[ \vec{n}_i, \frac{\vec{\mathfrak{D}}_{3\vec{n}_i}^{(o_K)}}{\mathfrak{D}_{3\vec{n}_i}^{(o_K)}} \right] \quad i=1,2 \quad K=2$$

$$\vec{\mathfrak{N}}_{4_i} = \dot{\omega}_i \frac{\vec{\mathfrak{D}}_{4\vec{n}_i}^{(o_K)}}{\mathfrak{D}_{4\vec{n}_i}^{(o_K)}} + \omega_i^2 \left[ \vec{n}_i, \frac{\vec{\mathfrak{D}}_{4\vec{n}_i}^{(o_K)}}{\mathfrak{D}_{4\vec{n}_i}^{(o_K)}} \right] \quad i=1,2 \quad K=2$$

$$\vec{\mathfrak{N}}_{5_i} = \dot{\omega}_i \frac{\vec{\mathfrak{D}}_{5\vec{n}_i}^{(o_K)}}{\mathfrak{D}_{5\vec{n}_i}^{(o_K)}} + \omega_i^2 \left[ \vec{n}_i, \frac{\vec{\mathfrak{D}}_{5\vec{n}_i}^{(o_K)}}{\mathfrak{D}_{5\vec{n}_i}^{(o_K)}} \right] \quad i=1,2,3 \quad K=3$$

$$\vec{\mathfrak{N}}_{6_i} = \dot{\omega}_i \frac{\vec{\mathfrak{D}}_{6\vec{n}_i}^{(o_K)}}{\mathfrak{D}_{6\vec{n}_i}^{(o_K)}} + \omega_i^2 \left[ \vec{n}_i, \frac{\vec{\mathfrak{D}}_{6\vec{n}_i}^{(o_K)}}{\mathfrak{D}_{6\vec{n}_i}^{(o_K)}} \right] \quad i=1,2,3 \quad K=3$$

## THEOREM 2.

Vector expression of angular momentum derivatives of the rigid N bodies, multi coupled rotations, around no intersecting axis in all cases, placed bodies on the each axis, between other terms, contain sum of products by intensity of rigid N bodies mass deviation moment vectors

$$\left| \mathfrak{S}_{\mathbf{i}\vec{n}_j}^{(o_K)} \right| = \left\| \vec{n}_j, \left[ \iiint_{V_i} [\vec{n}_j, \vec{\rho}_i] dm_i, \vec{n}_j \right] \right\|, \quad i = 1, 2, 3 \dots N, \quad j = 1, 2, 3 \dots K$$

for the axes oriented by unit vectors of component coupled rotation axes through pole on the rigid N bodies self-rotation axis and vector rotators defined by:

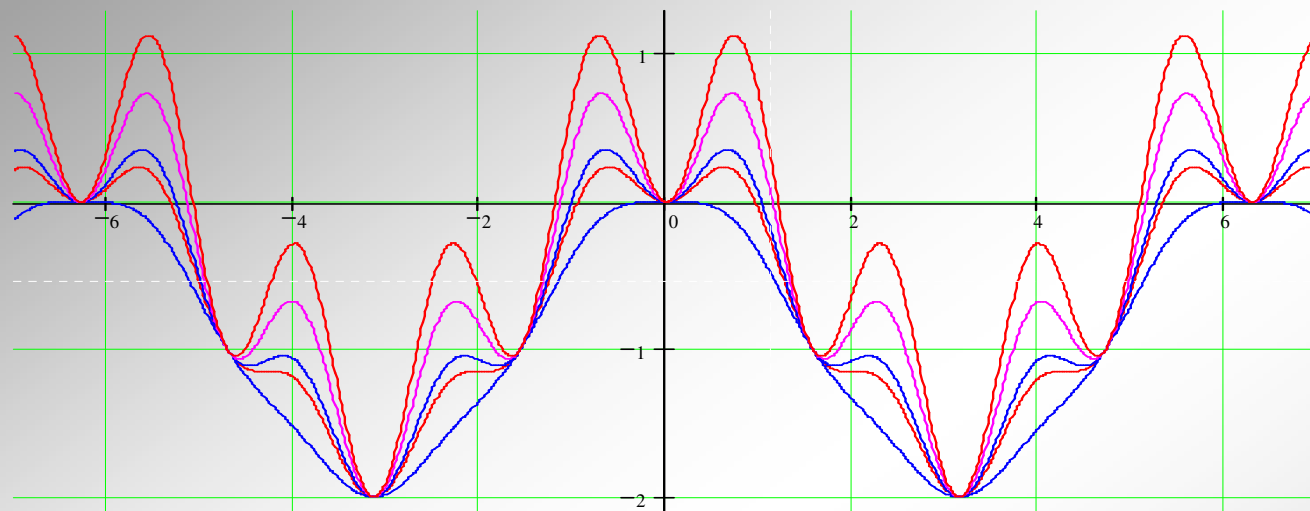
$$\vec{\mathfrak{R}}_{\mathbf{i}_i} = \dot{\omega}_j \frac{\vec{\mathfrak{S}}_{\mathbf{i}\vec{n}_j}^{(o_K)}}{\left| \mathfrak{S}_{\mathbf{i}\vec{n}_j}^{(o_K)} \right|} + \omega_j^2 \left[ \vec{n}_j, \frac{\vec{\mathfrak{S}}_{\mathbf{i}\vec{n}_j}^{(o_K)}}{\left| \mathfrak{S}_{\mathbf{i}\vec{n}_j}^{(o_K)} \right|} \right] \quad i = 1, 2, 3 \dots N, \quad j = 1, 2, 3 \dots K$$





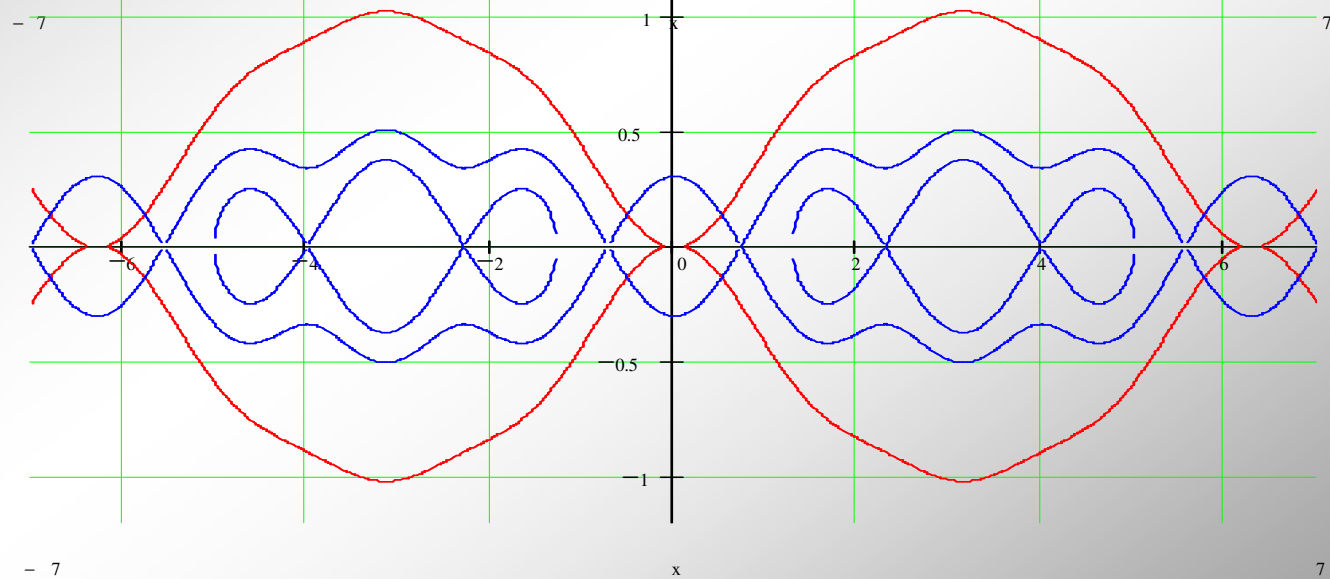
1.3

f1 (x)  
f2 (x)  
f3 (x)  
f4 (x)  
f6 (x)



- 2.3  
 1.2

b1 (x)  
b2 (x)  
b3 (x)  
b4 (x)  
b5 (x)  
b6 (x)



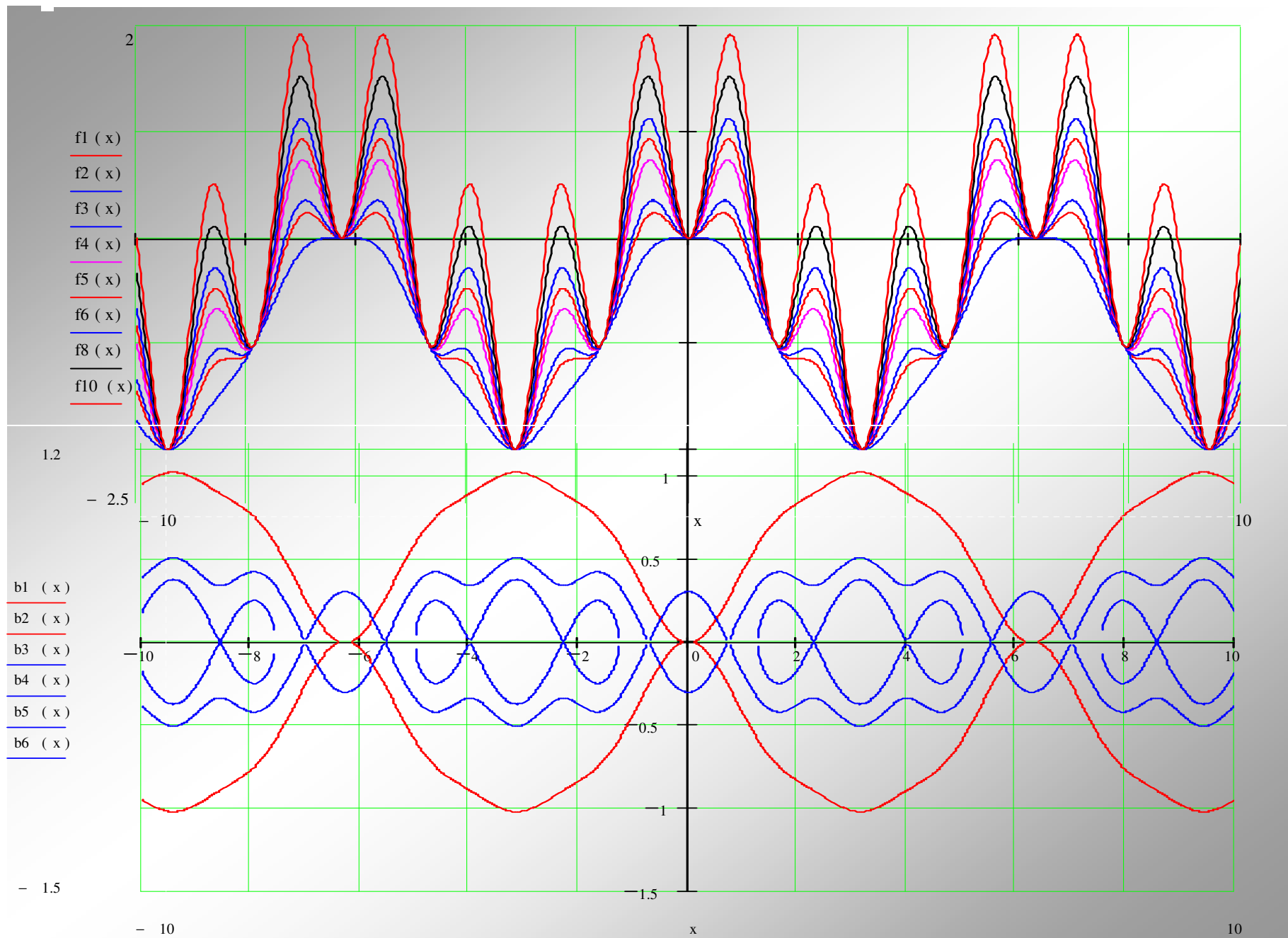
- 1.2

- 7

x

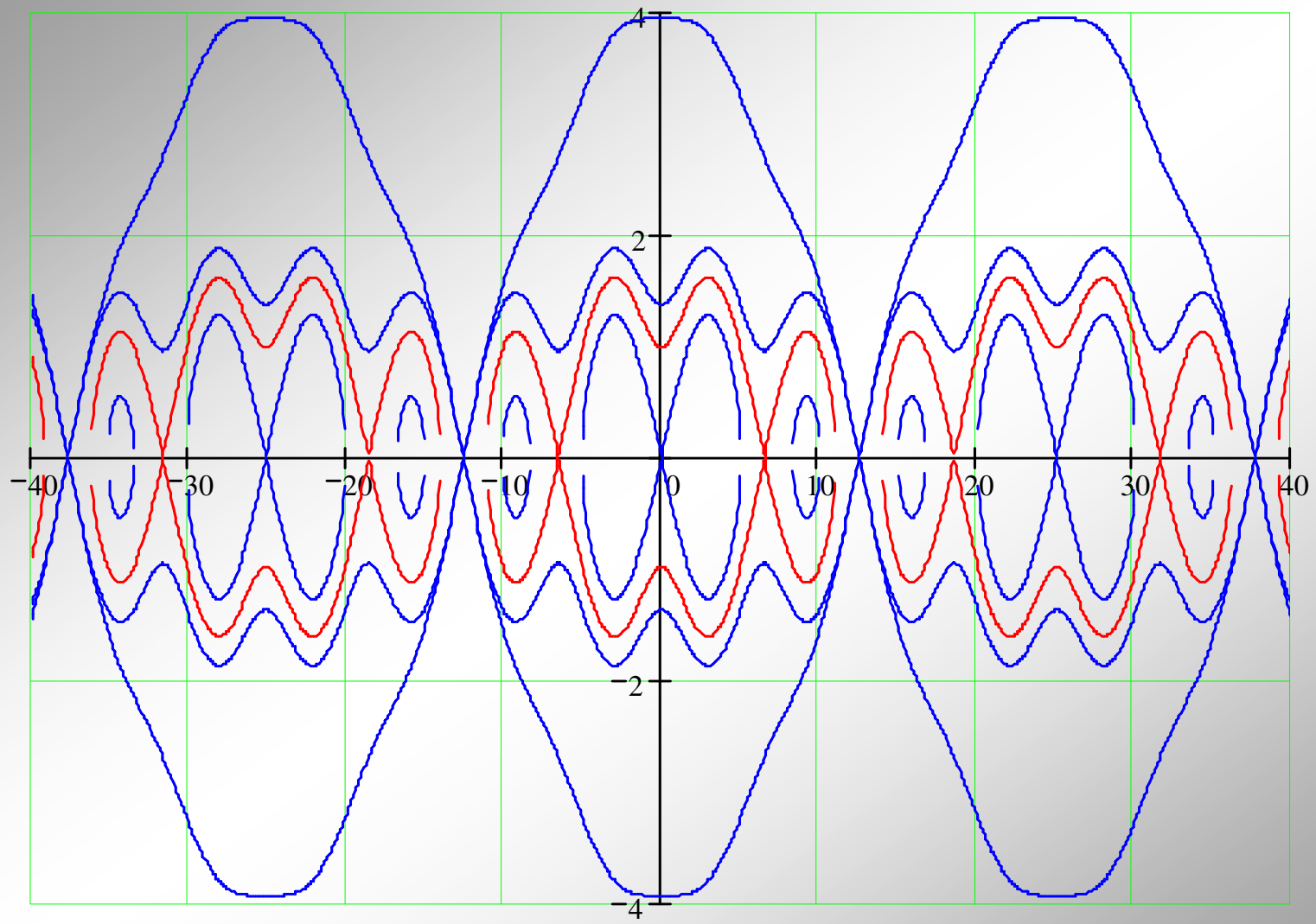
7





4

- a1(x)
- a2(x)
- a3(x)
- a4(x)
- a5(x)
- a6(x)
- a7(x)
- a8(x)

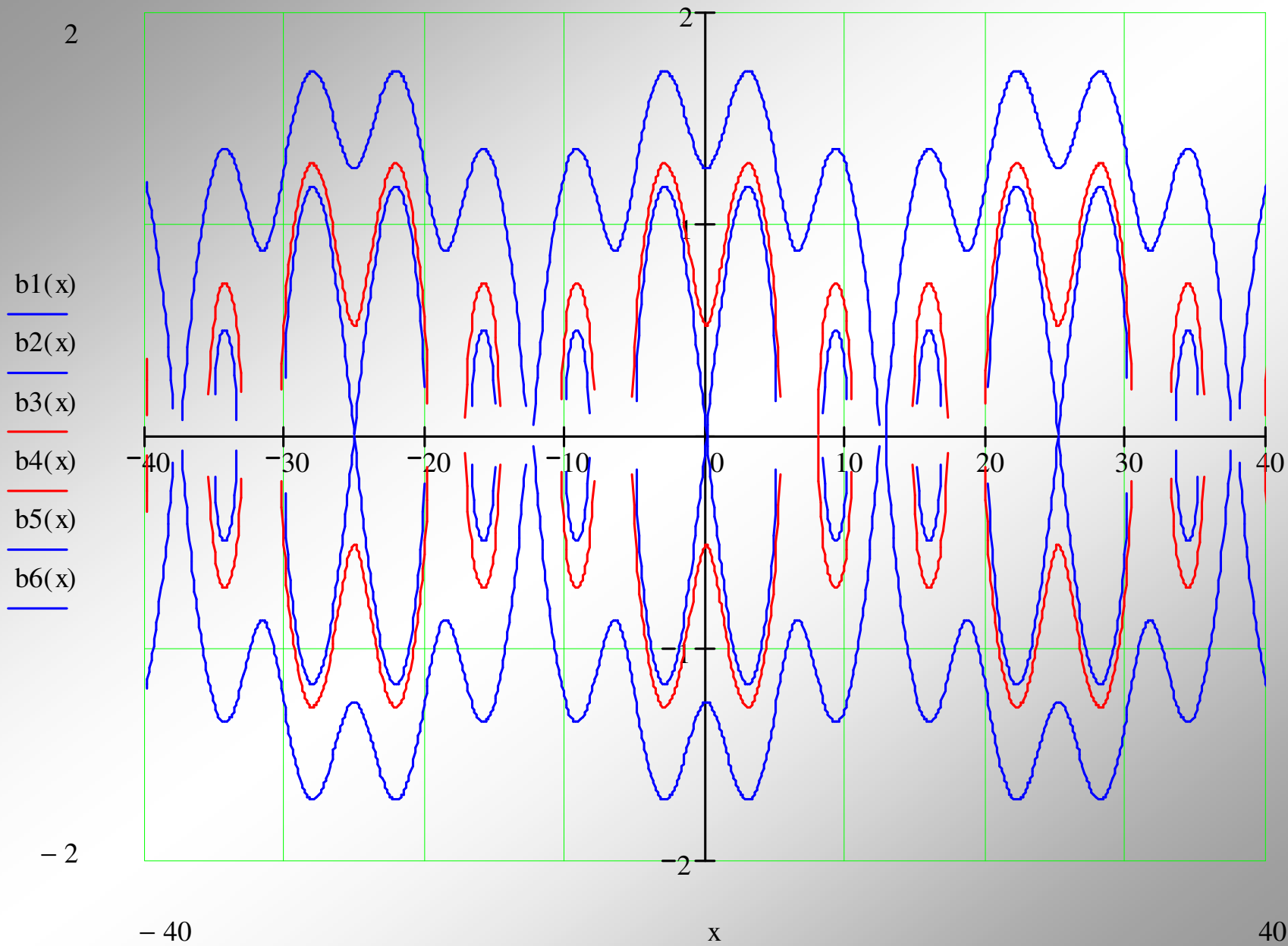


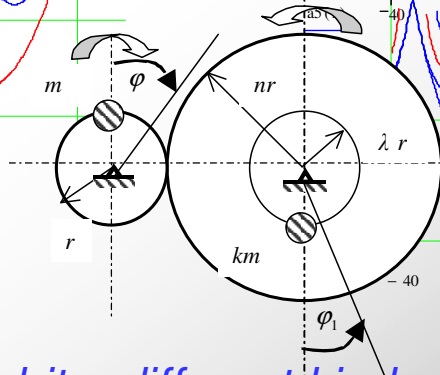
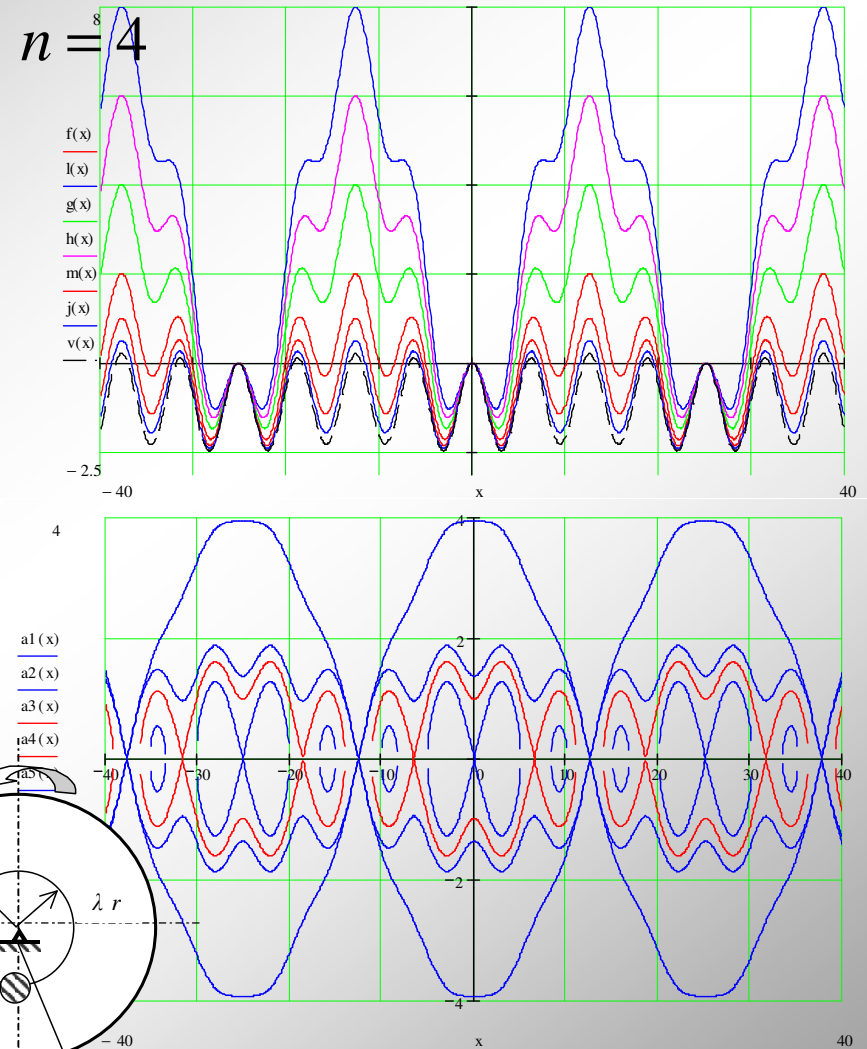
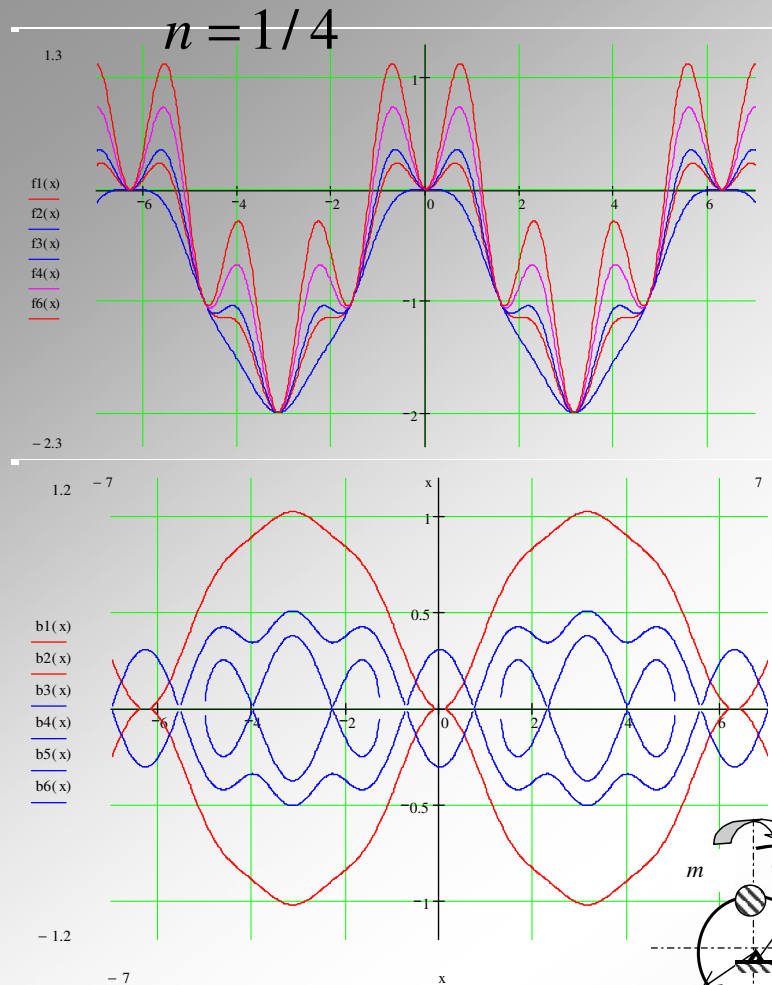
- 4

- 40

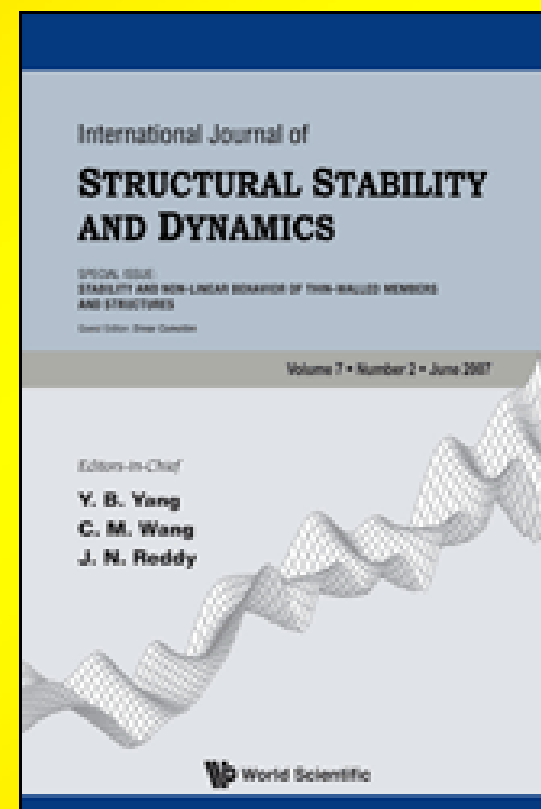
x

40





*Different type of homoclinic orbits: different kind of separatrix for inverse problem of coupled rotation nonlinear dynamics and corresponding families of potential energies.*



**Hedrih (Stevanović) K., (2013), Vector analysis of kinetic parameters of mass particle separation dynamics from space discrete system, Dynamical Systems – Theory, Edited by JAN AWREJCIEWICZ, MAREK KAŻMIERCZAK, PAWEŁ OLEJNIK, JERZY MROZOWSKI, 2013, pp. 343-354. ISBN 978-83-7283-588-6, Printed by: Wydawnictwo Politechniki Łódzkiej, [www.wydawnictwa.p.lodz.pl](http://www.wydawnictwa.p.lodz.pl). Tematski Zbornik**

Print ISSN: 0219-4554  
Online ISSN: 1793-6764

**From:** [computer](#)

**Sent:** Friday, March 14, 2014 3:18 AM

**To:** [khedrih](#)

**Subject:** Call for Papers -- Computer Technology and Application

## Call for Papers

Dear Katica R. Stevanovic Hedrih,

This is ***Computer Technology and Application*** (ISSN: 1934-7332) published across the **United States by David Publishing Company, New York, USA.**

We are glad to know you have a presentation titled '**Vector analysis of kinetic parameters of mass particle separation dynamics from space discrete system**' in '**12th Conference on Dynamical Systems - Theory and Applications**'. We are very interested in your research fields and would like to publish your paper in our journal. If the mentioned paper has not been published or you have other unpublished papers or books in hand and have the idea of making our journal a vehicle for your research interests, please free send the electronic version of your papers or books in MS word format to us.

.....  
**David Publishing Company**

**Site: 240 Nagle Avenue #15C, New York, NY 10034, USA**

**Website: <http://www.davidpublishing.com/>**







Hedrih (Stevanović) K., (213), **LINEAR AND NONLINEAR DYNAMICS OF HYBRID SYSTEM**, *Invited Plenary Lecture, (To memory of my Serbian professors: Draginja Nikolić, Danilo P. Rašković and Tatomir P. Andjelić and to academicians supported my international scientific activity in nonlinear dynamics: Yuri Alekseevich Mitropolskiy, Vladimir Metodievich Matrosov and Valentin Vitalevich Rumaantsev and President of IFNA Professor dr V. Lakshminatham )*, Proceedings of Fourth Serbian (29th Yu) Congress on Theoretical and Applied Mechanics, Vrnjačka Banja, Serbia, 4-7 June 2013, pp. 43-58. ISBN 978-86-909973-5-0  
**ISBN 978-86-909973-5-0. COBISS.SR-ID 198308876**



# THE 13<sup>th</sup> INTERNATIONAL CONFERENCE OF TENSOR SOCIETY ON DIFFERENTIAL GEOMETRY AND ITS APPLICATIONS, AND INFORMATICS BESIDES



Faculty of Mathematics  
Alexandru Ioan Cuza  
University of Iași

The 86<sup>th</sup> Anniversary of Radu MIRON's birthday

テンゾル学会  
( TENSOR SOCIETY )

September 3-7, 2013, Iași, Romania



Institute of Mathematics  
"Octav Mayer"  
Romanian Academy

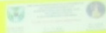
**Hedrih (Stevanović) K., (2013), Generalized function of fractional order dissipation of system energy and extended Lagrange differential equation in matrix form, *Dedicated to 86th Anniversary of Radu MIRON'S Birth. 30 minutes Plenary Lecture, Abstracts of THE 13th INTERNATIONAL CONFERENCE OF TENSOR SOCIETY ON DIFFERENTIAL GEOMETRY AND ITS APPLICATIONS, AND INFORMATICS BESIDES., The 86th Anniversary of Radu MIRON'S Birth. September 3rd (Tuesday) to September 7th (Saturday) in 2013. Faculty of Mathematics, Alexandru Ioan Cuza University and Mathematical Institute "O.Mayer" in Iași Romania And Tensor Society, Japan, 2013, p.3. (paper submitted for publishing in the journal Tensor of Tensor Society, Japan.)***

**<http://www.math.uaic.ro/~tensorconference2013/>**

**In Romaina Academy of Sciences (Depatmeny in lashi)**



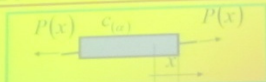


**Generalized function of fractional order  
dissipation of system energy**

$$2P_{\alpha \neq 0} = (D_t^\alpha \{x\}) C_\alpha^* (D_t^\alpha \{x\}), \quad \text{for } \alpha \neq 0$$

$$D_t^\alpha [x(t)] = \frac{d^\alpha x(t)}{dt^\alpha} = x^{(\alpha)}(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{x(\tau)}{(t-\tau)^\alpha} d\tau$$

$$0 < \alpha \leq 1$$











# TENSOR

*Edited by*

**Tomoaki KAWAGUCHI**

*With the cooperation of*

S. AMARI  
S. IGARASHI  
R. MIRON  
Y. SATO  
L. TAMÁSSY

W.-G. BOSKOFF  
H. KAWAGUCHI  
M. PRVANOVIĆ  
H. SHIMADA

K. (STEVANOVIĆ) HEDRIH  
K. MATSUMOTO  
M. SATO  
M. SHIMBO

**NEW SERIES**

**Volume 74, Number 1**

April 2013

PUBLISHED BY

**THE TENSOR SOCIETY**

**CHIGASAKI, JAPAN**

# Elements of mathematical phenomenology in dynamics of multi-body system with fractional order discrete continuum layers

**Катица Р. (Стевановић) Хедрих**

Одељење за механику Математичког института САНУ у Београду

и Машински факултет Универзитета у Нишу

Прив. адреса: 18000- Ниш, Србија, ул. Војводе Танкосића

e-mail: [khedrih@eunet.rs](mailto:khedrih@eunet.rs)

Hedrih (Stevanović) K., (2013), Two mass particle fractional order plane system dynamics, Dynamical Systems – Theory, Edited by JAN AWREJCEWICZ, MAREK KAŻMIERCZAK, PAWEŁ OLEJNIK, JERZY MROZOWSKI, 2013, pp. 403-4012. . ISBN 978-83-7283-588-8, Printed by: Wydawnictwo Politechniki Łódzkiej, [www.wydawnictwa.p.lodz.pl](http://www.wydawnictwa.p.lodz.pl).

Tematski Zbornik

Hedrih (Stevanović) K., (213), Fractional order differential equations of dynamics of two mass particles, constrained by a fractional order element, Plenary Lecture, Two pages Extended abstract, 8th INTERNATIONAL SYMPOSIUM ON CLASSICAL AND CELESTIAL MECHANICS (CCMECH'8), September 25 – 29, 2013, Siedlce, Poland, ( The Russian Academy of Sciences, A. A. Dorodnicyn Computing Centre of RAS, Moscow State University, Moscow State Aviation Institute, Warsaw University of Life Sciences, and Collegium Mazovia in Siedlce (Poland) will hold the Eighth International Symposium on Classical and Celestial Mechanics (CCMECH'2013) at Collegium Mazovia in Siedlce, Poland) Book of Abstracts, pp. 27-28. ISBN 078-83-63169-4, @Copyright by Collegium Mazovia <http://agora.guru.ru/display.php?conf=CCMECH7>, <http://www.msz.gov.pl/en>

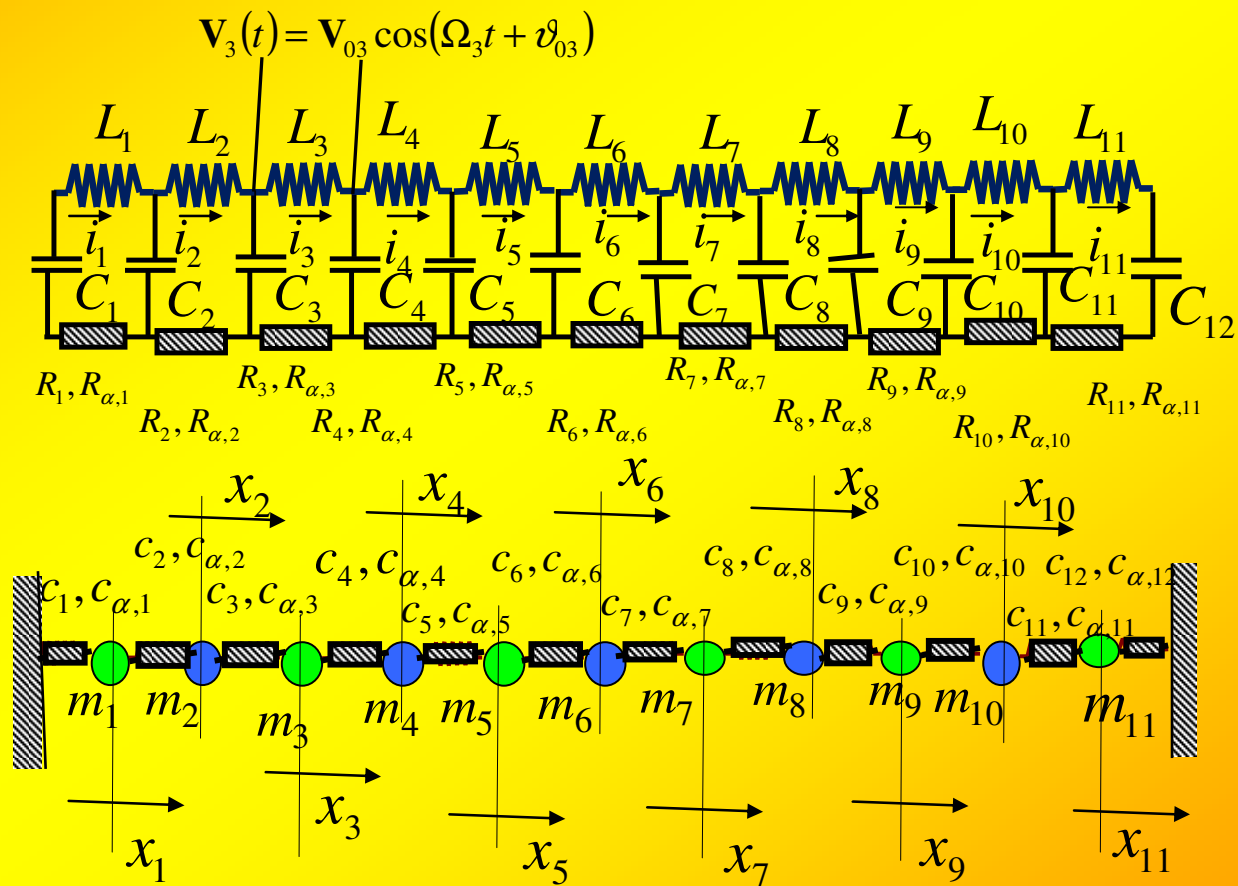
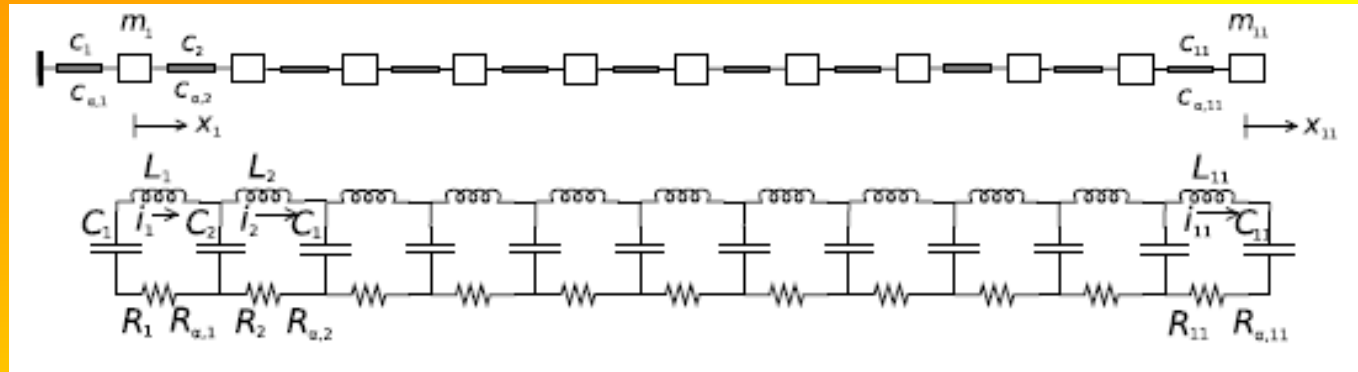
**Hedrih (Stevanović) K., (2013), A generalization of Lagrange method of variation constants, SCIENTIFIC REVIEW, Special Issue Nonlinear Dynamics, Dedicated to Milutin Milanković (1879- 1958), Guest Editors: Katica R. (Stevanović) Hedrih and Žarko Mijajlović, S2 (2013) pp. 37-66, Serbian Scientific Society, YU ISSN 0350-2910**

<http://afrodita.rcub.bg.ac.rs/~nds/>, <http://afrodita.rcub.bg.ac.rs/~nds/indexe.html>

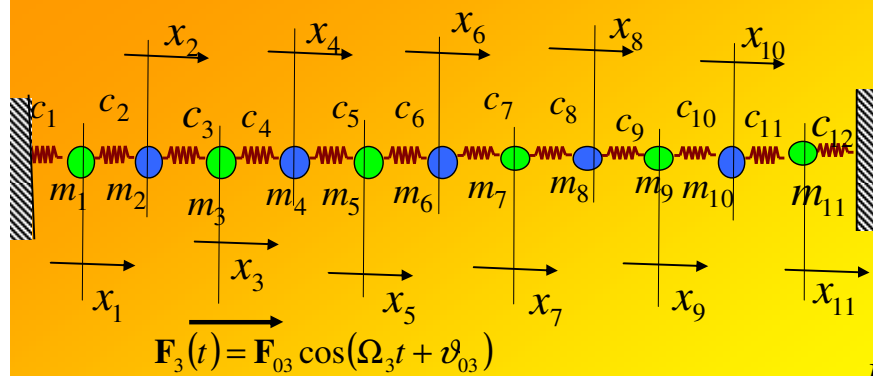
**Katica R. (Stevanović) Hedrih, (201`3), Fractional order hybrid system dynamics, PAMM, Proc. Appl. Math. Mech. 13, 25 – 26 (2013) / DOI 10.1002/pamm.201310008**

<http://onlinelibrary.wiley.com/doi/10.1002/pamm.v13.1/issuetoc>

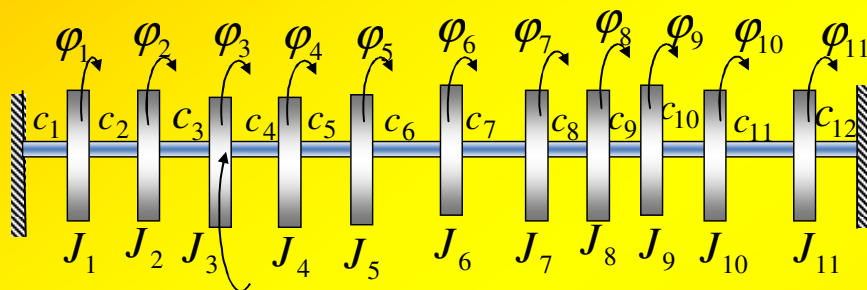
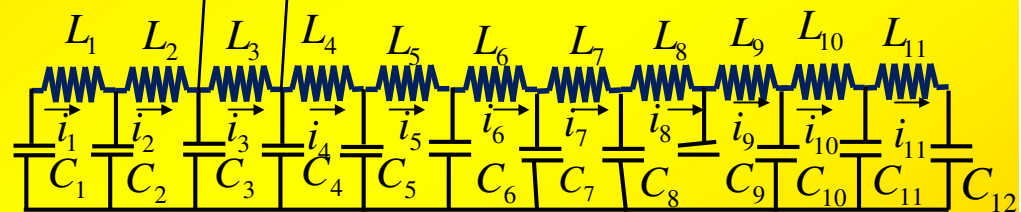




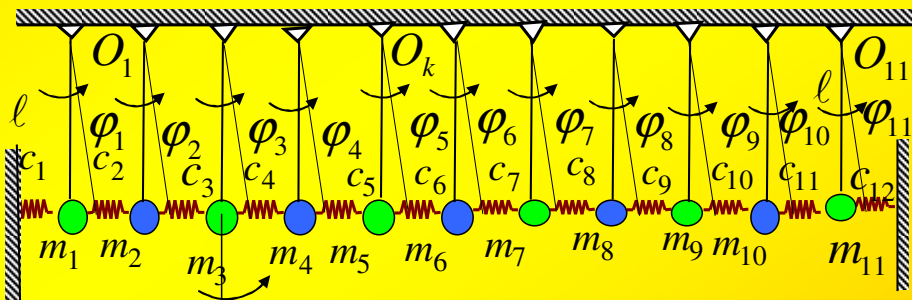




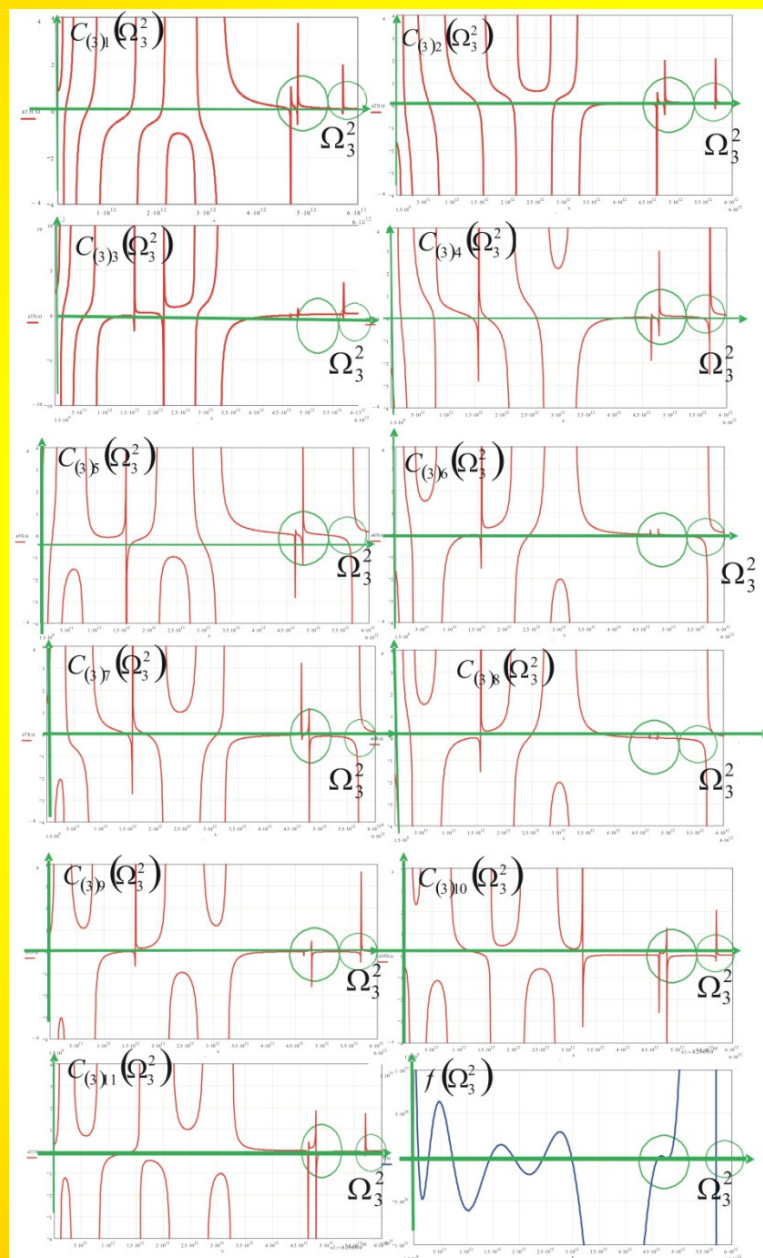
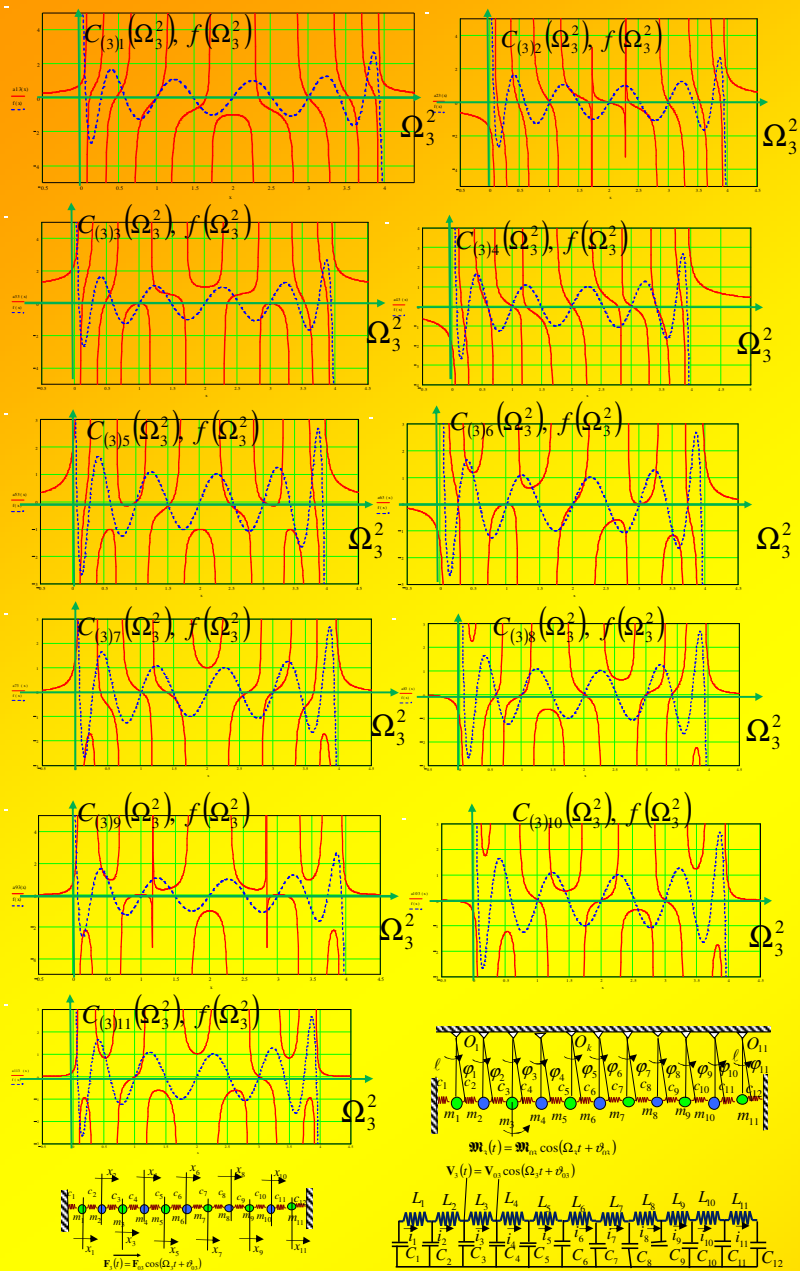
$$V_3(t) = V_{03} \cos(\Omega_3 t + \vartheta_{03})$$



$$\mathfrak{M}_3(t) = \mathfrak{M}_{03} \cos(\Omega_3 t + \vartheta_{03})$$



$$\mathfrak{M}_3(t) = \mathfrak{M}_{03} \cos(\Omega_3 t + \vartheta_{03})$$



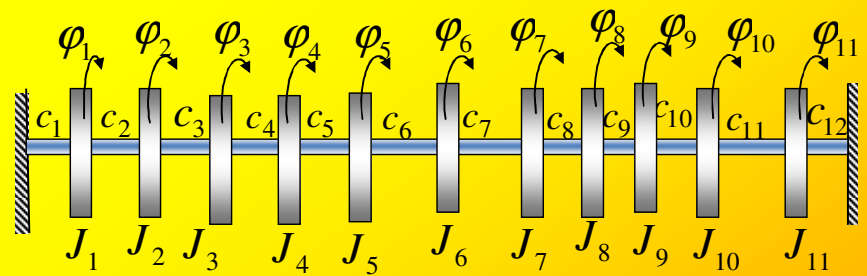
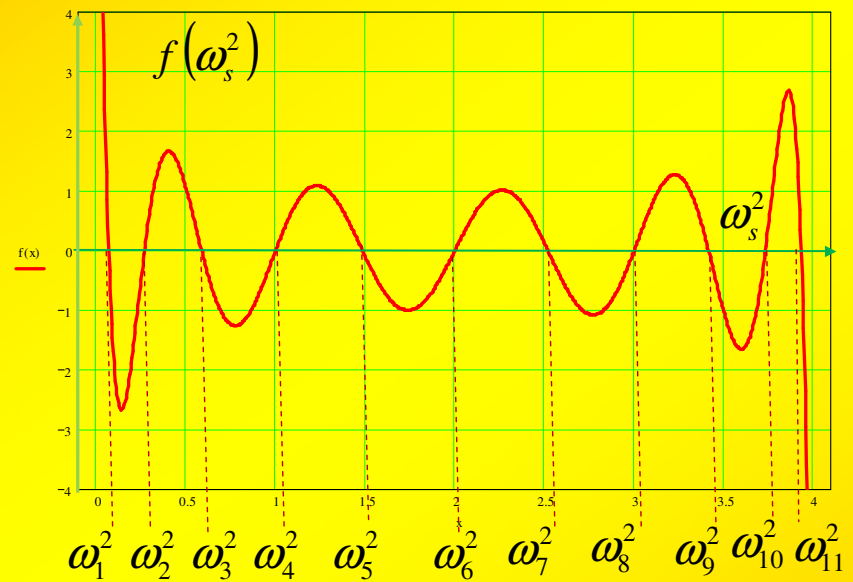
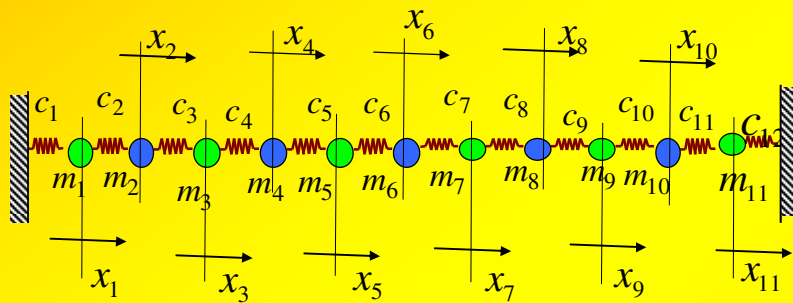
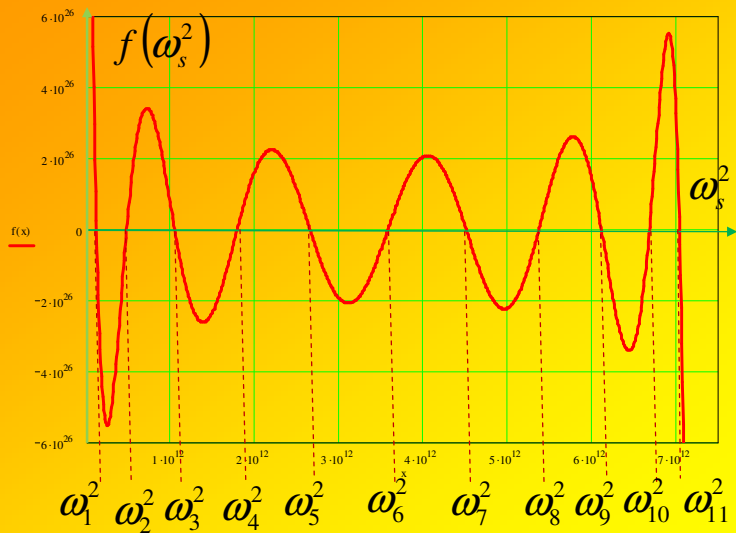


Table 1: Mathematical analogy between kinetic and material parameters of linear mechanical system and electrical linear system with one dof

| Linear differential equations                               | Generalized system coordinate | Inertial coefficient | Elastic rigidity coefficient | Dissipation coefficient         | External input                | Eigen circular frequency<br>Eigen resonant frequency |
|-------------------------------------------------------------|-------------------------------|----------------------|------------------------------|---------------------------------|-------------------------------|------------------------------------------------------|
| 1 $m\ddot{x} + b\dot{x} + cx = F_0 \cos \Omega t$           | $x$ , displacement            | $m$ , mass           | $c$ , rigidity               | $b$ , damping force coefficient | $F_0 \cos \Omega t$ , force   | $\Omega_{res} = \omega = \sqrt{\frac{c}{m}}$         |
| 2 $L\ddot{q} + R\dot{q} + \frac{1}{C}q = V_0 \cos \Omega t$ | $q$ , charge                  | $L$ , inductance     | $\frac{1}{C}$ , capacitance  | $R$ , resistance                | $V_0 \cos \Omega t$ , voltage | $\Omega_{res} = \omega = \sqrt{\frac{1}{CL}}$        |

Table 2: Mathematical analogy between kinetic and material parameters of FO inertia less mechanical visco-elastic element and FO inertia less electrical resistance element,  $0 \leq \alpha \leq 1$

| Constitutive relation of the FO element                       | Generalized system coordinate | Inertial coefficient | Elastic rigidity coefficient | FO coefficients                               | FO differential equations                                       |
|---------------------------------------------------------------|-------------------------------|----------------------|------------------------------|-----------------------------------------------|-----------------------------------------------------------------|
| 1 $P(t) = -[c_0 x(t) + c_\alpha D_t^\alpha [x(t)]]$           | $x$ , displacement            | $m$ , mass           | $c_0$ , rigidity             | $c_{(\alpha)}$ , damping FO force coefficient | $m\ddot{x} + c_{(\alpha)} D_t^\alpha [x] + cx = 0$              |
| 2 $V(t) = -[\frac{1}{C_0} q(t) + R_\alpha D_t^\alpha [q(t)]]$ | $q$ , charge                  | $L$ , inductance     | $C_0$ , capacitance          | $R_{(\alpha)}$ , fractional resistance        | $L\ddot{q} + R_{(\alpha)} D_t^\alpha [q] + \frac{1}{C_0} q = 0$ |

Table 3: Mathematical and qualitative analogies between energies and measurement of FO energy dissipations (degradations) and eigen characteristic numbers of a FO mechanical oscillator and an electrical FO oscillator with one dof, and phenomenological mapping to corresponding kinetic parameters of eigen FO modes in FO mechanical chain and electrical FO chain system  $0 < \alpha \leq 1$

| Kinetic energy                                      | Potential energy                                                | Generalized function of fractional order energy dissipation              | Characteristic numbers for eigen FO normal modes                                   |
|-----------------------------------------------------|-----------------------------------------------------------------|--------------------------------------------------------------------------|------------------------------------------------------------------------------------|
| 1 $E_k = \frac{1}{2} m [\dot{x}(t)]^2$ , mass       | $E_{p,\alpha} = \frac{1}{2} c_0 [x(t)]^2$ , elastic element     | $\Phi_\alpha = \frac{1}{2} c_\alpha \langle D_t^\alpha [x(t)] \rangle^2$ | $\omega_s^2 = \frac{c}{m}$ , $\omega_{(\alpha)}^2 = \kappa_\alpha \frac{c}{m}$     |
| 2 $E_k = \frac{1}{2} L [\dot{q}(t)]^2$ , inductance | $E_{p,\alpha} = \frac{1}{2} \frac{1}{C_0} [q(t)]^2$ , capacitor | $\Phi_\alpha = \frac{1}{2} R_\alpha \langle D_t^\alpha [q(t)] \rangle^2$ | $\omega^2 = \frac{1}{LC_0}$ , $\omega_{(\alpha)}^2 = \kappa_\alpha \frac{1}{LC_0}$ |

Table 4: Mathematical and qualitative analogies between matrix FODE of FO dynamics on a mechanical chain system with finite number of dof and an electrical chain system with corresponding finite number of loops and their eigen FO modes and eigen characteristic numbers and corresponding constitutive relations of inertia less standard FO mechanical viscoelastic element and FO electrical resistor-capacitive element included in the corresponding analogous systems:  $0 < \alpha \leq 1$ ,  $s = 1, \dots, n$ . Phenomenological mapping between eigen FO modes of mechanical and electrical chains

| Constitutive relation of the FO element                                    | Matrix FODE                                                  | Independent eigen fractional order normal oscillators                           | Characteristic numbers for eigen FO normal modes:                                                                                                                                 |
|----------------------------------------------------------------------------|--------------------------------------------------------------|---------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1 $P(t) = -\{c_0 x(t) + c_\alpha D_t^\alpha [x(t)]\}$                      | $A\{\ddot{x}\} + C_\alpha\{D_t^\alpha[x]\} + C[x] = \{0\}$   | $\ddot{\xi}_s + \omega_{(\alpha)s}^2 D_t^\alpha [\xi_s] + \omega_s^2 \xi_s = 0$ | $\omega_s^2 = 2\frac{c}{m} \left(1 - \cos \frac{(2s-1)\pi}{2n+1}\right)$ , $\omega_{(\alpha)s}^2 = 2\kappa_\alpha \frac{c}{m} \left(1 - \cos \frac{(2s-1)\pi}{2n+1}\right)$       |
| 2 $V(t) = -\left\{\frac{1}{C_0} q(t) + R_\alpha D_t^\alpha [q(t)]\right\}$ | $L\{\ddot{q}\} + R_\alpha\{D_t^\alpha[q]\} + C^*[q] = \{0\}$ | $\ddot{\xi}_s + \omega_{(\alpha)s}^2 D_t^\alpha [\xi_s] + \omega_s^2 \xi_s = 0$ | $\omega_s^2 = 2\frac{1}{LC_0} \left(1 - \cos \frac{(2s-1)\pi}{2n+1}\right)$ , $\omega_{(\alpha)s}^2 = 2\kappa_\alpha \frac{1}{LC_0} \left(1 - \cos \frac{(2s-1)\pi}{2n+1}\right)$ |

Table 5: Mathematical and qualitative analogies between energies and measurement of FO energy dissipations (degradations) and eigen characteristic numbers of a FO mechanical chain system with finite number of dof and a electrical FO system with corresponding finite number loops, and phenomenological mapping to corresponding kinetic parameters of eigen FO modes in FO mechanical chain and electrical FO chain system  $0 < \alpha \leq 1$ ,  $\eta_s$ ,  $s = 1, \dots, n$

| Kinetic energy                                                                | Potential energy                                                       | Generalized function of fractional order energy dissipation                                                                                         | Characteristic numbers for eigen FO normal modes                                                                                                                                   |
|-------------------------------------------------------------------------------|------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1 $2E_k = (\dot{x}) A\{\dot{x}\}$<br>$2E_k = \sum_{s=1}^{s=n} \dot{\eta}_s^2$ | $2E_p = (x) C\{x\}$<br>$2E_p = \sum_{s=1}^{s=n} \omega_s^2 \eta_s^2$   | $2P_{\alpha \neq 0} = (D_t^\alpha \{x\}) C_\alpha\{D_t^\alpha [x]\}$<br>$2P_\alpha = \sum_{s=1}^{s=n} \omega_{(\alpha)s}^2 (D_t^\alpha [\eta_s])^2$ | $\omega_s^2 = 2\frac{c}{m} \left(1 - \cos \frac{(2s-1)\pi}{2n+1}\right)$<br>$\omega_{(\alpha)s}^2 = 2\kappa_\alpha \frac{c}{m} \left(1 - \cos \frac{(2s-1)\pi}{2n+1}\right)$       |
| 2 $2E_k = (\dot{q}) L\{\dot{q}\}$<br>$2E_k = \sum_{s=1}^{s=n} \dot{\eta}_s^2$ | $2E_p = (q) C^*\{q\}$<br>$2E_p = \sum_{s=1}^{s=n} \omega_s^2 \eta_s^2$ | $2P_{\alpha \neq 0} = (D_t^\alpha \{q\}) R_\alpha\{D_t^\alpha [q]\}$<br>$2P_\alpha = \sum_{s=1}^{s=n} \omega_{(\alpha)s}^2 (D_t^\alpha [\eta_s])^2$ | $\omega_s^2 = 2\frac{1}{LC_0} \left(1 - \cos \frac{(2s-1)\pi}{2n+1}\right)$<br>$\omega_{(\alpha)s}^2 = 2\kappa_\alpha \frac{1}{LC_0} \left(1 - \cos \frac{(2s-1)\pi}{2n+1}\right)$ |





## Energy and Nonlinear Dynamics of Hybrid Systems

Katica R. (Stevanović) Hedrih

Katica R. (Stevanović) Hedrih, (2012), Energy and Nonlinear Dynamics of Hybrid Systems, Chapter in Book: Dynamical Systems and Methods, 2012, Part 1, 29-83, DOI: 10.1007/978-1-4614-0454-5\_2

## APPLIED MECHANICS REVIEWS



Yuriy A. Rossikhin

Marina V. Shitikova<sup>1</sup>

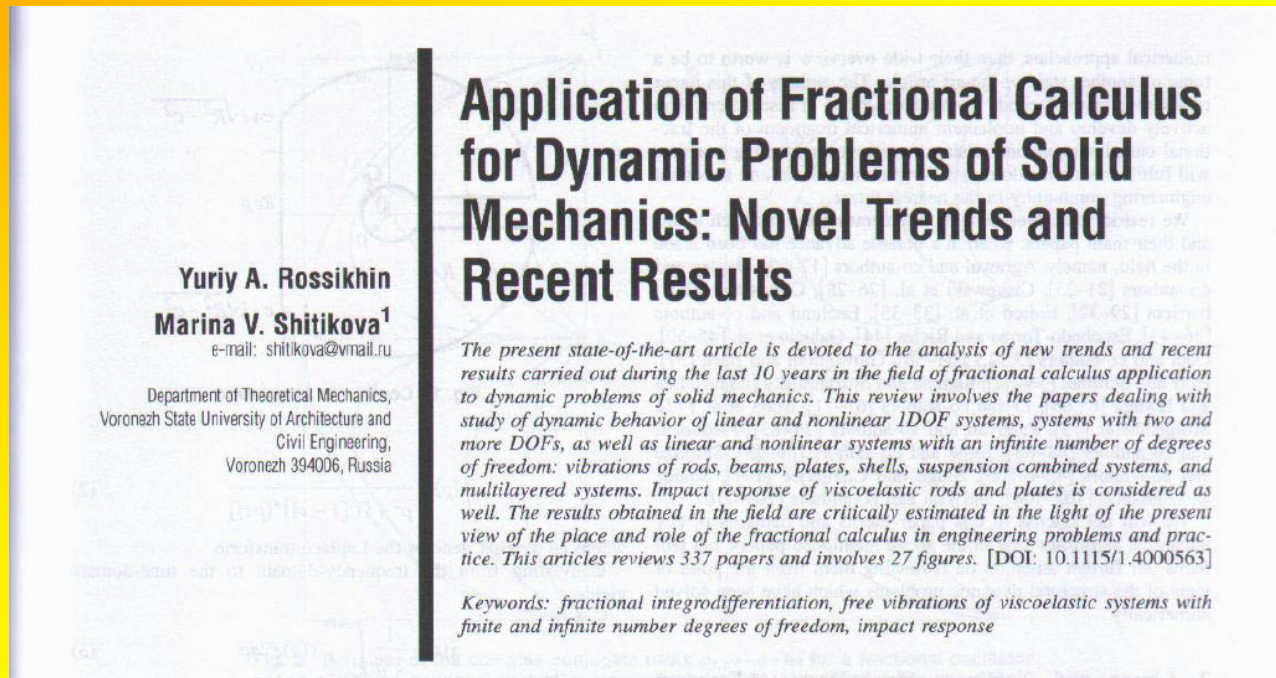
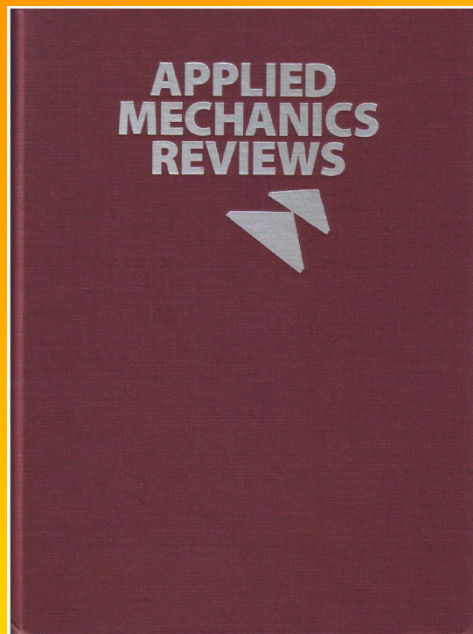
e-mail: shitikova@mail.ru

Department of Theoretical Mechanics,  
Voronezh State University of Architecture and  
Civil Engineering,  
Voronezh 394006, Russia

## Application of Fractional Calculus for Dynamic Problems of Solid Mechanics: Novel Trends and Recent Results

*The present state-of-the-art article is devoted to the analysis of new trends and recent results carried out during the last 10 years in the field of fractional calculus application to dynamic problems of solid mechanics. This review involves the papers dealing with study of dynamic behavior of linear and nonlinear 1DOF systems, systems with two and more DOFs, as well as linear and nonlinear systems with an infinite number of degrees of freedom: vibrations of rods, beams, plates, shells, suspension combined systems, and multilayered systems. Impact response of viscoelastic rods and plates is considered as well. The results obtained in the field are critically estimated in the light of the present view of the place and role of the fractional calculus in engineering problems and practice. This article reviews 337 papers and involves 27 figures. [DOI: 10.1115/1.4000563]*

**Keywords:** fractional integrodifferentiation, free vibrations of viscoelastic systems with finite and infinite number degrees of freedom, impact response



Hedtrih (Stevanović) K., (2008), Dynamics of coupled systems, *Journal Nonlinear Analysis: Hybrid Systems*, Volume 2, Issue 2, June 2008, Pages 310-334. [doi:10.1016/j.nahs.2006.06.003](https://doi.org/10.1016/j.nahs.2006.06.003)  
in line at <http://www.sciencedirect.com/science/journal/1751570X>

Hedrih (Stevanović K., Vibration Modes of a axially moving double belt system with creep layer ,*Journal of Vibration and Control*, (2008), 14(10-Sep): 1333-1347.  
<http://nainfo.nbs.bg.ac.vu.nainfo.nbs.bg.ac.vu:2048/Kobson/service/jcr.aspx?ISSN=1077-5463>

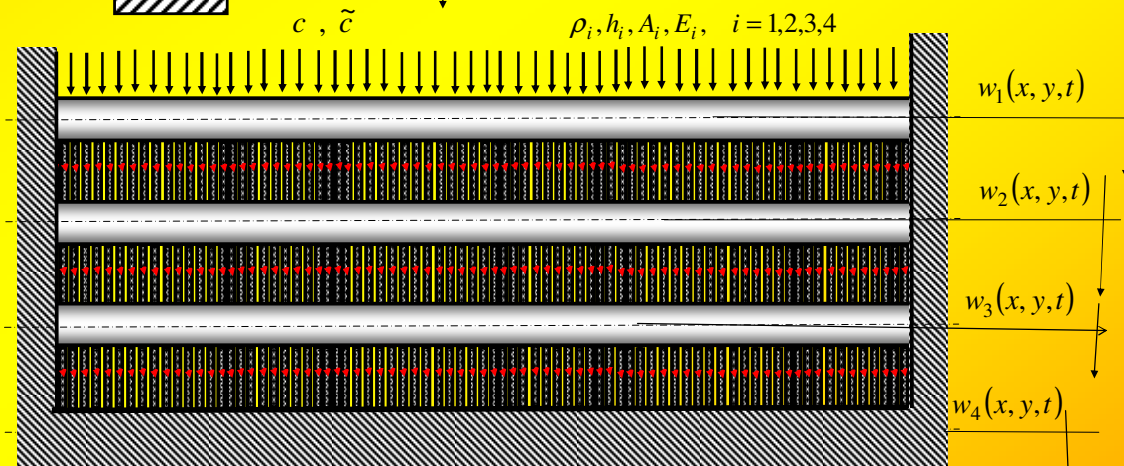
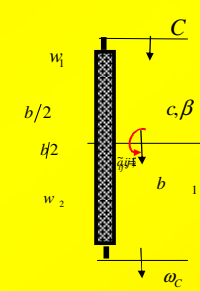
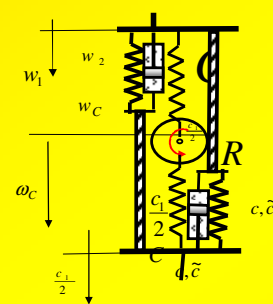
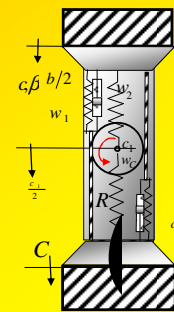
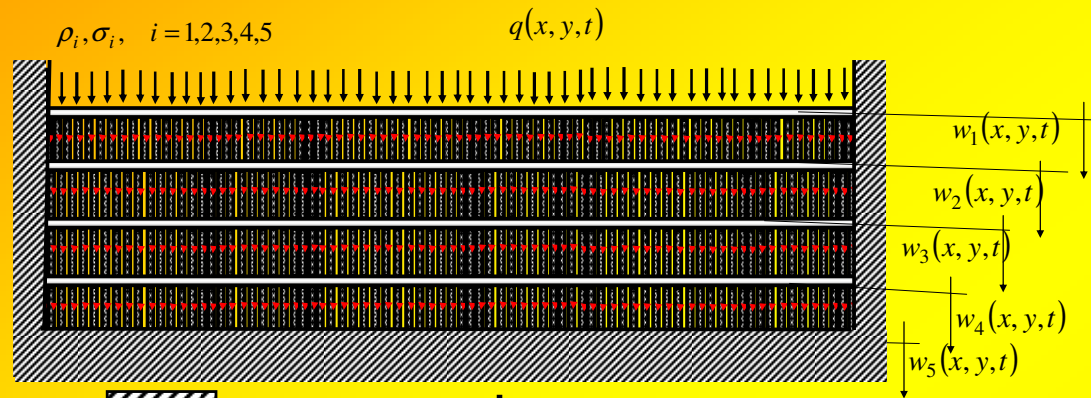
Hedrih (Stevanović), K., (2006), The transversal creeping vibrations of a fractional derivative order constitutive relation of nonhomogeneous beam, *Mathematical Problems in Engineering*, Special issue : Nonlinear Dynamics and their Applications in engineering sciences, Geust Editor: Jose Manoel Barhesar, Volume 2006 (2006), Article ID 46236, 18 pages, [www.hindawi.com](http://www.hindawi.com)  
[doi:10.1155/MPE/2006/46236](https://doi.org/10.1155/MPE/2006/46236), Volume 2006, No. 5, pp. 61-78.

Hedrih (Stevanović) K., Filipovski A., (2002), *Longitudinal Vibration of a Fractional Derivative Order Rheological Rod with Variable Cross Section*, *Facta Universitatis, Series Mechanics, Automatic Control and Robotics*, Vol. 3 No. 12, 2002. pp.327-350. YU ISSN 0534-2009. <http://facta.junis.ni.ac.vu/facta/macar/macar2002/macar2002-02.html>





Mathematical  
Institute  
SANU



March  
18-22



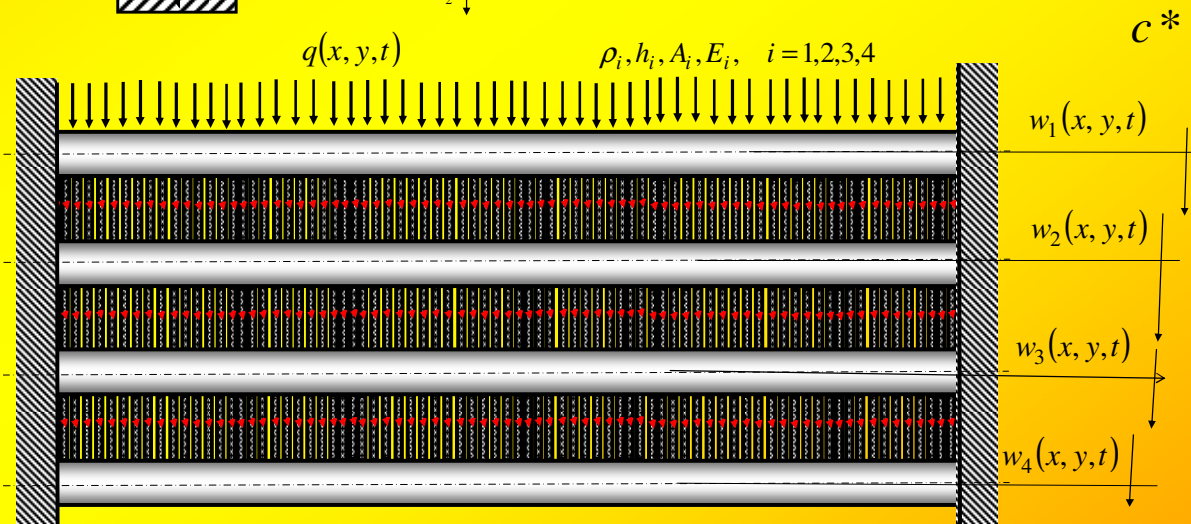
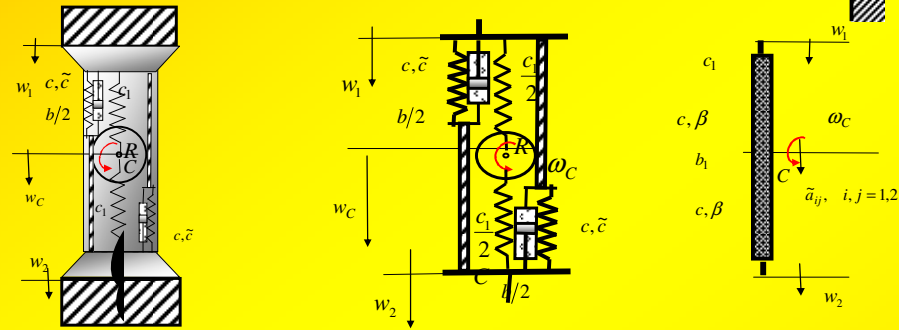
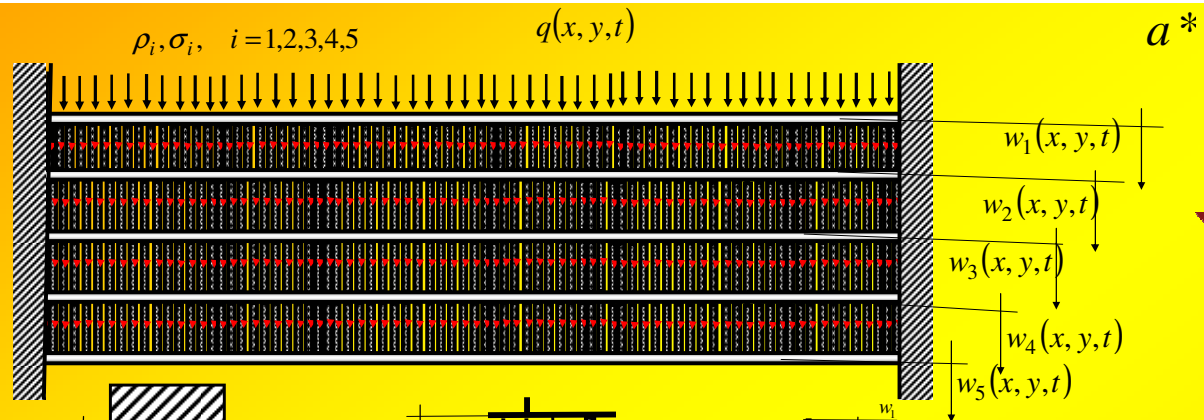
Mathematical  
Institute  
SANU



March  
18-22

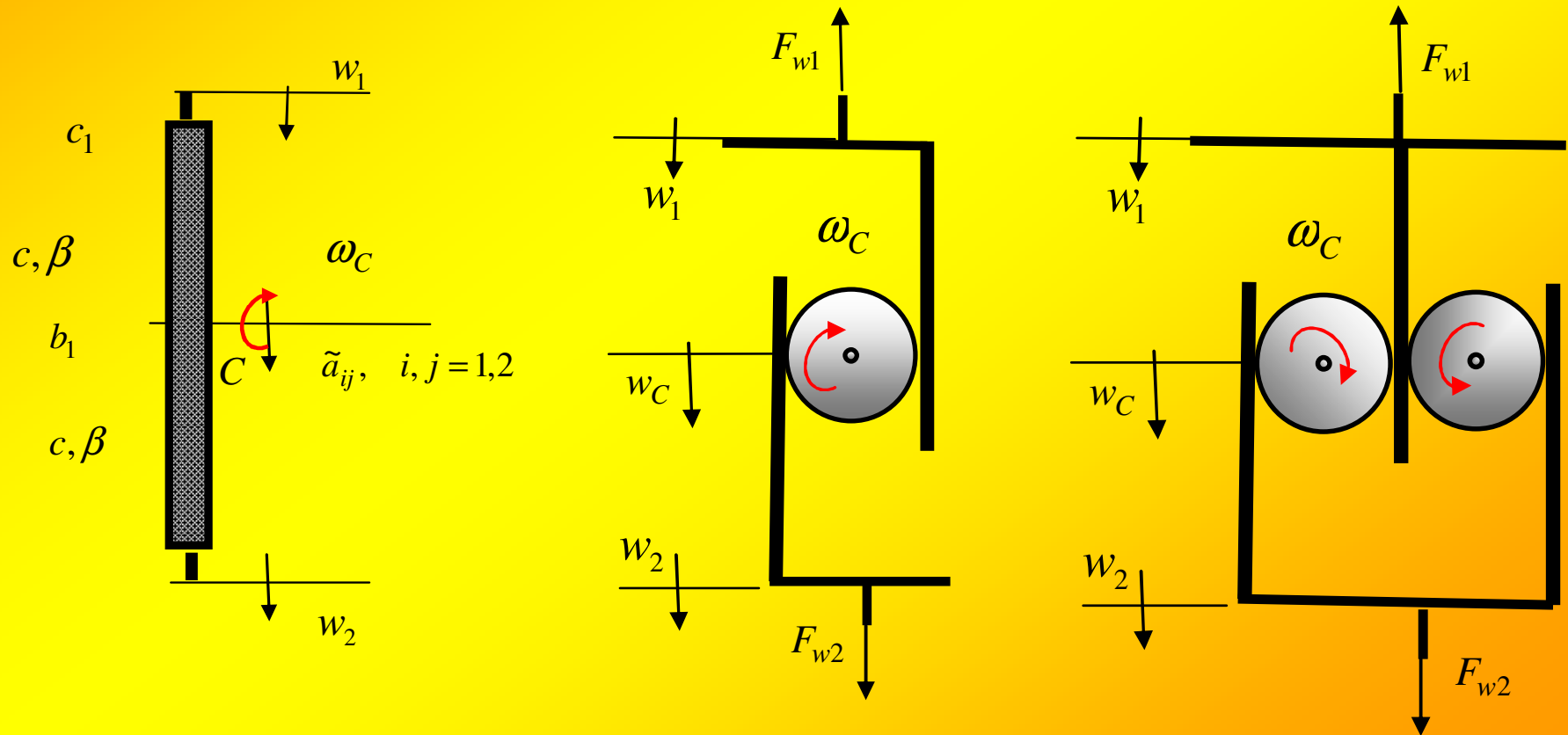


March  
18-22



# Standard element with translator and rotator inertia properties

We introduced standard element with **translator and rotator inertia properties**, taking into account mass and mass inertia moments and realized by a rolling disk or sphere



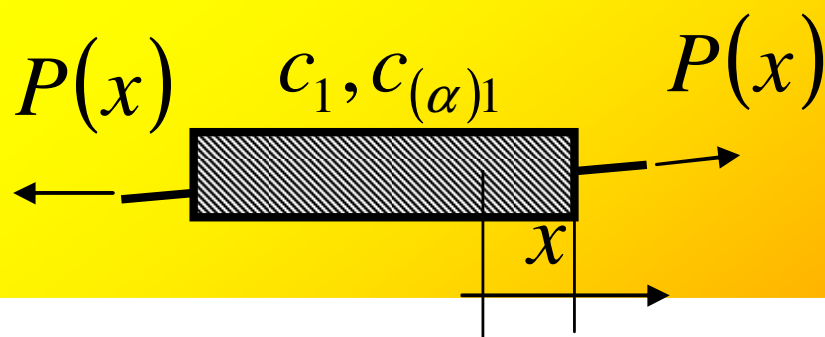


## Standard light fractional order element

$$P(t) = -\left\{c_0 x(t) + c_\alpha \mathfrak{D}_t^\alpha [x(t)]\right\} \quad 0 < \alpha \leq 1$$

$$\mathfrak{D}_t^\alpha [x(t)] = \frac{d^\alpha x(t)}{dt^\alpha} = x^{(\alpha)}(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{x(\tau)}{(t-\tau)^\alpha} d\tau$$

$$0 < \alpha \leq 1$$

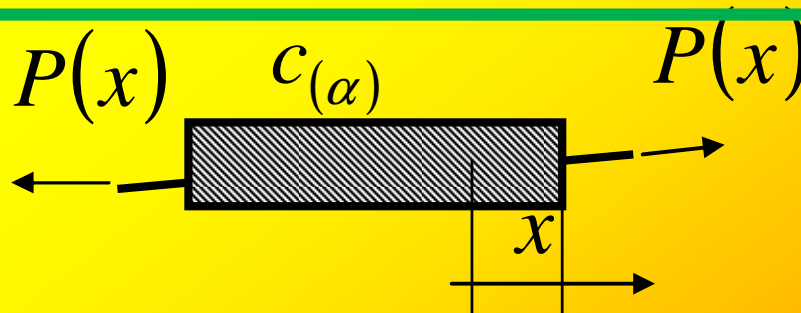


# Generalized function of fractional order dissipation of system energy

$$2P_{\alpha \neq 0} = \left( \mathfrak{D}_t^\alpha \{x\} \right) C_\alpha \{ \mathfrak{D}_t^\alpha \{x\} \}, \quad \text{for } \alpha \neq 0$$

$$\mathfrak{D}_t^\alpha [x(t)] = \frac{d^\alpha x(t)}{dt^\alpha} = x^{(\alpha)}(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{x(\tau)}{(t-\tau)^\alpha} d\tau$$

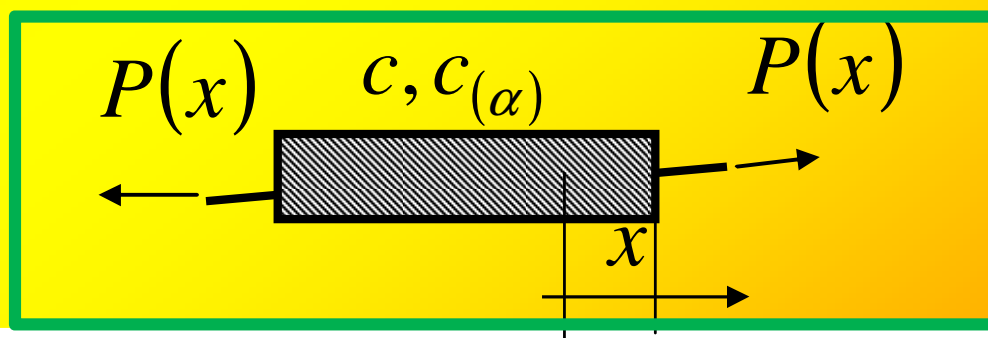
$$0 < \alpha \leq 1$$



Generalized forces  $\{\mathbf{F}_{\alpha \neq 0}\}$  of system no ideal viscoelastic creep fractional order disipaon of system energy for  $0 < \alpha \leq 1$   $\alpha \neq 0$  for generalied coordinates  $\{x\}$

$$\{\mathbf{F}_{\alpha \neq 0}\} = - \frac{\partial \mathbf{P}_{\alpha}}{\partial (\mathfrak{D}_t^{\alpha} \{x\})} = -C_{\alpha} \{\mathfrak{D}_t^{\alpha} \{x\}\}, \quad \text{for } \alpha \neq 0$$

$$0 < \alpha \leq 1$$



Generalized forces  $\{\mathbf{F}_{\alpha \neq 0}\}$  of system no ideal viscoelastic creep fractional order dissipation of system energy for  $0 < \alpha \leq 1$   $\alpha \neq 0$

for generalied coordinates  $\{x\}$

$$\{\mathbf{F}_{\alpha \neq 0}\} = -\frac{\partial \mathbf{P}_{\alpha}}{\partial (\mathfrak{D}_t^{\alpha} \{x\})} = -C_{\alpha} \{\mathfrak{D}_t^{\alpha} \{x\}\}, \quad \text{for } \alpha \neq 0$$

$$0 < \alpha \leq 1$$

Generalized dissipative forces  $\{\mathbf{F}_{\alpha=1}\}$  of system dissipative (no conservative) properties for  $\alpha = 1$   
for generalied coordinates  $\{x\}$

$$\{\mathbf{F}_{\alpha=1}\} = -\frac{\partial \Phi}{\partial \{\dot{x}\}} = -\frac{\partial \mathbf{P}_{\alpha=1}}{\partial \{\dot{x}\}} = -C_{\alpha=1} \{\dot{x}\}, \quad \text{for } \alpha = 1$$

# Generalized function of fractional order dissipation of system energy

$$\{x\} \quad x_k \quad k = 1, 2, 3, \dots, n$$

$$\mathbf{A} = \left( a_{kj} \right)_{\substack{\downarrow k=1,2,3,\dots,n \\ \rightarrow j=1,2,3,\dots,n}}$$

Matrix of coefficients of system mass  
inertia properties

$$\mathbf{C} = \left( c_{kj} \right)_{\substack{\downarrow k=1,2,3,\dots,n \\ \rightarrow j=1,2,3,\dots,n}}$$

Matrix of coefficients of system  
rigidity properties

$$\mathbf{C}_\alpha = \left( c_{\partial,kj} \right)_{\substack{\downarrow k=1,2,3,\dots,n \\ \rightarrow j=1,2,3,\dots,n}}$$

Matrix of coefficients of system  
viscoelastic creep fractional order  
properties



$$2\mathbf{E}_k = (\dot{x})\mathbf{A}\{\dot{x}\}$$

$$2\mathbf{E}_p = (x)C\{x\}$$

$$2\mathbf{P}_{\alpha \neq 0} = (\mathfrak{D}_t^\alpha \{x\})C_\alpha \{\mathfrak{D}_t^\alpha \{x\}\}, \quad \text{for } \alpha \neq 0$$

$$0 < \alpha \leq 1$$

$$2\Phi = 2\mathbf{P}_{\alpha=1} = (\dot{x})C_{\alpha=1}\{\dot{x}\}, \quad \text{for } \alpha=1$$

$$2\mathbf{E}_{p,\alpha} = 2\mathbf{P}_{\alpha=0} = (x)C_{\alpha=0}\{x\}, \quad \text{for } \alpha=0$$

Generalized forces of system inertia  $\{\mathbf{F}_j\}$  for generalized coordinates  $\{x\}$

$$\{\mathbf{F}_j\} = - \left( \frac{d}{dt} \frac{\partial \mathbf{E}_k}{\partial \{\dot{x}\}} - \frac{\partial \mathbf{E}_k}{\partial \{x\}} \right) = -\mathbf{A} \{\ddot{x}\}$$

Generalized forces  $\{\mathbf{F}_c\}$  of system ideal elastic (conservative) properties for generalized coordinates  $\{x\}$

$$\{\mathbf{F}_c\} = - \frac{\partial \mathbf{E}_p}{\partial \{x\}} = -C \{x\}$$

Generalized forces  $\{\mathbf{F}_{\alpha \neq 0}\}$  of system no ideal viscoelastic creep fractional order dissipation of system energy for  $0 < \alpha \leq 1$   $\alpha \neq 0$

for generalied coordinates  $\{x\}$

$$\{\mathbf{F}_{\alpha \neq 0}\} = -\frac{\partial \mathbf{P}_{\alpha}}{\partial (\mathfrak{D}_{\dagger}^{\alpha} \{x\})} = -C_{\alpha} \{\mathfrak{D}_{\dagger}^{\alpha} \{x\}\}, \quad \text{for } \alpha \neq 0$$

$$0 < \alpha \leq 1$$

Generalized dissipative forces  $\{\mathbf{F}_{\alpha=1}\}$  of system dissipative (no conservative) properties for  $\alpha=1$   
for generalied coordinates  $\{x\}$

$$\{\mathbf{F}_{\alpha=1}\} = -\frac{\partial \Phi}{\partial \{\dot{x}\}} = -\frac{\partial \mathbf{P}_{\alpha=1}}{\partial \{\dot{x}\}} = -C_{\alpha=1} \{\dot{x}\}, \quad \text{for } \alpha=1$$

**Generalized forces  $\{\mathbf{F}_{\alpha=0}\}$  of system with additional elastic (conservative) properties for  $\alpha=0$  for generalized coordinates  $\{x\}$**

$$\{\mathbf{F}_{\alpha=0}\} = -\frac{\partial \mathbf{E}_{p,\alpha=0}}{\partial \{x\}} = -\frac{\partial \mathbf{P}_{\alpha=0}}{\partial \{x\}} = -C_{\alpha=0} \{x\}, \quad \text{for } \alpha = 0$$

## Matrix fractional order differential equation of discrete fractional order system free vibrations

$$\{\mathbf{F}_j\} + \{\mathbf{F}_c\} + \{\mathbf{F}_\alpha\} = \{0\}$$

$$\frac{d}{dt} \frac{\partial \mathbf{E}_k}{\partial \{\dot{x}\}} - \frac{\partial \mathbf{E}_k}{\partial \{x\}} + \frac{\partial \mathbf{E}_p}{\partial \{x\}} + \frac{\partial \mathbf{P}_\alpha}{\partial (\mathfrak{D}_t^\alpha \{x\})} + \frac{\partial \Phi}{\partial \{\dot{x}\}} = 0$$

$$\mathbf{A} \{\ddot{x}\} + C_\alpha \{\mathfrak{D}_t^\alpha \{x\}\} + C \{x\} = \{0\}$$

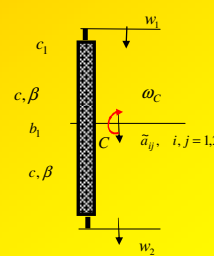
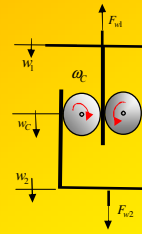




Mathematical  
Institute  
SANU



March  
18-22



March  
18-22



Mathematical  
Institute  
SANU

Generalized forces for generalized coordinates

$w_k(x, y, t)$  and  $w_{k+1}(x, y, t)$

$$Q_{w_1}^{elem-sloja} = [F_{j1} + F_{e1} + F_{w1}]_{w_1^{elem-sloja}} = - \left\langle \frac{d}{dt} \frac{\partial \mathbf{E}_k^{elem-sloja}}{\partial \left( \frac{\partial w_1}{\partial t} \right)} - \frac{\partial \mathbf{E}_k^{elem-sloja}}{\partial w_1} \right\rangle - \frac{\partial \mathbf{E}_p^{elem-sloja}}{\partial w_1} - \frac{\partial \Phi^{elem-sloja}}{\partial \left( \frac{\partial w_1(x, y, t)}{\partial t} \right)} = Q_{w_1}^{elem-ploca}$$

$$Q_{w_2}^{elem-sloja} = [F_{j2} + F_{e2} + F_{w2}]_{w_2^{elem-sloja}} = - \left\langle \frac{d}{dt} \frac{\partial \mathbf{E}_k^{elem-sloja}}{\partial \left( \frac{\partial w_2}{\partial t} \right)} - \frac{\partial \mathbf{E}_k^{elem-sloja}}{\partial w_2} \right\rangle - \frac{\partial \mathbf{E}_p^{elem-sloja}}{\partial w_2} - \frac{\partial \Phi^{elem-sloja}}{\partial \left( \frac{\partial w_2(x, y, t)}{\partial t} \right)} = Q_{w_2}^{elem-ploca}$$

$$Q_{w_k}^{elem} = -\frac{1}{4}m \left[ \left( \frac{\partial^2 w_{k+1}}{\partial t^2} + \frac{\partial^2 w_k}{\partial t^2} \right) - \frac{i_C^2}{R^2} \left( \frac{\partial^2 w_{k+1}}{\partial t^2} - \frac{\partial^2 w_k}{\partial t^2} \right) \right] -$$

$$- \{ c_{0k,k+1} [w_{k+1}(x, y, t) - w_k(x, y, t)] + c_{0N,k,k+1} [w_{k+1}(x, y, t) - w_k(x, y, t)]^3 \} - b_{k,k+1} \left[ \frac{\partial w_{k+1}(x, y, t)}{\partial t} - \frac{\partial w_k(x, y, t)}{\partial t} \right]$$

$$Q_{w_{k+1}}^{elem} = -\frac{1}{4}m \left[ \left( \frac{\partial^2 w_{k+1}}{\partial t^2} + \frac{\partial^2 w_k}{\partial t^2} \right) + \frac{i_C^2}{R^2} \left( \frac{\partial^2 w_{k+1}}{\partial t^2} - \frac{\partial^2 w_k}{\partial t^2} \right) \right] +$$

$$+ \{ c_{0,k,k+1} [w_{k+1}(x, y, t) - w_k(x, y, t)] + c_{0N,k,k+1} [w_{k+1}(x, y, t) - w_k(x, y, t)]^3 \} + b_{k,k+1} \left[ \frac{\partial w_{k+1}(x, y, t)}{\partial t} - \frac{\partial w_k(x, y, t)}{\partial t} \right]$$

**Governing partial fractional order differential equations of a hybrid multi deformable beam system transversal oscillations on a discrete continuum layer with visco-elastic and translator and rotator inertia properties**

$$\rho_1 A_1 \frac{\partial^2 w_1(x,t)}{\partial t^2} = -\mathfrak{B}_1 \frac{\partial^4 w_1(x,t)}{\partial x^4} - \frac{1}{4} m_1 \left[ \left( \frac{\partial^2 w_2(x,t)}{\partial t^2} + \frac{\partial^2 w_1(x,t)}{\partial^2 t} \right) - \kappa \left( \frac{\partial^2 w_2(x,t)}{\partial t^2} - \frac{\partial^2 w_1(x,t)}{\partial^2 t} \right) \right] +$$

$$+ c_{0(1,2)} [w_2(x,t) - w_1(x,t)] + c_{\alpha(1,2)} \mathfrak{D}_t^\alpha [w_2(x,t) - w_1(x,t)] + b \left[ \frac{\partial w_2(x,t)}{\partial t} - \frac{\partial w_1(x,t)}{\partial t} \right] + q_1(x,t)$$

$$\rho_2 A_2 \frac{\partial^2 w_2(x,t)}{\partial t^2} = -\mathfrak{B}_2 \frac{\partial^4 w_2(x,t)}{\partial x^4} - \frac{1}{4} m_1 \left[ \left( \frac{\partial^2 w_2(x,t)}{\partial t^2} + \frac{\partial^2 w_1(x,t)}{\partial^2 t} \right) + \kappa \left( \frac{\partial^2 w_2(x,t)}{\partial t^2} - \frac{\partial^2 w_1(x,t)}{\partial^2 t} \right) \right] - c_{0(1,2)} [w_2(x,t) - w_1(x,t)] -$$

$$- c_{\alpha(1,2)} \mathfrak{D}_t^\alpha [w_2(x,t) - w_1(x,t)] - b \left[ \frac{\partial w_2(x,t)}{\partial t} - \frac{\partial w_1(x,t)}{\partial t} \right] - \frac{1}{4} m_2 \left[ \left( \frac{\partial^2 w_3(x,t)}{\partial t^2} + \frac{\partial^2 w_2(x,t)}{\partial^2 t} \right) - \kappa \left( \frac{\partial^2 w_3(x,t)}{\partial t^2} - \frac{\partial^2 w_2(x,t)}{\partial^2 t} \right) \right] -$$

$$+ c_{0(2,3)} [w_3(x,t) - w_2(x,t)] + c_{\alpha(2,3)} \mathfrak{D}_t^\alpha [w_3(x,t) - w_2(x,t)] - b \left[ \frac{\partial w_2(x,t)}{\partial t} - \frac{\partial w_3(x,t)}{\partial t} \right] - q_2(x,t) = 0$$

$$\rho_3 A_3 \frac{\partial^2 w_3(x,t)}{\partial t^2} = -\mathfrak{B}_3 \frac{\partial^4 w_3(x,t)}{\partial x^4} - \frac{1}{4} m_2 \left[ \left( \frac{\partial^2 w_3(x,t)}{\partial t^2} + \frac{\partial^2 w_2(x,t)}{\partial^2 t} \right) + \kappa \left( \frac{\partial^2 w_3(x,t)}{\partial t^2} - \frac{\partial^2 w_2(x,t)}{\partial^2 t} \right) \right] -$$

$$- c_{0(2,3)} [w_3(x,t) - w_2(x,t)] - c_{\alpha(2,3)} \mathfrak{D}_t^\alpha [w_3(x,t) - w_2(x,t)] -$$

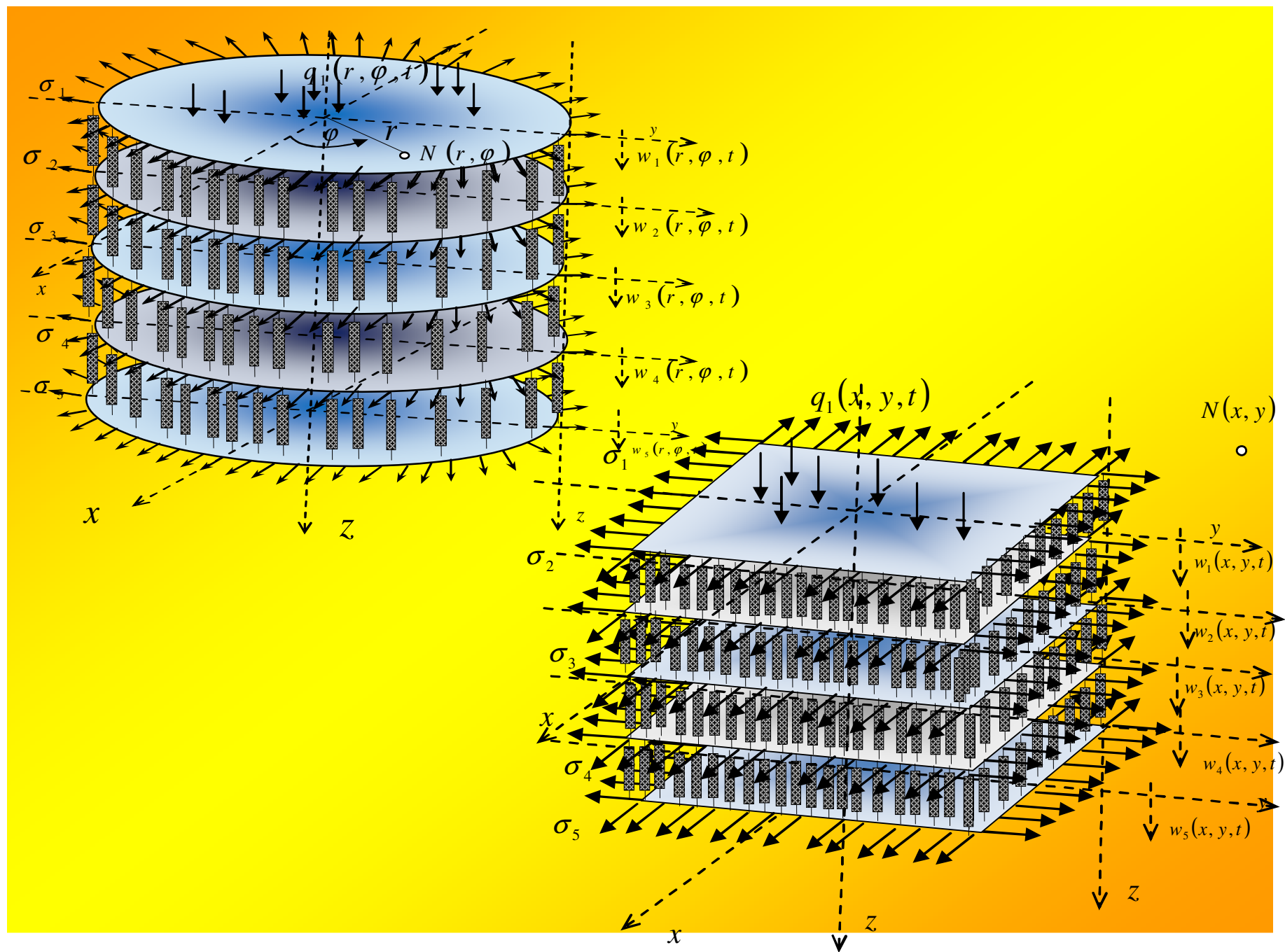
$$- b \left[ \frac{\partial w_3(x,t)}{\partial t} - \frac{\partial w_2(x,t)}{\partial t} \right] - \frac{1}{4} m_0 \left[ \left( \frac{\partial^2 w_4(x,t)}{\partial t^2} + \frac{\partial^2 w_3(x,t)}{\partial^2 t} \right) - \kappa \left( \frac{\partial^2 w_4(x,t)}{\partial t^2} - \frac{\partial^2 w_3(x,t)}{\partial^2 t} \right) \right] -$$

$$+ c_{0(1,2)} [w_4(x,t) - w_3(x,t)] + c_{\alpha(3,4)} \mathfrak{D}_t^\alpha [w_4(x,t) - w_3(x,t)] + c_{0N(3,4)} [w_4(x,t) - w_3(x,t)]^3 -$$

$$- b \left[ \frac{\partial w_3(x,t)}{\partial t} - \frac{\partial w_4(x,t)}{\partial t} \right] - q_3(x,t)$$

**Governing partial fractional order differential equations of a hybrid multi deformable plate system transversal oscillations on a discrete continuum layer with viscoelastic and translator and rotator inertia properties**

$$\begin{aligned}
 \rho_1 h_1 \frac{\partial^2 w_1(x, y, t)}{\partial t^2} = & -\mathfrak{D}_1 \Delta \Delta w_1(x, y, t) - \frac{1}{4} m_1 \left[ \left( \frac{\partial^2 w_2}{\partial t^2} + \frac{\partial^2 w_1}{\partial^2 t} \right) - \kappa \left( \frac{\partial^2 w_2}{\partial t^2} - \frac{\partial^2 w_1}{\partial^2 t} \right) \right] + \\
 & + c_{0(1,2)} [w_2(x, y, t) - w_1(x, y, t)] + c_{\alpha(1,2)} \mathfrak{D}_t^\alpha [w_2(x, y, t) - w_1(x, y, t)] + \\
 & + b \left[ \frac{\partial w_2(x, y, t)}{\partial t} - \frac{\partial w_1(x, y, t)}{\partial t} \right] + q_1(x, y, t) \\
 \rho_2 A_2 \frac{\partial^2 w_2(x, y, t)}{\partial t^2} = & -\mathfrak{D}_2 \Delta \Delta w_2(x, y, t) - \frac{1}{4} m_1 \left[ \left( \frac{\partial^2 w_2(x, y, t)}{\partial t^2} + \frac{\partial^2 w_1(x, y, t)}{\partial^2 t} \right) + \kappa \left( \frac{\partial^2 w_2(x, y, t)}{\partial t^2} - \frac{\partial^2 w_1(x, y, t)}{\partial^2 t} \right) \right] - \\
 & - b \left[ \frac{\partial w_2(x, y, t)}{\partial t} - \frac{\partial w_1(x, y, t)}{\partial t} \right] - \frac{1}{4} m_2 \left[ \left( \frac{\partial^2 w_3(x, y, t)}{\partial t^2} + \frac{\partial^2 w_2(x, y, t)}{\partial^2 t} \right) - \kappa \left( \frac{\partial^2 w_3(x, y, t)}{\partial t^2} - \frac{\partial^2 w_2(x, y, t)}{\partial^2 t} \right) \right] - \\
 & - c_{0(1,2)} [w_2(x, y, t) - w_1(x, y, t)] - c_{\alpha(1,2)} \mathfrak{D}_t^\alpha [w_2(x, y, t) - w_1(x, y, t)] + \\
 & + c_{0(2,3)} [w_3(x, t) - w_2(x, t)] + c_{\alpha(2,3)} \mathfrak{D}_t^\alpha [w_3(x, t) - w_2(x, t)] - \\
 & - b \left[ \frac{\partial w_2(x, y, t)}{\partial t} - \frac{\partial w_3(x, y, t)}{\partial t} \right] - q_2(x, y, t) \\
 \rho_3 A_3 \frac{\partial^2 w_3(x, y, t)}{\partial t^2} = & -\mathfrak{D}_3 \Delta \Delta w_3(x, y, t) - \frac{1}{4} m_2 \left[ \left( \frac{\partial^2 w_3(x, y, t)}{\partial t^2} + \frac{\partial^2 w_2(x, y, t)}{\partial^2 t} \right) + \kappa \left( \frac{\partial^2 w_3(x, y, t)}{\partial t^2} - \frac{\partial^2 w_2(x, y, t)}{\partial^2 t} \right) \right] - \\
 & - b \left[ \frac{\partial w_3(x, y, t)}{\partial t} - \frac{\partial w_2(x, y, t)}{\partial t} \right] - \frac{1}{4} m_0 \left[ \left( \frac{\partial^2 w_0(x, y, t)}{\partial t^2} + \frac{\partial^2 w_3(x, y, t)}{\partial^2 t} \right) - \kappa \left( \frac{\partial^2 w_0(x, y, t)}{\partial t^2} - \frac{\partial^2 w_3(x, y, t)}{\partial^2 t} \right) \right] - \\
 & - c_{0(2,3)} [w_3(x, y, t) - w_2(x, y, t)] - c_{\alpha(2,3)} \mathfrak{D}_t^\alpha [w_3(x, y, t) - w_2(x, y, t)] + \\
 & + c_{0(1,2)} [w_4(x, y, t) - w_3(x, y, t)] + c_{\alpha(3,4)} \mathfrak{D}_t^\alpha [w_4(x, y, t) - w_3(x, y, t)] + c_{0N(3,4)} [w_4(x, y, t) - w_3(x, y, t)]^3 - \\
 & - b \left[ \frac{\partial w_3(x, y, t)}{\partial t} - \frac{\partial w_0(x, y, t)}{\partial t} \right] - q_3(x, y, t)
 \end{aligned}$$



**Governing partial fractional order differential equations of a hybrid multi deformable membrane system transversal oscillations on a discrete continuum layer with visco-elastic and translator and rotator inertia properties**

$$\begin{aligned} \rho_1 \frac{\partial^2 w_1(x, y, t)}{\partial t^2} = & \rho_1 c_1^2 \Delta w_1(x, y, t) - \frac{1}{4} m_1 \left[ \left( \frac{\partial^2 w_2}{\partial t^2} + \frac{\partial^2 w_1}{\partial^2 t} \right) - \kappa \left( \frac{\partial^2 w_2}{\partial t^2} - \frac{\partial^2 w_1}{\partial^2 t} \right) \right] + \\ & + c_{0(1,2)} [w_2(x, y, t) - w_1(x, y, t)] + c_{\alpha(1,2)} \mathfrak{D}_t^\alpha [w_2(x, y, t) - w_1(x, y, t)] + \\ & + b \left[ \frac{\partial w_2(x, y, t)}{\partial t} - \frac{\partial w_1(x, y, t)}{\partial t} \right] + q_1(x, y, t) \end{aligned}$$

$$\begin{aligned} \rho_2 \frac{\partial^2 w_2(x, y, t)}{\partial t^2} = & \rho_2 c_2^2 \Delta w_2(x, y, t) - \frac{1}{4} m_1 \left[ \left( \frac{\partial^2 w_2}{\partial t^2} + \frac{\partial^2 w_1}{\partial^2 t} \right) + \kappa \left( \frac{\partial^2 w_2}{\partial t^2} - \frac{\partial^2 w_1}{\partial^2 t} \right) \right] - \\ & - b \left[ \frac{\partial w_2(x, t)}{\partial t} - \frac{\partial w_1(x, t)}{\partial t} \right] - \frac{1}{4} m_2 \left[ \left( \frac{\partial^2 w_3}{\partial t^2} + \frac{\partial^2 w_2}{\partial^2 t} \right) - \kappa \left( \frac{\partial^2 w_3}{\partial t^2} - \frac{\partial^2 w_2}{\partial^2 t} \right) \right] - \\ & - c_{0(1,2)} [w_2(x, y, t) - w_1(x, y, t)] - c_{\alpha(1,2)} \mathfrak{D}_t^\alpha [w_2(x, y, t) - w_1(x, y, t)] + \\ & + c_{0(2,3)} [w_3(x, t) - w_2(x, t)] + c_{\alpha(2,3)} \mathfrak{D}_t^\alpha [w_3(x, t) - w_2(x, t)] - b \left[ \frac{\partial w_2(x, y, t)}{\partial t} - \frac{\partial w_3(x, y, t)}{\partial t} \right] - q_2(x, y, t) \end{aligned}$$

$$\begin{aligned} \rho_3 \frac{\partial^2 w_3(x, y, t)}{\partial t^2} = & \rho_2 c_3^2 \Delta w_3(x, y, t) - \frac{1}{4} m_2 \left[ \left( \frac{\partial^2 w_3}{\partial t^2} + \frac{\partial^2 w_2}{\partial^2 t} \right) + \kappa \left( \frac{\partial^2 w_3}{\partial t^2} - \frac{\partial^2 w_2}{\partial^2 t} \right) \right] - \\ & - b \left[ \frac{\partial w_3(x, t)}{\partial t} - \frac{\partial w_2(x, t)}{\partial t} \right] - \frac{1}{4} m_0 \left[ \left( \frac{\partial^2 w_0}{\partial t^2} + \frac{\partial^2 w_3}{\partial^2 t} \right) - \kappa \left( \frac{\partial^2 w_0}{\partial t^2} - \frac{\partial^2 w_3}{\partial^2 t} \right) \right] - b \left[ \frac{\partial w_3(x, y, t)}{\partial t} - \frac{\partial w_0(x, y, t)}{\partial t} \right] - q_3(x, y, t) - \\ & - c_{0(2,3)} [w_3(x, y, t) - w_2(x, y, t)] - c_{\alpha(2,3)} \mathfrak{D}_t^\alpha [w_3(x, y, t) - w_2(x, y, t)] + \\ & + c_{0(1,2)} [w_4(x, y, t) - w_3(x, y, t)] + c_{\alpha(3,4)} \mathfrak{D}_t^\alpha [w_4(x, y, t) - w_3(x, y, t)] + c_{0N(3,4)} [w_4(x, y, t) - w_3(x, y, t)]^3 - \end{aligned}$$



## Solution of the governing partial fractional order differential equations of a hybrid multi deformable membrane system transversal oscillations on a discrete continuum layer with visco-elastic and translator and rotator inertia properties

$$\begin{aligned} & \left[ 1 + \frac{1}{4} \mu_{1,2} (1 + \kappa) \right] \ddot{T}_{1(nm)}(t) + c_1^2 k_{nm}^2 T_{1(nm)}(t) + a_{0(1,2)}^2 T_{1(nm)}(t) + 2\delta_{1,2} \dot{T}_{1(nm)}(t) - \\ & + a_{\alpha(1,2)}^2 \mathfrak{D}_t^\alpha [T_{1(nm)}(t)] - a_{0(1,2)}^2 T_{2(nm)}(t) - a_{\alpha(1,2)}^2 \mathfrak{D}_t^\alpha [T_{2(nm)}(t)] - \\ & - 2\delta_{1,2} \dot{T}_{2(nm)}(t) - \frac{1}{4} \mu_{1,2} (1 - \kappa) \ddot{T}_{2(nm)}(t) = h_{01,nm} \sin(\Omega_{1,nm} t + \vartheta_{1,nm}) \end{aligned}$$

$$\begin{aligned} & \left[ 1 + \frac{1}{4} \tilde{\mu}_{1,2} (1 + \kappa) + \frac{1}{4} \mu_{2,3} (1 + \kappa) \right] \ddot{T}_{2(nm)}(t) + c_2^2 k_{nm}^2 T_{2(nm)}(t) + [\tilde{a}_{0(1,2)}^2 + a_{0(2,3)}^2] T_{2(nm)}(t) + 2[\tilde{\delta}_{1,2} + 2\delta_{2,3}] \dot{T}_{2(nm)}(t) + \\ & + [\tilde{a}_{\alpha(1,2)}^2 + a_{\alpha(2,3)}^2] \mathfrak{D}_t^\alpha [T_{2(nm)}(t)] - \tilde{a}_{0(1,2)}^2 T_{1(nm)}(t) - \tilde{a}_{\alpha(1,2)}^2 \mathfrak{D}_t^\alpha [T_{1(nm)}(t)] - a_{0(2,3)}^2 T_{3(nm)}(t) - a_{\alpha(2,3)}^2 \mathfrak{D}_t^\alpha [T_{3(nm)}(t)] + \\ & + \frac{1}{4} \tilde{\mu}_{1,2} (1 - \kappa) \ddot{T}_{1(nm)}(t) - 2\tilde{\delta}_{1,2} \dot{T}_{1(nm)}(t) + \frac{1}{4} \mu_{2,3} (1 - \kappa) \ddot{T}_{3(nm)}(t) - 2\delta_{2,3} \dot{T}_{3(nm)}(t) = \\ & = h_{02,nm} \sin(\Omega_{2,nm} t + \vartheta_{2,nm}) \end{aligned}$$

$$\begin{aligned} & \left[ 1 + \frac{1}{4} \tilde{\mu}_{2,3} (1 + \kappa) + \frac{1}{4} \mu_{3,4} (1 + \kappa) \right] \ddot{T}_{3(nm)}(t) + c_3^2 k_{nm}^2 T_{3(nm)}(t) + [\tilde{a}_{0(2,3)}^2 + a_{0(3,4)}^2] T_{3(nm)}(t) + \\ & + 2[\tilde{\delta}_{2,3} + 2\delta_{3,4}] \dot{T}_{3(nm)}(t) + \frac{1}{4} \tilde{\mu}_{2,3} (1 - \kappa) \ddot{T}_{2(nm)}(t) - \tilde{a}_{0(2,3)}^2 T_{2(nm)}(t) - 2\tilde{\delta}_{2,3} \dot{T}_{2(nm)}(t) + \\ & + [\tilde{a}_{\alpha(2,3)}^2 + a_{\alpha(3,4)}^2] \mathfrak{D}_t^\alpha [T_{3(nm)}(t)] - \tilde{a}_{\alpha(2,3)}^2 \mathfrak{D}_t^\alpha [T_{2(nm)}(t)] \approx \\ & \approx -a_{NL,3,4}^2 \tilde{\delta}_{nm} [T_{3(nm)}(t)]^\beta + \frac{1}{4} \mu_{3,4} (1 - \kappa) \Omega_{0nm}^2 w_{0nm} \sin(\Omega_{0nm} t + \vartheta_{0nm}) + a_{0(3,4)}^2 w_{0nm} \sin(\Omega_{0nm} t + \vartheta_{0nm}) + \\ & + a_{\alpha(3,4)}^2 w_{0nm} \mathfrak{D}_t^\alpha [\sin(\Omega_{0nm} t + \vartheta_{0nm})] + 2\delta_{3,4} \Omega_{0nm} w_{0nm} \cos(\Omega_{0nm} t + \vartheta_{0nm}) + h_{03,nm} \sin(\Omega_{3nm} t + \vartheta_{3,nm}) \end{aligned}$$



$$T_{1(nm)}(t) = \sum_{s=1}^{s=3} A_{(nm)1}^{(s)} \cos(\tilde{\omega}_{(nm)s} t + \alpha_{(nm)s}) = \sum_{s=1}^{s=3} \mathbf{K}_{(nm)31}^{(s)}(\tilde{\omega}_{nm(s)}^2) C_{(nm)s} \cos(\tilde{\omega}_{(nm)s} t + \alpha_{(nm)s})$$

$$T_{2(nm)}(t) = \sum_{s=1}^{s=3} A_{(nm)2}^{(s)} \cos(\tilde{\omega}_{(nm)s} t + \alpha_{(nm)s}) = \sum_{s=1}^{s=3} \mathbf{K}_{(nm)32}^{(s)}(\tilde{\omega}_{nm(s)}^2) C_{(nm)s} \cos(\tilde{\omega}_{(nm)s} t + \alpha_{(nm)s})$$

$$T_{3(nm)}(t) = \sum_{s=1}^{s=3} A_{(nm)3}^{(s)} \cos(\tilde{\omega}_{(nm)s} t + \alpha_{(nm)s}) = \sum_{s=1}^{s=3} \mathbf{K}_{(nm)33}^{(s)}(\tilde{\omega}_{nm(s)}^2) C_{(nm)s} \cos(\tilde{\omega}_{(nm)s} t + \alpha_{(nm)s})$$

$$T_{1(nm)}(t) = \sum_{s=1}^{s=3} \mathbf{K}_{(nm)31}^{(s)}(\tilde{\omega}_{nm(s)}^2) \xi_{(nm)s}(t)$$

$$T_{2(nm)}(t) = \sum_{s=1}^{s=3} \mathbf{K}_{(nm)32}^{(s)}(\tilde{\omega}_{nm(s)}^2) \xi_{(nm)s}(t)$$

$$T_{3(nm)}(t) = \sum_{s=1}^{s=3} \mathbf{K}_{(nm)33}^{(s)}(\tilde{\omega}_{nm(s)}^2) \xi_{(nm)s}(t)$$

$$T_{k(nm)}(t) = \sum_{s=1}^{s=3} \mathbf{K}_{(nm)3k}^{(s)} C_{(nm)s} \cos(\tilde{\omega}_{(nm)s} t + \alpha_{(nm)s}) = \sum_{s=1}^{s=3} \mathbf{K}_{(nm)3k}^{(s)} \xi_{(nm)s}(t)$$

$$\left\langle \ddot{\xi}_{(nm)1}(t) + \tilde{\omega}_{nm(1)}^2 \xi_{(nm)1}(t) + \tilde{\omega}_{\alpha(nm)(1)}^2 \mathfrak{D}_t^\alpha [\xi_{(nm)1}(t)] \right\rangle = \frac{\begin{vmatrix} h_{01,nm} \sin(\Omega_{1,nm} t + \vartheta_{1,nm}) & \mathbf{K}_{(nm)31}^{(2)} & \mathbf{K}_{(nm)31}^{(3)} \\ h_{02,nm} \sin(\Omega_{2,nm} t + \vartheta_{2,nm}) & \mathbf{K}_{(nm)32}^{(2)} & \mathbf{K}_{(nm)32}^{(3)} \\ h_{03,nm} \sin(\Omega_{3,nm} t + \vartheta_{3,nm}) & \mathbf{K}_{(nm)33}^{(2)} & \mathbf{K}_{(nm)33}^{(3)} \end{vmatrix}}{\begin{vmatrix} \mathbf{K}_{(nm)31}^{(1)} & \mathbf{K}_{(nm)31}^{(2)} & \mathbf{K}_{(nm)31}^{(3)} \\ \mathbf{K}_{(nm)32}^{(1)} & \mathbf{K}_{(nm)32}^{(2)} & \mathbf{K}_{(nm)32}^{(3)} \\ \mathbf{K}_{(nm)33}^{(1)} & \mathbf{K}_{(nm)33}^{(2)} & \mathbf{K}_{(nm)33}^{(3)} \end{vmatrix}}$$

$$\left\langle \ddot{\xi}_{(nm)2}(t) + \tilde{\omega}_{nm(2)}^2 \xi_{(nm)2}(t) + \tilde{\omega}_{\alpha(nm)(2)}^2 \mathfrak{D}_t^\alpha [\xi_{(nm)2}(t)] \right\rangle = \frac{\begin{vmatrix} \mathbf{K}_{(nm)31}^{(1)} & h_{01,nm} \sin(\Omega_{1,nm} t + \vartheta_{1,nm}) & \mathbf{K}_{(nm)31}^{(3)} \\ \mathbf{K}_{(nm)32}^{(1)} & h_{02,nm} \sin(\Omega_{2,nm} t + \vartheta_{2,nm}) & \mathbf{K}_{(nm)32}^{(3)} \\ \mathbf{K}_{(nm)33}^{(1)} & h_{03,nm} \sin(\Omega_{3,nm} t + \vartheta_{3,nm}) & \mathbf{K}_{(nm)33}^{(3)} \end{vmatrix}}{\begin{vmatrix} \mathbf{K}_{(nm)31}^{(1)} & \mathbf{K}_{(nm)31}^{(2)} & \mathbf{K}_{(nm)31}^{(3)} \\ \mathbf{K}_{(nm)32}^{(1)} & \mathbf{K}_{(nm)32}^{(2)} & \mathbf{K}_{(nm)32}^{(3)} \\ \mathbf{K}_{(nm)33}^{(1)} & \mathbf{K}_{(nm)33}^{(2)} & \mathbf{K}_{(nm)33}^{(3)} \end{vmatrix}}$$

$$\left\langle \ddot{\xi}_{(nm)3}(t) + \tilde{\omega}_{nm(3)}^2 \xi_{(nm)3}(t) + \tilde{\omega}_{\alpha(nm)(3)}^2 \mathfrak{D}_t^\alpha [\xi_{(nm)3}(t)] \right\rangle = \frac{\begin{vmatrix} \mathbf{K}_{(nm)31}^{(1)} & \mathbf{K}_{(nm)31}^{(2)} & h_{01,nm} \sin(\Omega_{1,nm} t + \vartheta_{1,nm}) \\ \mathbf{K}_{(nm)32}^{(1)} & \mathbf{K}_{(nm)32}^{(2)} & h_{02,nm} \sin(\Omega_{2,nm} t + \vartheta_{2,nm}) \\ \mathbf{K}_{(nm)33}^{(1)} & \mathbf{K}_{(nm)33}^{(2)} & h_{03,nm} \sin(\Omega_{3,nm} t + \vartheta_{3,nm}) \end{vmatrix}}{\begin{vmatrix} \mathbf{K}_{(nm)31}^{(1)} & \mathbf{K}_{(nm)31}^{(2)} & \mathbf{K}_{(nm)31}^{(3)} \\ \mathbf{K}_{(nm)32}^{(1)} & \mathbf{K}_{(nm)32}^{(2)} & \mathbf{K}_{(nm)32}^{(3)} \\ \mathbf{K}_{(nm)33}^{(1)} & \mathbf{K}_{(nm)33}^{(2)} & \mathbf{K}_{(nm)33}^{(3)} \end{vmatrix}}$$

## Theorem 1:

Generalized forces  $Q_{w_k}^{elem-sloja}$  and  $Q_{w_{k+1}}^{elem-sloja}$  of interaction between two deformable bodies coupled by standard discrete continuum layer with known kinetic and potential energies  $E_k^{elem-sloja}$  and  $E_p^{elem-sloja}$  and known function of energy dissipation  $\Phi^{elem-sloja}$  for generalized coordinates  $w_k(x, y, t)$  and displacement of deformable bodies at the point of contacts with discrete continuum layer are in the following forms:

$$Q_{w_k}^{elem-sloja} = - \left\langle \frac{d}{dt} \frac{\partial E_k^{elem-sloja}}{\partial \left( \frac{\partial w_k(x, y, t)}{\partial t} \right)} - \frac{\partial E_k^{elem-sloja}}{\partial w_k(x, y, t)} \right\rangle - \frac{\partial E_p^{elem-sloja}}{\partial w_k(x, y, t)} - \frac{\partial \Phi^{elem-sloja}}{\partial \left( \frac{\partial w_k(x, y, t)}{\partial t} \right)} = Q_{w_k(x, y, t)}^{elem-ploca}$$

$$Q_{w_{k+1}}^{elem-sloja} = - \left\langle \frac{d}{dt} \frac{\partial E_k^{elem-sloja}}{\partial \left( \frac{\partial w_{k+1}(x, y, t)}{\partial t} \right)} - \frac{\partial E_k^{elem-sloja}}{\partial w_{k+1}(x, y, t)} \right\rangle - \frac{\partial E_p^{elem-sloja}}{\partial w_{k+1}(x, y, t)} - \frac{\partial \Phi^{elem-sloja}}{\partial \left( \frac{\partial w_{k+1}(x, y, t)}{\partial t} \right)} = Q_{w_{k+1}(x, y, t)}^{elem-ploca}$$

expressed by energies and energy dissipation which posses discrete continuum layer.

## Theorem 2.

Dynamics of hybrid system which contain deformable bodies (beams, plates or membranes) coupled by discrete continuum fractional order layers with equal boundary conditions and with displacements

$$w_k(x, y, t)$$

and  $w_{k+1}(x, y, t)$  is described by corresponding system of coupled partial fractional order differential equations. Systems of coupled partial fractional order differential equations for the cases of the hybrid systems containing coupled beams, or coupled plates or coupled membranes by discrete continuum fractional order layers are in mathematical analogy.

### Theorem 3.

Dynamics of hybrid system which contain deformable bodies (beams, plates or membranes) coupled by discrete continuum fractional order layers with equal boundary conditions and with displacements

$$w_k(x, y, t)$$

and  $w_{k+1}(x, y, t)$  and described by corresponding system of coupled partial fractional order differential equations, in each eigen amplitude mode from set of infinite number is described by corresponding like -frequency eigen time functions

$N$

$$T_{k(nm)}(t) = \sum_{s=1}^{s=N} \mathbf{K}_{(nm)Nk}^{(s)} \xi_{(nm)s}(t)$$

where  $\xi_{(nm)s}(t)$ ,  $s = 1, 2, 3, \dots, N$  are normal main coordinates of corresponding subsystem in eigen amplitude mode

These normal coordinates are analogous to the corresponding normal coordinates of corresponding linear system dynamics in same eigen amplitude mode.

$$W_{nm}(x, y)$$

## Theorem 4.

Dynamics of hybrid system which contain deformable bodies (beams, plates or membranes) coupled by discrete continuum fractional order layers with equal boundary conditions and with displacements

$$w_k(x, y, t)$$

and  $w_{k+1}(x, y, t)$  and described by corresponding system of coupled partial fractional order differential equations, in each eigen amplitude mode

$$W_{nm}(x, y)$$

from set of infinite number is described by corresponding like -frequency eigen time functions

$$N$$

$$T_{k(nm)}(t) = \sum_{s=1}^{s=N} \mathbf{K}_{(nm)Nk}^{(s)} \xi_{(nm)s}(t)$$

where  $\xi_{(nm)s}(t)$ ,  $s = 1, 2, 3, \dots, N$  are normal main coordinates of corresponding subsystem in eigen amplitude mode

These normal coordinates are analogous to the corresponding normal coordinates of corresponding linear system dynamics in same eigen amplitude mode.

$$W_{nm}(x, y)$$



These normal fractional order time modes  $\xi_{(nm)s}(t)$   $s = 1, 2, 3, \dots, N$  are described by system of independent ordinary fractional order differential equations in the forms:

$$\ddot{\xi}_{(nm)s}(t) + \tilde{\omega}_{nm(s)}^2 \xi_{(nm)s}(t) + \tilde{\omega}_{\alpha(nm)(s)}^2 \mathfrak{D}_t^\alpha [\xi_{(nm)s}(t)] = 0$$

with two sets of characteristic numbers:  $\tilde{\omega}_{nm(s)}^2$  and  $\tilde{\omega}_{\alpha(nm)(s)}^2$

First set  $\tilde{\omega}_{nm(s)}^2$  of characteristic numbers are square of eigen circular frequencies, same as for corresponding linear system, and seconds set  $\tilde{\omega}_{\alpha(nm)(s)}^2$  of characteristic numbers correspond to fractional properties of eigen like one frequency fractional order mode.

## Theorem 5.

In dynamics of hybrid system which contain deformable bodies (beams, plates or membranes) coupled by discrete continuum fractional order layers with equal boundary conditions and with displacements

$$w_k(x, y, t)$$

and  $w_{k+1}(x, y, t)$  and described by corresponding system of coupled partial fractional order differential equations, in each eigen amplitude mode  $W_{nm}(x, y)$  from set of infinite number, described by corresponding like -frequency eigen time functions

$N$

$$T_{k(nm)}(t) = \sum_{s=1}^{s=N} \mathbf{K}_{(nm)Nk}^{(s)} \xi_{(nm)s}(t) \quad W_{nm}(x, y)$$

where  $\xi_{(nm)s}(t)$ ,  $s = 1, 2, 3, \dots, N$  displacements of partial, independent fractional order oscillators described by system of independent ordinary fractional order differential equations in the forms:

$$\ddot{\xi}_{(nm)s}(t) + \tilde{\omega}_{nm(s)}^2 \xi_{(nm)s}(t) + \tilde{\omega}_{\alpha(nm)(s)}^2 \mathfrak{D}_t^\alpha [\xi_{(nm)s}(t)] = 0$$

with two sets of characteristic numbers:  $\tilde{\omega}_{nm(s)}^2$  and  $\tilde{\omega}_{\alpha(nm)(s)}^2$

and two complement modes: first  $[\xi_{(nm)s}(t)]_{\text{Like } \cos(\tilde{\omega}_{nm(s)}t + \alpha_{nm(s)})}$   
fractional order like cos mode and second  $[\xi_{(nm)s}(t)]_{\text{Like } \sin(\tilde{\omega}_{nm(s)}t + \alpha_{nm(s)})}$   
fractional order like sin mode expressed by series along time in the following analytical forms:

$$[\xi_{(nm)s}(t)]_{\text{Like } \cos(\tilde{\omega}_{nm(s)}t + \alpha_{nm(s)})} = \sum_{k=0}^{\infty} (-1)^k \tilde{\omega}_{\alpha(nm)(s)}^{2k} t^{2k} \sum_{j=0}^k \binom{k}{j} \frac{(\mp 1)^j \tilde{\omega}_{\alpha(nm)(s)}^{2j} t^{-\alpha j}}{\tilde{\omega}_{nm(s)}^{2j} \Gamma(2k+1-\alpha j)}$$

$$[\xi_{(nm)s}(t)]_{\text{Like } \sin(\tilde{\omega}_{nm(s)}t + \alpha_{nm(s)})} = \sum_{k=0}^{\infty} (-1)^k \tilde{\omega}_{\alpha(nm)(s)}^{2k} t^{2k+1} \sum_{j=0}^k \binom{k}{j} \frac{(\mp 1)^j \tilde{\omega}_{\alpha(nm)(s)}^{-2j} t^{-\alpha j}}{\tilde{\omega}_{nm(s)}^{2j} \Gamma(2k+2-\alpha j)}$$

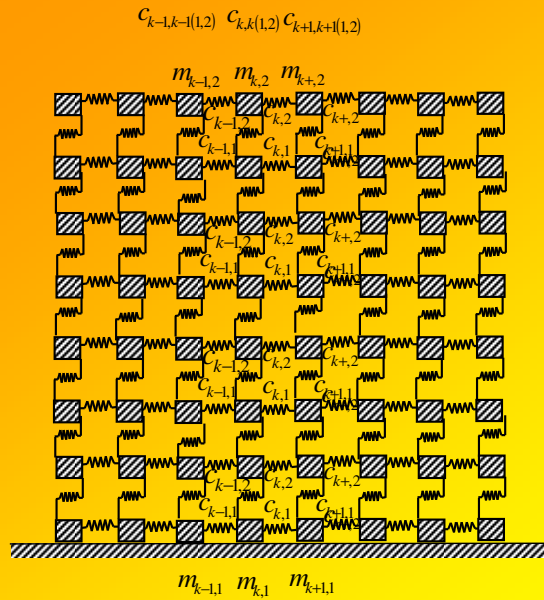
$$\begin{aligned} \xi_{(nm)s}(t) = & \xi_{(nm)s}(0) \sum_{k=0}^{\infty} (-1)^k \tilde{\omega}_{\alpha(nm)(s)}^{2k} t^{2k} \sum_{j=0}^k \binom{k}{j} \frac{(\mp 1)^j \tilde{\omega}_{\alpha(nm)(s)}^{2j} t^{-\alpha j}}{\tilde{\omega}_{nm(s)}^{2j} \Gamma(2k+1-\alpha j)} + \\ & + \dot{\xi}_{(nm)s}(0) \sum_{k=0}^{\infty} (-1)^k \tilde{\omega}_{\alpha(nm)(s)}^{2k} t^{2k+1} \sum_{j=0}^k \binom{k}{j} \frac{(\mp 1)^j \tilde{\omega}_{\alpha(nm)(s)}^{-2j} t^{-\alpha j}}{\tilde{\omega}_{nm(s)}^{2j} \Gamma(2k+2-\alpha j)} \end{aligned}$$

### **Theorem 6:**

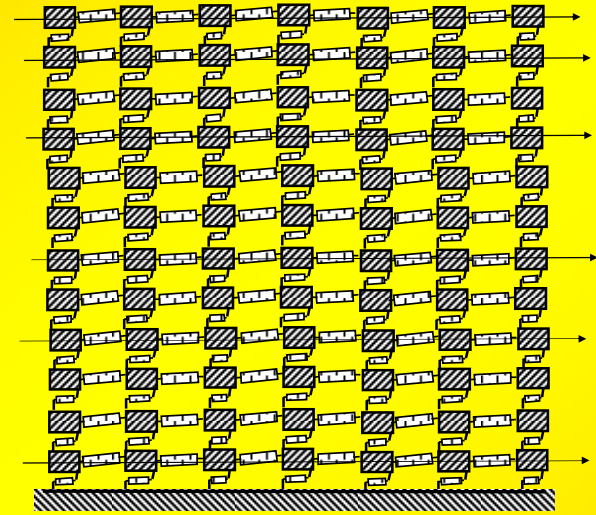
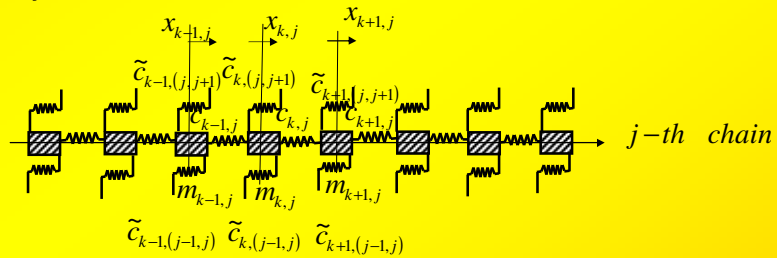
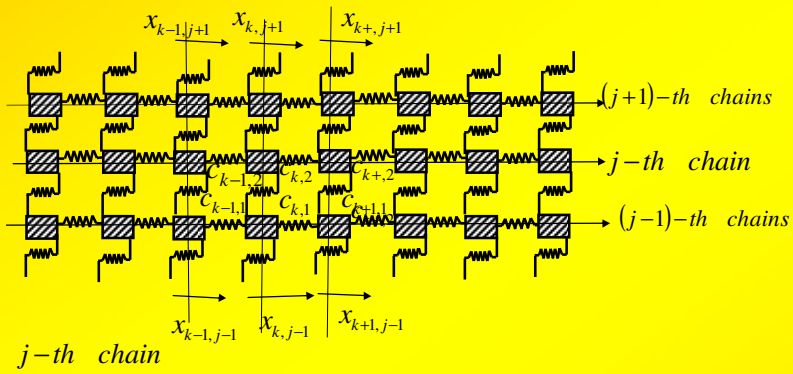
Considered system of fractional order differential equation is linear and main coordinates of corresponding system of linear differential equations are analogous to normal coordinates of fractional order differential equations.

### **Theorem 7:**

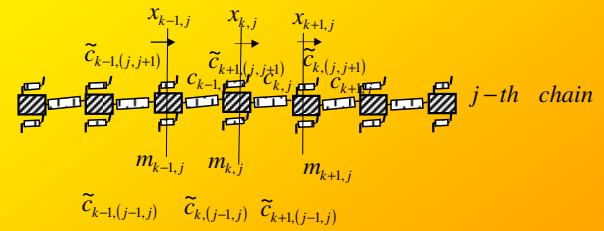
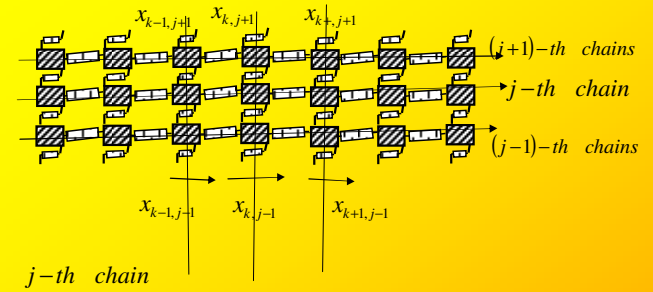
Considered system of fractional order differential equations described by like  $N$  frequency fractional order modes with corresponding eigen circular frequencies and corresponding characteristic numbers describing fractional order properties of fractional order like one frequency vibrations.



$(j-1, j, j+1)$ -th chains

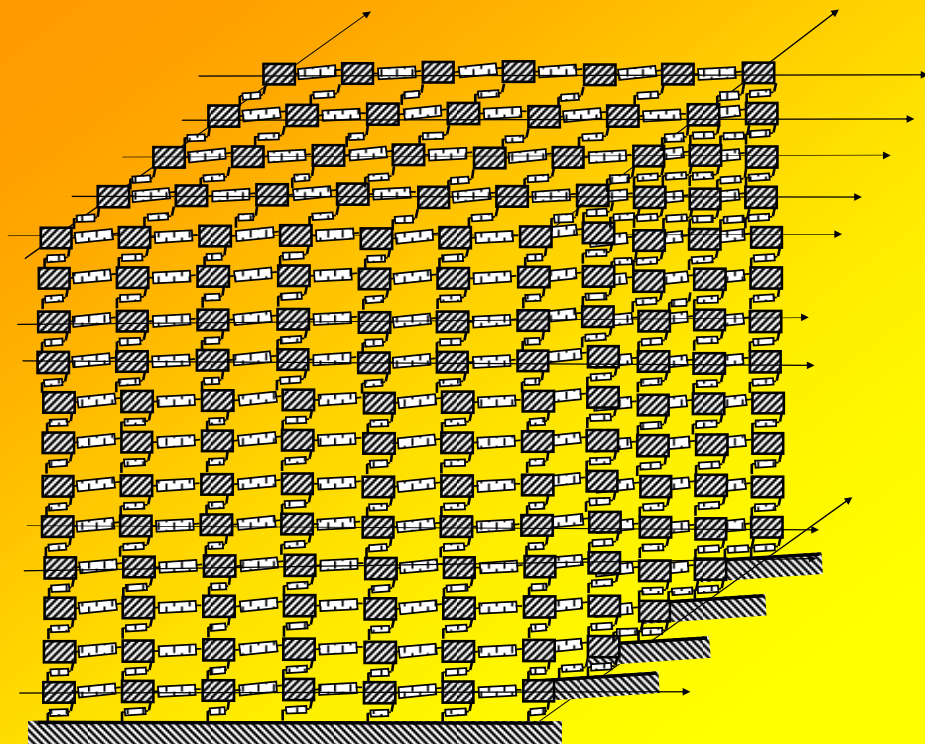


$(j-1, j, j+1)$ -th chains

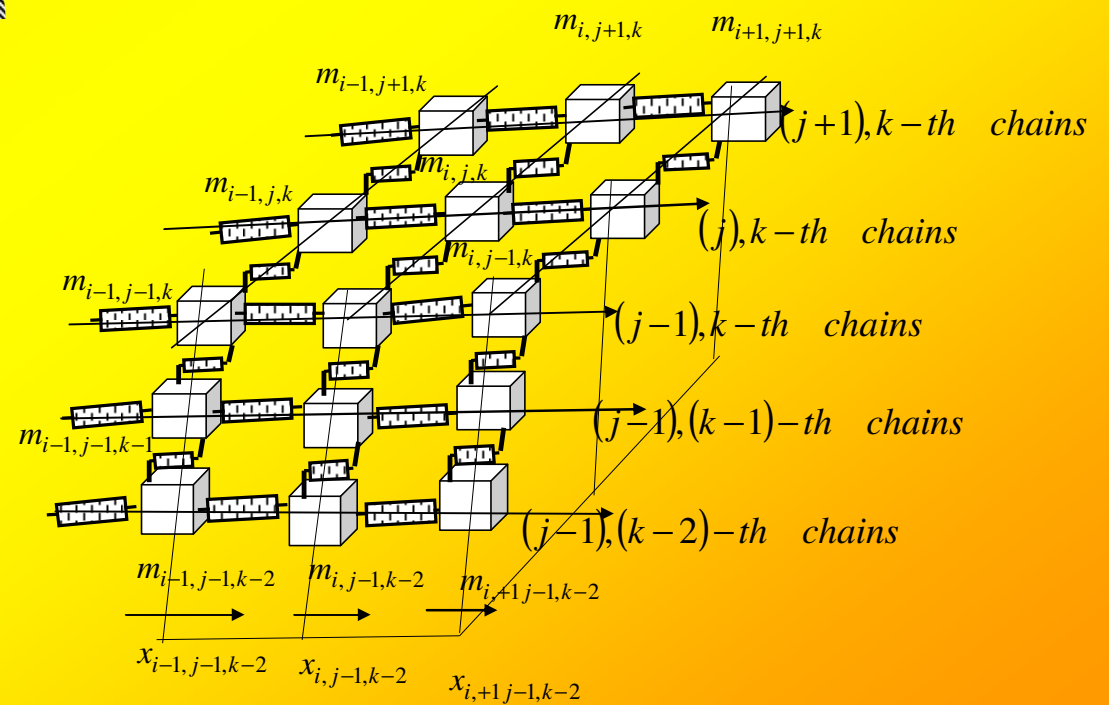


$$\tilde{c}_{k-1,(j-1,j)}\alpha \quad \tilde{c}_{k,(j-1,j)}\alpha \quad \tilde{c}_{k+1,(j-1,j)}\alpha \quad c_{k-1,j}\alpha \quad c_{k,j}\alpha \quad c_{k+1,j}\alpha$$

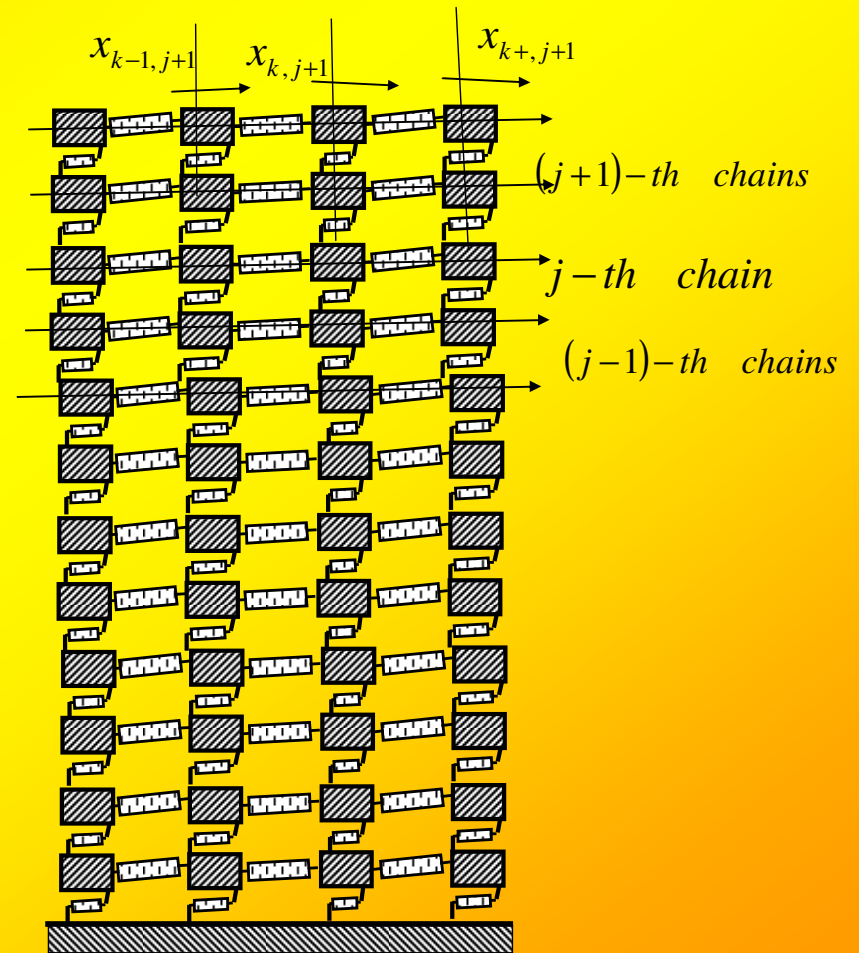
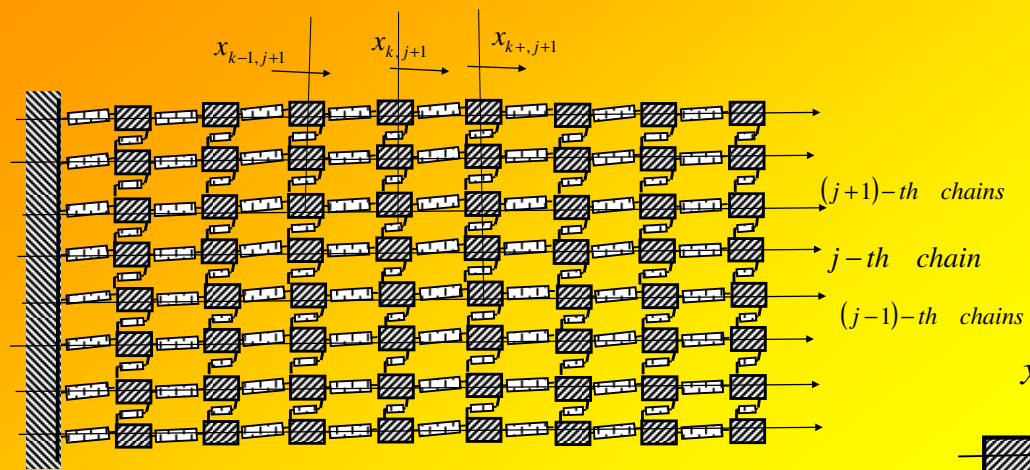
$$\tilde{c}_{k-1,(j,j+1)}\alpha \quad \tilde{c}_{k,(j,j+1)}\alpha \quad \tilde{c}_{k+1,(j,j+1)}\alpha$$

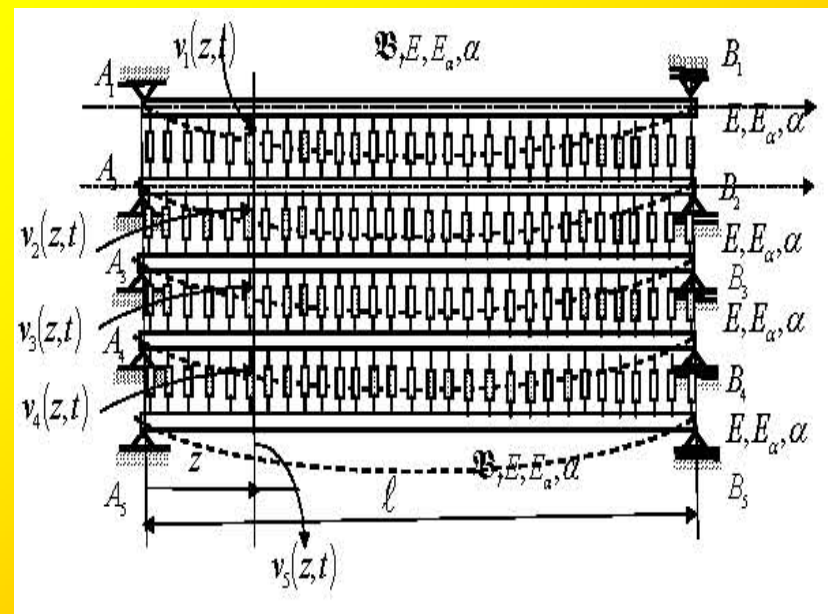
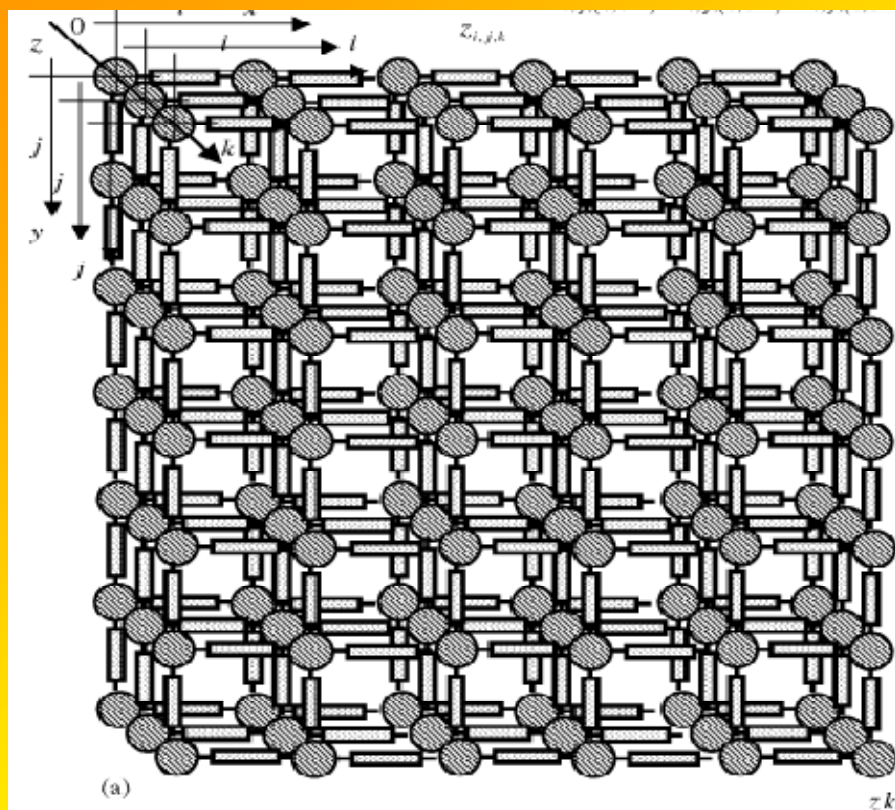


$$\ddot{\zeta}^{(s)(r)(p)} + \omega_{(s)(r)(p)}^2 \zeta^{(s)(r)(p)} + \omega_{\alpha(s)(r)(p)}^2 \mathfrak{D}_t^\alpha \left[ \zeta^{(s)(r)(p)} \right] = 0$$









**Theorem:** Let fractional order system dynamics with finite number of degrees of freedom is defined by  $\mathbf{A}$  matrix of coefficients of system inertia properties,  $\mathbf{C}$  matrix of coefficients of system rigidity properties and  $\mathbf{C}_\alpha$  matrix of coefficients of system viscoelastic creep fractional order properties. In the case that modal matrix

$$\mathbf{R} = \left( \left\{ K_{nk}^s \right\} \right) = \left( K_{nk}^s \right)_{\substack{\downarrow k=1,2,3,\dots,n \\ \rightarrow s=1,2,3,\dots,n}}$$

of corresponding linear system produce diagonalization of matrix  $\mathbf{C}_\alpha$  of coefficients of system viscoelastic creep fractional order

properties using product  $\mathfrak{C}_s = \mathbf{R}' \mathbf{C}_\alpha \mathbf{R} = (c_{(\alpha)ss})$   
 then this system possess eigen main  
 independent fractional order modes  $\xi_s$   
 $s = 1, 2, 3, \dots, n$  governed by ordinary fractional  
 order differential equations:

$$a_{ss} \ddot{\xi}_s + c_{(\alpha)ss} \mathfrak{D}_t^\alpha \{\xi_s\} + c_{ss} \xi_s = 0$$

or eigen normal modes  $\zeta_s$   $s = 1, 2, 3, \dots, n$

governed by  $\ddot{\zeta}_s + \omega_{(\alpha)s}^2 \mathfrak{D}_t^\alpha \{\zeta_s\} + \omega_s^2 \zeta_s = 0$  where

$$\omega_s^2 = \frac{c_{ss}}{a_{ss}} \quad \text{and} \quad \omega_{(\alpha)s}^2 = \frac{c_{(\alpha)ss}}{a_{ss}}$$

are two sets of  
 characteristic numbers of fractional order

system oscillations, first set contain square of eigen circular frequencies same as for corresponding linear system, a second contain characteristic numbers expressing fractional order system properties.

# Elements of mathematical phenomenology in dynamics of multi-body system with fractional order discrete continuum layers

Dedicated to Centennial jubilee of Russian Academician Yury N. Rabotnov

Katica R. (Stevanović) Hedrih

# Mathematical Problems in Engineering

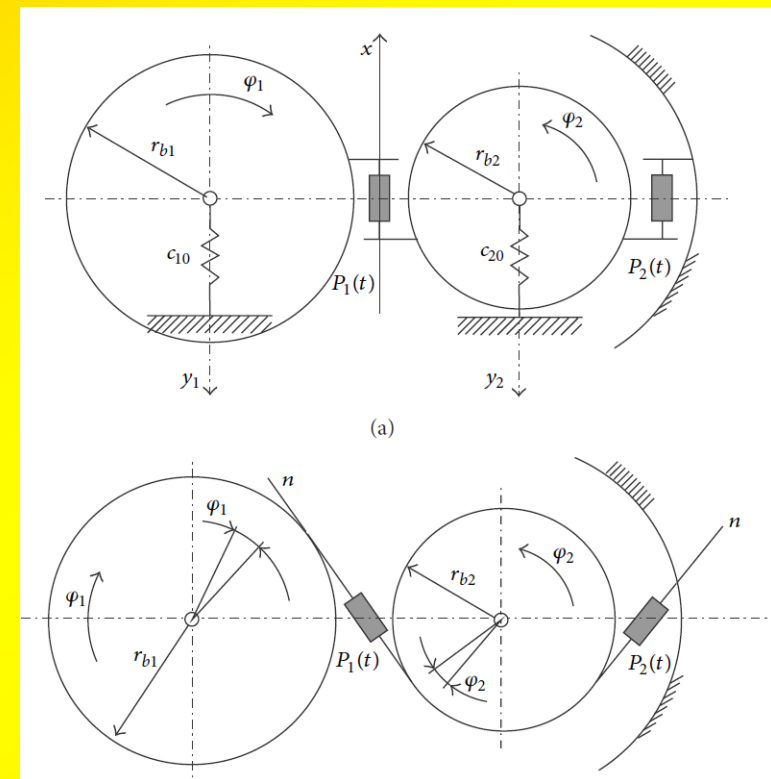
Theory, Methods, and Applications

Editors-in-Chief: V. Lakshmikantham and Semyon M. Meerkerk



Hindawi Publishing Corporation  
<http://www.hindawi.com>

Volume 2006  
Number 1



**Nikolic-Stanojevic Vera B Veljovic Ljiljana V Dolicanin Cemal B , A New Model of the Fractional Order Dynamics of the Planetary Gears (Article), MATHEMATICAL PROBLEMS IN ENGINEERING, (2013), vol. br. , str. -**



- Hedrih (Stevanović) K., Milosavljević D., **Veljović Lj.**,.: *Multi-parameter Analysis of a Rigid Body Nonlinear Coupled Rotations around No Intersecting Axes Based on the Vector Method*, Adv. Theor. Appl. Mech., Vol. 6, 2013, no. 2, 49 - 70, HIKARI Ltd, ISSN 1313-6250 pp. 49-70, 2013
- *Vector expressions of the kinetic parameters of the nonlinear dynamics of a rigid body coupled rotations around two no intersecting axes, on the basis of the three parametric analysis of the vector rotators, and transformations of the phase trajectories, show that vector method as well as applications of the mass moment vectors and vector rotators give a simplest way and expressions for analysis characteristic vector structures of coupled rotation kinetic properties, especially angular velocities of the vector rotators which are in directions of the kinetic pressures on shaft bearings or their reactions.*
- *Vektorski izrazi kinetičkih parametara nelinearne dinamike spregnutih rotacija oko dve ose koje se mimoilazu, epokazuju da vektorska metoda zasnovana na vektorima momenata mase, predstavlja veoma pogodan način za proučavanje karakteristika kinetičkih parametara spregnutih rotacija. Posebno treba istaći direktnu zavisnost kinetičkih pritisaka od ugaone brzine i vektora rotatora.*
- **Lj. Veljovic.**, *ANALYSIS OF A RIGID BODY ROTATION AROUND TWO NO INTERSECTING AXES – VECTOR METHOD AND PARAMETER ANALYSIS OF PHASE TRAJECTORIES*, SCIENTIFIC REVIEW (2013) Series: Scientific and Engineering - Special Issue Nonlinear Dynamics S2 (2013) pp. 319-324 YU ISSN 0350-2910
- *Vector expressions for linear momentum and angular momentum and their corresponding derivatives with respect to time describe rigid body nonlinear dynamics with coupled rotations around axes without intersection. Analysis of rotation of a heavy gyro-rotor show us that in graphical presentations of the system kinetic parameters exists a set of the fixed points not depending of change of rigid body eccentricity or angle of inclination or of the orthogonal distance between axes of rigid body coupled rotations*

- Katica R. (Stevanović) Hedrih, **Ljiljana Veljović.:** *New Vector Description of Kinetic Pressures on Shaft Bearings of a Rigid Body Nonlinear Dynamics with Coupled Rotations around No Intersecting Axes* , *Acya Polytechnica Hungarica*, ISSN 1785-8860, Vol.10, No 7, pp 151-170, 2013

– New vector description of kinetic pressures on shaft bearings of a rigid body nonlinear dynamics with coupled rotations around no intersecting axes is first main result presented in this paper. As an example of defined dynamics, we take into consideration a heavy gyro-rotordisk with one degree of freedom and coupled rotations when one component of rotation is programmed by constant angular velocity.

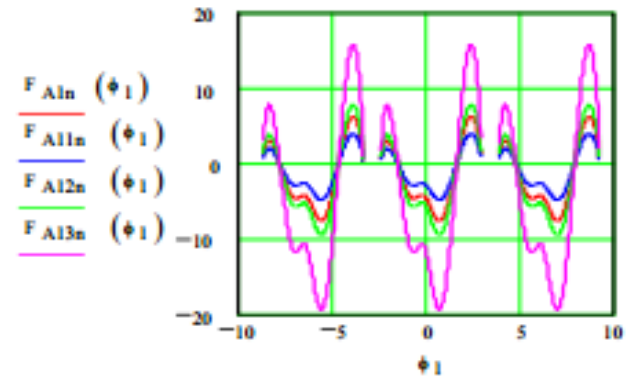
– Kinetički pritisci na ležišta vratila tela koje se obrće oko mimoilaznih osa predstavljani su na nov vektorski način. Kao primer korišćen je model teškog rotora

$\ddot{\varphi}_2 + \Omega^2 (\lambda - \cos \varphi_2) \sin \varphi_2 + \Omega^2 \psi \cos \varphi_2 = 0$  a stepena pokretljivosti tj. sa jednom

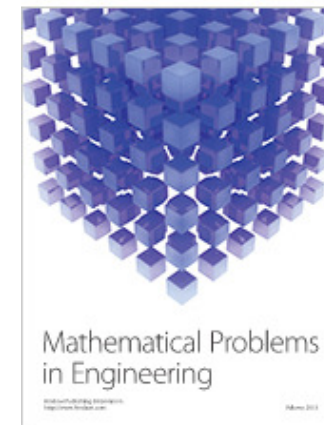
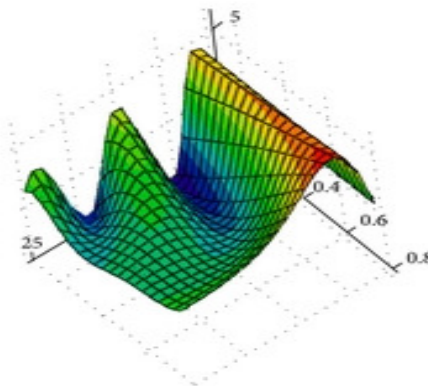
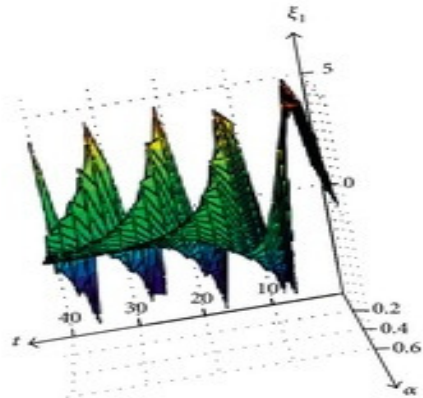
$$\vec{F}_{A\vec{n}_2} = \left[ \vec{S}_{\vec{n}_1}^{(O)} \left( \vec{R}_{1, \vec{n}_2} \right) + \left| \vec{S}_{\vec{n}_2}^{(O)} \right| \left( \vec{R}_{21, \vec{n}_2} \right) - \left( \vec{G}, \vec{n}_2 \right) \right] \vec{n}_2$$

$$\begin{aligned} \vec{F}_{B2} = & \frac{1}{2} \left\{ \left| \vec{S}_{\vec{n}_2}^{(O_2)} \right| \left[ \vec{n}_2, \left[ \vec{R}_2, \vec{n}_2 \right] \right] + \left| \vec{S}_{\vec{n}_1}^{(O_2)} \right| \left[ \vec{n}_2, \left[ \vec{R}_1, \vec{n}_2 \right] \right] + \left| \vec{S}_{\vec{n}_2}^{(O_2)} \right| \left[ \vec{n}_2, \left[ \vec{R}_{21}, \vec{n}_2 \right] \right] - \left[ \vec{n}_2, \left[ \vec{G}, \vec{n}_2 \right] \right] \right\} + \\ & + \frac{1}{2\ell} \left\{ \left| \vec{D}_{\vec{n}_2}^{(O_2)} \right| \left[ \vec{R}_{02}^*, \vec{n}_2 \right] + \left| \vec{D}_{\vec{n}_1}^{(O_2)} \right| \left[ \vec{R}_{01}^*, \vec{n}_2 \right] + \left[ \vec{n}_1, \vec{n}_2 \right] \dot{\omega}_1 \left( \vec{n}_1, \vec{J}_{\vec{n}_1}^{(O_2)} \right) - \left[ \vec{\rho}_C, \vec{G} \right], \vec{n}_2 \right\} + \\ & + \frac{1}{2\ell} \left\{ \left[ \left[ \vec{n}_2, \vec{J}_{\vec{n}_1}^{(O_2)} \right] + \left[ \vec{n}_1, \vec{J}_{\vec{n}_2}^{(O_2)} \right] + \vec{J}^{(O_2)} \left[ \vec{n}_1, \vec{n}_2 \right] \right\}, \vec{n}_2, \vec{n}_1 \right\} \end{aligned}$$

$$\begin{aligned} \vec{F}_{AT2} = & \frac{1}{2} \left\{ \left| \vec{S}_{\vec{n}_2}^{(O_2)} \right| \left[ \vec{n}_2, \left[ \vec{R}_2, \vec{n}_2 \right] \right] + \left| \vec{S}_{\vec{n}_1}^{(O_2)} \right| \left[ \vec{n}_2, \left[ \vec{R}_1, \vec{n}_2 \right] \right] + \left| \vec{S}_{\vec{n}_2}^{(O_2)} \right| \left[ \vec{n}_2, \left[ \vec{R}_{21}, \vec{n}_2 \right] \right] - \left[ \vec{n}_2, \left[ \vec{G}, \vec{n}_2 \right] \right] \right\} - \\ & - \frac{1}{2\ell} \left\{ \left| \vec{D}_{\vec{n}_2}^{(O_2)} \right| \left[ \vec{R}_{02}^*, \vec{n}_2 \right] + \left| \vec{D}_{\vec{n}_1}^{(O_2)} \right| \left[ \vec{R}_{01}^*, \vec{n}_2 \right] + \left[ \vec{n}_1, \vec{n}_2 \right] \dot{\omega}_1 \left( \vec{n}_1, \vec{J}_{\vec{n}_1}^{(O_2)} \right) - \left[ \vec{\rho}_C, \vec{G} \right], \vec{n}_2 \right\} - \\ & - \frac{1}{2\ell} \left\{ \left[ \left[ \vec{n}_2, \vec{J}_{\vec{n}_1}^{(O_2)} \right] + \left[ \vec{n}_1, \vec{J}_{\vec{n}_2}^{(O_2)} \right] + \vec{J}^{(O_2)} \left[ \vec{n}_1, \vec{n}_2 \right] \right\}, \vec{n}_2, \vec{n}_1 \right\} \end{aligned}$$



- Vera Nikolic-Stanojevic, **Ljiljana Veljovic**, Cemal Dolicanin,.: *A New Model of the Fractional Order Dynamics of the Planetary Gears*, Mathematical Problems in Engineering Volume 2013 (2013), Article ID 932150, 14 pages <http://dx.doi.org/10.1155/2013/932150>
  - *Dynamic model of the planetary gears with four degrees of freedom is used. Applying the basic principles of analytical mechanics and taking the initial and boundary conditions into consideration, the system of equations representing physical meshing process between the two or more gears is obtained. This investigation was focused to a new model of the fractional order dynamics of the planetary gear.*
  - *Razmatra se dinamički model planetarnog prenosičnika sa četiri stepena slobode. Primenom osnovnih principa mehanike dobijen je sistem diferencijalnih jednačina kojima je opisano sprezanje zupčanika. Prikazan je nov model frakcionih jednačina dinamike*



- Gordana Bogdanović, Dragan Milosavljević, **Ljiljana Veljović**, Aleksandar Radaković, Mirjana Lazić.: *COMPOSITE MATERIALS IN AUTOMOTIVE ENGINEERING – MECHANICAL BEHAVIOR OF ANISOTROPIC MEDIA*, Mobility and Vehicle Mechanics, University of Kragujevac, Faculty of Engineering, ISBN 1450-5304 vol. 39, br. 1, str. 39-49, 2013.,
  - *In recent engineering practice composite materials are widely used. In such materials two or more materials are combined to obtain a new one with new properties, while individual properties of constituents remain distinguished. Such materials have notable feature that are anisotropic, having different mechanical properties in different directions. Here is special attention devoted to their mechanical behavior. Small changes of preferred direction have significant influence to stress strain relations in fibre reinforced layers.*
  - *U inženjerskoj praksi kompozitni materijali su u širokoj upotrebi. U takvim materijalima dva ili više materijala se kombinuju i daju posebne osobine novoformiranom materijalu. Posebna pažnja posvećena je mehaničkom ponašanju ovih anizotropnih materijala.*

$$\rho \frac{\partial^2 u_i}{\partial t^2} = C_{ijkl} \frac{\partial^2 u_l}{\partial x_j \partial x_k}$$

$$u_i = U_i e^{i(kn_j x_j - \omega t)} = U_i e^{i\varphi}$$

- Gordana Bogdanović, Dragan Milosavljević, **Ljiljana Veljović**, Aleksandar Radaković.: *COMPOSITE MATERIALS – MECHANICAL BEHAVIOR OF ANISOTROPIC*, Proceedings of papers 11 International Conference on Accomplishments in Electrical and Mechanical Engineering and Information Technology DEMI 2013, PP. 111-114
- **Ljiljana Veljović**, Dragan Milosavljević, Gordana Bogdanović, Aleksandar Radaković.: *MODELING AND ANALYSIS FOR THE VIBRATION OF A GYROROTOR*, , Proceedings of papers 11 International Conference on Accomplishments in Electrical and Mechanical Engineering and Information Technology DEMI 2013,PP. 149-153
- Aleksandar Radaković, Dragan Milosavljević, Gordana Bogdanović, **Ljiljana Veljović**, Srba Aleksandrović.: *SECOND-ORDER FAILURE CRITERIA IN LAMINATE INCLUDING*, Proceedings of papers 11 International Conference on Accomplishments in Electrical and Mechanical Engineering and Information Technology DEMI 2013,PP. 149-153
- M. Matejic, **Lj. Veljovic**, V. Marjanovic, M. Blagojevic, N. Marjanovic, *DYNAMIC BEHAVIOR OF PLANETARY GEARBOX NEW CONCEPT.*.,DEMI 2013, Banja Luka, 2013, 30.5.-1.6., pp. 121-126, ISBN 978-99938-39-45-3

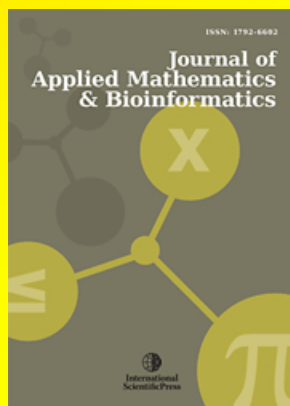
- **Ljiljana Veljović.:** *ABOUT KINEMATICAL VECTOR ROTATORS DEFINED FOR RIGID BODY DYNAMICS WITH COUPLED ROTATIONS AROUND AXES WITHOUT INTERSECTION*, Proceedings of papers 4TH International Congress of Serbian Society of Mechanics June 4-7, 2013, Vrnjačka Banja, PP. 199-2002, ISBN 978-86-909973-5-0
- Aleksandar Radaković , Dragan Milosavljević, Gordana Bogdanović, **Ljiljana Veljović.:** *FAILURE ANALYSIS OF A COMPOSITE LAMINATE MODELED USING THE HIGHER ORDER DEFORMATION THEORY* , Proceedings of papers 4TH International Congress of Serbian Society of Mechanics June 4-7, 2013, Vrnjačka Banja, PP. 523-528, ISBN 978-86-909973-5-0
- Dragan Milosavljević, Gordana Bogdanović, **Ljiljana Veljović**, Aleksandar Radaković.: *BULK WAVES IN FIBRE REINFORCED MATERIALS*, Proceedings of papers 4TH International Congress of Serbian Society of Mechanics June 4-7, 2013, Vrnjačka Banja, PP. 921-926, ISBN 978-86-909973-5-0
- Gordana Bogdanović, Dragan Milosavljević, **Ljiljana Veljović**, Aleksandar Radaković.: *WAVE PROPAGATION IN ORTHOTROPIC MATERIALS*, , Proceedings of papers 4TH International Congress of Serbian Society of Mechanics June 4-7, 2013, Vrnjačka Banja, PP. 927-932, ISBN 978-86-909973-5-0
- **Ljiljana Veljović**, Dragan Milosavljević, Gordana Bogdanović, Aleksandar Radaković .:, *ABOUT RIGID BODY OSCILLATIONS AROUND TWO INCLINED AXES WITHOUT INTERSECTION*, Proceedings of papers 4TH International Congress of Serbian Society of Mechanics June 4-7, 2013, Vrnjačka Banja, PP. 933-936, ISBN 978-86-909973-5-0



**A.Hedrih, K.(Stevanovic) Hedrih, B. Bugarski. Oscillatory Spherical net model of Mouse Zona Pellucida. Journal of Applied Mathematics and Bioinformatics. 2013, vol.3, no.4, 225-268. ISSN: 1792-6602 (print), 1792-6939 (online) Scienpress Ltd, 2013.**

[http://www.scienpress.com/journal\\_focus.asp?main\\_id=57&Sub\\_id=IV](http://www.scienpress.com/journal_focus.asp?main_id=57&Sub_id=IV)

**Katica (Stevanović) R Hedrih, Andjelka N Hedrih, Phenomenological mapping and dynamical absorptions in chain systems with multiple degrees of freedom, Journal of Vibration and Control (M21=8, in press)**



ISSN: 1792-6939 (Online version)  
1792-6602 (Print version)

[http://www.scienpress.com/journal\\_focus.asp?Main\\_Id=57](http://www.scienpress.com/journal_focus.asp?Main_Id=57)



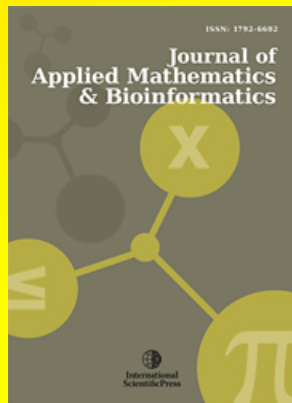
**A.Hedrih, K.(Stevanovic) Hedrih, B. Bugarski. Oscillatory Spherical net model of Mouse Zona Pellucida. Journal of Applied Mathematics and Bioinformatics. 2013, vol.3, no.4, 225-268. ISSN: 1792-6602 (print), 1792-6939 (online) Scienpress Ltd, 2013.**

**[http://www.scienpress.com/journal\\_focus.asp?main\\_id=57&Sub\\_id=IV](http://www.scienpress.com/journal_focus.asp?main_id=57&Sub_id=IV)**

**Katica (Stevanović) R Hedrih, Andjelka N Hedrih, Phenomenological mapping and dynamical absorptions in chain systems with multiple degrees of freedom, Journal of Vibration and Control (M21=8, in press)**

**1077546314525984,**

**doi:10.1177/1077546314525984**

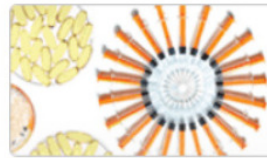


ISSN: 1792-6939 (Online version)

1792-6602 (Print version)

[http://www.scienpress.com/journal\\_focus.asp?Main\\_Id=57](http://www.scienpress.com/journal_focus.asp?Main_Id=57)





**Your Campaigns. Our Solutions.**

article reprints • print advertising • online advertising  
• sponsored supplements • custom publishing

**SAGE**

Sign In to gain access to subscriptions and/or My Tools.

Sign In

My Tools

Contact Us

HELP

**SAGE journals**

Search all journals

Advanced Search

Search History

Browse Journals

# Journal of Vibration and Control

Home

OnlineFirst

All Issues

Subscribe

RSS

Email Alerts

« Return to Search Results | Edit My Last Search

Search this journal

Advanced Journal Search »

**Impact Factor:** 1.966 | **Ranking:** 6/31 in Acoustics | 15/125 in Engineering, Mechanical | 19/134 in Mechanics

Source: 2012 Journal Citation Reports®  
(Thomson Reuters, 2013)

## Phenomenological mapping and dynamical absorptions in chain systems with multiple degrees of freedom

Katica R Stevanović Hedrih<sup>1,2</sup>

Andjelka N Hedrih<sup>3</sup>

<sup>1</sup>Department for Mechanics, Mathematical Institute SANU Belgrade, Serbia

<sup>2</sup>Faculty of Mechanical Engineering, University of Niš, Serbia

<sup>3</sup>Department for Bio-medical Science, State University of Novi Pazar, Serbia

Andjelka N Hedrih, Department for Bio-medical Science, State University of Novi Pazar, Vuka Karadzica bb, 36 300 Novi Pazar, Serbia. Email: [handjelka@hm.co.rs](mailto:handjelka@hm.co.rs)

### Abstract

### This Article

Published online before print March 19, 2014, doi:

10.1177/1077546314525984

Journal of Vibration and Control March 19, 2014 1077546314525984

» Abstract Free

Full Text (PDF)

References

Services

- ▶ Email this article to a colleague
- ▶ Alert me when this article is cited
- ▶ Alert me if a correction is posted
- ▶ Similar articles in this journal
- ▶ Download to citation manager
- ▶ Request Permissions

### Current Issue

▶ April 2014, 20 (6)

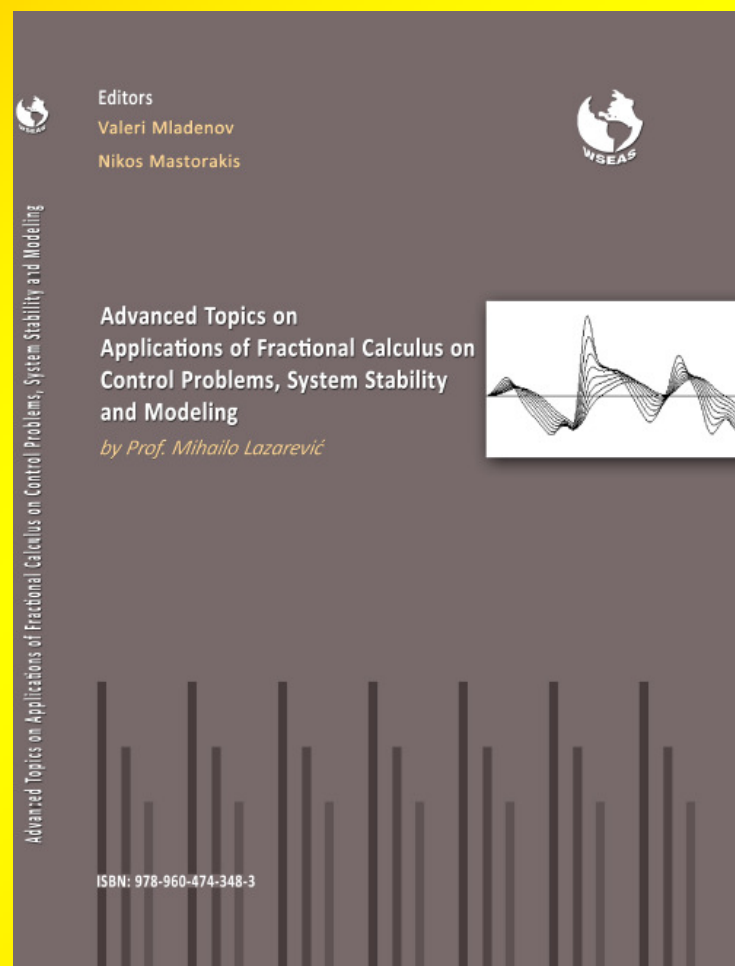


▶ Alert me to new issues of Journal of Vibration and Control

Submit a Manuscript

Free Sample Copy

**Hedrih, N. Andjelka and Hedrih(Stevanovvić) K. , (2013), Modeling Double DNA Helix Main Chains of the Free and Forced Fractional Order Vibrations, Chapter in Book Advanced topics on fractional calculus on control problem, modeling, system stability and modeling, Editor M. Lazarević, (2013), pp. 145-183 and Appendix pp. 192-200. WORLD SCIENTIFIC PUBLISHING COMPANY PTE LTD**





ENOC 2011  
7th European Nonlinear Dynamics Conference  
July 24-29, 2011 – Rome, ITALY

On the occasion of ENOC 2011, the EUROMECH Nonlinear Oscillations Conference Committee (ENOCC) has conferred an

## ENOC 2011 Young Scientist Prize

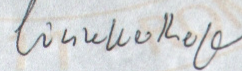
to Mrs. **Andjelka N. Hedrih**

for the best presentation given at the 7th European Nonlinear Dynamics Conference, Rome, 24-29 July 2011,  
within a Regular Session on **"Biosystems"**:

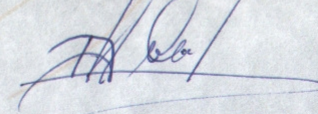
**"Modeling oscillations of Zona Pelucida before and after fertilization"**

In addition to the requirements for the ENOC 2011 Proceedings the prize recipient is invited to write a contribution with respect to his (her) presentation, to be published in the EUROMECH Newsletter

Giuseppe Rega  
Chairman of ENOC 2011



Dick H. van Campen  
Chairman of ENOCC

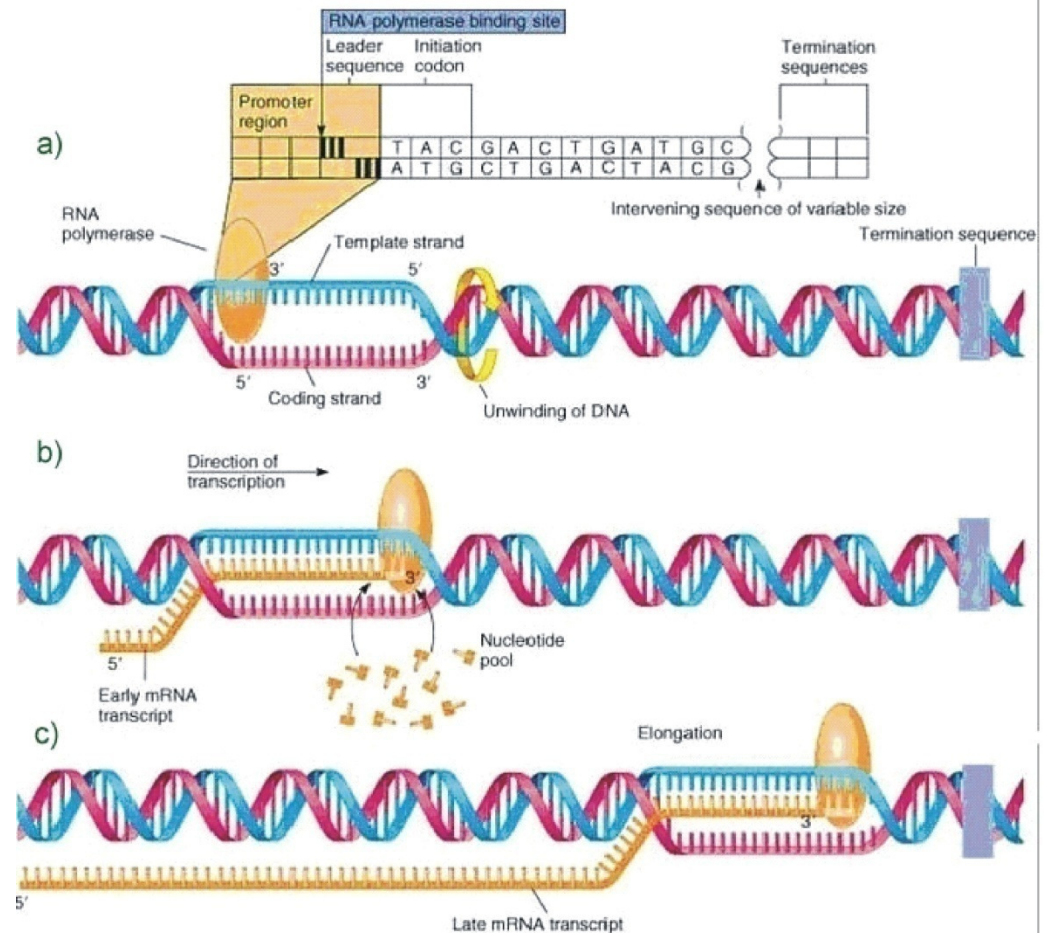
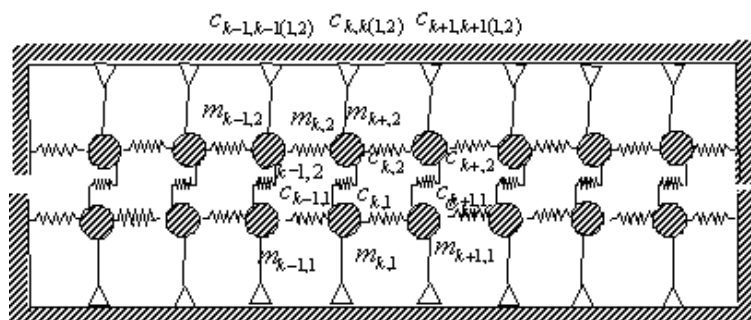
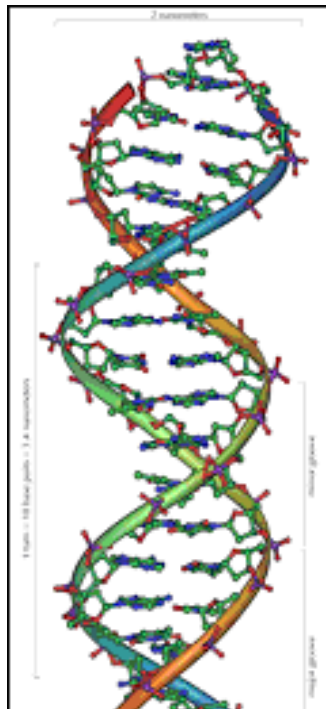


SAPIENZA  
UNIVERSITÀ DI ROMA

ENOC 2011  
AN ATTRACTOR FROM HISTORY



# Modelling Double DNA Helix Main Chains of the Free and Forced Fractional Order Vibrations



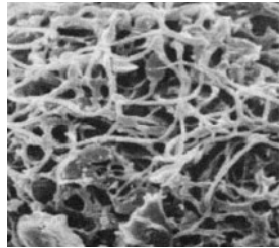
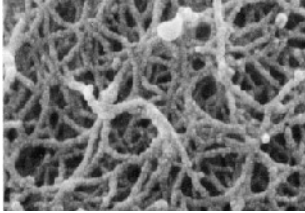
## Transkripcija DNK

Andjelka N. Hedrih, Katica R. (Stevanović) Hedrih. (2014) Modeling Double DNA Helix Main Chains of the Free and Forced Fractional Order Vibrations. Ch 7 In: Advanced Topics on Applications of Fractional Calculus on Control Problems, System Stability and Modeling (eds: Mihailo Lazarevic, Nikos Mastorakis. (2014), pp. 145-183. and Appendix E pp. 192-200. WSEAS Press. ISBN: 978-960-474-348-3



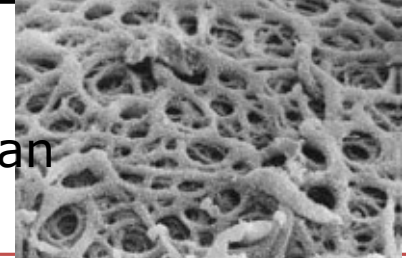
# Changes in 3D structure of mZP-SEM

Mouse



human

human



## Mouse mature oocyte

- ZP has mesh-like structure on scanning electron microscopy
- **ZP has more pores with less diameter than in embryo**
- ZP is penetrable for one certain spermatozoa
- **Young module has 2.5 times higher value compare to ZP of mouse embryo**
- **Diameter of oocyte and its ZP is slightly lower than in embryo**

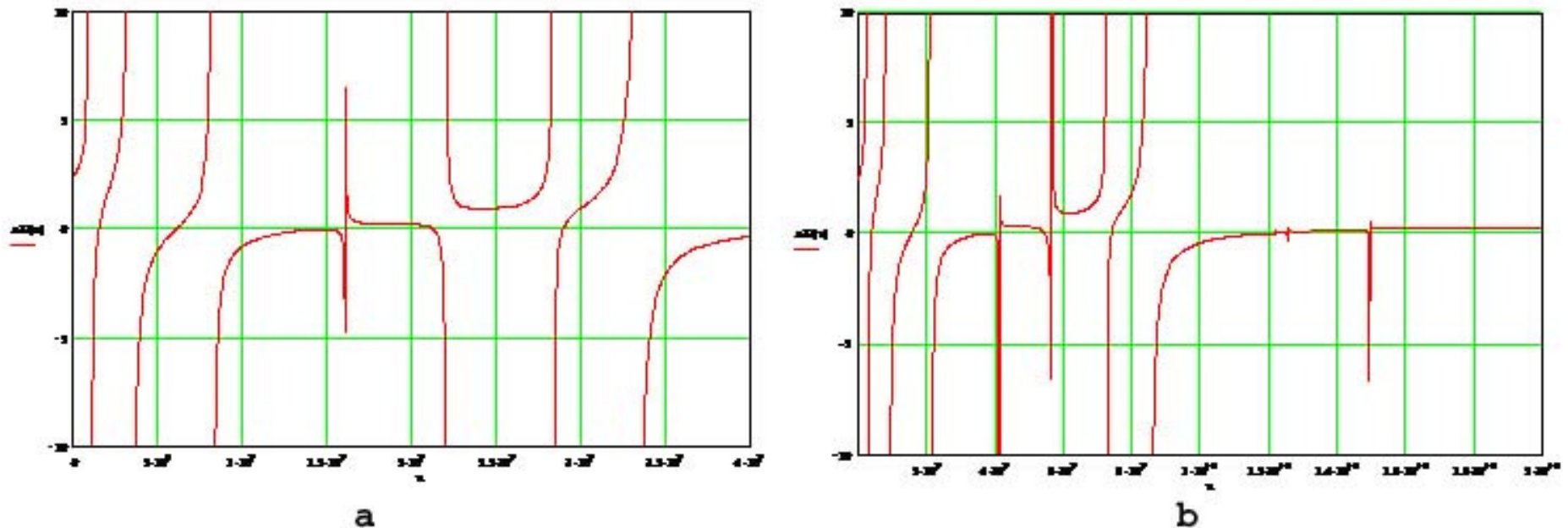
## Mouse embryo

- **ZP is not penetrable for spermatozoa**
- ZP has less pores with larger diameter than in oocyte
- **Young module has 2.5 times lower value compare to ZP of mouse oocyte**
- Diameter of embryo and its ZP is slightly higher than in oocyte

Source: Familiari G, Relucenti M, Heyn R, Micara G, and Correr S. (2006) Three-Dimensional Structure of the Zona Pellucida at Ovulation. *Microscopy research and technique* 69:415–426

## Transition in oscillatory behavior in mouse oocyte and mouse embryo

through oscillatory spherical net model of mouse Zona Pellucida



- Amplitude-frequency stationary forced regimes for forced oscillations of **third** material particle in chain excited by external excitation force with amplitude and frequency applied to third mass particle in chain;  $x=\Omega^2$ . a. oocyte b. embryo

**Andjelka Hedrih.** Transition in oscillatory behavior in mouse oocyte and mouse embryo through oscillatory spherical net model of mouse Zona Pellucida" (LIF138) in Applied Non-Linear Dynamical Systems/DSTA 2013; Series title: Proceedings in Mathematics and Statics, Edited by Jan Awrejcewicz, Springer, in press.

# Fertilization as a biomechanical oscillatory phenomenon in mammals<sup>B</sup>

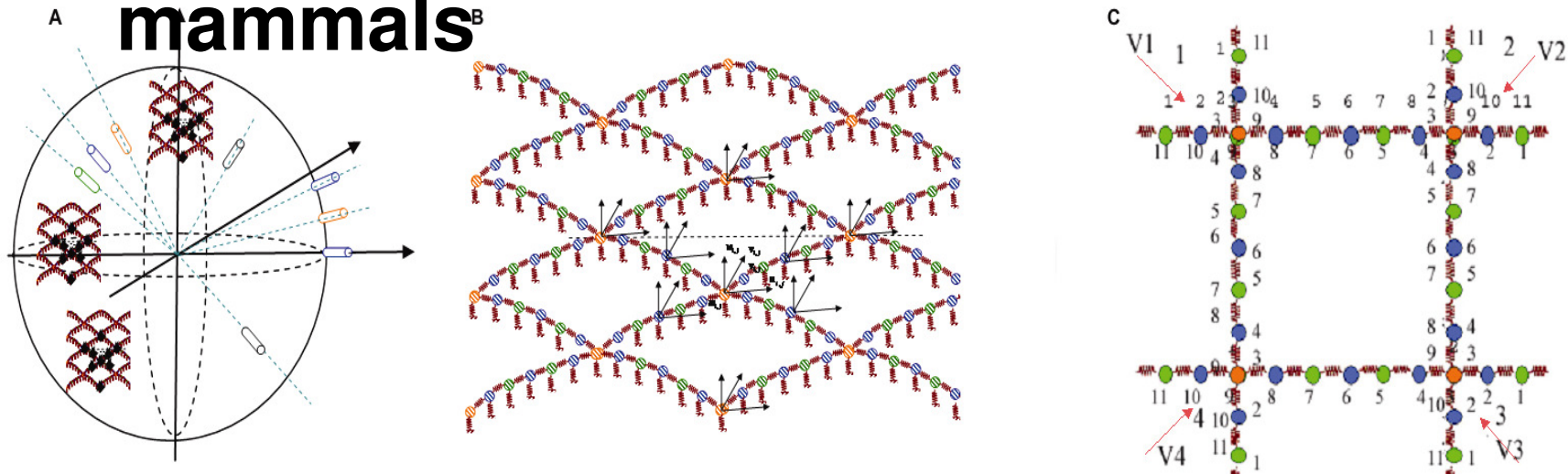
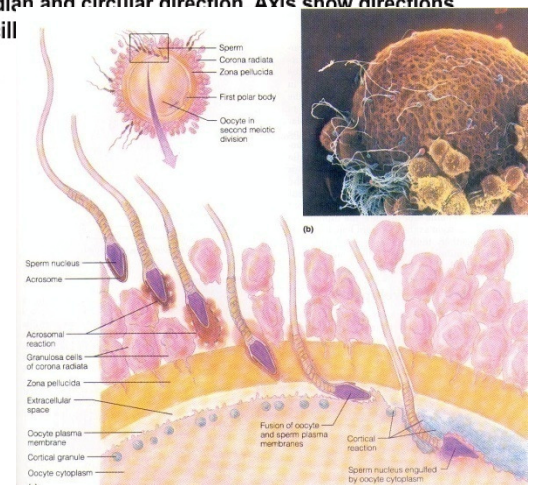
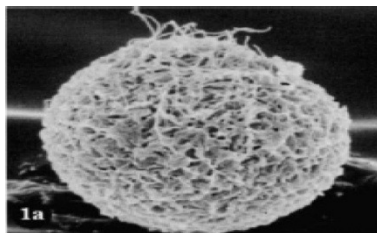
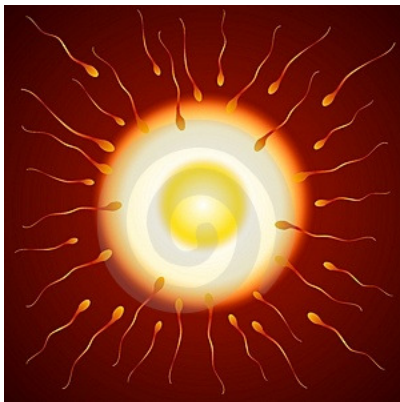
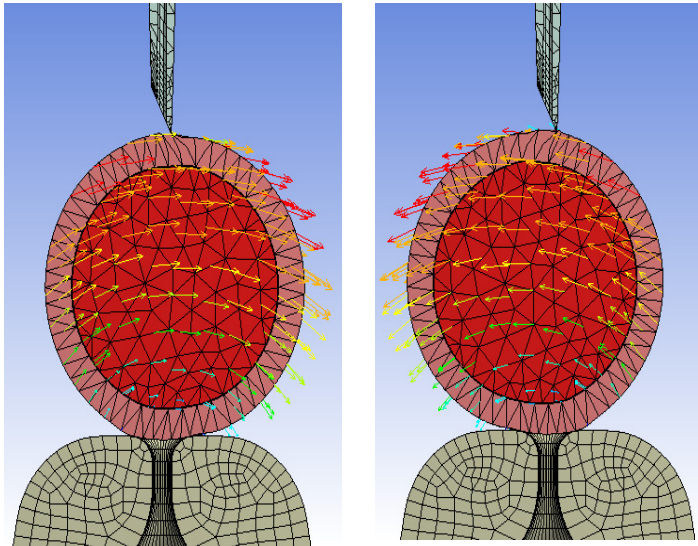
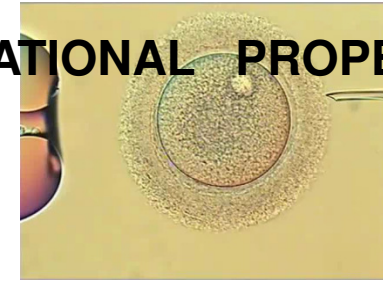


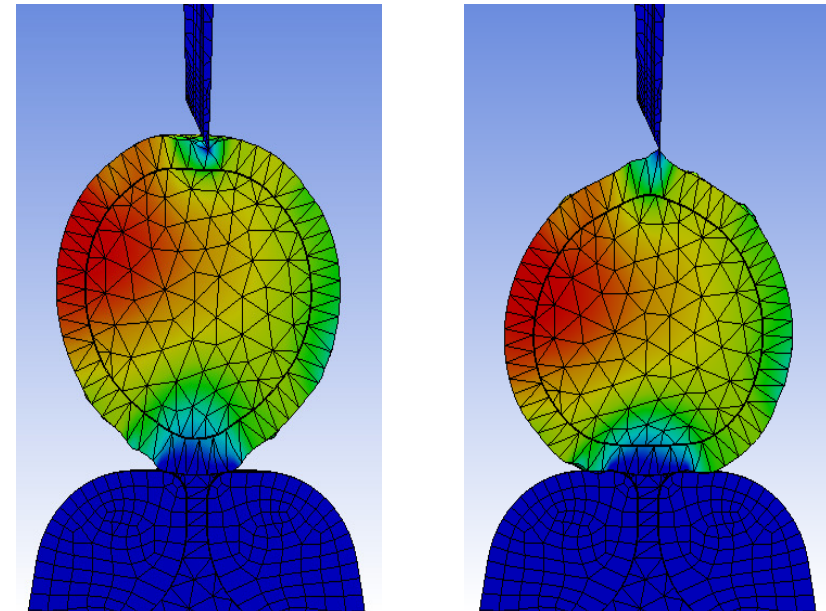
Figure1. A. Model of ZP spherical surface that shows radial direction of axis of constructive elements of the model-ZP molecules. B. Part of the ZP network on a part of the sphere-oocyte: orange (ZP1), blue (ZP2) and green (ZP3) represents molecules of ZP proteins. Chains of spherical net are identical in meridian and circular direction. Axis show directions of movements of molecules of ZP proteins. Each ZP protein in the model is connected to the sphere with elastic spring that can oscillate. C. Surface net model of mZP: v1-v4 are sperm cells with effective velocities. Red arrows denote sperm cell impact on a knot molecule.



# THE USE OF FINITE ELEMENTS METHOD IN VIBRATIONAL PROPERTIES CHARACTERIZATION OF MOUSE EMBRYO



**Figure 6.** Shape and particle velocities distribution in extreme points of embryo vibrations in mode 2.



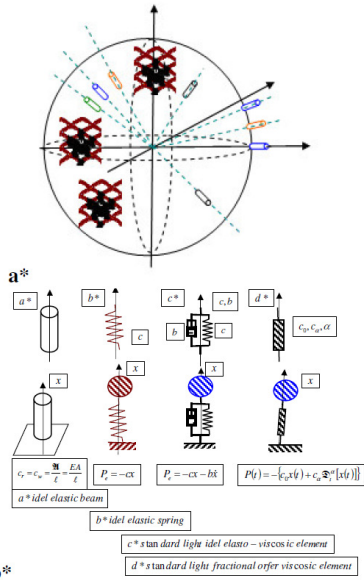
**Figure 11.** Shape and deformations distribution in extreme points of embryo oscillations in mode 2.



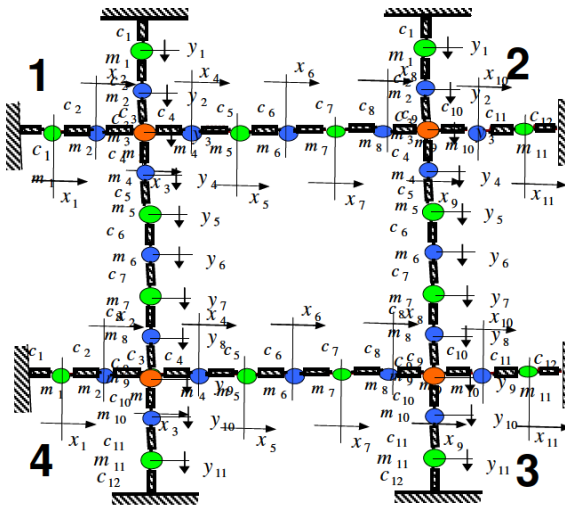
# Analysis of energy state of discrete fractional order spherical net of mouse *zona pellucida* before and after fertilization

Andjelka N. Hedrih and Katica (Stevanović) Hedrih

$$\begin{aligned} \Phi_{\text{Rer.Part}, 0 \leq \alpha < 1} = & \sum_{j=1}^{j=3} \sum_{k=0}^{k=12} \frac{1}{2} c_{0 < \alpha < 1(k, k+1), j} \left\langle \mathfrak{D}_t^\alpha [u_{k+1, j}(t) - u_{k, j}(t)] \right\rangle^2 + \\ & + \sum_{j=2}^{j=4} \sum_{k=0}^{k=12} \frac{1}{2} c_{0 < \alpha < 1(k, k+1), j} \left\langle \mathfrak{D}_t^\alpha [v_{k+1, j}(t) - v_{k, j}(t)] \right\rangle^2 + \\ & + \sum_{k=1}^{k=11} \sum_{j=1}^{j=11} \tilde{c}_{0 < \alpha < 1(k, k), j} \left\langle \mathfrak{D}_t^\alpha [w_{k, k}(t)] \right\rangle^2 \end{aligned} \quad (18)$$



**Figure 1.** a\* Part of the ZP spherical net model on a sphere (oocyte). Orange (ZP1), blue (ZP2) and (ZP3) represents ZP proteins. Net is identical in circular meridian direction. Axis shows directions of movement ZP proteins. Each ZP protein is connected to the sphere standard light visco-elastic elements of fractional (SLFOE) that can oscillate in radial direction. Schematically presentation of fractional order visco-element from part of Zona Pelucida to the model.



$$\frac{d}{dt} \left[ \frac{\partial E_{\text{Kin, Re pr.}}}{\partial \langle \dot{u}_{k, j}(t) \rangle} \right] - \frac{\partial E_{\text{Kin, Re pr.}}}{\partial \langle u_{k, j}(t) \rangle} + \frac{\partial E_{p, \text{Re pr. Part}, \alpha=0}}{\partial \langle u_{k, j}(t) \rangle} + \frac{\partial \Phi_{\text{Rer.Part}, 0 \leq \alpha \leq 1}}{\partial \langle \mathfrak{D}_t^\alpha [u_{k, j}(t)] \rangle} = 0$$

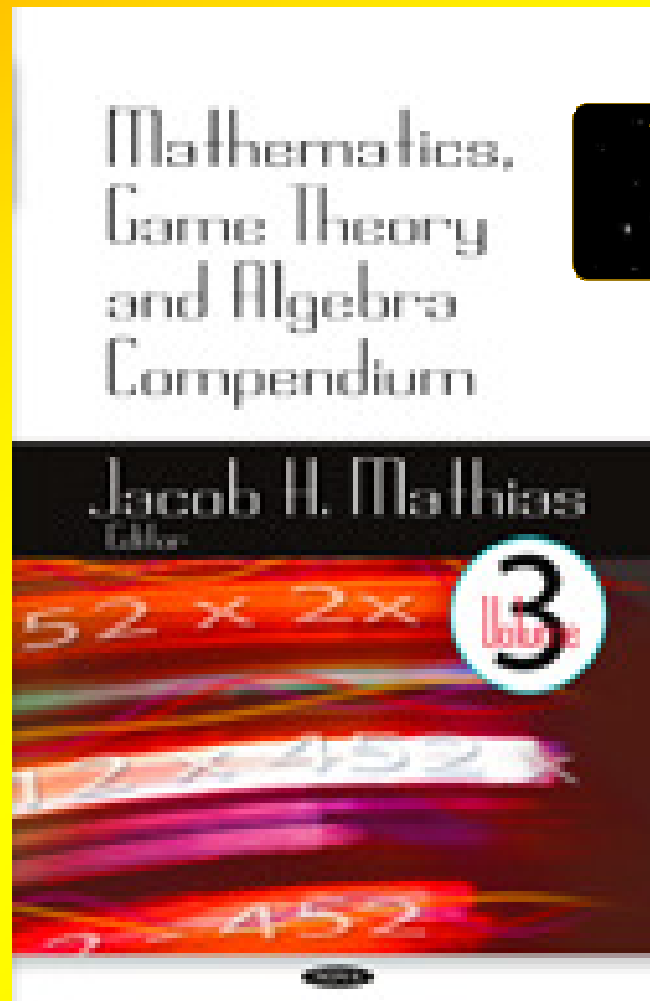
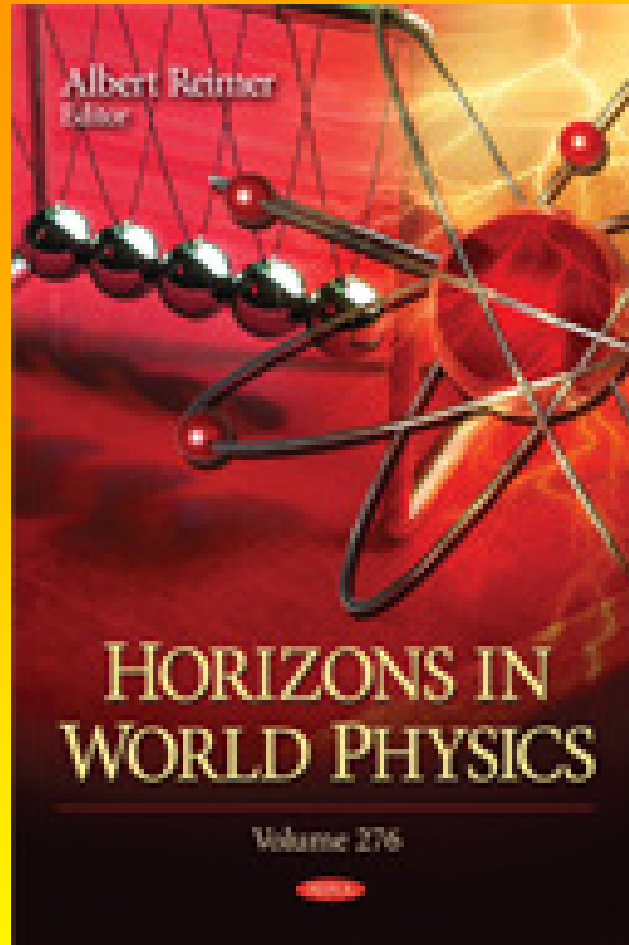
$k = 1, 2, 3, \dots, 11, j = 1, 3, 0 \leq \alpha \leq 1$  (23)

$$\frac{d}{dt} \left[ \frac{\partial E_{\text{Kin, Re pr.}}}{\partial \langle \dot{v}_{k, j}(t) \rangle} \right] - \frac{\partial E_{\text{Kin, Re pr.}}}{\partial \langle v_{k, j}(t) \rangle} + \frac{\partial E_{p, \text{Re pr. Part}, \alpha=0}}{\partial \langle v_{k, j}(t) \rangle} + \frac{\partial \Phi_{\text{Rer.Part}, 0 \leq \alpha \leq 1}}{\partial \langle \mathfrak{D}_t^\alpha [v_{k, j}(t)] \rangle} = 0$$

$k = 1, 2, 3, \dots, 11, j = 2, 4, 0 \leq \alpha \leq 1$  (24)

$$\frac{d}{dt} \left[ \frac{\partial E_{\text{Kin, Re pr.}}}{\partial \langle \dot{w}_{k, j}(t) \rangle} \right] - \frac{\partial E_{\text{Kin, Re pr.}}}{\partial \langle w_{k, j}(t) \rangle} + \frac{\partial E_{p, \text{Re pr. Part}, \alpha=0}}{\partial \langle w_{k, j}(t) \rangle} + \frac{\partial \Phi_{\text{Rer.Part}, 0 \leq \alpha \leq 1}}{\partial \langle \mathfrak{D}_t^\alpha [w_{k, j}(t)] \rangle} = 0$$

$k, j = 1, 2, 3, \dots, 11, k \neq 3, k \neq 9, 0 \leq \alpha \leq 1$  (25)



***Katica (Stevanović) Hedrih and Tijana Ivancevic, RIGOROUS KINETIC ANALYSIS OF THE RACKET FLICK-MOTION IN TENNIS FOR GENERATING TOPSPIN AND BACKSPIN, In: International Journal of Mathematics, Game Theory and Algebra , Volume 20, Issue 2, pp. 1–26 © 2011 Nova Science Publishers, Inc., ISSN: 1060-9881***



Mathematics,  
Game Theory  
and Algebra  
Compendium

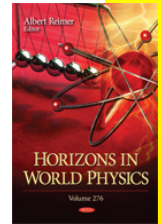
Jacob H. Mathias  
Editor

52 x 2x

3

1000

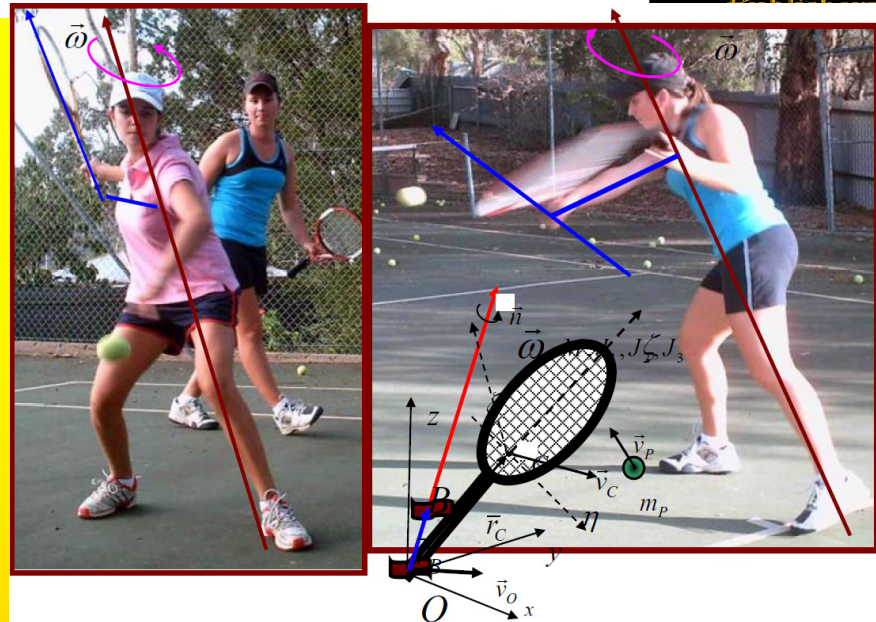
152

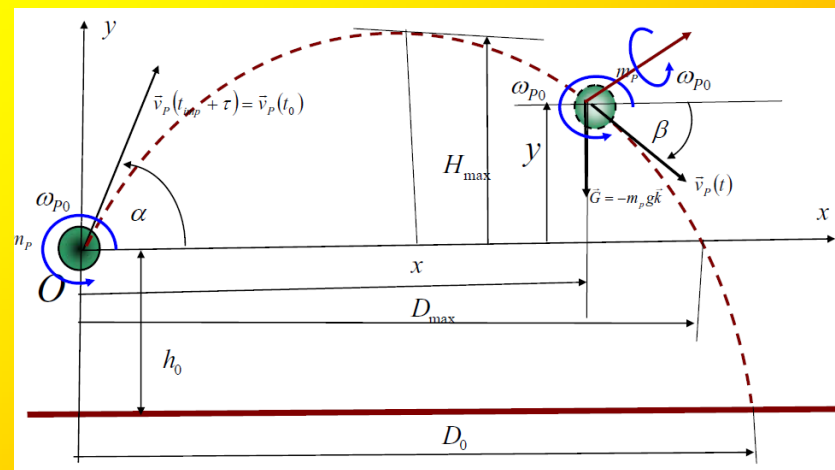
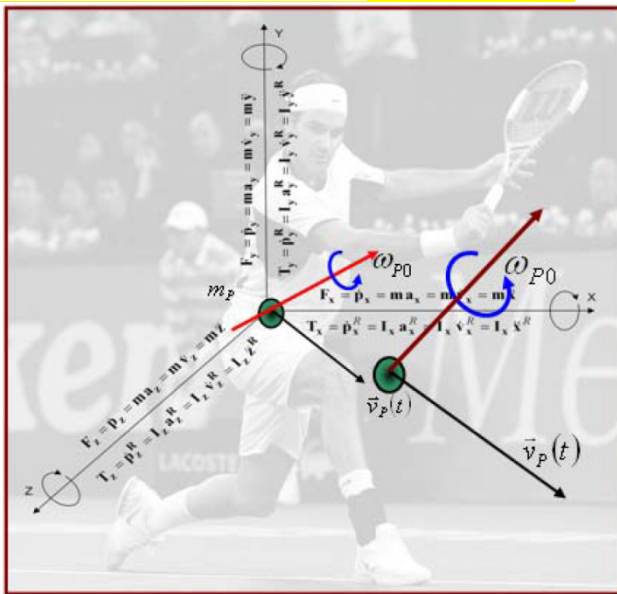
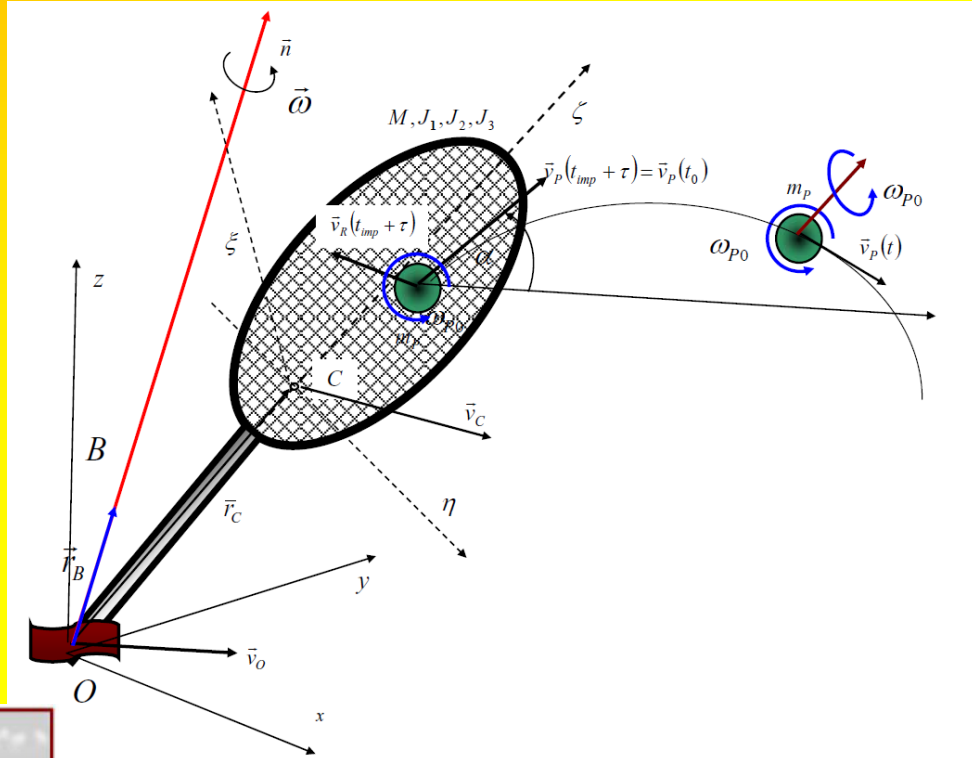
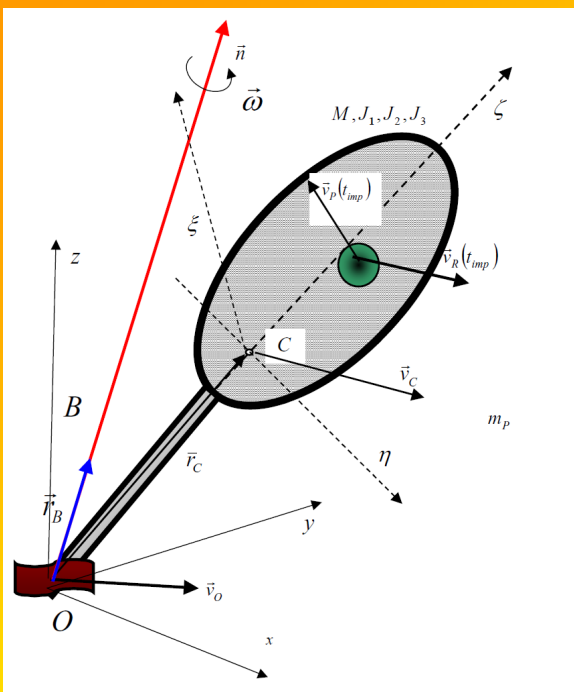


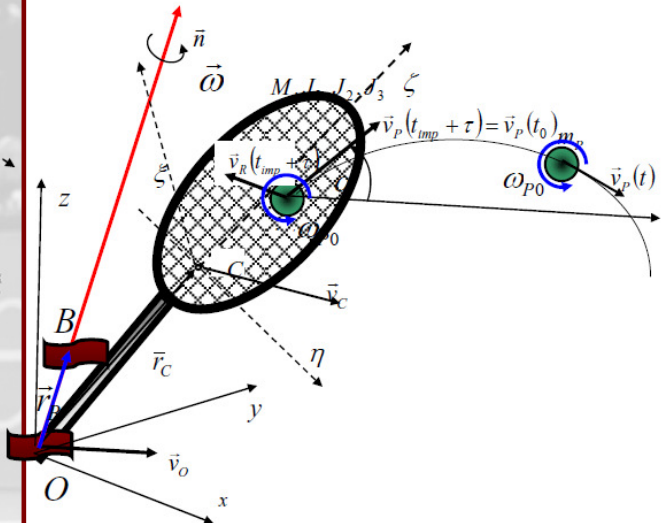
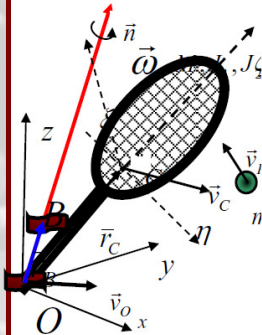
**NOVA**  
Public

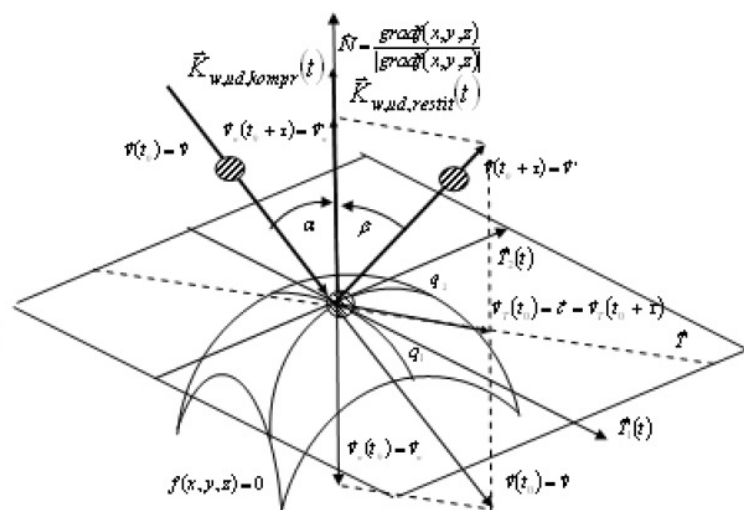
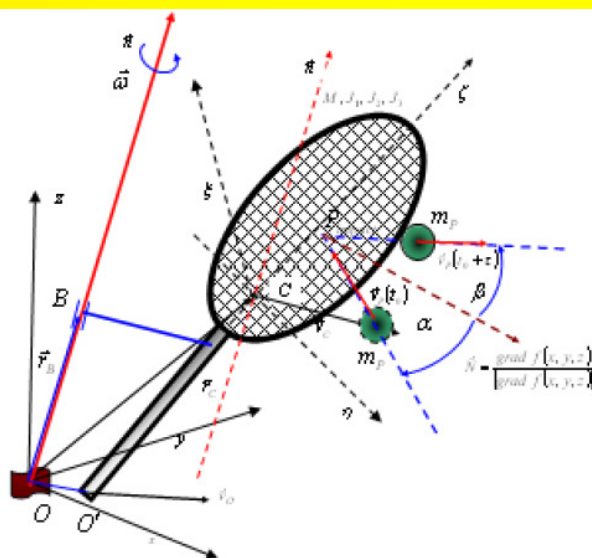
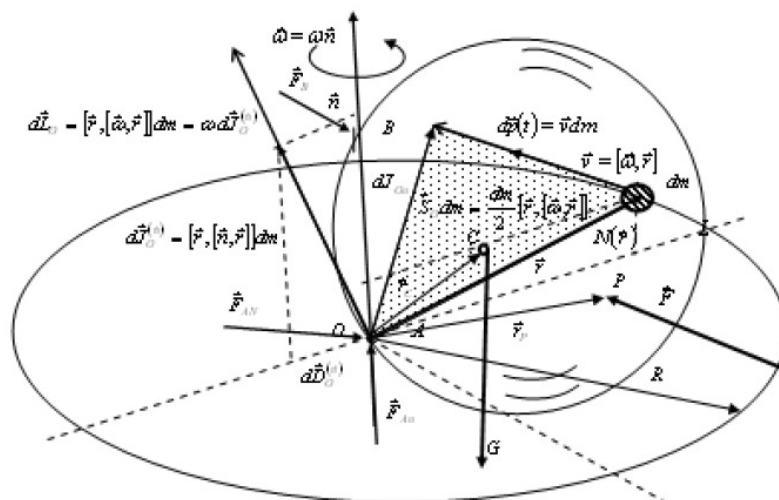
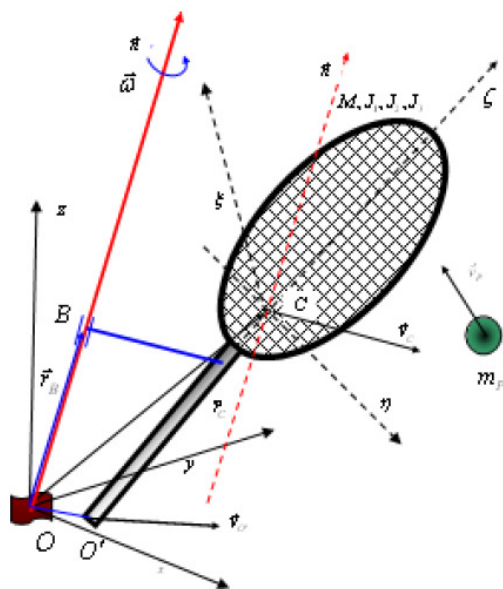
The figure consists of four sub-diagrams arranged in a 2x2 grid, each showing a gray sphere with a black 'W' logo.

- Top Left:** A horizontal arrow labeled "Linear Velocity" points to the right. A curved arrow labeled "Self-Rotation" indicates clockwise rotation around a vertical axis.
- Top Right:** A horizontal arrow labeled "Linear Velocity" points to the right. A curved arrow labeled "Self-Rotation" indicates counter-clockwise rotation around a vertical axis.
- Bottom Left:** A dashed red line passes through the center of the sphere. A blue arrow labeled "Translator velocity of the ball mass center C" points to the right. A red curved arrow indicates rotation around the dashed axis. A black curved arrow indicates self-rotation around a vertical axis.
- Bottom Right:** A dashed red line passes through the center of the sphere. A blue arrow labeled "Translator velocity of the ball mass center C" points to the right. A red curved arrow indicates rotation around the dashed axis. A black curved arrow indicates self-rotation around a vertical axis.





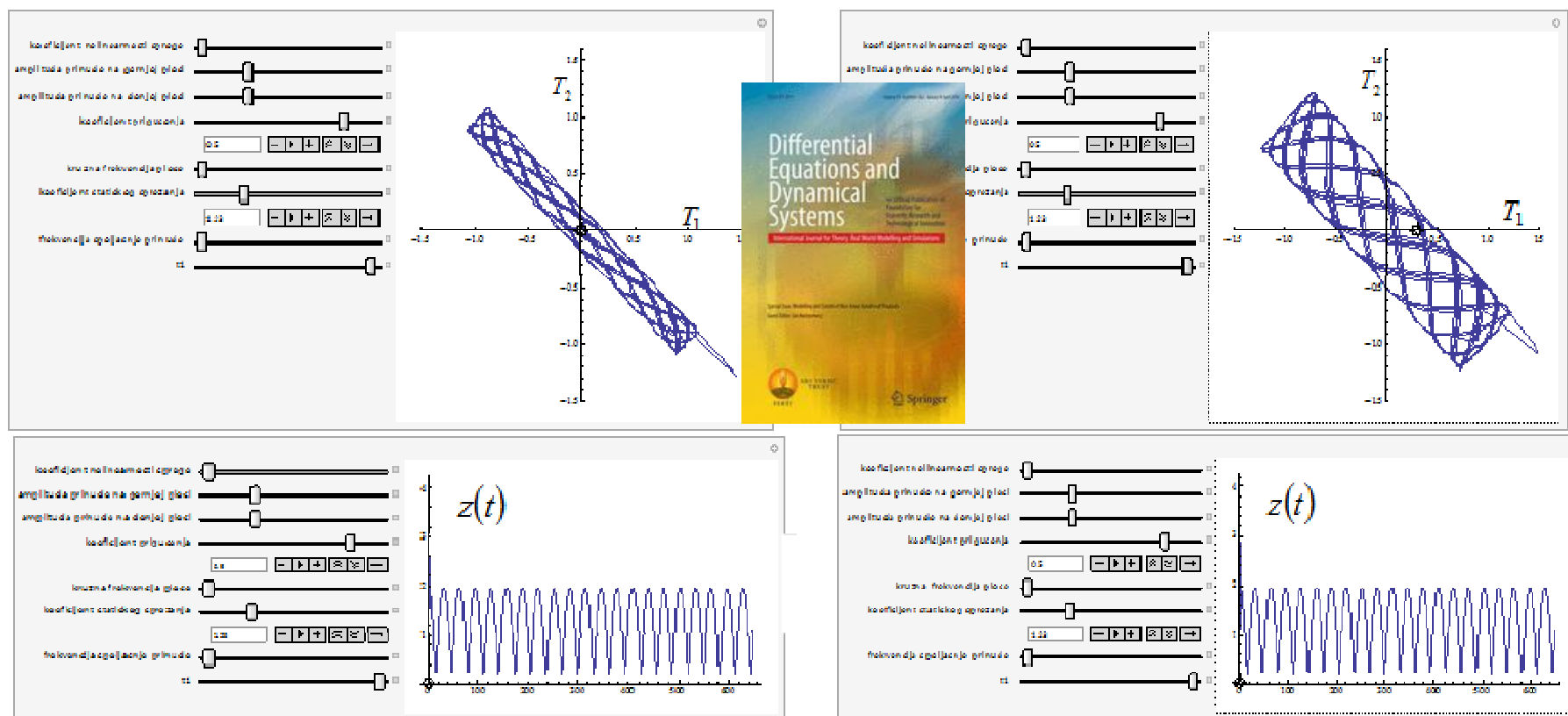








*Simonović J., (2013), Synchronization in Coupled Systems with Different Type of Coupling Elements, Differential Equations and Dynamical Systems, Volume 21, Issue 1 (2013), Pp. 141-148, © Springer 2013.*



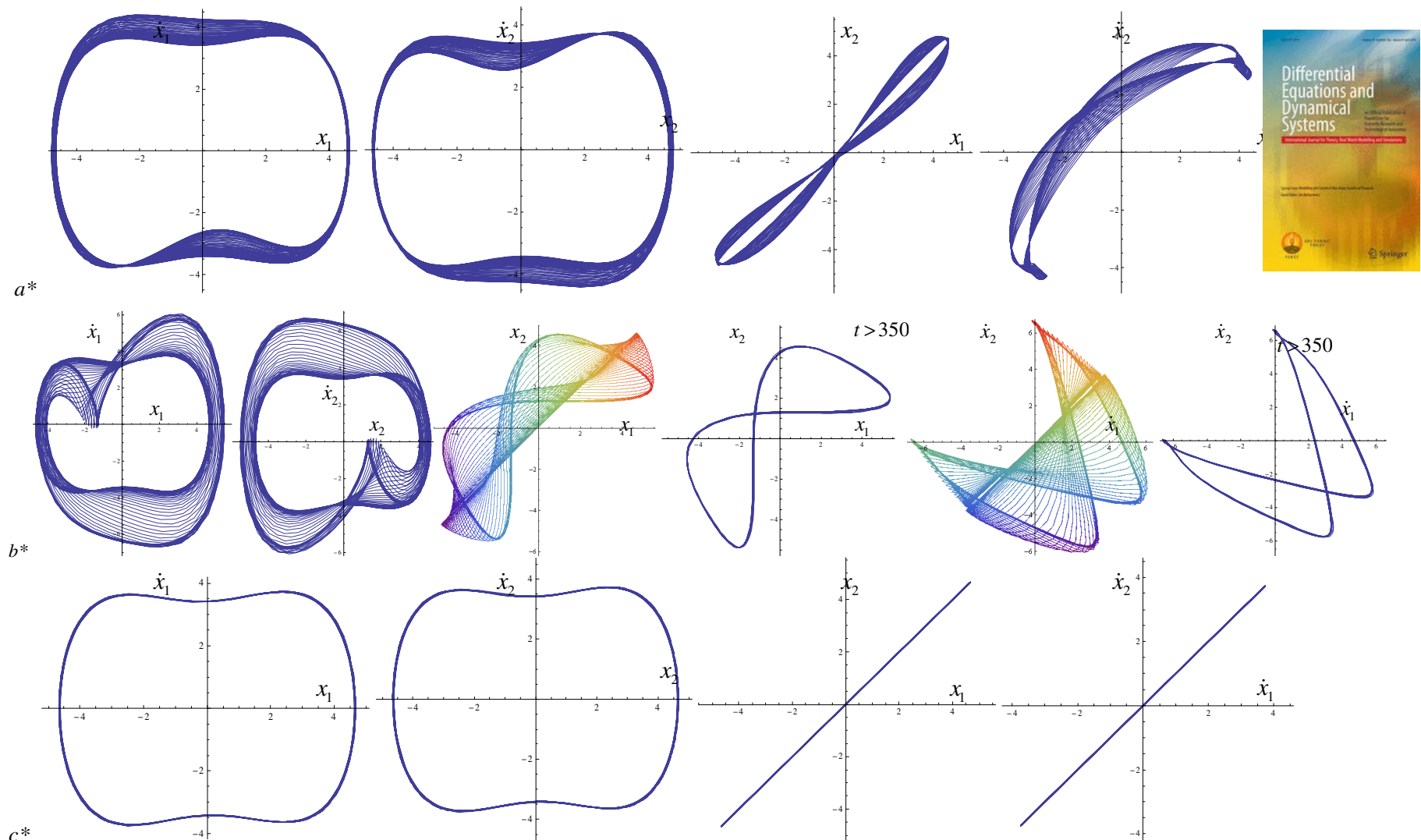
a)

b)

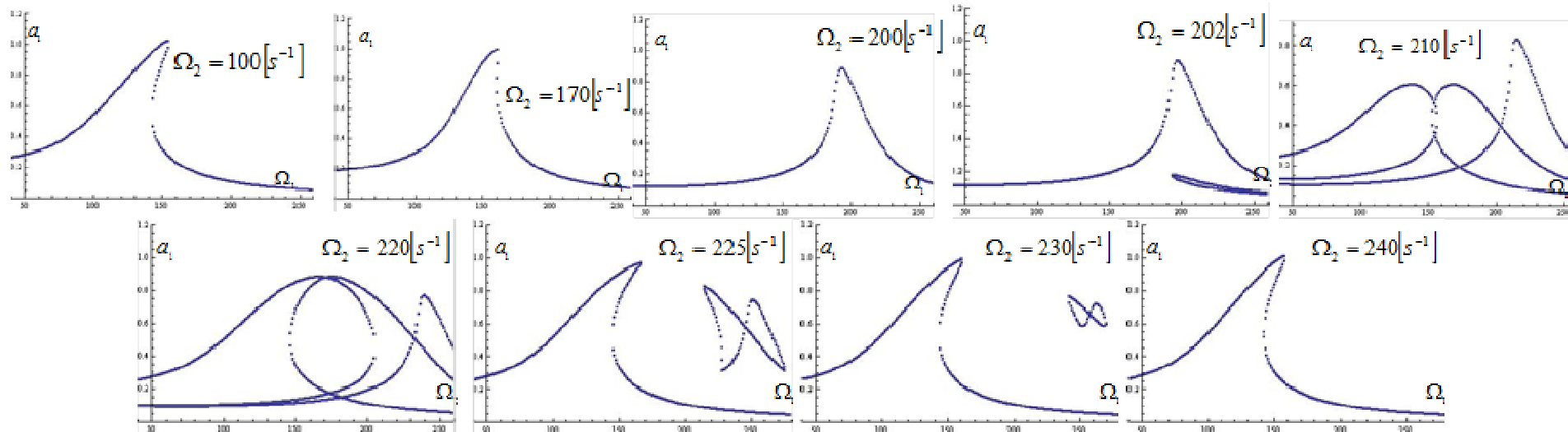
**Karakteristični atraktori asinhronizacije u sistemu vremenskih funkcija oscilovanja dve ploče spregnute slojem visko-elastičnih nelinearnih elemenata sa prinudama istih amplituda za različite početne uslove i  $\alpha_{(t)}^2 = 1.23$  i  $\delta_{(1)} = \delta_{(2)} = 0.25$  a)**

$$T_1(t) = 0; T_2(t) = 0.2; \dot{T}_1(t) = \dot{T}_2(t) = 0; \text{ b) } T_1(t) = 0.3; T_2(t) = 0.2; \dot{T}_1(t) = \dot{T}_2(t) = 0$$

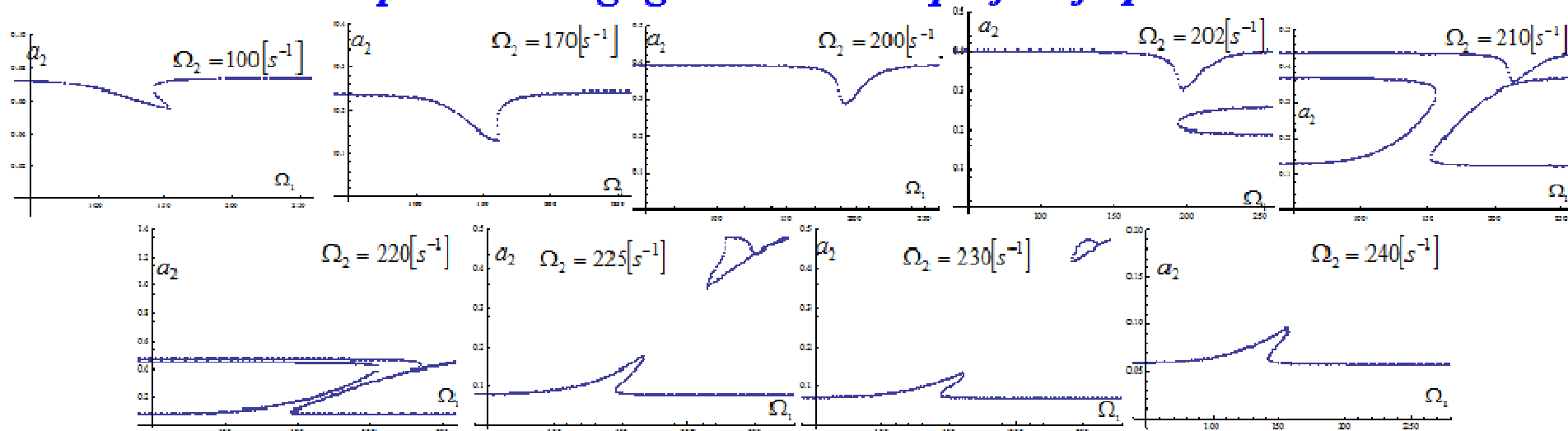




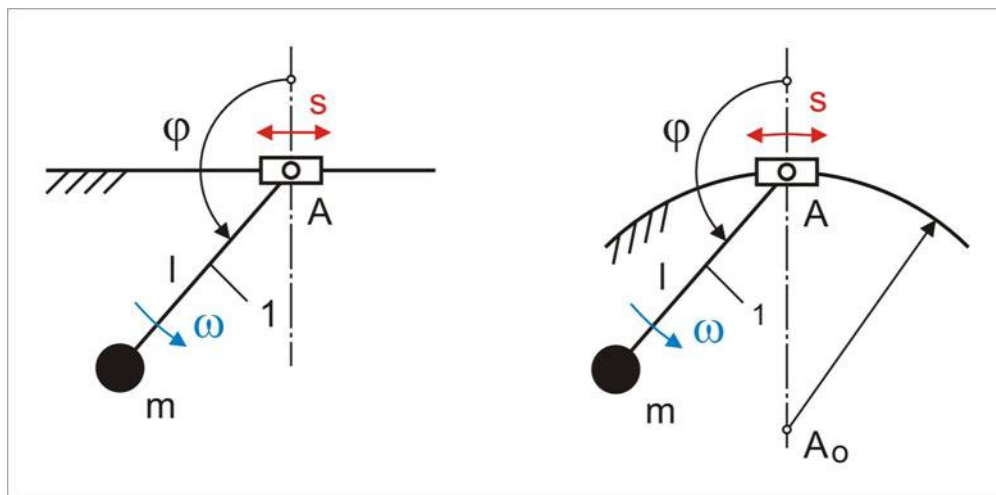
*Fazni dijagrami hibridnih nelinearnih podsistema spregnutih statičkim vezama sa spoljašnjom pobudom.  $a^*$  koeficijent krutosti statičke sprege je  $a_1^2 = 0.6$ ,  $b^* a_1^2 = 0.8$  i  $c^* a_1^2 = 0.87$  a u sva tri slučaja početni uslovi su isti  $x_1(0) = 2.49, \dot{x}_1(0) = -0.2$  i  $x_2(0) = 2.5, \dot{x}_2(0) = -0.2$*



**Amplitudno-frekventna kriva prvog harmonika  $a_{1nm} = f_1(\Omega_{1nm})$  za fiksirane vrednosti druge kružne frekvencije prinudne sile  $\Omega_{2nm}$ , i povećana vrednost amplitude drugog harmonika spoljašnje pobude**



**Amplitudno-frekventna kriva drugog harmonika  $a_{2nm} = f_2(\Omega_{1nm})$**

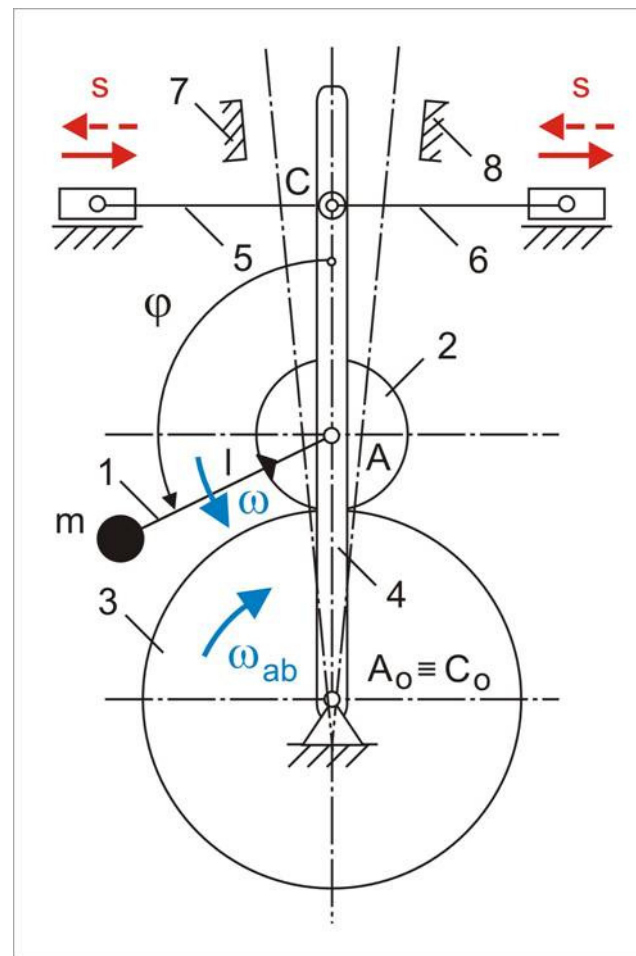


**Tomislav Petrovic:** Na slici 1. su dati analizirani modeli fizičkog **klatna sa pokretnim osloncem** . Klatno dužine  $l$  i mase  $m$  osciluje pod dejstvom sile u pokretnoj tački oslonca **A** .

Kretanje tačke **A** može biti pravolinijsko ili kružno .

Analizirajući mogućnosti praktične primene povoljniji je slučaj oscilovanja klatna čiji se pokretni oslonac ( tačka **A** ) kreće po kružnoj putanji oko nepokretne tačke  $A_o$ . Saglasno tome koncipirana je struktura i način funkcionisanja novog mehanizma za transformaciju oscilatornog u jednosmerno kružno kretanje slika 2.

U dosadašnjem radu su pored napred rečenog koncipirana nova konstrukciona rešenja ovog mehanizma koja bi u praksi mogla imati primenu.



Ostvarene teorijske rezultate nažalost nisam u mogućnosti da proverim eksperimentalnim putem zbog nedostatka finansijskih sredstava za realizaciju dosta složenog i prilično zahtevnog opitnog stola i skupe laboratorijske opreme.



АКАДЕМІЯ НАУК ВИЩОЇ ШКОЛИ УКРАЇНИ (АН ВШ України)

Р/р 26003875 в ВАТ "Кредитпромбанк" м. Києва, МФО 300863

Код ЗКПО 00062426. E-mail: [ANVSU@ukr.net](mailto:ANVSU@ukr.net)

01135, м. Київ, вул. Ісаакяна, 18; тел. 236-02-39; факс 236-01-28

# Phase Trajectory Portrait of the Vibro-impact Forced Dynamics of Two Heavy Mass Particles Motions along Rough Circle



**Katica R. (Stevanović) Hedrih**  
**Mathematical Institute SANU Belgrade**

ul. Vojvode Tankosića 3/22, 18000 Niš, Serbia

E-mail: [khedrih@eunet.rs](mailto:khedrih@eunet.rs)

**Vladimir Raičević and Srdjan Jović**

*\*Faculty of Technical Sciences, Kosovska Mitrovica, University of Priština*

*38 220 Kosovska Mitrovica, Ul. Kralja Petra I br. 149/12, Serbia,*

*e-mail: [jovic003@yahoo.com](mailto:jovic003@yahoo.com)*



**Serbian Scientific Society**



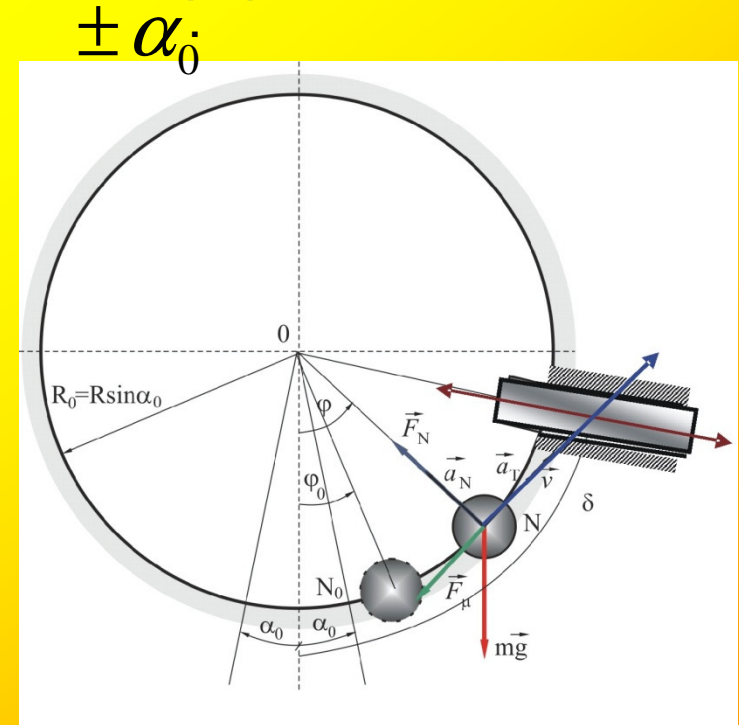
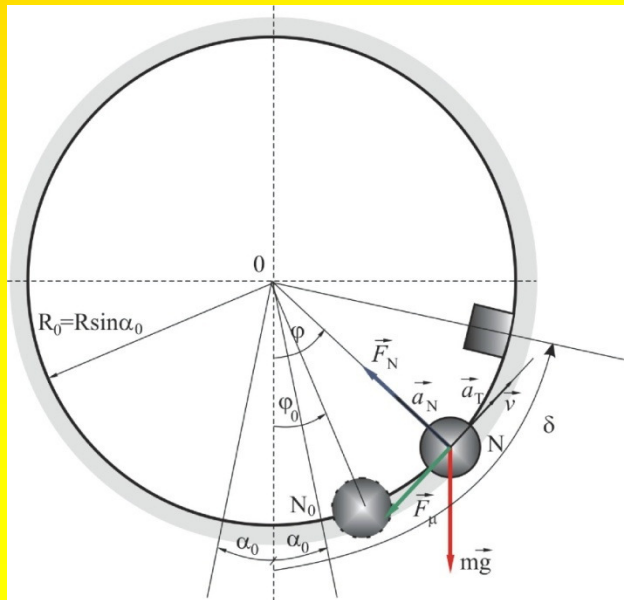


$$\varphi(0) = \varphi_0 \quad \dot{\varphi}(0) = \dot{\varphi}_0$$

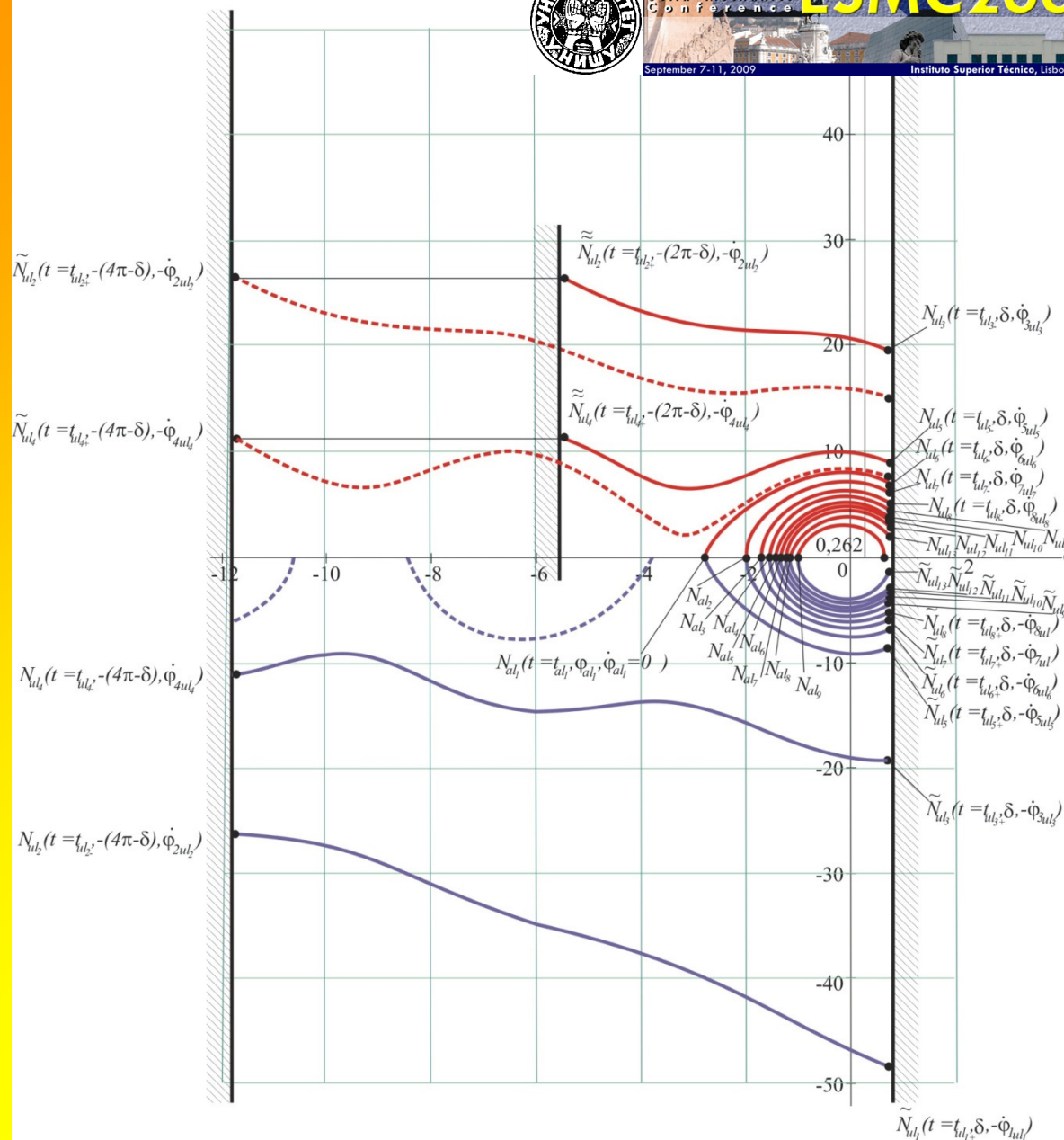
$$\varphi_0 < \delta$$

**Figure 8.** Heavy material particle oscillations along rough circle with moving limiter in radial direction as a limiter with double functions both sides impact limitation of the elongation; Positions of the limiter (a\*) „on“ and (b\*) „off“. Generalized coordinate  $\varphi$  and plan of the active and reactive forces; The „double relative“ equilibrium positions with properties of the alternations  $\pm \alpha_0$

$$\sqrt{2 \frac{g}{R} (1 - \cos \delta)} < \dot{\varphi}_0 < \sqrt{4 \frac{g}{R} - 2 \frac{g}{R} (1 - \cos \delta)}$$







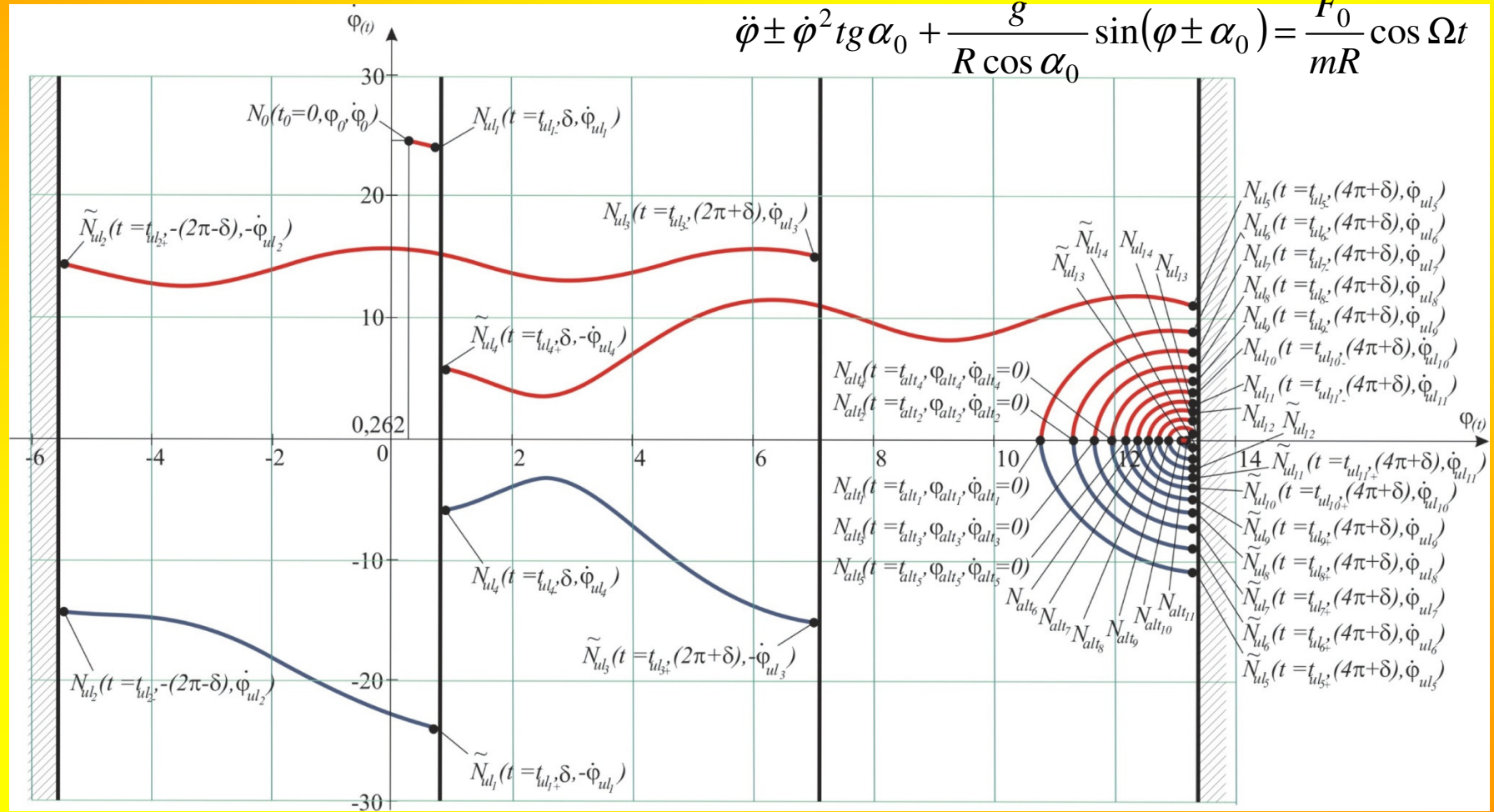
**Figure 9.** Phase trajectory branches  $(\varphi, \dot{\varphi})$ , of the vibroimpact system on the basis of the heavy material particle motions along rough circle with one radially moving limiter and with both side limited angular elongation for the case that coefficient of dry Coulomb's type friction  $\mu = 0,05$ .

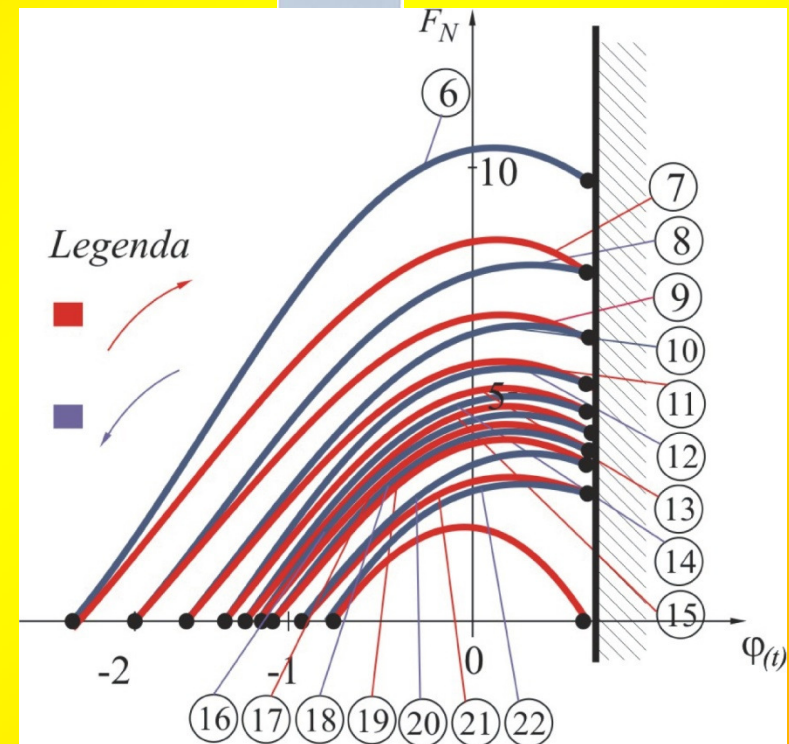
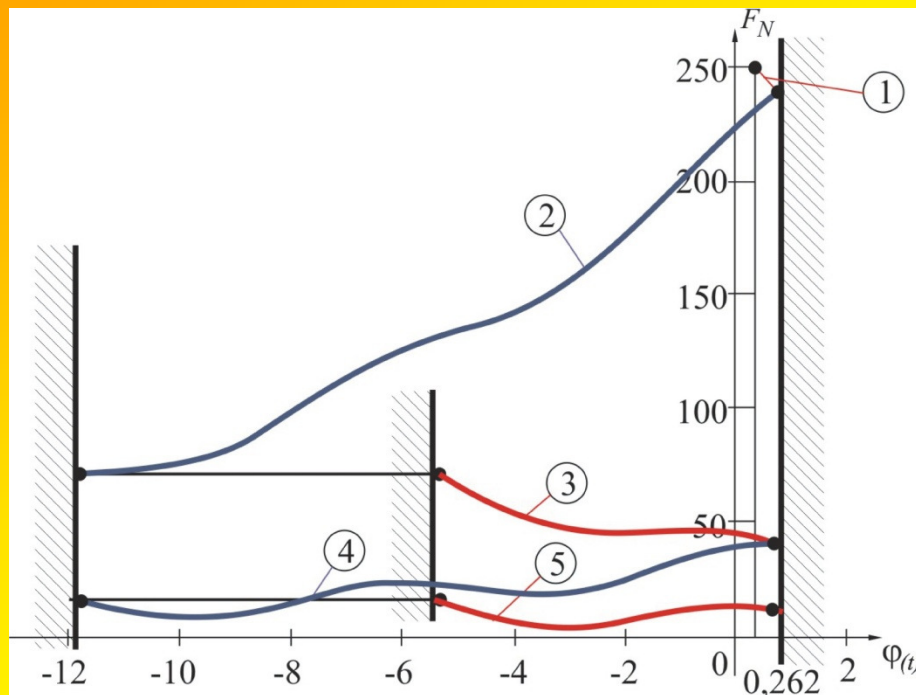


# FORCED VIBROIMPACT SYSTEM DYNAMICS: HEAVY MATERIAL PARTICLE OSCILLATIONS ALONG ROUGH CIRCLE WITH TWO SIDE MOVING IMPACT LIMITERS

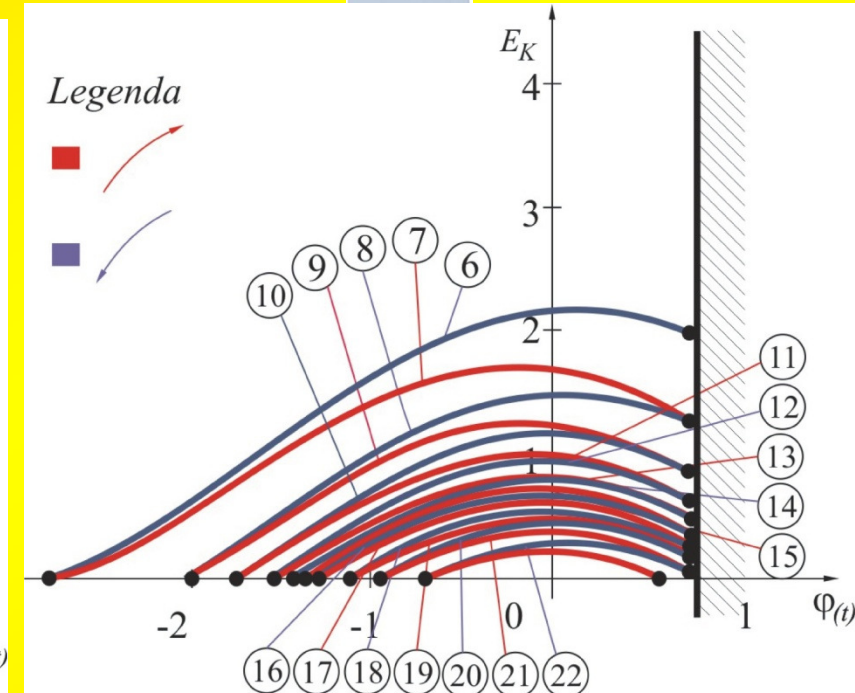
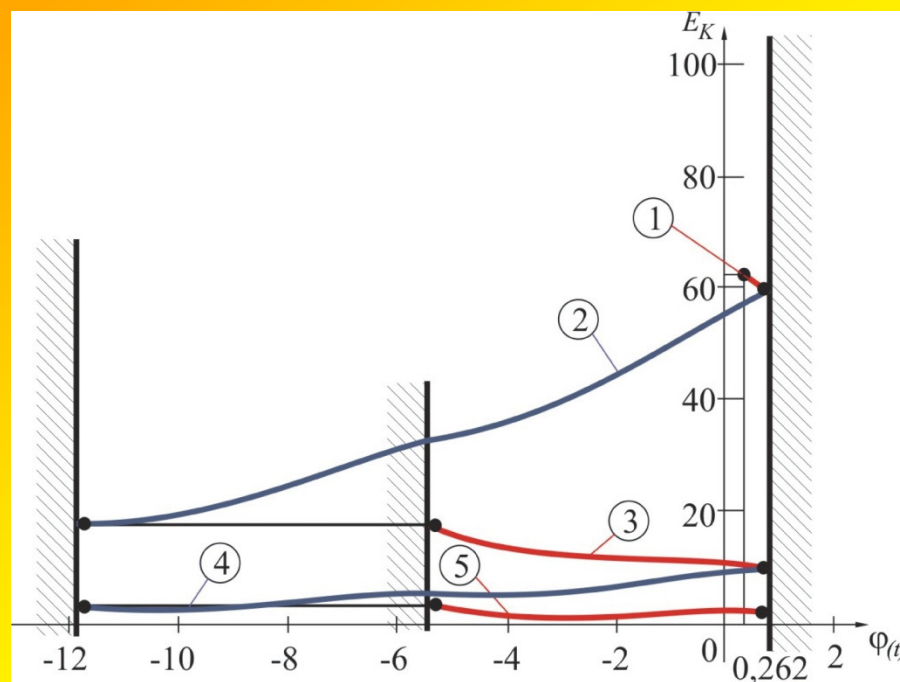
Katica R. (Stevanović) Hedrih\*, Vladimir Raičević\*\*, Srdjan Jovičić\*\*

$$\ddot{\varphi} \pm \dot{\varphi}^2 \operatorname{tg} \alpha_0 + \frac{g}{R \cos \alpha_0} \sin(\varphi \pm \alpha_0) = \frac{F_0}{mR} \cos \Omega t$$



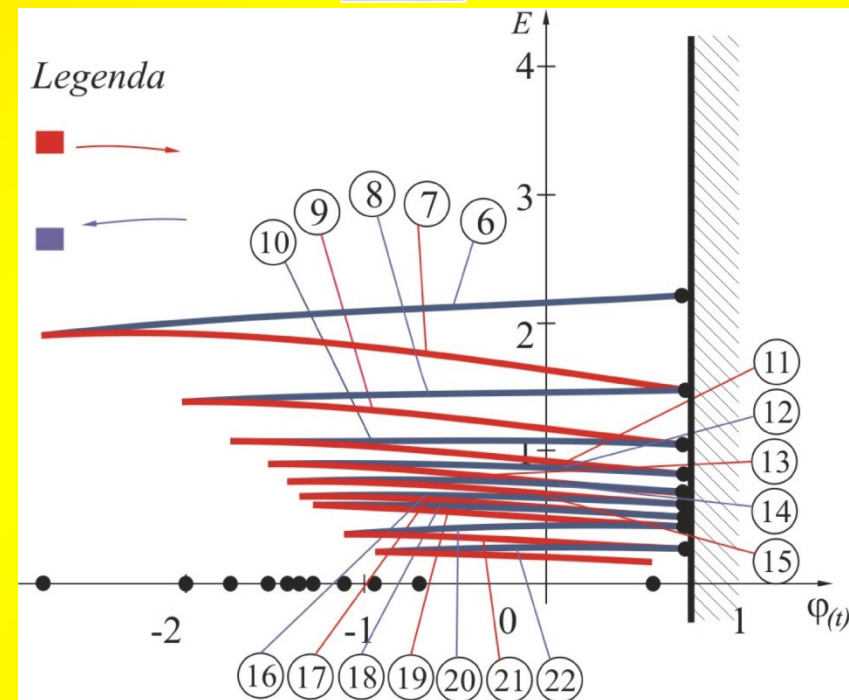
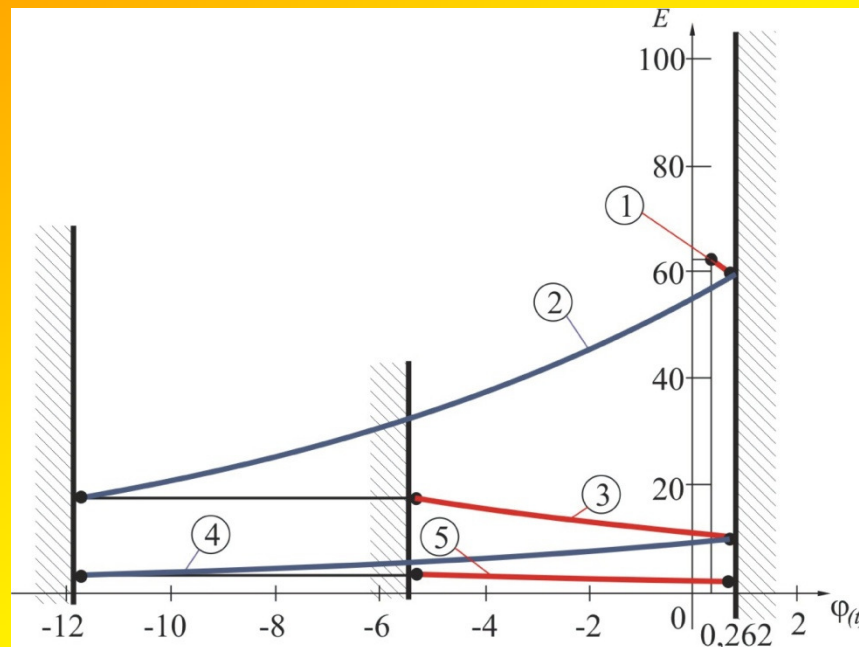


**Figure 10.** Graphical presentation of the constraint normal reaction,  $(F_N, \varphi)$ , or preasure of the material particle on the rogh circle of the vibroimpact system on the basis of the heavy material particle motions along rough circle with one radially moving limiter and with both side limited angular elongation for the case that coefficient of dry Coulomb's type friction  $\mu = 0,05$ .  $b^*$  is detail of the main graphical presentation in  $a^*$ .

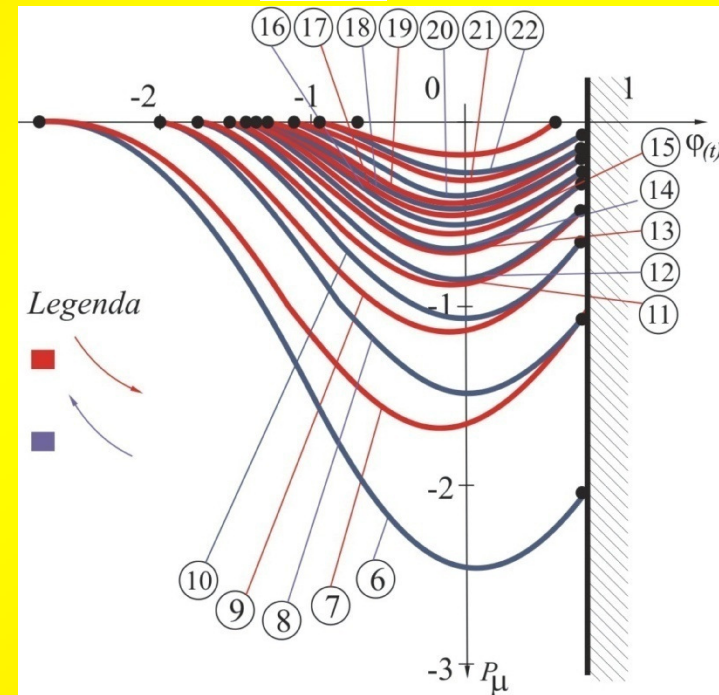
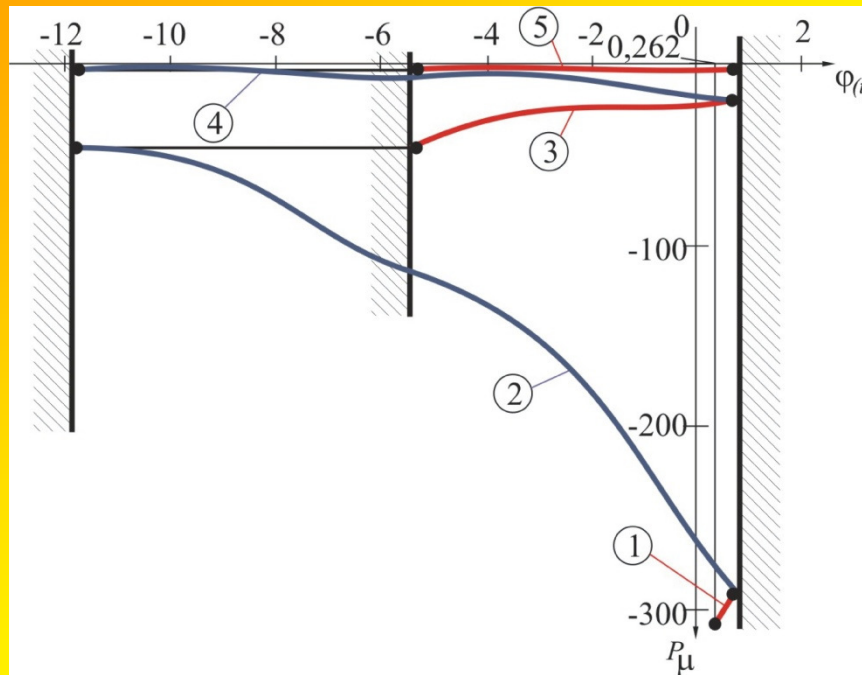


**Figure 11.** Graphical presentation of the kinetic energy  $(E_k, \varphi)$  of the material particle on the rough circle of the vibroimpact system on the basis of the heavy material particle motions along rough circle with one radially moving limiter and with both side limited angular elongation for the case that coefficient of dry Coulomb's type friction  $\mu = 0,05$ .  $b^*$  is detail of the main graphical presentation in  $a^*$ .

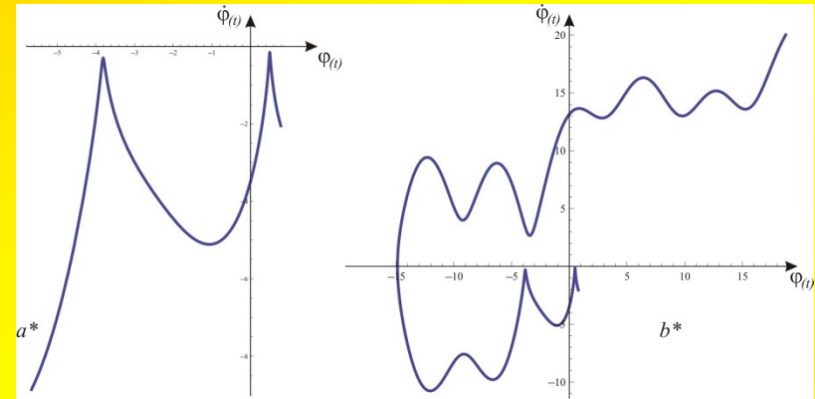
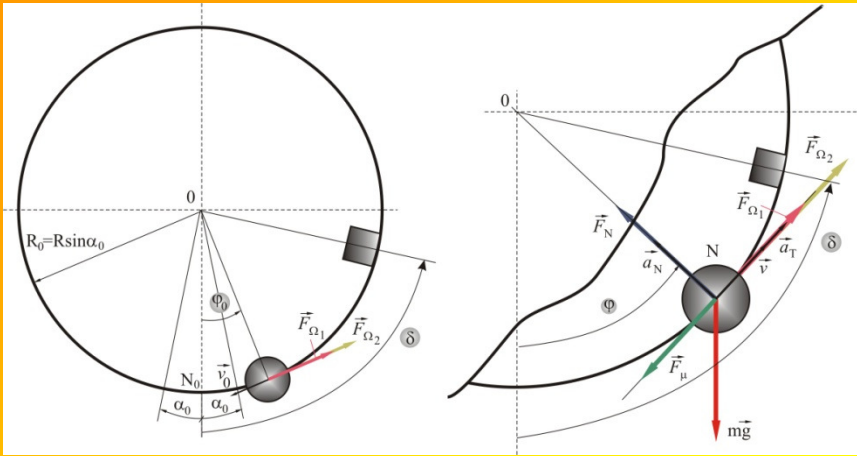




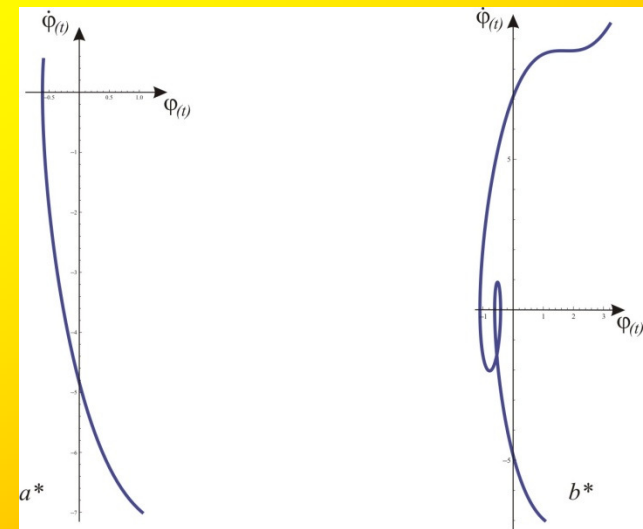
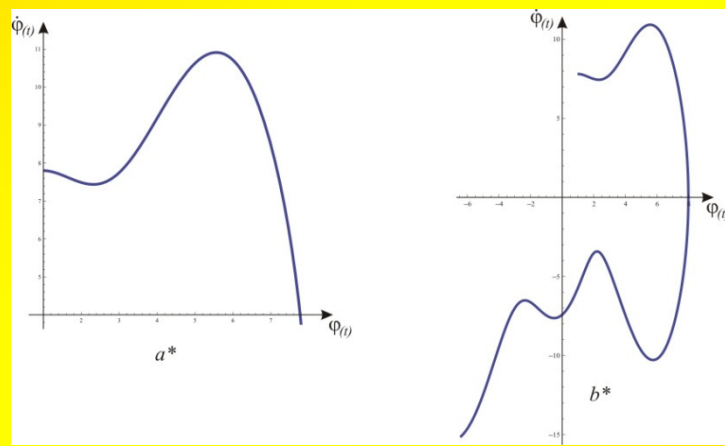
**Figure 13.** Graphical presentation of the system total mechanical energy  $E$ ,  $(E, \varphi)$  of the material particle on the rough circle of the vibroimpact system on the basis of the heavy material particle motions along rough circle with one radially moving limiter and with both side limited angular elongation for the case that coefficient of dry Coulomb's type friction  $\mu = 0,05$ .  $b^*$  is detail of the main graphical presentation in  $a^*$ .



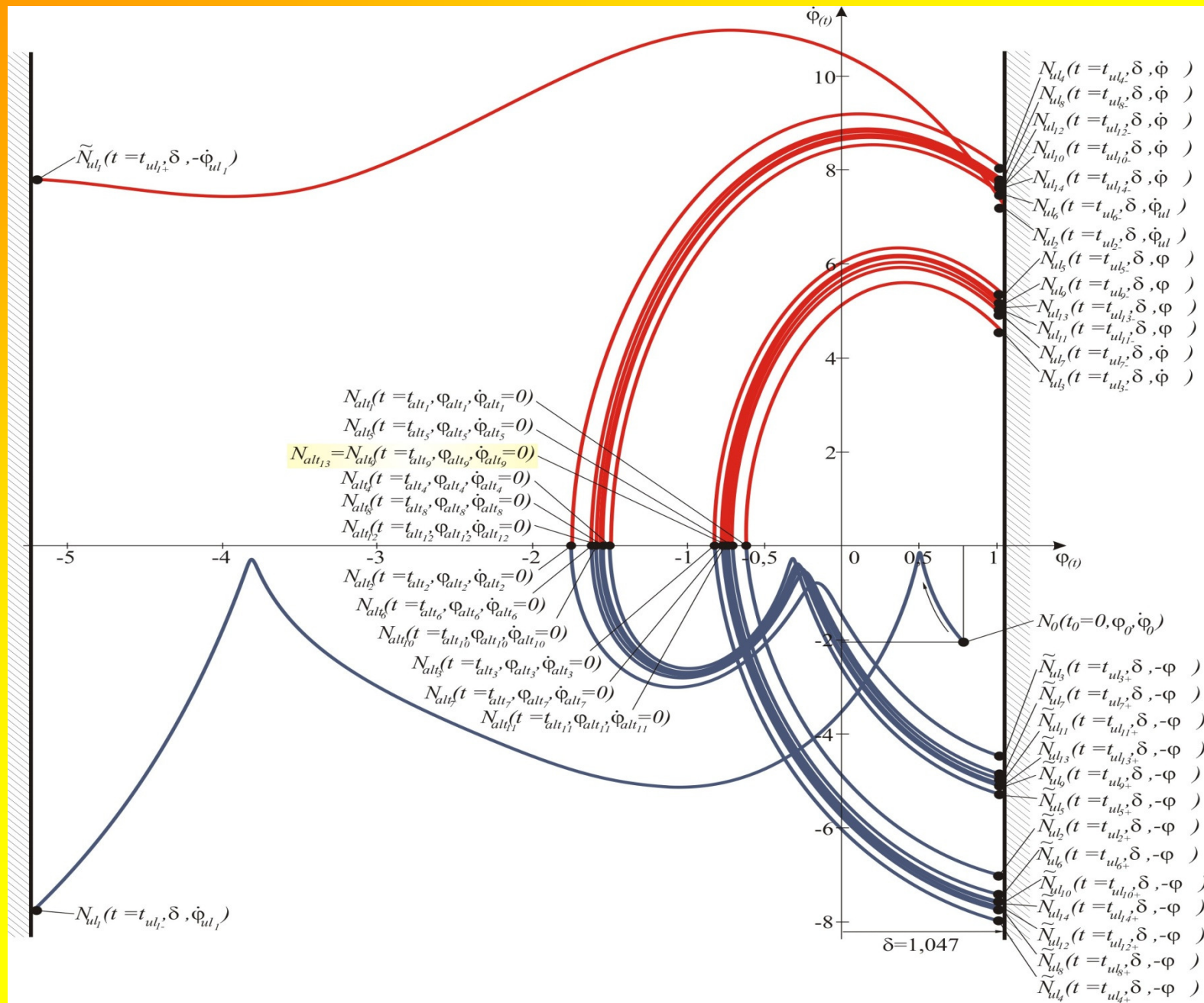
**Figure 14.** Graphical presentation in the plane  $(P_\mu, \varphi)$  power of the friction force of the material particle moyion reaction on the rogh circle of the vibroimpact system on the basis of the heavy material particle motions along rough circle with one radially moving limiter and with both side limited angular elongation for the case that coefficient of dray Coulomb's type friction  $\mu = 0,05$  . b\* is detail of the main graphical presentation in a\*.

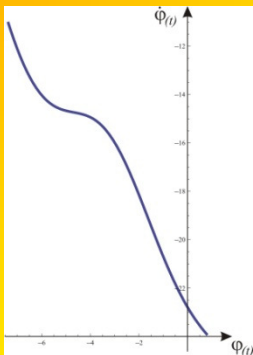
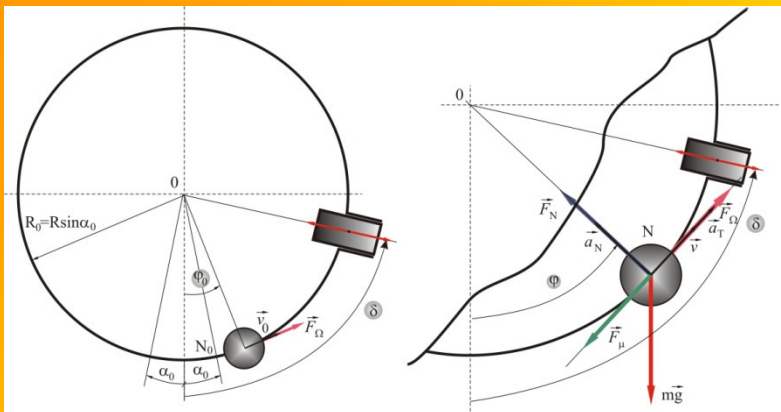


$$\ddot{\phi} \pm \dot{\phi}^2 \operatorname{tg} \alpha_0 + \frac{g}{R \cos \alpha_0} \sin(\phi \pm \alpha_0) = \frac{1}{mR} (F_{10} \cos \Omega_1 t + F_{20} \cos \Omega_2 t)$$

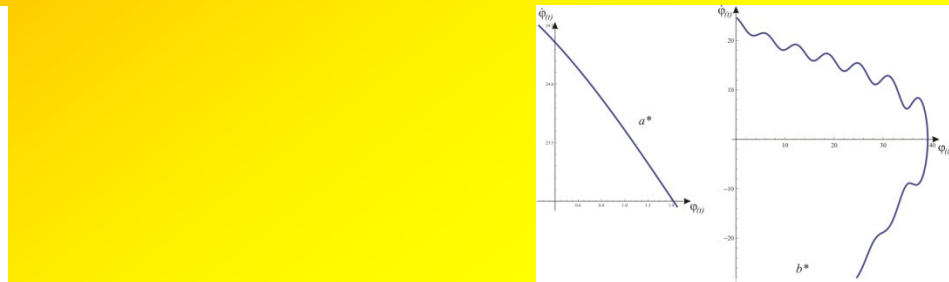
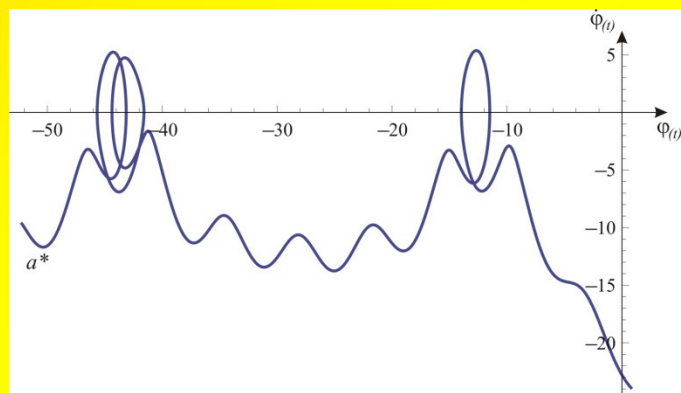




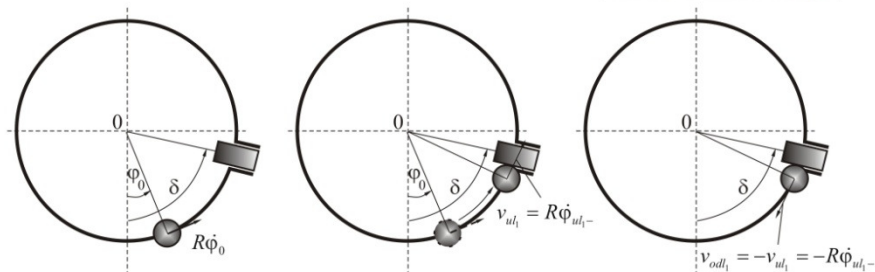




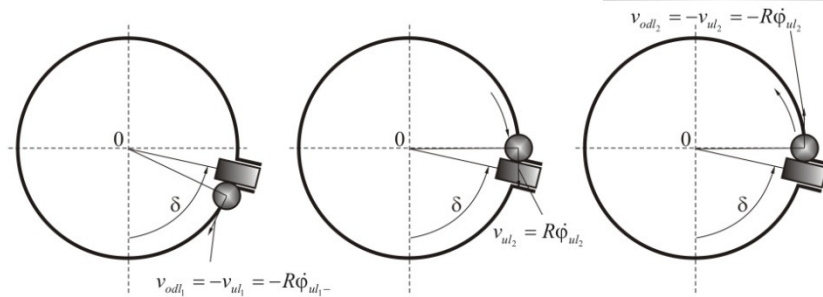
$$\ddot{\phi} \pm \dot{\phi}^2 \tan \alpha_0 + \frac{g}{R \cos \alpha_0} \sin(\phi \pm \alpha_0) = \frac{F_0}{mR} \cos \Omega t$$



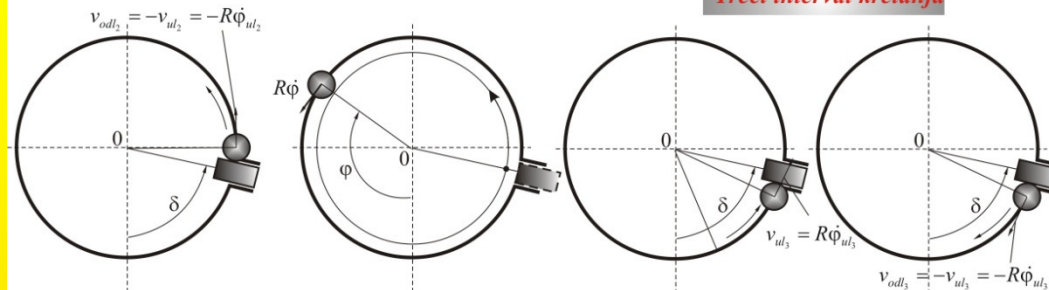
Prvi interval kretanja



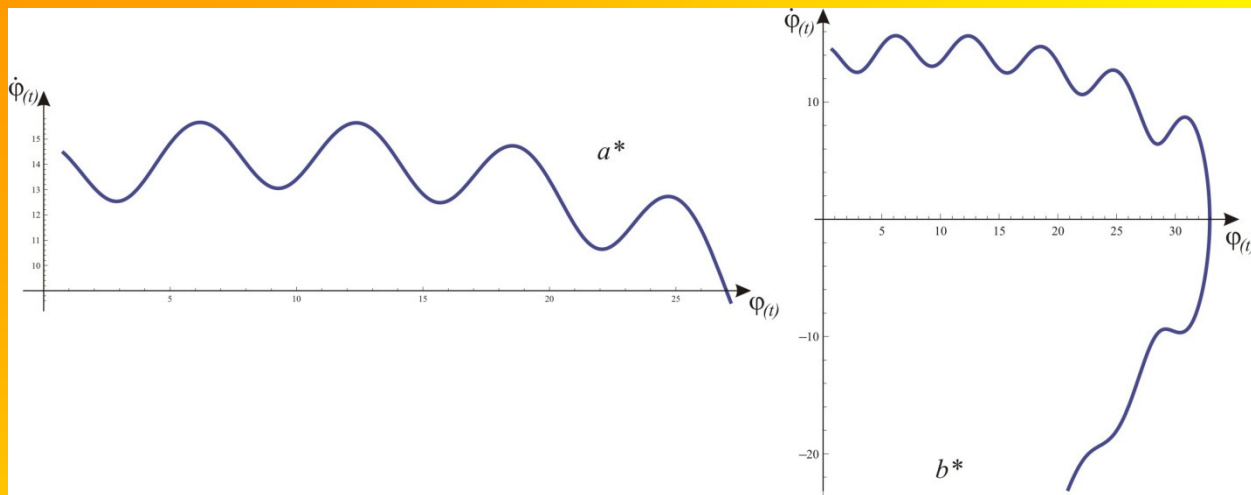
Drugi interval kretanja



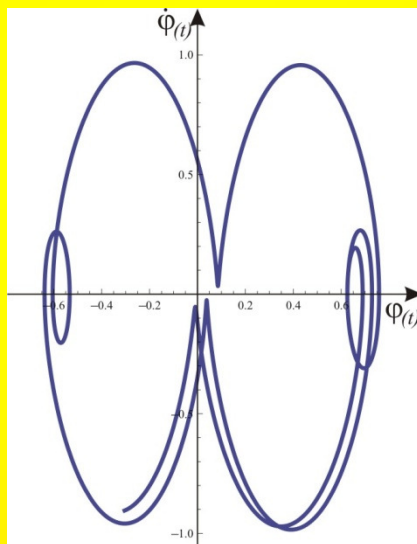
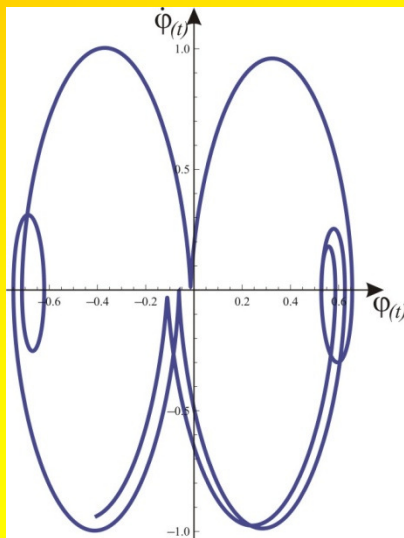
Treći interval kretanja



...



$$\ddot{\phi} \pm \dot{\phi}^2 \operatorname{tg} \alpha_0 + \frac{g}{R \cos \alpha_0} \sin(\phi \pm \alpha_0) = \frac{F_0}{mR} \cos \Omega t$$

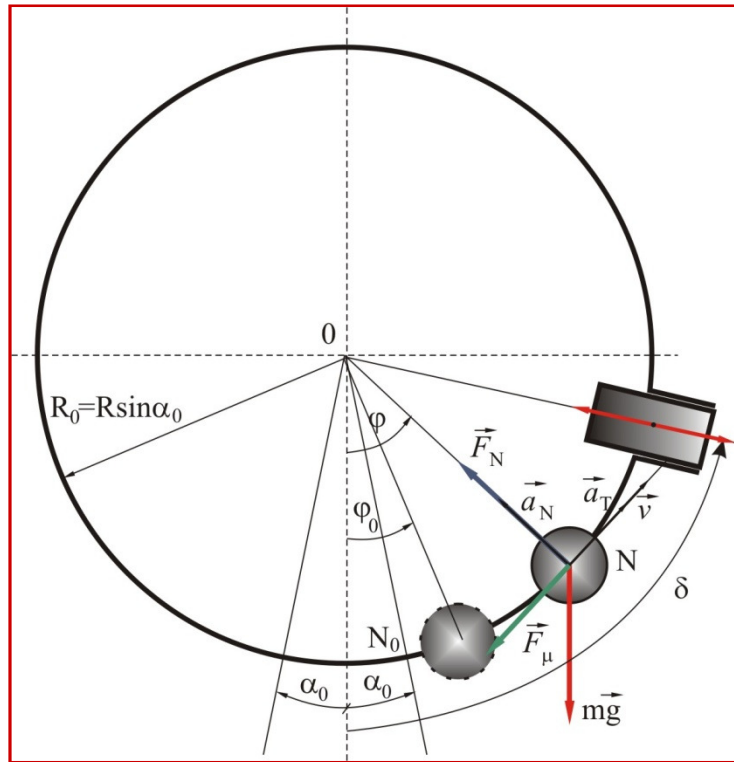


---

*Katica R. (Stevanović) Hedrih and Srdjan V. Jović*

**VIBROIMPACT SYSTEM DYNAMICS:  
HEAVY MATERIAL PARTICLE  
OSCILLATIONS ALONG ROUGH CIRCLE  
WITH ONE SIDE IMPACT LIMIT**

# Introduction



Let us, to use principle of dynamical equilibrium and on the basis of this principle, we can write vector equation of the heavy material particle motion along rough line in the following form:

$$\vec{I}_F + \vec{G} + \vec{F}_N + \vec{F}_\mu = 0$$

***Heavy material particle oscillations along rough circle with one side impact limit elongation***

By use natural coordinate system:

$$\left(-m\dot{v}\vec{T}\right) + \left(-m\frac{v^2}{R_k}\vec{N}\right) + mg\left(-\sin\varphi\vec{T} - \cos\varphi\vec{N}\right) + F_N\vec{N} - \mu|\vec{F}_N|\frac{\vec{v}}{|\vec{v}|} = 0$$

where  $R_k$  is the radius of the path line curvature at the point of the material particle terminate position.  $\vec{T}$  and  $\vec{N}$  are unit vectors of the tangent and normal to the path line at terminate position.



## Governing differential equation of the heavy material particle oscillations along rough circle with one side impact limit elongation

Vector equation contains two scalar equations, and after elimination normal and tangential components of the rough circle line reaction, we obtain a ordinary nonlinear differential equation expressed by generalized coordinate  $\varphi$  in the following form:

$$\ddot{\varphi}(\pm)\varphi^2 \operatorname{tg} \alpha_0 + \frac{g}{R \cos \alpha_0} \sin(\varphi(\pm) \alpha_0) = 0$$

in which upper sign is for  $\dot{\varphi} > 0$  and lower sign for  $\dot{\varphi} < 0$ , according alternations of the friction force alternations, correspond to the opposite direction of the material particle motion. For complete conditions of the material particle motion it is necessary to add **initial conditions**:  $\dot{\varphi}(0) = \dot{\varphi}_0$  and  $\varphi(0) = \varphi_0$ , as well as **one side impact limit condition**:  $\varphi(t_{ul.i\mp}) = \delta$

$\dot{\varphi}(t_{ul.i-}) = \dot{\varphi}_{ul.i-}$  and  $\dot{\varphi}(t_{ul.i+}) = -k \dot{\varphi}_{ul.i-}$ ,  $i = 1, 2, 3, \dots, n$ , where  $n$  is total number of the impacts before appear no impact oscillations or state of the rest of the material particle.

# Phase trajectory equation of the heavy material particle oscillations along rough circle with one side impact limit elongation

By introducing the following  $\dot{\varphi}^2 = u$  a transformation of the nonlinear differential equation give a *first order differential equation with corresponding integral* in the form:

$$[\dot{\varphi}(\varphi)]^2 = \frac{2g}{(1 + 4tg^2\alpha_0)R \cos \alpha_0} [\cos(\varphi \pm \alpha_0) - 2tg\alpha_0 \sin(\varphi \pm \alpha_0)] + Ce^{\mp 2\varphi g\alpha_0}$$

where  $C$  is integral constant depending of initial conditions for the corresponding interval of the material particle motion and in which upper sign is for  $\dot{\varphi} > 0$  and lower sign for  $\dot{\varphi} < 0$ , according alternations of the friction force alternations. This previous equation is equation of the phase trajectory in the phase plane  $(\varphi, \dot{\varphi})$  containing representative phase point

$N(t, \varphi, \dot{\varphi})$  presenting kinetic state of the material particle oscillations. For first phase trajectory branch, from initial kinetic state to the first impact moment, we use equation with upper sign, for the case  $\dot{\varphi} > 0$ :

$$\dot{\varphi}_1^2(\varphi) = \frac{2g}{(1 + 4tg^2\alpha_0)R \cos \alpha_0} [\cos(\varphi + \alpha_0) - 2tg\alpha_0 \sin(\varphi + \alpha_0)] + C_1(\varphi_0, \dot{\varphi}_0)e^{2\varphi g\alpha_0}$$

# ***Phase trajectory equation of the heavy material particle oscillations along rough circle with one side impact limit elongation***

where  $C_1(\varphi_0, \dot{\varphi}_0)$  integral constant defined by the following expression:

$$C_1(\varphi_0, \dot{\varphi}_0) = e^{+2\varphi_0 t g \alpha_0} \left\{ \dot{\varphi}_0^2 - \frac{2g}{(1 + 4tg^2 \alpha_0)R \cos \alpha_0} [\cos(\varphi_0 + \alpha_0) - 2tg \alpha_0 \sin(\varphi_0 + \alpha_0)] \right\}$$

First impact appear at the moment:  $t = t_{ul,1-}$ , corresponding to one side impact limit  $\varphi(t_{ul,1-}) = \delta$  and corresponding impact angular velocity:

$$\dot{\varphi}(t_{ul,1-}) = \dot{\varphi}_{ul,1-}$$

$$\dot{\varphi}_{ul,1} = \sqrt{\frac{2g}{(1 + 4tg^2 \alpha_0)R \cos \alpha_0} [\cos(\delta + \alpha_0) - 2tg \alpha_0 \sin(\delta + \alpha_0)] + C_1(\varphi_0, \dot{\varphi}_0) e^{-2\delta g \alpha_0}}$$

at the moment  $t = t_{ul,1-}$  :

$$t_{ul_1} = \int_{\varphi_0}^{\delta} \frac{d\varphi}{\sqrt{\frac{2g}{(1 + 4tg^2 \alpha_0)R \cos \alpha_0} [\cos(\varphi + \alpha_0) - 2tg \alpha_0 \sin(\varphi + \alpha_0)] + C_1(\varphi_0, \dot{\varphi}_0) e^{-2\varphi g \alpha_0}}}$$

## ***Phase trajectory equation of the heavy material particle oscillations along rough circle with one side impact limit elongation***

The normal constraint reaction  $F_N(\varphi)$  in the function of the generalized coordinate  $\varphi$ , as a normal pressure of the heavy material particle to the rough circle line in the first time interval motion, before first impact, and corresponding for the force of friction  $F_\mu(\varphi) = -\mu F_N(\varphi)$  is defined by following expression:

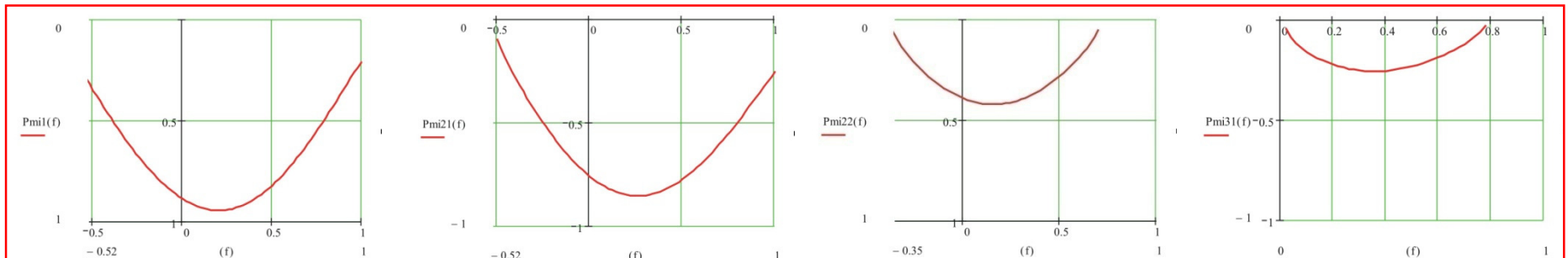
$$F_N = mg \cos \varphi + mR \left( \frac{2g}{(1 + 4tg^2 \alpha_0)R \cos \alpha_0} [\cos(\varphi + \alpha_0) - 2tg \alpha_0 \sin(\varphi + \alpha_0)] + C_1(\varphi_0, \dot{\varphi}_0) e^{-2\varphi g \alpha_0} \right)$$

Second interval of the material particle motion after first impact, and between first and second impacts, we split to the two subinterval. First subinterval is between first impact and first state of the alternation of the friction force, and second subinterval is between first state of the alternation of the friction force, and second impact.

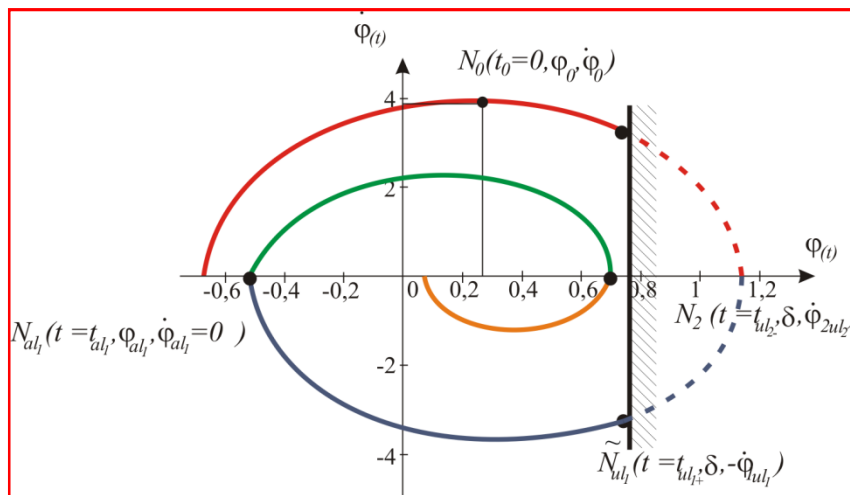
For second phase trajectory branch in the first subinterval, from first impact kinetic state to the first alternation of the direction of the friction force, we use equation with lower sign, for the case  $\dot{\varphi} < 0$  :

# Phase trajectory equation of the heavy material particle oscillations along rough circle with one side impact limit elongation

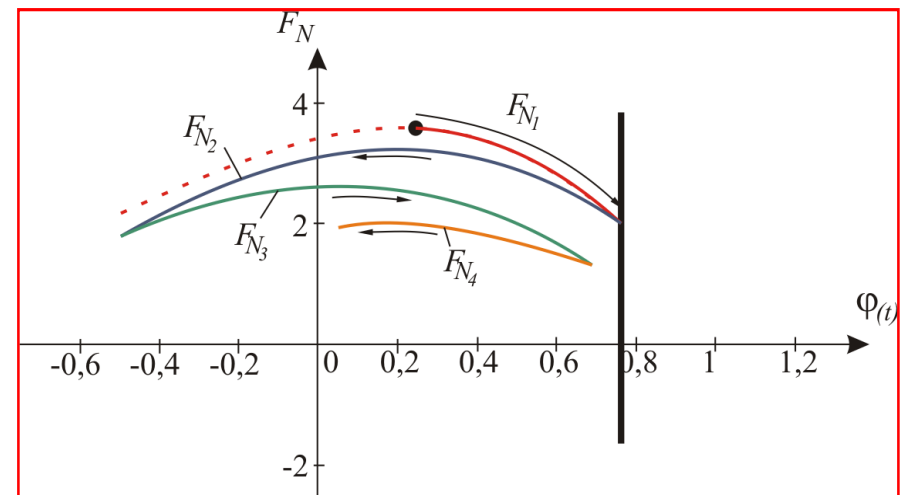
## Graphical presentation



**Power of the work of the friction force between impact and alternations of the friction force directions**



**An example of the phase trajectory**



**The normal constraint reaction  $F_N(\varphi)$**



# Numerical example and visualization

---

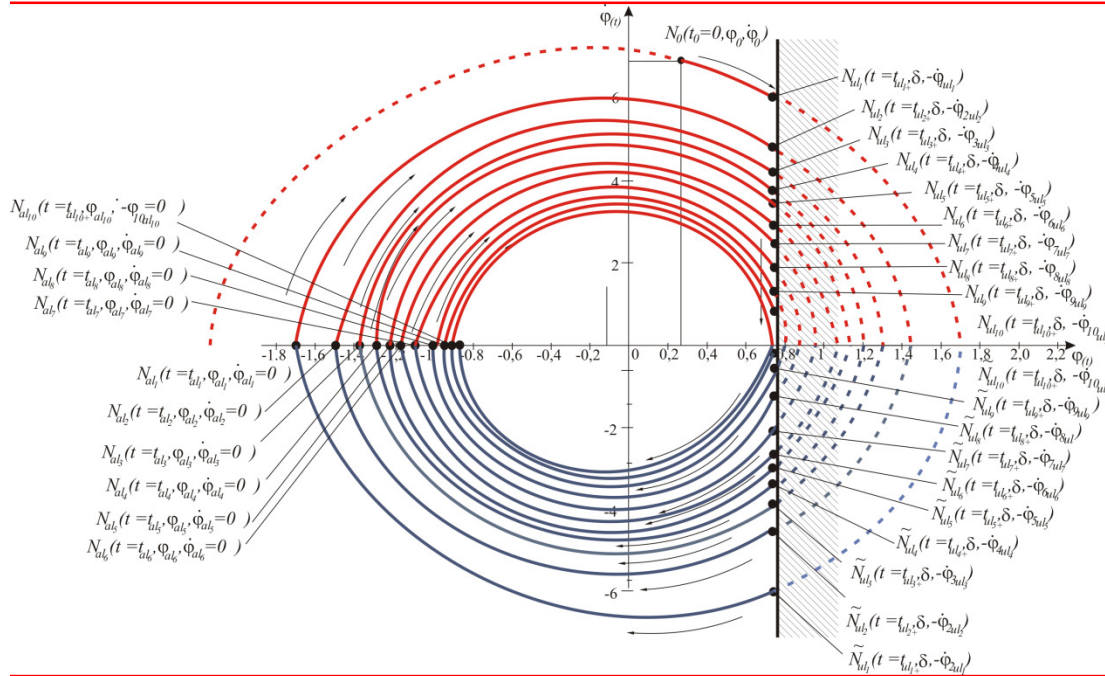
By use *MathCad program* for the geometrical and kinetic parameters of the material particle motion along rough circle with one side impact limit elongation:  $m = 0,2[kg]$ ,  $R = 0,5[m]$ ,  $\mu = tg \alpha_0 = 0.05$  (dimension less),

$$\delta = \frac{\pi}{4}[rad], \varphi_0 = \frac{\pi}{12}[rad], \dot{\varphi}_0 = 7\left[\frac{rad}{s}\right], g = 9,81\left[\frac{m}{s^2}\right], k = 1,$$

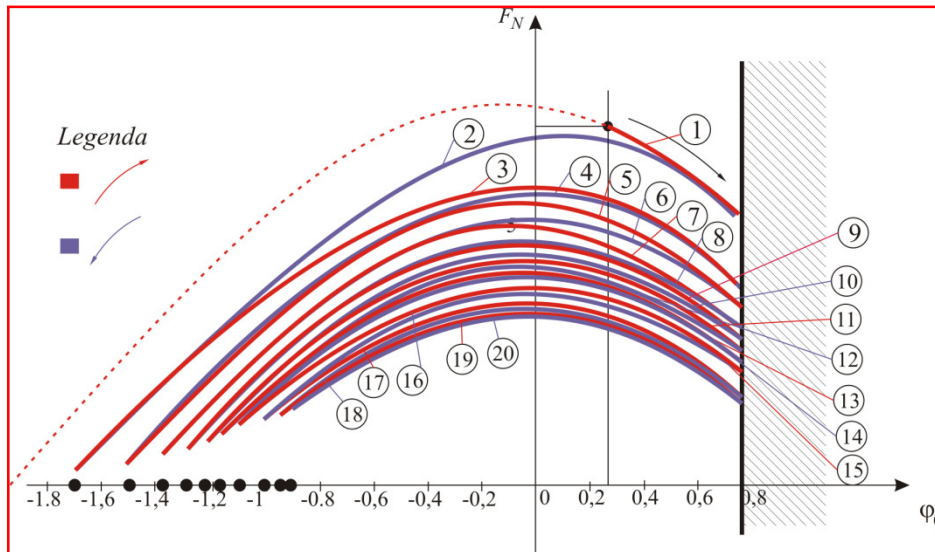
we obtain a phase trajectory presentation of the impact system dynamics with initial conditions determining ten impacts. *Thus phase trajectory is presented in Figure 1.*

After ten impacts osilations of the materijal particle is without impacts. For same example a graphical presentation of the *normal constraint reaction in the functions of tis presented in Figure 2.*

# Numerical example and visualization

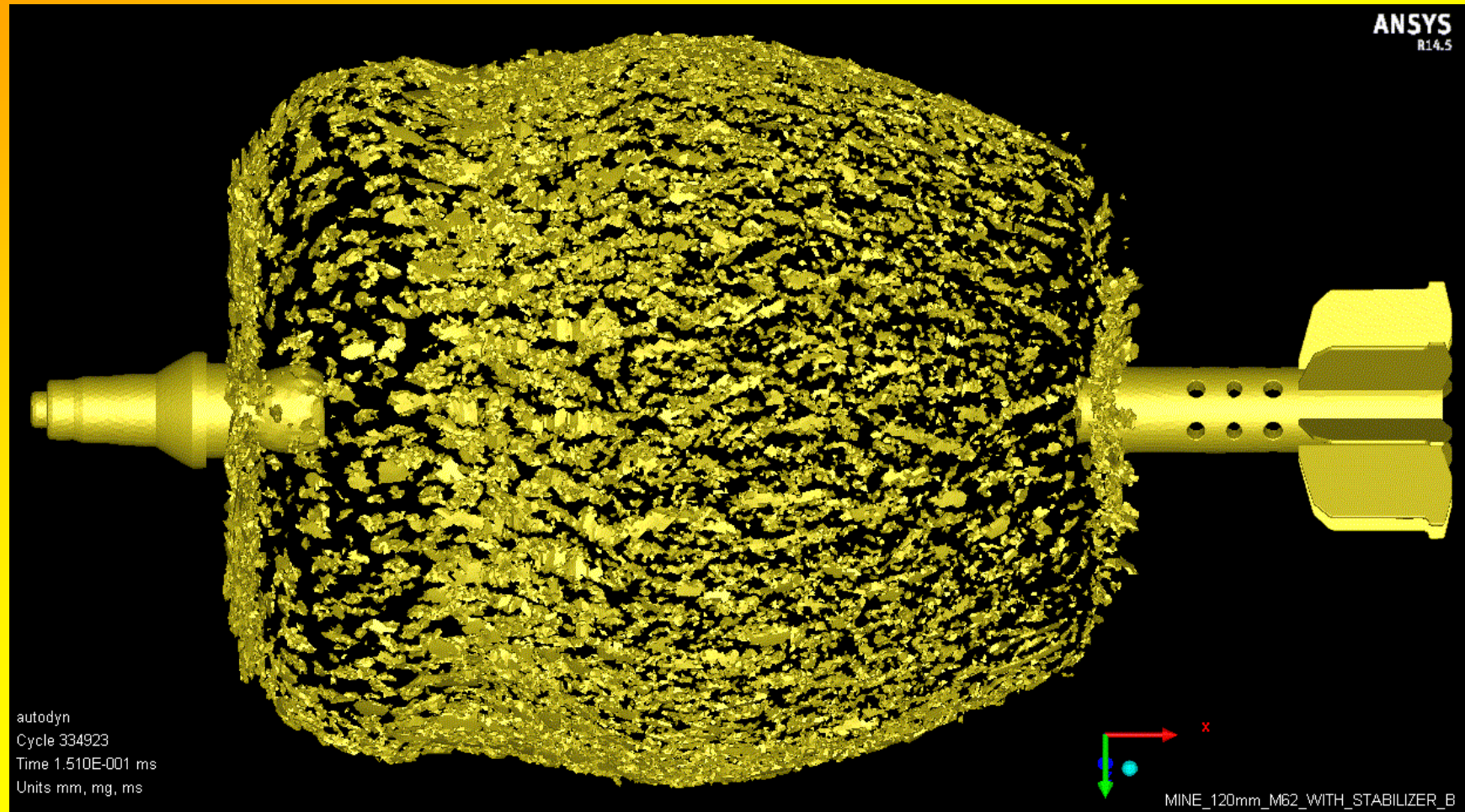


**Figure 1.** An example of the phase trajectory of the vibroimpact system with initial conditions determining ten impacts

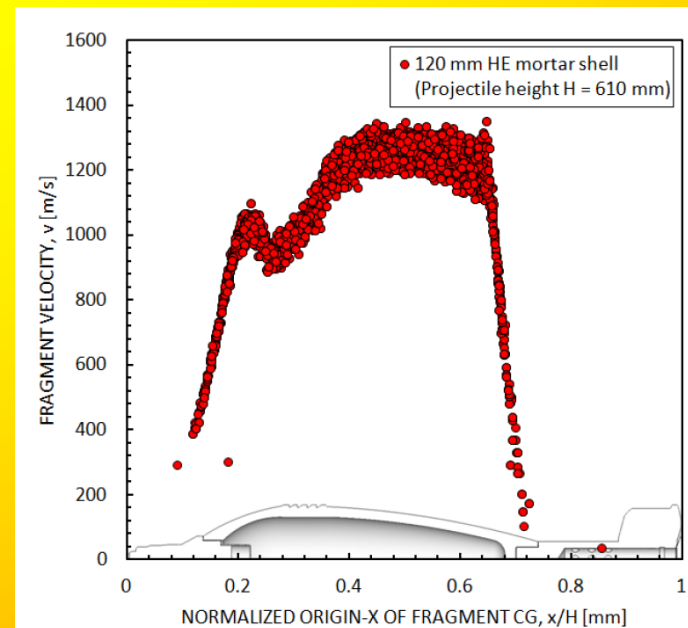
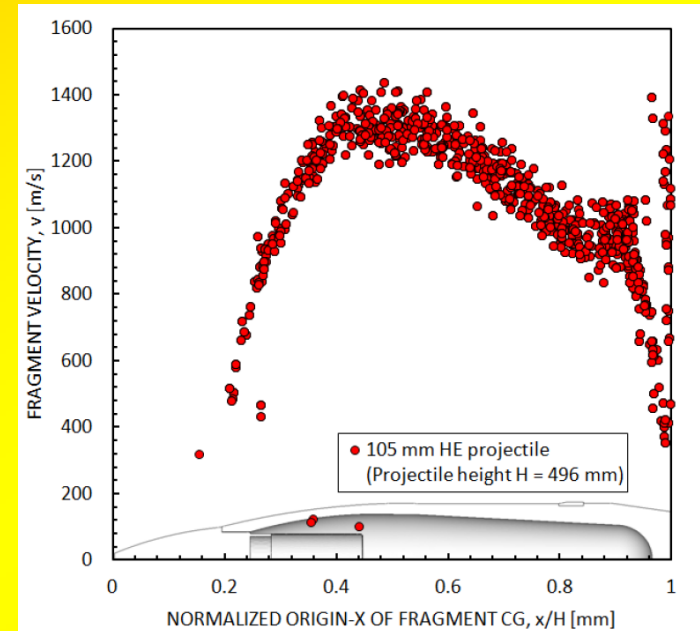
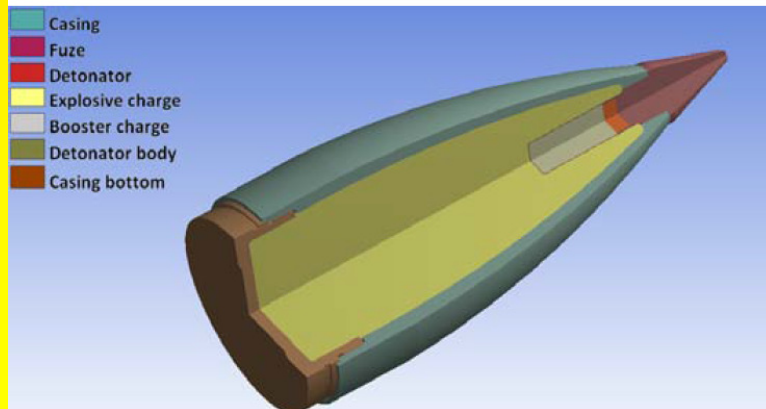
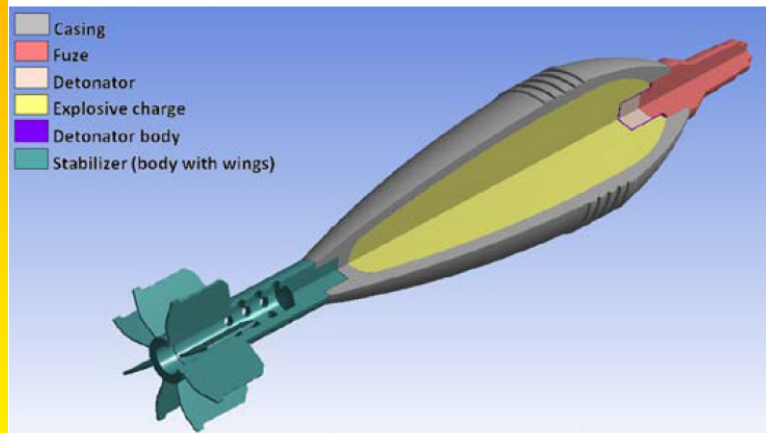
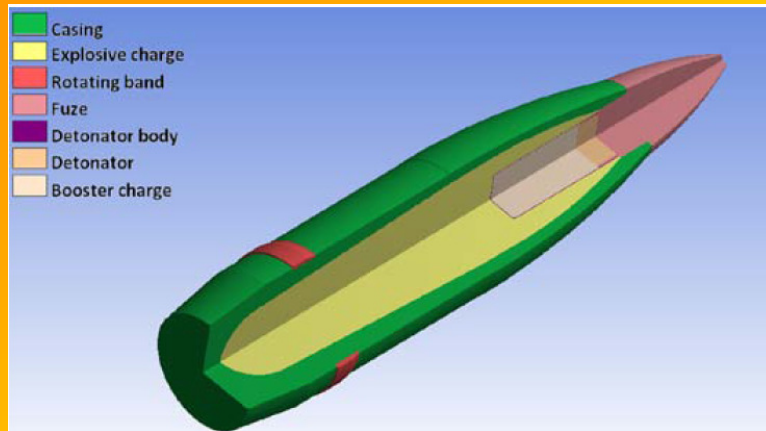


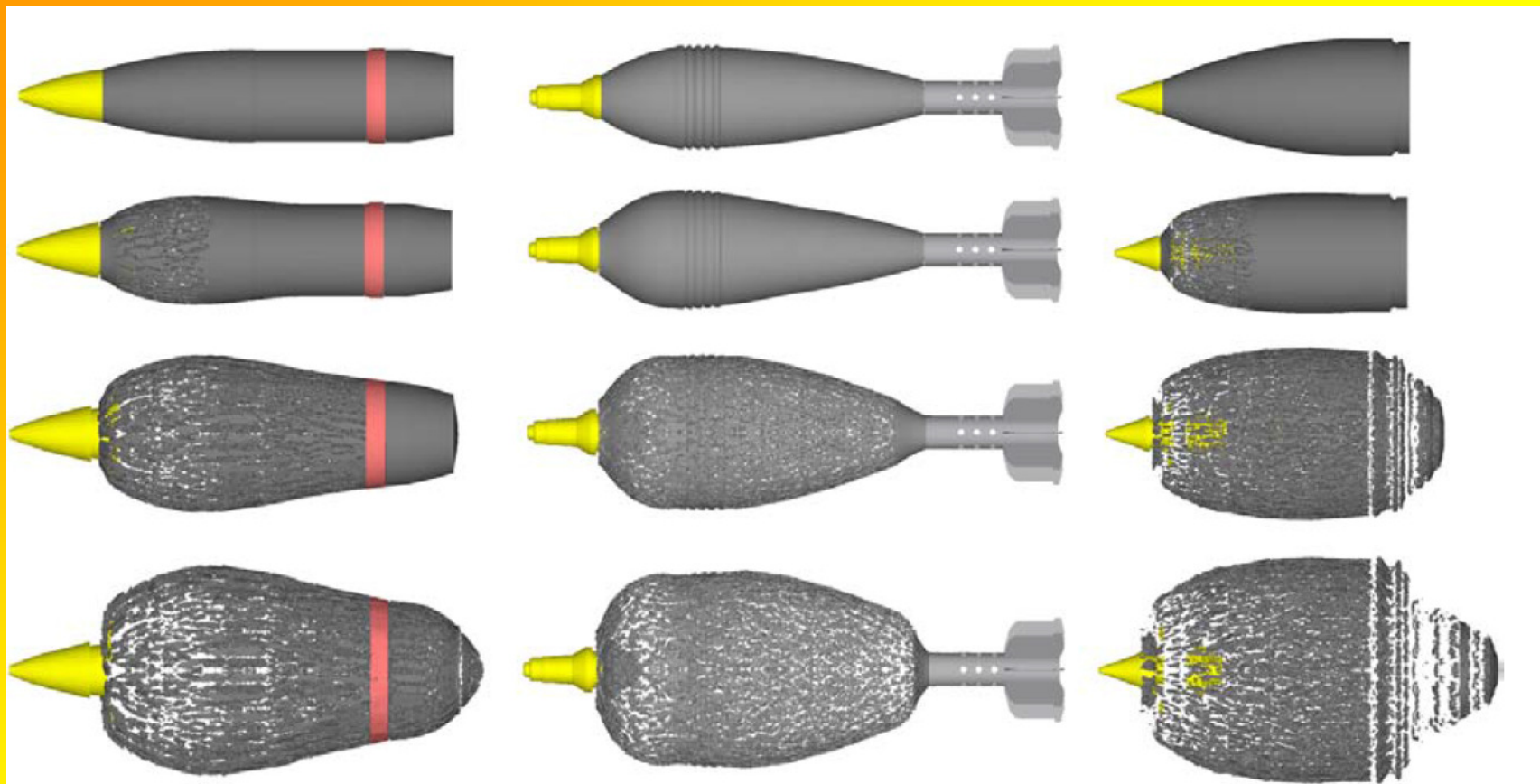
**Figure 2.** An example of the normal constraint reaction  $F_N(\varphi)$  in the functions of  $\varphi$  of the vibroimpact system with initial conditions determining ten impacts

Marinko Ugrčić and Miodrag Ivanišević, Characterization of the Natural Fragmentation of High Explosive Projectiles Using the Numerical Techniques based on the FEM, (to appear)



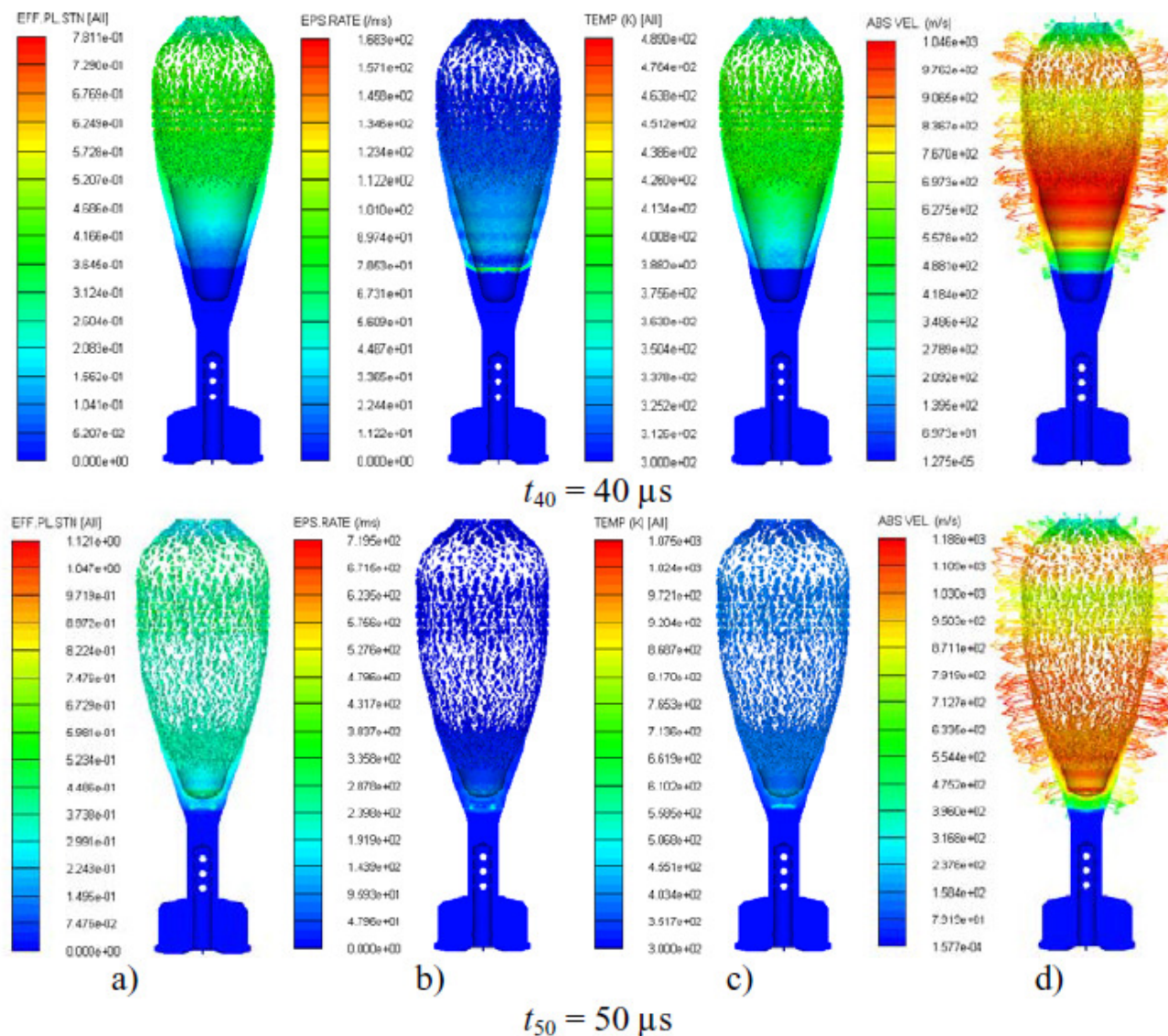






**Figure 7.** Fragmentation of the 105 mm HE projectile M1, 120 mm HE mortar shell M62 and 128 mm HE warhead M63:  $t = 0, 30, 60$  and  $90 \mu s$





**Figure 21.** Physical parameters of the 120 mm HE mortar shell fragmentation: a) Plastic strain, b) Strain rate, c) Temperature and d) Fragments velocity

**PROJEKAT 174001**

**DINAMIKA HIBRIDNIH SISTEMA  
SLOŽENIH STRUKTURA. MEHANIKA  
MATERIJALA**

**PODTEMA:**

**Analiza stabilnosti i upravljanje sistemima  
sa čistim vremenskim kašnjenjem**

**ISTRAŽIVAČKI TIM**

Prof. **Dr Dragutin Debeljković**, Mašinski fakultet u Beogradu

V. prof. **Dr Sreten Stojanović**, Tehnološki fakultet u Leskovcu

Dr **Nebojša Dimitrijević**, profesor, Visoka škola  
primenjenih strukovnih studija u Vranju

Dr **Goran Simeunović**, istraživač saradnik, Inovacioni centar  
Mašinskog fakulteta u Beogradu

## KATEGORIJE RADOVA U OBJAVLJENIH U 2013.

| Kategorija radova | Broj radova | Klasa sistema sa kašnjenjem | Koncept stabilnosti                                             |
|-------------------|-------------|-----------------------------|-----------------------------------------------------------------|
| M22               | 1           | regularni                   | FTS                                                             |
| M23               | 3           | regularni, singularni       | FTS,<br>praktična stabilnost                                    |
| M24               | 1           | regularni                   | FTS                                                             |
| M33               | 5           | regularni, singularni       | FTS,<br>praktična stabilnost,<br>FTB                            |
| M42               | 5           | regularni, singularni       | asimptotska stabilnost,<br>FTS,<br>praktična stabilnost,<br>FTB |

## ANALIZA NAJZNAČAJNIJIH REZULTATA

- U navedenim radovima razmatrana je stabilnost na konačnom i beskonačnom vremenskom intervalu sistema sa kašnjenjem sa ili bez prisustva neodređenosti u modelu.
- Pored regularne klase sistema, razmatrana je i klasa singularnih (deskriptivnih) sistema sa kašnjenjem.
- Problem upravljanja ovim sistemima realizovan je u povratnoj sprezi koristeći metodu stabilizacije sistema.



# M22

- Na osnovu kvazi Ljapunove metode i svojstva matričnih nejednakosti, izvedeni su dovoljni uslovi robusne stabilnosti i stabilizacije na konačnom vremenskom intervalu sistema sa kašnjenjem i prisutnim neodređenostima.

**Theorem 2.** There exists a memoryless state feedback controller such that the closed-loop system is FTS with respect to  $(c_1, c_2, T)$ ,  $c_1 < c_2$ , if there exist a nonnegative scalar  $\alpha$ , positive scalars  $\lambda, \delta, \varepsilon, \beta_1, \beta_2, \beta_3$ , positive-definite symmetric matrices  $X, Y$  and matrix  $Z$  such that the conditions hold:

$$\Omega = \begin{bmatrix} A_0 X + X A_0^T + B Z + Z^T B^T + Y - \alpha X & A_1 X & X E_0^T & 0 & Z^T E_B^T \\ + \lambda D_0 D_0^T + \delta D_1 D_1^T + \varepsilon D_B D_B^T & & & & \\ * & -Y & 0 & X E_1^T & 0 \\ * & * & -\lambda I & 0 & 0 \\ * & * & * & -\delta I & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix} < 0$$

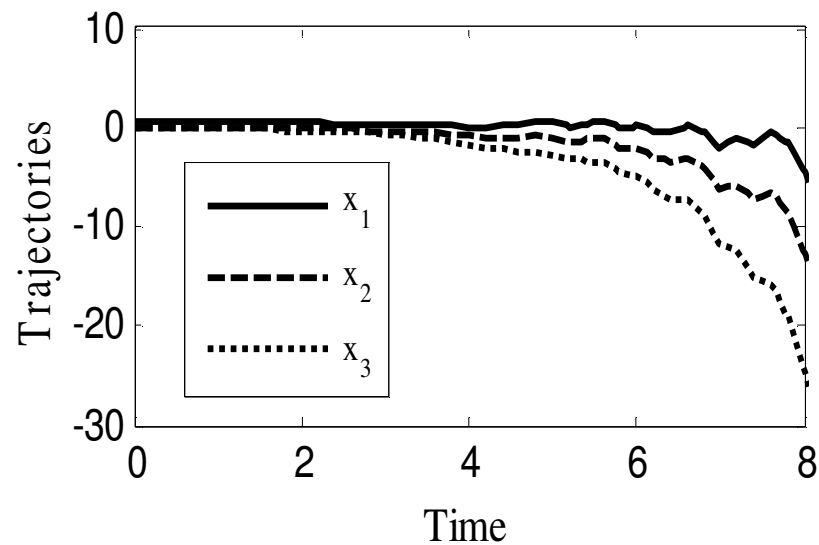
$$-c_2 e^{-\alpha T} \beta_1 + c_1 \beta_2 + \tau c_1 \beta_3 < 0$$

$$\begin{bmatrix} X & I \\ I & \beta_2 I \end{bmatrix} > 0, \quad \begin{bmatrix} \beta_1^{-1} I & I \\ I & X^{-1} \end{bmatrix} > 0, \quad \begin{bmatrix} Y^{-1} & X^{-1} \\ X^{-1} & \beta_3 I \end{bmatrix} > 0$$

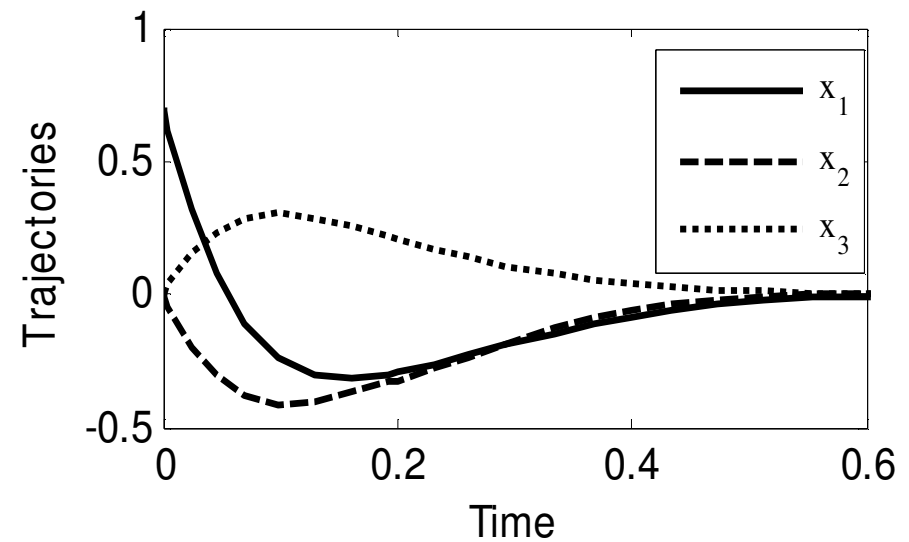
The memoryless state finite-time controller gain is given by  $K = Z X^{-1}$ .

- Problem projektovanja kontrolera u povratnoj sprezi rešen je pomoću linearnih matričnih nejednačina i uz primenu CCLI algoritma.
- Simulacija projektovanog sistema upravljanja

otvoreni sistem



zatvoreni sistem



# M23

- U radu je razmatrana stabilnost na konačnom vremenskom intervalu za klasu linearnih kontinualnih sistema sa kašnjenjem. Koristeći podesan kvazi Ljapunov funkcional, kao i Jensenovu i Kopelovu nejednakost, izveden je uslov stabilnosti na konačnom vremenskom intervalu u obliku skupa algebarskih nejednakosti.

**Theorem 1.** The time-delayed system with is finite-time stable with respect to  $\{\alpha, \beta, T\}$  if there exists a positive scalar  $\wp$  such that:

$$x^T(t - \vartheta)x(t - \vartheta) < q x^T(t)x(t), \quad q > 0, \quad \vartheta \in [-\tau, 0], \quad \forall t \in [0, T]$$

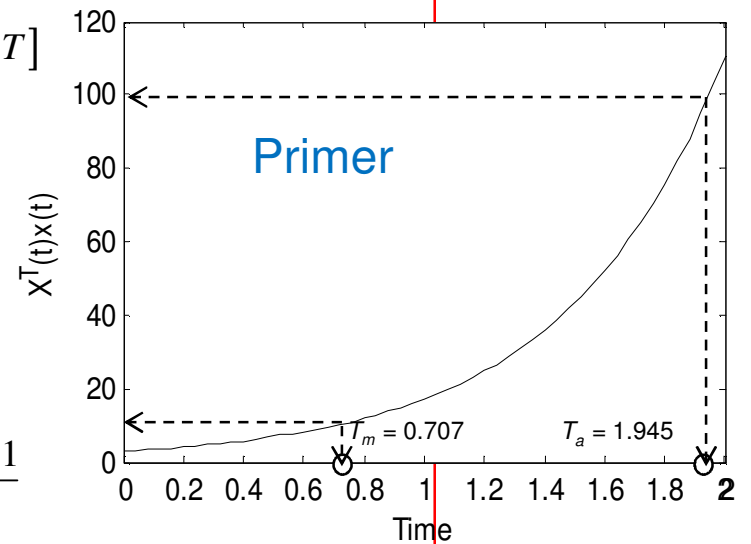
$$(1 + \tau)(1 + \psi) \left( 1 - \wp \psi - \frac{q\tau}{\wp} \right)^{-1} e^{\lambda_{\max}(\Xi)T} < \frac{\beta}{\alpha}$$

$$\wp \in (\max\{\wp_1, 0\}, \wp_2), \quad \wp_{1,2} = \frac{1 \pm \sqrt{1 - 4\psi\tau q}}{2\psi}, \quad 4\psi\tau q < 1$$

where:

$$R = A_0 + Q(0), \quad \Xi = R^T + R, \quad \psi = \lambda_{\max} \left( Q(0)Q^T(0) \right) \frac{e^{2\mu_2(R)\tau} - 1}{2\mu(R)}$$

$\mu_2(R)$  being matrix measure of matrix  $R$  and  $Q(0)$  is any solution of the following nonlinear transcendental matrix equation:  $e^{A_0 + Q(0)\tau} Q(0) = A_1$ .



## M23

- Koristeći kvazi Ljapunove funkcije izvedeni su novi, dovoljni uslovi stabilnosti, od kojih neki zavise od kašnjenja, a drugi su nezavisni. Pri analizi koncepta praktične stabilnosti, prethodno pomenuti prilaz kombinovan je sa klasičnom Ljapunovom metodom kako bi se obezbedila atraktivna praktična stabilnost razmatranog dinamičkog ponašanja sistema. Prilaz sa stanovišta LMI metode je takođe primenjen sa ciljem da se oslabe neki od ograničavajućih uslova iz prethodnih rezultata.

**Theorem 3.** Singular time delayed system is regular, impulse free and finite time stable with respect to  $\{\alpha, \beta, T\}$ ,  $\alpha < \beta$ , if for some fixed nonnegative scalar  $\wp$  there exist positive scalars  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$ , nonsingular matrix  $P$ , positive definite matrices  $\Pi$  and  $Q$ , such that the following conditions hold:

$$PE = E^T P^T \geq 0, \quad PE = E^T \Pi E$$

$$\begin{bmatrix} A_0^T P^T + PA_0 + Q - \wp PE & PA_1 \\ * & -Q \end{bmatrix} < 0$$

$$\lambda_1 I < \Pi, \quad \lambda_2 I > PE, \quad \lambda_3 I > Q, \quad -\beta e^{-\alpha T} \lambda_1 + \alpha \lambda_2 + \alpha \tau \lambda_3 < 0$$

PROJEKAT 174001  
DINAMIKA HIBRIDNIH SISTEMA SLOŽENIH  
STRUKTURA. MEHANIKA MATERIJALA

- Radovi (2013)
- Istraživač Nataša Trišović, Mašinski fakultet u Beogradu



- **Ужа категорија M23: Научни радови у међународним часописима (1 рад)**
- 
- Ezedine Allaboudi, Tasko Maneski, Natasa Trisovic, Todor Ergić, „Improving structure dynamic behaviour using a reanalysis procedures technique“, Journal: [Tehnički vjesnik](#), Vol.20, No.2, Str. 297 - 304, April, 2013, ISSN 1330-3651, (ISI-SCI list, IF: 0,347)
- <http://book.scribd.com/indonesian2behavioral>

- **Ужа категорија М33: Радови саопштени на скупу међународног значаја, штампани у целини (1 радова)**
- N. Trišović, Wavelet Families – A Primer, the Forth (29th Yu) International Congress of Serbian Society of Mechanics held in Vrnjačka Banja, 4th – 7th June, 2013. Pp.1005-1011.ISBN 978-86-909973-5-0

- **Ужа категорија M24: Рад у часопису међународног значаја верификованог посебном одлуком (1 рад)**
- Trišović Nataša, Maneski Taško, Golubović Zorana, Segla Štefan, **Elements of Dynamic Parameters Modification and Sensitivity**, FME Transactions, Volume 41, No 2, pp. 146-152, Beograd, 2013, (ISSN, 1451-2092)
- [http://www.mas.bg.ac.rs/transactions/Vol\\_41](http://www.mas.bg.ac.rs/transactions/Vol_41)

Ужа категорија М51: Рад у часопису националног значаја (1 рад)

**Ужа категорија М51: Рад у часопису националног значаја (1 рад)**

Nataša Trišović, Wei Li, Taško Maneski, Mirjana Misita, Ljubica Milović,  
**Elements of Dynamic Modifications and Sensitivity Considering the Effect of  
Structural Parameters Uncertainty**

SCIENTIFIC REVIEW, ISSN 0350-2910, BELGRADE (2013), Serbian Scientific Society,  
p.p. 389-404

<http://afrodita.rcub.bg.ac.rs/~nds/>

<http://afrodita.rcub.bg.ac.rs/~nds/indexe.html>

Editor-in-Chief: Slobodan Perović

Series: Scientific and Engineering

Special Issue Nonlinear Dynamics S2 (2013)

Dedicated to Milutin Milanković (1879- 1958)

Guest Editors:

Katica R. (Stevanović) Hedrih and Žarko Mijajlović

- **Ужа категорија (М36): Уређивање зборника саопштења међународног научног скупа**
- 
- Proceedings, 1<sup>st</sup> International Congress of Serbian Society of Mechanics, Vrnjačka Banja, June 3-7, 2013, ISBN 978-86-909973-5-0, Editors: S. Maksimović, T. Igić, N. Trišović;

# Napomena uz radove

- Godine 2013 napravljen je pomak uvođenjem teorije verovatnoće u predhodna istraživanja koja su se odnosila na reanalizu konstrukcija. Analizirana su moguća odstupanja ulaznih parametara, i pri tome je korišćena Monte Karlo metoda. U nastavku autor će se baviti daljim istraživanjima iz ove oblasti.
- Osim toga dat je jedan pregledni rad iz oblasti Wavelets na kongresu mehanike, što takođe



PROJEKAT 174001  
DINAMIKA HIBRIDNIH SISTEMA SLOŽENIH STRUKTURA.  
MEHANIKA MATERIJALA

Radulović Radoslav, Mašinski fakultet u Beogradu

# Spisak položenih ispita u 2013. god

| Re. Br. | Predmet                                             | Profesor                      | Ocena |
|---------|-----------------------------------------------------|-------------------------------|-------|
| 1       | O metodama naučno istraživačkog rada i komunikacije | prof. Dr Miloš Nedeljković    | 10    |
| 2       | Viši kurs matematike                                | prof. Dr Slobodan Radojević   | 10    |
| 3       | Numeričke metode                                    | prof. Dr Miodrag Spalević     | 10    |
| 4       | Odabrana poglavlja iz mehanike                      | prof. Dr Zoran Mitrović       | 10    |
| 5       | Tenzorski račun                                     | prof. Dr Zoran Stokić         | 10    |
| 6       | Analitička Mehanika                                 | prof. Dr Olivera Jeremić      | 10    |
| 7       | Epistemologija nauke i tehnike                      | prof. Dr Zoran Stokić         | 10    |
| 8       | Oscilacije mehaničkih sistema-linearne              | prof. Dr Aleksandar Obradović | 10    |
| 9       | Oscilacije mehaničkih sistema-nelinearne            | prof. Dr Zoran Mitrović       | 10    |
| 10      | Stabilnost kretanja sistema                         | prof. Dr Zoran Mitrović       | 10    |
| 11      | Dinamika sistema krutih tela                        | prof. Dr. Mihailo Lazarević   | 10    |
| 12      | Upravljanje kretanjem mehaničkih sistema            | prof. Dr Aleksandar Obradović | 10    |
| 13      | Mehanika kontinuuma                                 | prof. Dr Zoran Stokić         | 10    |
| 14      | Mehanika sistema promenljive mase                   | prof. Dr Olivera Jeremić      | 10    |
| 15      | Mehanika neholonomnih sistema                       | prof. Dr Dragomir Zeković     | 10    |
| 16      | Mehanika udara                                      | prof. Dr Mirko Pavišić        | 10    |

# Objavljeni radovi i nagrade u 2013. godini

Spisak prezentovanih radova na kongresima i simpozijumima:

- [1] Radulović, R.: *Shooting method in determining global minimum time of brachistochronic motion*, in: *Proceedings of the 4<sup>th</sup> International Congress of Serbian Society of Mechanics*, 04-07.06.2013, Vrnjačka Banja, pp. 159-164.
- [2] Radulović, R., Obradović, A. and Jeremić, B.: *Brachistochronic Motion of a Nonholonomic Mechanical System with Limited Reactions of Constraints*, in: *Proceedings of the 4<sup>th</sup> International Congress of Serbian Society of Mechanics*, 04-07.06.2013, Vrnjačka Banja, pp. 903-908.

Spisak radova u časopisima:

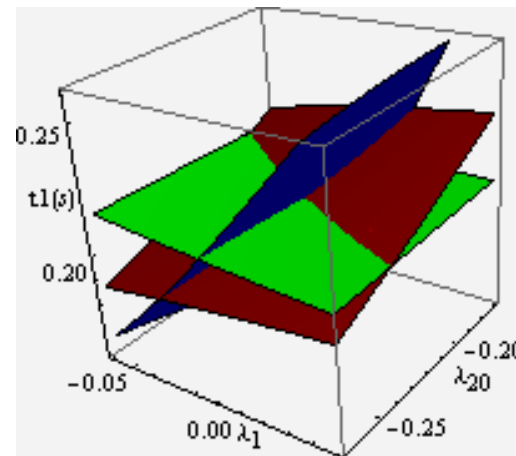
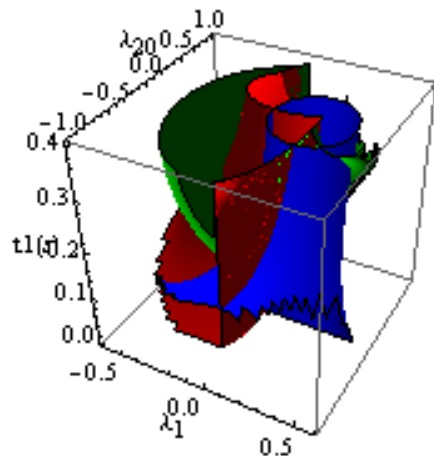
- [3] Radulović, R., Obradović, A. and Jeremić, B.: *Analysis of the minimum required coefficient of sliding friction at brachistochronic motion of a nonholonomic mechanical system*, FME Transactions, 2013.

Istraživač Radulović Radoslav je nagrađen prestižnom nagradom „*Rastko Stojanović*“ za rad [1] kao samostalni autor na međunarodnom kongresu Srpskog društva za mehaniku koji je održan u Vrnjačkoj Banji od 04.-07. juna 2013. godine.

# Analiza najznačajnijih rezultata

- U radu [1] razmatra se problem brahistohronog kretanja za opšti slučaj holonomnog skleronomnog mehaničkog sistema.
- Daje se postupak određivanja globalnog minimuma za sisteme sa 3 DOF , gde je moguće dati i grafičke prezentacije u trodimenzionom prostoru, gde je treća dimenzija krajnji trenutak.
- **Globalni minimum vremena** kod brahistohronog kretanja krutog tela može se dati na osnovu grafičkog prikaza rešenja sistema nelinearnih jednačina, kao i globalne procene koordinata spregnutog vektora.

$$\psi_1 = f_{\psi}(\lambda_1, \lambda_{20}, t_1), \theta_1 = f_{\theta}(\lambda_1, \lambda_{20}, t_1) \text{ i } \varphi_1 = f_{\varphi}(\lambda_1, \lambda_{20}, t_1)$$



➤ U radovima [2] i [3] analizira se problem brahistohronog kretanja mehaničkog sistema, na primeru jednog uprošćenog modela vozila.

➤ Sistem se kreće vrednosti mehaničke

➤ Numeričko rešava metodom šutinga.

➤ Na osnovu tako aktivne upravljačke si

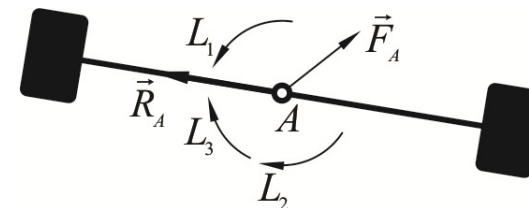
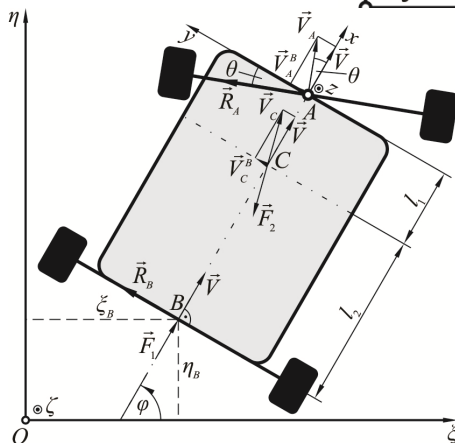
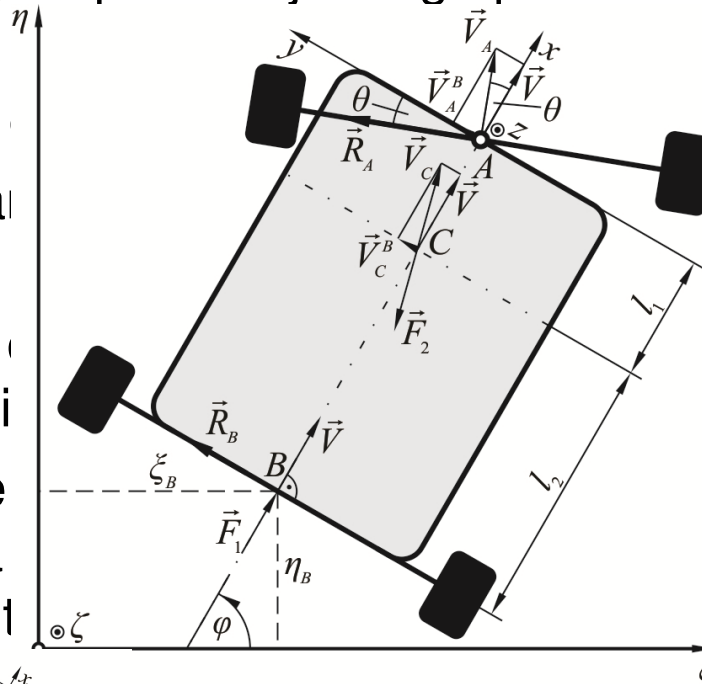
➤ Koristeći Kulonove vrednost koeficijenta vozila u tačkama kon

ja pri neizmenjenoj

problema vrši se

etanja određuju se

eđuje se minimalna  
šlo do proklizavanja



PROJEKAT 174001  
DINAMIKA HIBRIDNIH SISTEMA SLOŽENIH  
STRUKTURA. MEHANIKA MATERIJALA

Prof dr Zeković Dragomir, Mašinski fakultet u Beogradu



DRAGOMIR N. ZEKOVIĆ, DIFFERENTIAL EQUATIONS OF MOTION FOR MECHANICAL SYSTEMS WITH NONLINEAR NONHOLONOMIC CONSTRAINTS - VARIOUS FORMS AND THEIR EQUIVALENCE, SCIENTIFIC REVIEW, ISSN 0350-2910, BELGRADE (2013), Serbian Scientific Society, p.p. 179-196

<http://afrodita.rcub.bg.ac.rs/~nds/>

<http://afrodita.rcub.bg.ac.rs/~nds/indexe.html>

Editor-in-Chief: Slobodan Perović

Series: Scientific and Engineering

Special Issue Nonlinear Dynamics S2 (2013)

Dedicated to Milutin Milanković (1879- 1958)

Guest Editors:

Katica R. (Stevanović) Hedrih and Žarko Mijajlović

PROJEKAT 174001  
DINAMIKA HIBRIDNIH SISTEMA SLOŽENIH STRUKTURA.  
MEHANIKA MATERIJALA

Veljić Vladimir, Mašinski fakultet u Beogradu

# Objavljeni radovi i nagrade u 2013. godini

Uža kategorija M33: Radovi saopšteni na skupu međunarodnog značaja, štampani u celini

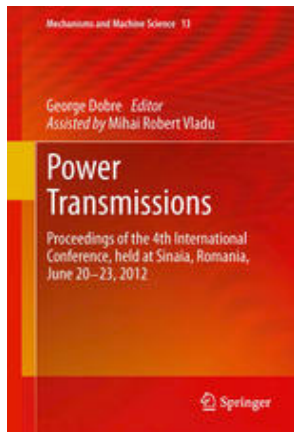
|                                                                                                                               |                                                                                                                                |                                     |      |                              |             |                        |
|-------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------|-------------------------------------|------|------------------------------|-------------|------------------------|
| MECHANICAL PROPERTIES INVESTIGATION OF COMMERCIAL AND AND NANOPHOTONICS SOFT CONTACT LENSES                                   | 4 <sup>th</sup> (29th Yu) International Congress of Serbian Society of Mechanics held in Vrnjačka Banja, 4th – 7th June, 2013. | V. Veljić, A. Debeljković, Đ.Koruga | 2013 | Serbian Society of Mechanics | pp. 591-597 | ISBN 978-86-909973-5-0 |
| STUDY OF MECHANICAL PROPERTIES OF COMMERCIAL AND NANOPHOTONICS MATERIALS FOR SOFT CONTACT LENSES BY OPTOMAGNETIC SPECTROSCOPY | 4 <sup>th</sup> (29th Yu) International Congress of Serbian Society of Mechanics held in Vrnjačka Banja, 4th – 7th June, 2013. | V. Veljić, A. Debeljković, Đ.Koruga | 2013 | Serbian Society of Mechanics | pp. 961-967 | ISBN 978-86-909973-5-0 |

Uža kategorija M24: Rad u časopisu međunarodnog značaja verifikovanog posebnom odlukom

|                                                                                                                    |                                             |                                                                      |      |                                                     |              |  |
|--------------------------------------------------------------------------------------------------------------------|---------------------------------------------|----------------------------------------------------------------------|------|-----------------------------------------------------|--------------|--|
| Characterization of materials for commercial and new nanophotonic soft contact lenses by Optomagnetic Spectroscopy | Journal: FME Transactions, Volume 42, No 2, | A. Debeljkovic, V. Veljic, V. Sijacki-Zeravcic, L. Matija,Dj. Koruga | 2014 | Faculty of Mechanical Engineering, Belgrade, Serbia | pp. 141-146, |  |
|--------------------------------------------------------------------------------------------------------------------|---------------------------------------------|----------------------------------------------------------------------|------|-----------------------------------------------------|--------------|--|

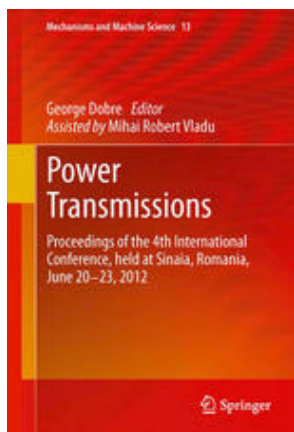
U radu „MECHANICAL PROPERTIES INVESTIGATION OF COMMERCIAL AND AND NANOPHOTONICS SOFT CONTACT LENSES“ korišćen je AFM skenirajuća tehnikom kojaom je potvrđeno da su mehaničke osobine mekih kontaktnih sočiva bolje kada se u komercijalna meka kontaktna sočiva implementiraju nanočestice

U radovima „ STUDY OF MECHANICAL PROPERTIES OF COMMERCIAL AND NANOPHOTONICS MATERIALS FOR SOFT CONTACT LENSES BY OPTOMAGNETIC SPECTROSCOPY“ i „ Characterization of materials for commercial and new nanophotonic soft contact lenses by Optomagnetic Spectroscopy“ korišćena je metoda optomegnetne spektroskopiije, koja je dala značajne kvalitativne rezultate u pogledu mehaničkih i optičkih karakteristika kontaktnih sočiva napravljenih od nanomaterijala



M. Jelić, **I. Atanasovska**: THE NEW APPROACH FOR CALCULATION OF TOTAL MESH STIFFNESS AND NONLINEAR LOAD DISTRIBUTION FOR HELICAL GEARS, **Mechanisms and Machine Science (Book Series)**, Series Ed.: Ceccarelli Marco, ISSN: 2211-0984, *Vol. 13: Power Transmissions (Proceedings of The 4th International Conference on Power Transmissions- PT 12, June 20 -23, 2012, Sinaia, Romania)*, Editor: G.Dobre, ISBN: 978-94-007-6557-3 (Print) 978-94-007-6558-0 (Online), Publisher: **Springer Science + Business Media Dordrecht** 2013, doi: 10.1007/978-94-007-6558-0\_52, pp. 645-654  
[http://link.springer.com/chapter/10.1007/978-94-007-6558-0\\_52#](http://link.springer.com/chapter/10.1007/978-94-007-6558-0_52#)

**I. Atanasovska**, M. Vukšić Popović: DYNAMICS OF GEAR-PAIR SYSTEMS WITH PERIODIC VARYING MESH STIFFNESS - SPUR GEARS VS HELICAL GEARS, Series: Scientific Review, Scientific and Engineering - Special Issue - Nonlinear Dynamics S2 (2013) Dedicated to Milutin Milanković (1879-1958), *Guest Editors: Katica R (Stevanović) Hedrih and Željko Mijajlović*, YU ISSN:0350-2910, UDK 001, Publisher: *Serbian Scientific Society*, 2013., pp. 373-388.  
<http://afrodita.rcub.bg.ac.rs/~nds/indexe.html>



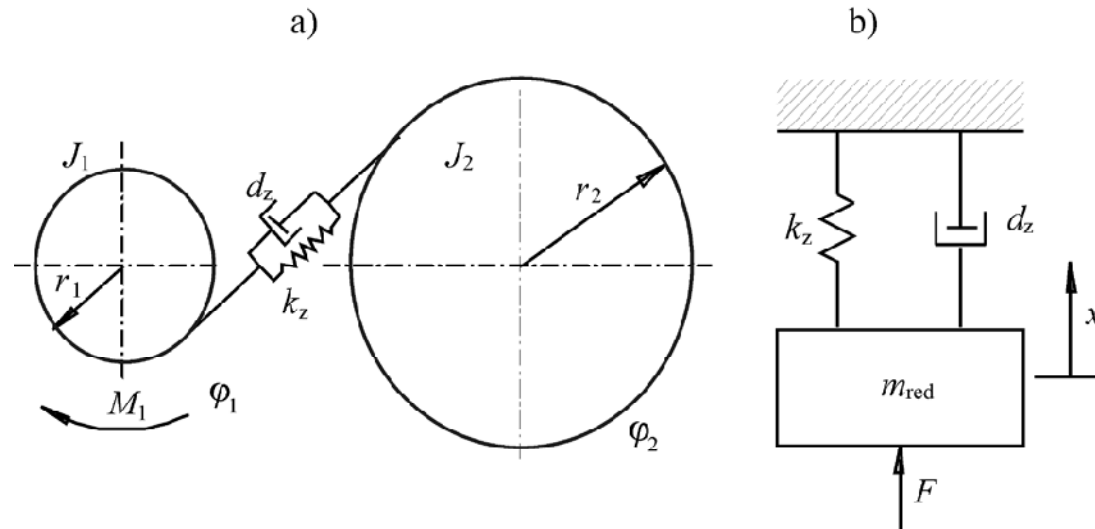
## Abstrakt.

Opisan je novi pristup rešavanju krutosti spregnutih zubaca i raspodele opterećenja za zupčanike sa kosim zupcima. Krutost para spregnutih zubaca je parameter koji varira u toku perioda sprežanja i takođe duž kontaktne linije spregnutih zubaca, pa se može precizno izračunati samo istovremenim rešavanjem oba navedena zadatka. Metoda konačnih elemenata korišćena je za proračun promene ukupne deformacije i raspodele opterećenja u vremenu i duž kontaktne linije.

Korišćen je model dinamičkog ponašanja zupčastog para u komparativnoj analizi zupčanika sa pravim i kosim zupcima sa aspekta njihove stabilnosti. Dobijeni rezultati prikazani kao fazni portreti potvrđuju da zupčanici sa kosim zupcima imaju stabilniji rad od zupčanika sa pravim zupcima iste geometrije i pri istim opterećenjem.



Pojednostavljeni dinamički model zupčastog para → zupčasti par se simulira sa dva diska između kojih se kontakt simulira nelinearno promenljivom krutosti sprege i prigušenjem sprege



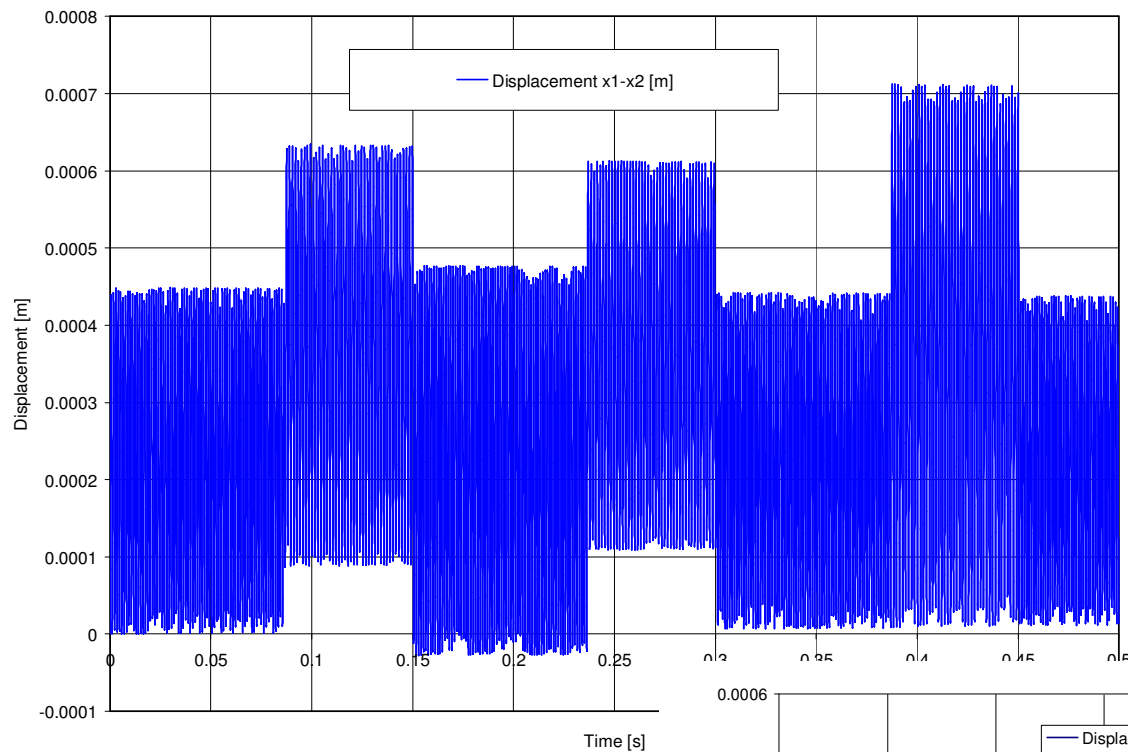
a) model II – sa dva stepena slobode, b) model I – sa jednim stepenom slobode

$$m_1 \ddot{x}_1 + d_z(t)(\dot{x}_1 - \dot{x}_2) + k_z(t)(x_1 - x_2) = F_n(t)$$

$$m_{red} \ddot{x} + d_z(t)\dot{x} + k_z(t)x = F_n(t)$$

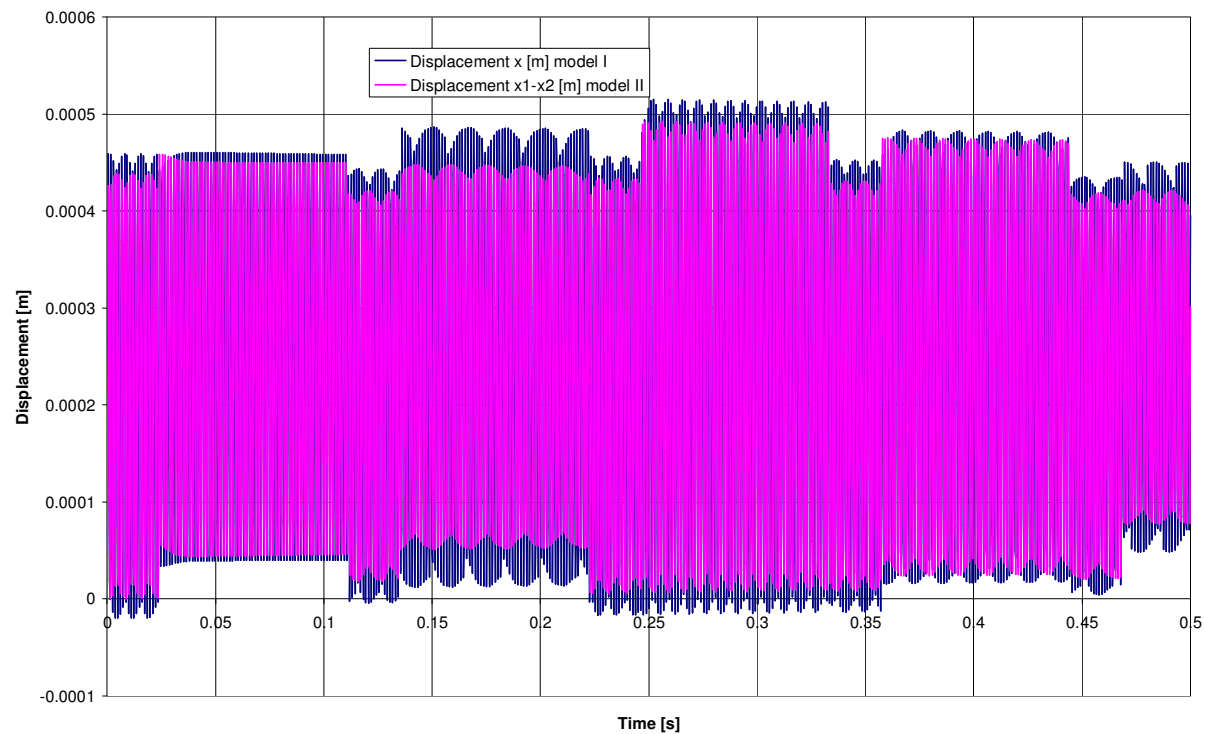
$$m_2 \ddot{x}_2 - d_z(t)(\dot{x}_1 - \dot{x}_2) - k_z(t)(x_1 - x_2) = -F_n(t)$$

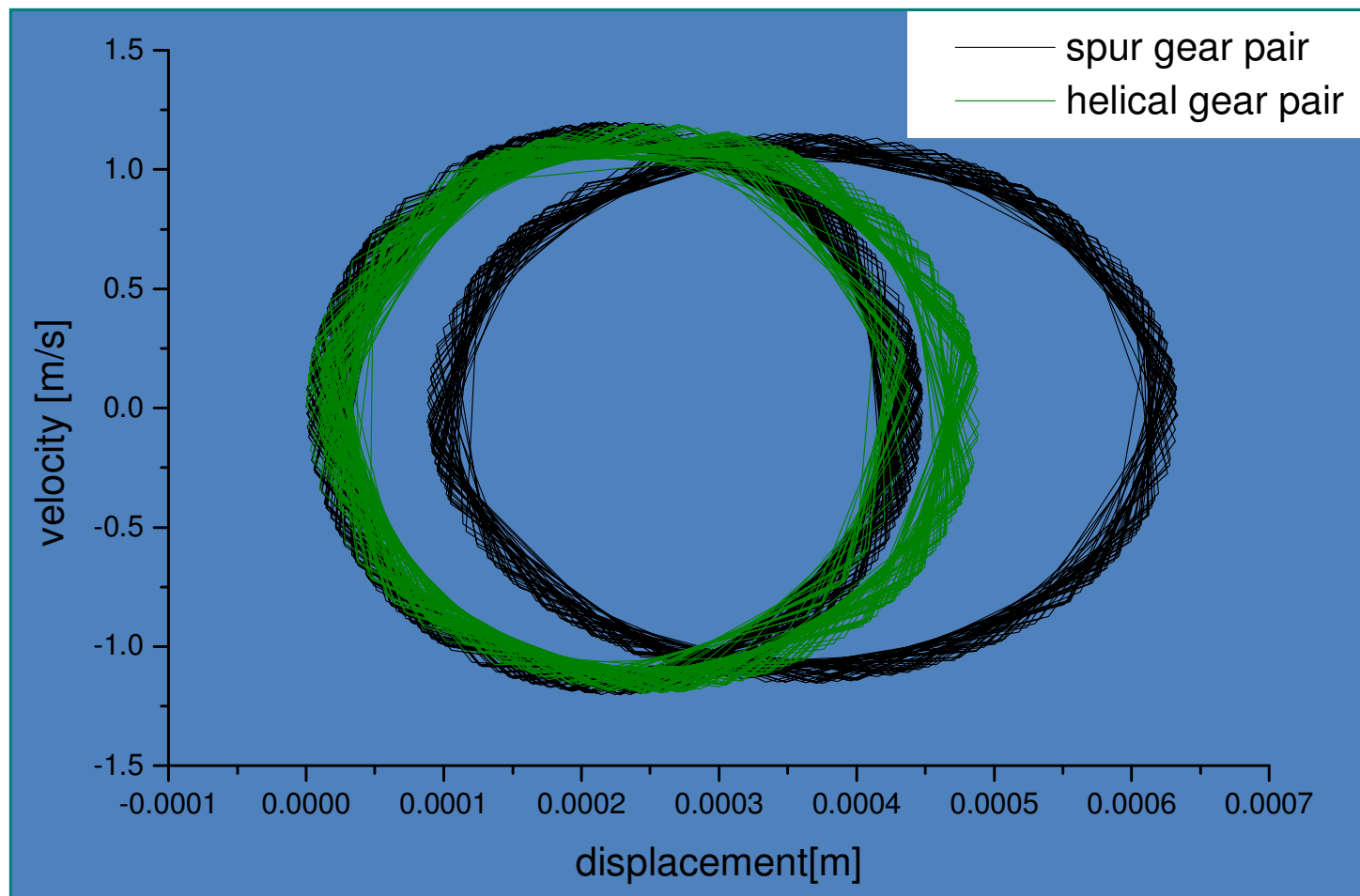
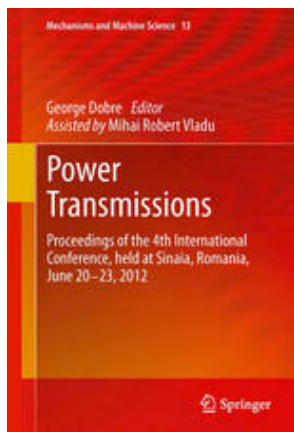
$m_i$  ( $i=1,2$ ) – ekvivalentna masa zupčanika 1 i 2;  $m_{red}$  – redukovana masa zupčanika;  $k_z(t)$  – krutost sprege;  $d_z(t)$  – prigušenje sprege;  $F_n(t)$  – funkcija raspodele spoljašnjeg opterećenja



**Zupčasti par sa  
pravim zupcima**

**Zupčasti par sa  
kosim zupcima**

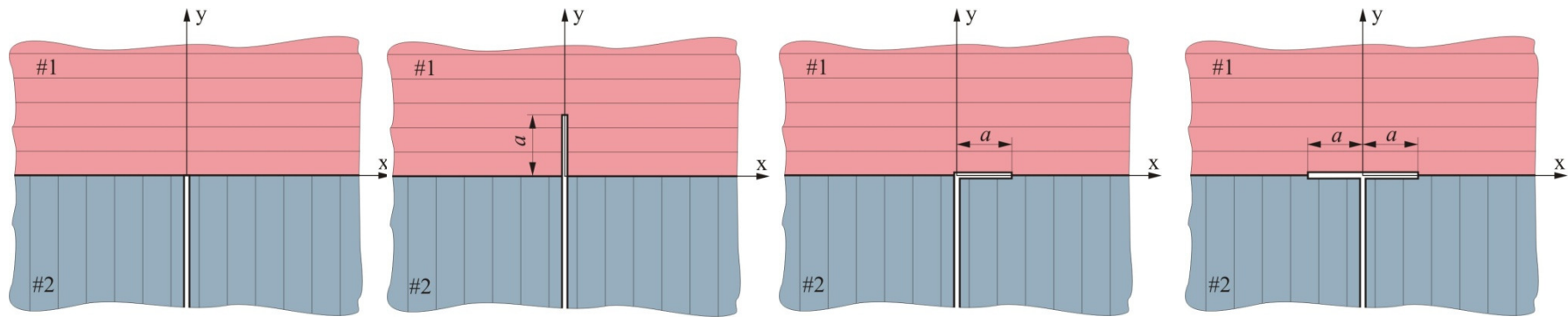




**Jelena M. Djoković**, Ružica R. Nikolić,

“CRACK DEFLECTION AT AN INTERFACE BETWEEN THE TWO ORTHOTROPIC MATERIALS”,  
Proceedings Fourth Serbian (29<sup>th</sup> Yu) Congress on Theoretical and Applied Mechanics,  
Vrnjačka Banja, Serbia, 4-7 June 2013, C43, pp.535-540, ISBN 978-86-909973-5-0

In this paper is presented studying of the problem of a crack that approaches interface between the two orthotropic materials at the right angle. The three possible cases of the crack attacking the interface were considered: crack penetrates the interface and continues to grow in the material above it; the crack deflects into the interface as a single branch and crack deflects into the interface in two branches (double deflection).



Based on results presented in this paper, it can be concluded that the ratio of energy release rates depends on variation of the anisotropic parameters  $\lambda_1$ ,  $\lambda_2$ ,  $\rho_1$  and  $\rho_2$ . It is noticed that the value of the  $G_d/G_p$  ratio changes within interval 0.2 to 5. When the value of this ratio exceeds this means that crack approaching interface at the right angle will deflect into it even if the interface toughness is higher than the toughness of the base material.

Results presented in this paper provide the possibility for comparison of the interface and base material (substrate) toughnesses, for the purpose of determination whether the incoming crack would deflect into the interface or would it penetrate the interface and continue to propagate in the second material above it. If the ratio of the interface fracture toughness to fracture toughness of the material into which the crack continues to propagate is less than the ratio of the energy release rate for the crack that deflects into the interface and the energy release rate for the crack that penetrates the interface, the crack will deflect into the interface. If this relation is reversed the crack will penetrate the interface.

**Jelena M. Djoković**, Ružica R. Nikolić, Katarina Z. Živković,

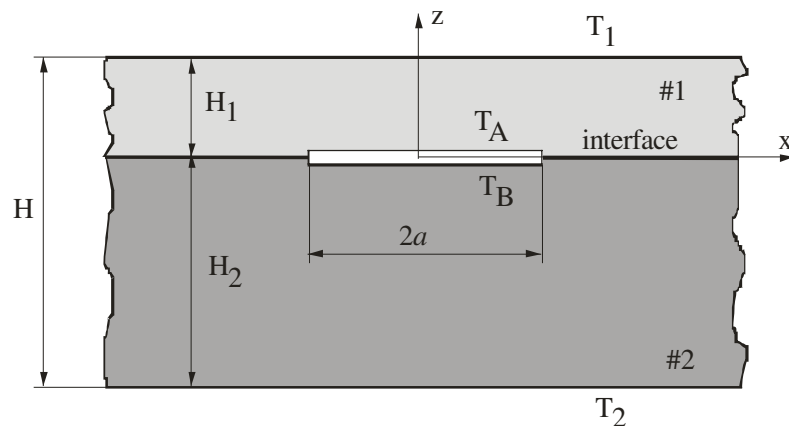
"INTERFACIAL CRACK BEHAVIOR IN THE STATIONARY TEMPERATURE FIELD CONDITIONS" ,

Thermal Science, 2013, Vol. 17, No.17, Suppl. 1, pp. S169-S178, ISSN 0354-9836.

DOI information: 10.2298/TSCI120828113D

In this paper the theoretical basis for determining the driving forces of interfacial crack propagation in a two-layer bimaterial specimen is presented, under conditions when the temperatures of the outer layers' surfaces are different.

The analysis is limited to the fact that the two-layer bi-material sample is exposed to a stationary temperature field.



The driving force of the interfacial fracture, in this case, is the energy release rate  $G$ , which is determined as a function of the temperature loading. It was noticed that the energy release rate tends to increase with increasing temperature difference. This relation can be used to predict the maximum temperature differences the two-layer sample can sustain without delamination.

The highest value of the energy release rate is for a crack located at approximately one fifth of the sample thickness, what means that it is the most likely that the crack, which causes the sample delamination lies at this distance.

Variation of the Biot's number imposes significantly higher influence on Mode II stress intensity factor and in the case of the mixed mode crack propagation the value of dimensionless stress intensity factor for Mode I is much smaller than the value of this factor for Mode II. The Biot's number  $B_c$  plays an important role in heat flow through the crack.

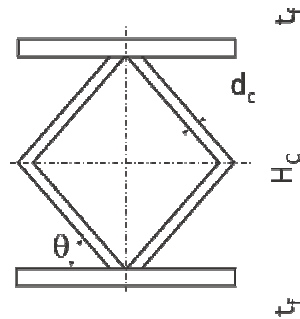
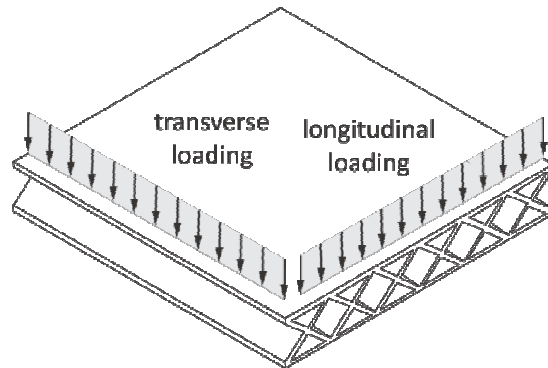
For large values of the Biot's number, the energy release rate  $G$  has a relatively lower value, which means that the crack opening is small. However, when  $B_c$  is small, the energy release rate is large, as well as the crack opening, namely crack is completely and totally isolated, and suitable for delamination of the sample if the temperature gradient is sufficiently large.

The threshold of the temperature difference also increases with temperature. If the threshold exceeds the imposed temperature difference, delamination of the sample should not be expected. The heat flow through the crack can be significant and the assumption of a perfectly isolated crack would be wrong.

**Jelena M. Djoković**, Ružica R. Nikolić, Katarina M. Veljković,

"OPTIMIZATION OF PLATES WITH PRISMATIC CORES",

Proceedings of ICET 2013 – International Congress on Engineering and Technology,  
25th – 27th June, 2013, Dubrovnik – Croatia, pp. 159-166. ISBN: 978-8-87670-08-8



In this paper are analyzed multifunctional sandwich plates with prismatic cores. The comparison of those plates to plates with honeycomb cores was performed. An estimate was done of optimal dimensions and minimal mass of plates with prismatic cores. The two loading directions were considered, longitudinal and transversal. The fracture mechanisms are derived that take into account mutual interaction of the core and the basic plate during buckling for each loading direction.

The dimensionless expressions were derived for the loading combination of transversal force and bending moment for each fracture mechanism.

Four variables were used in dimensions optimization, which reduces the sandwich plate mass. The plates with prismatic cores have best performances when loaded longitudinally, since the plate characteristics are restricted by buckling of a plate and not of a beam. Plates with the honeycomb cores are more efficient related to weight, than the plates with the prismatic cores for smaller loads. This advantage vanishes with load increase. Large plastic deformations of material used in manufacturing of the sandwich plates produce better performances for the plates with the prismatic cores. To emphasize advantages of those plates the comparison to sandwich plates with the truss cores was also performed

**Jelena M. Djoković** and Ružica R. Nikolić,

"COMPARATIVE OPTIMIZATION OF SANDWICH PLATES WITH PRISMATIC AND TRUSS CORES",

Proceedings 10-th European Conference of Young Researchers and Scientists, TRANSCOM 2013,  
24-26 June 2013, University of Žilina, Žilina, Slovak Republic, pp.71-74, ISBN 978-80-554-0695-4

In this paper have been analyzed multifunctional sandwich panels with prismatic and truss cores. Their behavior have been compared with panels designed using honeycomb cores. The optimal dimensions and the minimum weight of sandwich panels with prismatic and truss cores have been evaluated and their mutual comparison. Non-dimensional expressions are obtained for combination of bending moment and shear force for both cores. A quadratic optimizer is used to ascertain the dimensions that minimize the panel weight. Honeycomb core panels are more weight efficient than prismatic and truss core panels at low load capacity. The benefit diminishes as the load increases. The larger the yield strain of the material used to manufacture the panel, the greater the performance, and the larger the benefits of the prismatic core



**Jelena M. Djoković, Ružica R. Nikolić and Jan Bujnak,**  
**"FUNDAMENTAL PROBLEMS OF MODELING THE FRACTURE PROCESSES IN CONCRETE I:  
MICROMECHANICS AND LOCALIZATION OF DAMAGES",**  
**Procedia Engineering (2013), pp. 186-195 ISSN: 1877-7058.**  
**DOI information: 10.1016/j.proeng.2013.09.029**

This paper presents an attempt to review some of the most important works in the area of modeling the fracture processes in concrete. Here are considered two out of several major issues the researchers are confronted with when studying this field: micromechanics and localization of damages leading to concrete fracture. From the very first works of Vile and Bažant, numerous scientists have studied the mentioned problems. Plethora of papers was published on defining the constitutive law for concrete, with more or less success. Works of Bažant, Stroeve, Ferretti and others are major contributions in understanding the fracture process initiation and development in concrete. The problem of damage localization-the strain softening is also a very important one. This process starts from the beginning of loading and progresses with load increase. The interactions of the preexisting cracks due to acting load are yet to be fully explained and understood. All the works published on those subjects until now are explaining some of the aspects in searching for the solution how to explain, understand and prevent the concrete fracture. The complete and final answer is not in sight. There are also other major issues to be discussed, like the size effect of concrete particles on the fracture process.

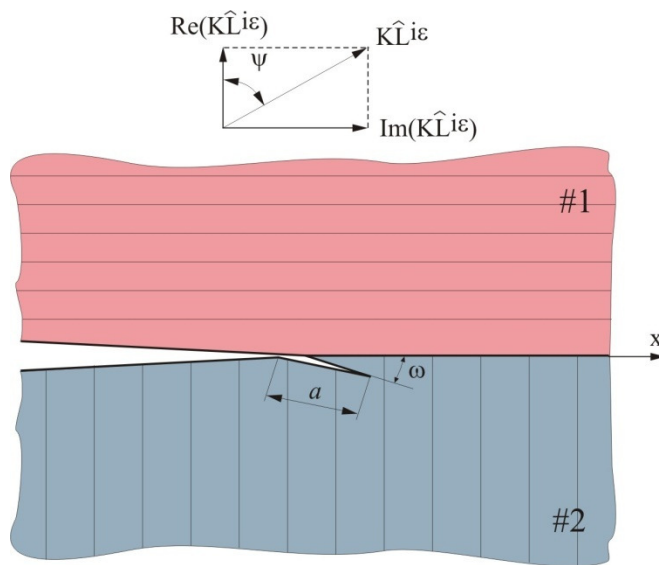
**Jelena M. Djoković, Ružica R. Nikolić and Jan Bujnak,**  
**" FUNDAMENTAL PROBLEMS OF MODELING THE FRACTURE PROCESSES IN CONCRETE II:  
SIZE EFFECT AND SELECTION OF THE SOLUTION APPROACH",**  
**Procedia Engineering (2013), pp. 196-205, ISSN: 1877-7058.**  
**DOI information: 10.1016/j.proeng.2013.09.030**

This paper presents the second part of the review of the most important works in the area of modeling the fracture processes in concrete. Here are considered two out of several major issues the researchers are confronted with when studying this field: size effect and scaling laws and phenomenological versus micromechanical approach to solving the fracture processes in concrete. Works of Vile, Bažant, Stroeve, Ferretti and others represent major contributions in understanding the fracture process initiation and development in concrete. All the works published on those subjects until now are explaining some of the aspects in searching for the solution how to explain, understand and prevent the concrete fracture. The complete and final answer is still not in sight.

**Jelena M. Djokovic,**

**"CRACK KINKING IN MATERIALS WITH THE PERPENDICULAR ANISOTROPY",  
Proceedings 45-th International October Conference on Mining and Metallurgy, IOC 2013,  
16-19 October 2013, Bor Lake, Bor (Serbia), pp.45-48, ISBN 978-86-6305-012-9**

In this work an attempt was made to analyze the crack kinking away from the interface between the two different anisotropic materials. The attention was focused on the kinking initiation and on the condition that the length of the segment, which is leaving the interface, is small with respect to the crack segment that remains on the interface. Based on this analysis, the stress intensity factors were obtained as well as the energy release rate for the kinking crack in terms of the corresponding variables for the crack prior to kinking



The practical use of this analysis is for the interface between the glued layers and also in application of the various protective coatings on metals. Certain special cases were considered like the crack kinking in orthotropic materials with mutually perpendicular main axes. The competition between the crack kinking away from the interface or propagating along it is measured by the ratio of the corresponding "driving forces",  $G^S$  and  $G$ .

The average energy needed for the crack to kink away from the interface increases abruptly with the change of the load phase angle and then it drops. The influence of the anisotropy, present within the two dimensionless parameters  $\lambda$  and  $\rho$ , is that the kinking will be easier, i.e., it is easier for the crack to kink away from the interface into the "softer" of the two materials.

The role of parameters  $\alpha$  and  $\beta$ , which reflect the mismatch of the elastic characteristics of the two materials, is shown in this paper. The parameter  $\alpha$  measures the relative stiffness of the two materials, while the parameter  $\beta$  defines the oscillatory nature of the field around the crack tip. For the majority of the practical problems, the coefficient  $\beta$  ranges between -0.25 and +0.25 and its influence on the crack behavior can be neglected. The influence of the parameter  $\alpha$ , on the other hand, is far more prominent. For the bimaterial combinations shown in this paper the highest probability that the crack would kink away from the interface is for the parameters combination  $\alpha = 0.5$  and  $\beta = 0$ , while the lowest probability of kinking is for the case  $\alpha = -0.5$  and  $\beta = 0$ . The load phase angle is based on the reference length which is being chosen in such a manner that its value with respect to kinking length  $a$ . The ratio  $G^S / G$  versus the load phase angle decreases with decrease of  $\alpha$ , with  $\beta$  being constant.

# **NUMERICKA PROCENA VEKA ELEMENATA KONSTRUKCIJA**

- analiza ponašanja konstrukcija pod dejstvom cikličnih opterećenja
- primena numeričkih metoda u razvoju aplikativnog softvera za procenu veka elemenata konstrukcija
- poređenje rezultata proračuna sa eksperimentalno dobijenim rezultatima

A1 Boljanović S., Maksimović S. *Mixed mode crack growth simulation with/without overloads*, International Journal of Fatigue ISSN 0142-1123. DOI:<http://dx.doi.org/10.1016/j.ijfatigue.2013.11.011>. (IF = 1.976 (1.974)).  $M_{21}=8$

A2 Boljanović S., Maksimović S. *Fatigue crack growth modeling of attachment lugs*, International Journal of Fatigue 58(1), 2014, ISSN 0142-1123, pp. 66-74. (IF = 1.976 (1.974)).  $M_{21}=8$

A3 Boljanović S., Maksimović S. *Analysis of the crack growth propagation process under mixed mode loading*. Engineering Fracture Mechanics 78(8), May 2011, ISSN 0013-7944, pp.1565-1576. (2011-IF = 1.353 (1.776)).  
 $M_{21}=8$

**Radovi štanpani u vodećim časopisima nacionalnog značaja ( $M_{51}$ )**

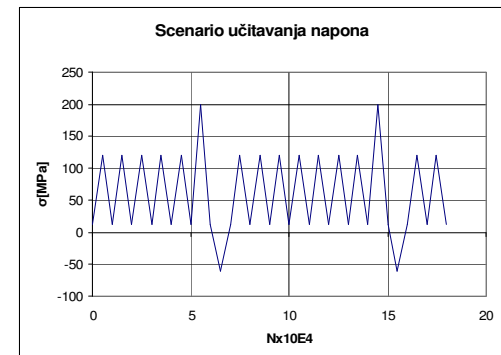
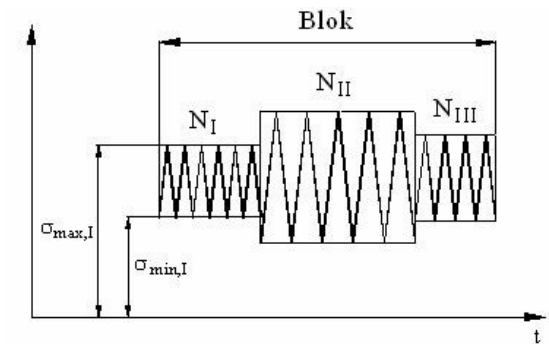
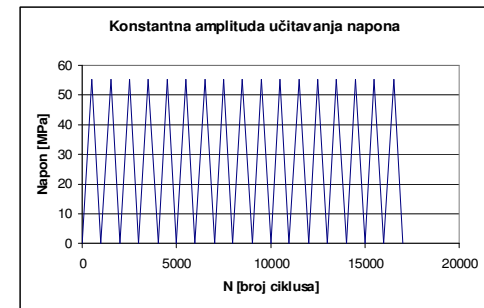
**B1 Boljanović S., Maksimović S., Carpinteri A. *Fatigue life evaluation of damaged aircraft lugs*. Scientific Technical Review 63(4), 2013, ISSN 1820- 0206, pp.3-9.  $M_{51} = 2$**

**B2 Boljanović S. *Fatigue strength analysis of a semi-elliptical surface crack*, Scientific Technical Review 62(1), 2012, ISSN 1820- 0206, pp. 10-16.  $M_{51} = 2$**

**B3 Boljanović S. Maksimović, S., Carpinteri A. *An analysis of crack propagation and a plasticity-induced closure effect*. Scientific Technical Review 60(2), 2010, ISSN 1820- 0206, pp. 14-19.  $M_{51} = 2$**

*Po svojoj prirodi ciklična opterećenja, bar kada se radi o opisivanju ponašanja aviona u letu, najčešće mogu biti razmatrana kao:*

- opterećenja konstantne amplitude
- opterećenja promenljive amplitude u vidu “stepenastog” spektra opterećenja
- oprerećenja promenljive amplitude sa pojedinačnim “pikovima”, tj. ciklusima sa povećanim ili smanjenim intezitetima opterećenja





# NUMERIČKE METODE ZA PROCENU VEKA ELEMENTA KONSTRUKCIJA DO POJAVE INICIJALNOG OŠTEĆENJA

## Krive malociklusnog zamora

- Morrow-ova kriva malociklusnog zamora je oblika

$$\frac{\Delta \epsilon}{2} = \frac{\sigma'_f - \sigma_m}{E} N_f^b + \epsilon'_f N_f^c$$

- Manson-Halford-ova kriva malociklusnog zamora:

$$\frac{\Delta \epsilon}{2} = \frac{\sigma'_f - \sigma_m}{E} N_f^b + \left( \frac{\sigma'_f - \sigma_m}{\sigma'_f} \right)^{\frac{c}{b}} \epsilon'_f N_f^c$$

- Smith-Watson-Topper -ova kriva malociklusnog zamora

$$P_{\text{SWT}} = \sqrt{\sigma_{\text{max}} \frac{\Delta \epsilon}{2} E} = \sqrt{(\sigma'_f)^2 (N_f)^{2b} + E \sigma'_f \epsilon'_f (N_f)^{b+c}}$$

$$\sigma_{\text{max}} = \sigma_m + \frac{\Delta \sigma}{2}$$


**Racunanje broja ciklusa do pojave oštećenja**

|                 |                                     |              |                                    |              |                                |              |                                 |           |                                |
|-----------------|-------------------------------------|--------------|------------------------------------|--------------|--------------------------------|--------------|---------------------------------|-----------|--------------------------------|
| <b>Sigmaf</b>   | <input type="text" value="241"/>    | <b>c</b>     | <input type="text" value="-0.83"/> | <b>Smin1</b> | <input type="text" value="1"/> | <b>Smax1</b> | <input type="text" value="10"/> | <b>n1</b> | <input type="text" value="1"/> |
| <b>Epsilonf</b> | <input type="text" value="0.158"/>  | <b>Kt</b>    | <input type="text" value="4.27"/>  | <b>Smin2</b> | <input type="text"/>           | <b>Smax2</b> | <input type="text"/>            | <b>n2</b> | <input type="text" value="1"/> |
| <b>E</b>        | <input type="text" value="10000"/>  | <b>Kprim</b> | <input type="text" value="101"/>   | <b>Smin3</b> | <input type="text"/>           | <b>Smax3</b> | <input type="text"/>            | <b>n3</b> | <input type="text" value="1"/> |
| <b>b</b>        | <input type="text" value="-0.149"/> | <b>nprim</b> | <input type="text" value="0.04"/>  | <b>Smin4</b> | <input type="text"/>           | <b>Smax4</b> | <input type="text"/>            | <b>n4</b> | <input type="text" value="1"/> |

SWT

Izracunaj

Briši

Broj blokova

|   | Maksimalan napon | Minimalan napon | sig1        | sig2         | BrojCiklusa      | Izabrani test |
|---|------------------|-----------------|-------------|--------------|------------------|---------------|
|   | 30               | 3               | 84.33660960 | -30.94525844 | 3757.88606793471 | Morrow        |
|   | 30               | 3               | 84.33660960 | -30.94525844 | 2212.61465106022 | SWT           |
|   | 25               | 2.5             | 82.11994485 | -13.95500068 | 9600.22607454948 | Morrow        |
|   | 25               | 2.5             | 82.11994485 | -13.95500068 | 4346.35856301142 | SWT           |
|   | 20               | 2               | 77.98038412 | 1.120371223  | 34620.7158227465 | Morrow        |
|   | 20               | 2               | 77.98038412 | 1.120371223  | 10708.5606854795 | SWT           |
|   | 15               | 1.5             | 63.99449822 | 6.346365517  | 270369.66677516  | Morrow        |
|   | 15               | 1.5             | 63.99449822 | 6.346365517  | 53704.1977763114 | SWT           |
|   | 10               | 1               | 42.70000301 | 4.269822605  | 5917231.26032748 | Morrow        |
| ▶ | 10               | 1               | 42.70000301 | 4.269822605  | 808129.28290633  | SWT           |
| * |                  |                 |             |              |                  |               |

**Program za računanje broja ciklusa do pojave oštećenja omogućava korisniku izbor kriterijuma za procenu veka. Ponuđeni su Morrow-ov I SWT kriterijum.**

# Rezultati računanja broja ciklusa pri širenju prskotine na epruveti **P1-CCT-2219** sa otvorom koji ima jednu prskotinu

## Direktno rešavanje diferencijalne jednačine

| N     | DeltaSigma       | DeltaK           | Kmax             | a                |
|-------|------------------|------------------|------------------|------------------|
| 800   | 55.16            | 140.38106221966  | 140.38106221966  | 2.06166767103115 |
| 1600  | 55.16            | 142.5355700402   | 142.5355700402   | 2.12542645962429 |
| 2400  | 55.16            | 144.727144432275 | 144.727144432275 |                  |
| 3200  | 55.16            | 146.956426708168 | 146.956426708168 |                  |
| 4000  | 55.16            | 149.224069233573 | 149.224069233573 |                  |
| 4800  | 55.16            | 151.530735618326 | 151.530735618326 |                  |
| 5600  | 55.16            | 153.877100910438 | 153.877100910438 |                  |
| 6400  | NUkupno          |                  | a0               | a1               |
| 7200  | 1282.95554721385 |                  | 2                | 2.1              |
| 8000  | 2502.7314072725  |                  | 2.1              | 2.2              |
| 8800  | 3665.18401541138 |                  | 2.2              | 2.3              |
| 9600  | 4775.39612887147 |                  | 2.3              | 2.4              |
| 10400 | 5837.80691121492 |                  | 2.4              | 2.5              |
| 11200 | 6856.31583815177 |                  | 2.5              | 2.6              |
| 12000 | 7834.36649258534 |                  | 2.6              | 2.7              |
| 12800 | 8775.01473865834 |                  | 2.7              | 2.8              |
| 13600 | 9680.98463842215 |                  | 2.8              | 2.9              |
| 14400 | 10554.7146599741 |                  | 2.9              | 3                |
| 15200 | 11398.3961289534 |                  | 3                | 3.1              |
| 16000 | 12214.0054328053 |                  | 3.1              | 3.2              |
| 16800 | 13003.3311556866 |                  | 3.2              | 3.3              |
| 17600 | 13767.9970709697 |                  | 3.3              | 3.4              |
| *     | 14509.4817266175 |                  | 3.4              | 3.5              |
|       | 15229.1352109794 |                  | 3.5              | 3.6              |
|       | 15928.1935717826 |                  | 3.6              | 3.7              |
|       | 16607.7912712171 |                  | 3.7              | 3.8              |
|       | 17268.9719891417 |                  | 3.8              | 3.9              |
|       | 17912.698030147  |                  | 3.9              | 4                |

**P1-CCT-2219-T851-Njutnovu pravilo za integraciju**

**Ulazni podaci**

E: 71000      ln': 3.067      W: 26

SigmaF: 613      Psi: 0.95152      a0: 2

EpsilonF': 0.35      DeltaKth0: 8      SigmaMax: 55.16

n': 0.121      Kc: 380      SigmaMin: 0

Korak: 0.1      Izracunaj integral      Brisi      b: 6

**Rezultati**

DeltaSigma: 55.16

R: 0

ac: 4

C1: 4.4755474028841072269E-09

Konstanta: 8

|                  | NUkupno | a0  | a1                | a/b             | Kl               | Klmax            | Alfa |
|------------------|---------|-----|-------------------|-----------------|------------------|------------------|------|
| 0                | 2       | 2.1 | 0.333333333333333 | 138.26561562864 | 138.265615628646 | 1.04104115226337 |      |
| 1316.72246764456 | 2.1     | 2.2 | 0.35              | 141.68009559718 | 141.680095597183 | 1.0428381729375  |      |
| 2567.04006539822 | 2.2     | 2.3 | 0.366666666666666 | 145.01420106901 | 145.014201069017 | 1.04411495130453 |      |
| 3757.24753992615 | 2.3     | 2.4 | 0.383333333333333 | 148.27335409622 | 148.273354096222 | 1.04509312497248 |      |
| 4892.79025502987 | 2.4     | 2.5 | 0.4               | 151.46239321429 | 151.462393214296 | 1.045981968      |      |
| 5978.40990586784 | 2.5     | 2.6 | 0.416666666666666 | 154.58565774825 | 154.585657748255 | 1.0469760003215  |      |
| 7018.26034244001 | 2.6     | 2.7 | 0.433333333333333 | 157.64705707598 | 157.647057075985 | 1.04825259543621 |      |
| 8016.00055821112 | 2.7     | 2.8 | 0.45              | 160.65012800485 | 160.650128004859 | 1.0499695924375  |      |
| 8974.87003251481 | 2.8     | 2.9 | 0.466666666666666 | 163.59808266369 | 163.598082663691 | 1.05226290225514 |      |
| 9897.75029176301 | 2.9     | 3   | 0.483333333333333 | 166.49384875872 | 166.493848758728 | 1.05524411823791 |      |
| 10787.2156028216 | 3       | 3.1 | 0.5               | 169.34010363098 | 169.340103630985 | 1.058998125      |      |
| 11645.5750186955 | 3.1     | 3.2 | 0.516666666666666 | 172.13930324307 | 172.139303243074 | 1.06358070755684 |      |
| 12474.907485585  | 3.2     | 3.3 | 0.533333333333333 | 174.89370698875 | 174.893706988758 | 1.06901616046091 |      |
| 13277.0913393467 | 3.3     | 3.4 | 0.55              | 177.60539903822 | 177.605399038227 | 1.0752948969375  |      |
| 14053.8292323637 | 3.4     | 3.5 | 0.566666666666666 | 180.27630679261 | 180.276306792619 | 1.08237105802058 |      |
| 14806.6693135058 | 3.5     | 3.6 | 0.583333333333333 | 182.90821691232 | 182.908216912321 | 1.09016012168853 |      |
| 15537.0233163017 | 3.6     | 3.7 | 0.6               | 185.50278929790 | 185.502789297906 | 1.098536512      |      |
| 16246.1820807452 | 3.7     | 3.8 | 0.616666666666666 | 188.06156933451 | 188.061569334518 | 1.10733120822968 |      |
| 16935.3289329735 | 3.8     | 3.9 | 0.633333333333333 | 190.58599865630 | 190.585998656302 | 1.11632935400412 |      |
| 17605.5512675243 | 3.9     | 4   | 0.65              | 193.07742464379 | 193.07742464379  | 1.1252678664375  |      |

$$\int_{x_0}^{x_3} f(x) dx = \frac{3h}{8} (f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)) - \frac{3h^5}{80} f^{(4)}(\xi); (x_0 < \xi < x_3)$$

## Gauss-Legendre-ova formula

## **Primer kablovski vođenog robotskog sistema: CABLE-SUSPENDED PARALLEL ROBOT TYPE C (CPR-C System)**

[www.zl50.com/Article.aspx?wd=Off...](http://www.zl50.com/Article.aspx?wd=Off...)

**Ljubinko Kevac, Innovation Center of School of Electrical Engineering,  
Bulevar Kralja Aleksandra 73, 11000 Belgrade, Serbia,  
[ljubinko.kevac@ic.etf.rs](mailto:ljubinko.kevac@ic.etf.rs), [ljubinko.kevac@gmail.com](mailto:ljubinko.kevac@gmail.com),**

**Mirjana Filipovic, Mihajlo Pupin Institute, The University of Belgrade,  
Volgina 15, 11060 Belgrade, Serbia, [mira@robot.imp.bg.ac.rs](mailto:mira@robot.imp.bg.ac.rs)**



## Matematički model sistema CPR-C

$p = \begin{bmatrix} x & y & z \end{bmatrix}^T$  spoljašnje koordinate, pozicije kamere u 3D prostoru

$\phi = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 \end{bmatrix}^T$  unutrašnje koordinate, angularne pozicije motora

- potrebno je naći vezu između unutr. i spolj. koord. što daje Jakobijeva matrica – vidi se da je matrica puna, što znači da postoji veliko kuplovanje između unutr. i spolj. koordinata

$$\dot{\phi} = J_c \cdot \dot{p}$$

$$J_c = \begin{bmatrix} J_{c11} & J_{c12} & J_{c13} \\ J_{c21} & J_{c22} & J_{c23} \\ J_{c31} & J_{c32} & J_{c33} \end{bmatrix}$$



## Matematički model sistema CPR-C

$$E_k = \frac{1}{2} \cdot m \cdot \dot{x}^2 + \frac{1}{2} \cdot m \cdot \dot{y}^2 + \frac{1}{2} \cdot m \cdot \dot{z}^2 \quad E_p = m \cdot g \cdot z$$

- Kinetička i potencijalna energija

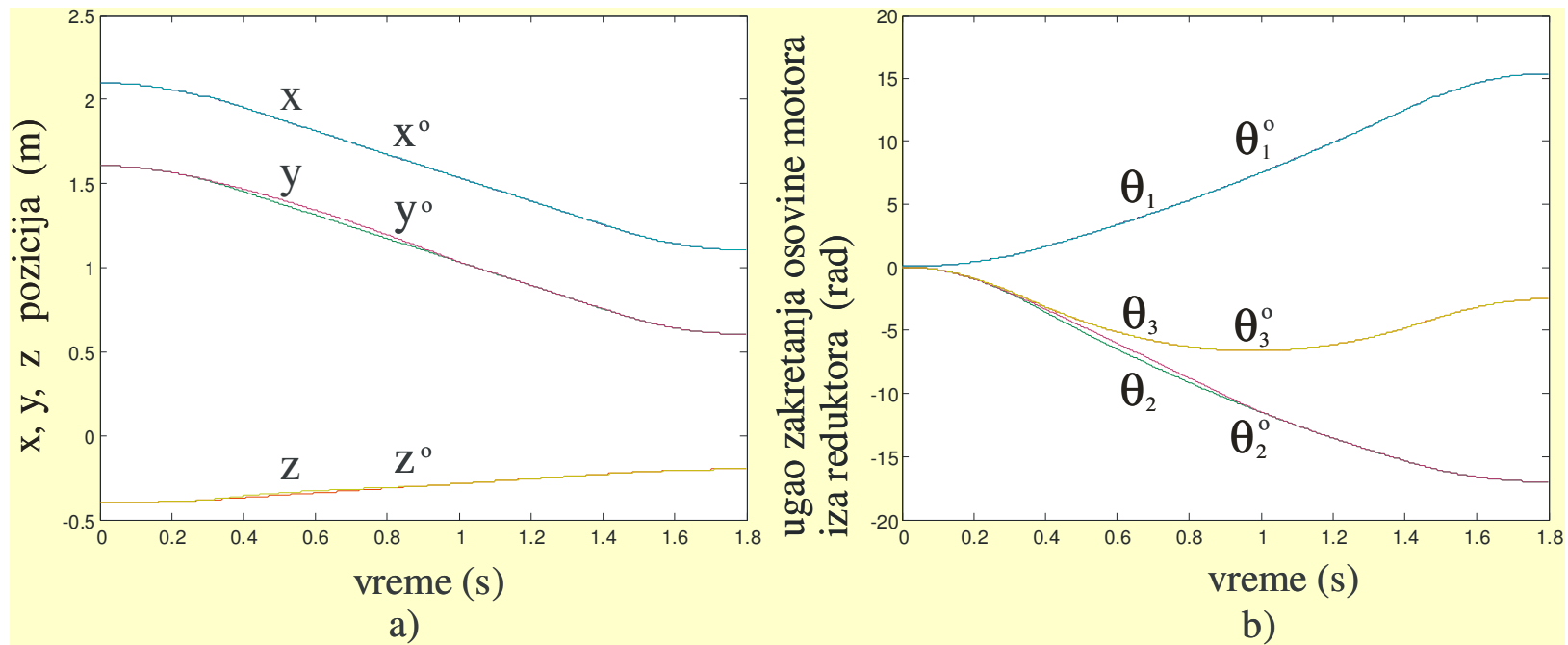
$$u = G_v \cdot \ddot{\phi} + L_v \cdot \dot{\phi} + S_v \cdot R \cdot C_c \cdot F$$

- Dinamički model sistema, gde su: u-vektor napona na motorima (upravljanja),  $G_v$  – dijagonalna matrica inercije tri motora,  $L_v$  – dijagonalna matrica prigušenja tri motora,  $S_v$  – dijagonalna matrica geometrijski karakteristika,  $F$  – spoljašnje sile,  $R$  – vektor poluprečnika tri čekrka,  $C_c$  – matrica koja povezuje spoljašnje i rezultatne sile.

$$u_i = K_{lpi} \cdot (\theta_i^o - \theta_i) + K_{lvi} \cdot (\dot{\theta}_i^o - \dot{\theta}_i)$$

- Za proračun upravljačkih signala se koriste PD regualtori

## Simulacioni rezultati



referentna i realna trajektorija

$$p_{start}^o = [2.1 \quad 1.6 \quad -0.4](m)$$

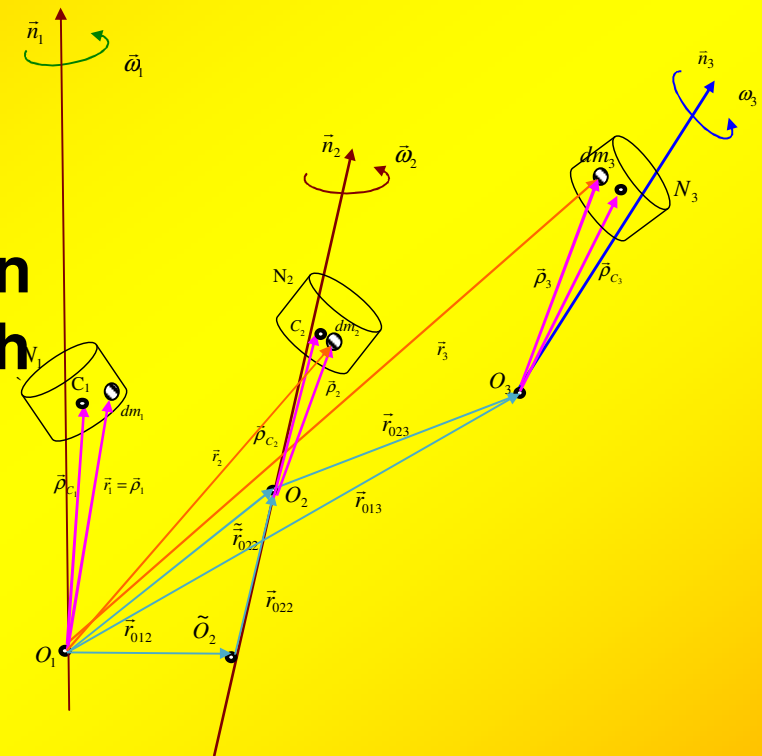
početna tačka

$$p_{end}^o = [1.1 \quad 0.6 \quad -0.2](m)$$

ciljana tačka

Katica R. (Stevanović) Hedrih,  
Marija B. Stamenković,

## Mass moment vector application to the multi body dynamics with coupled rotations about no intersecting axes

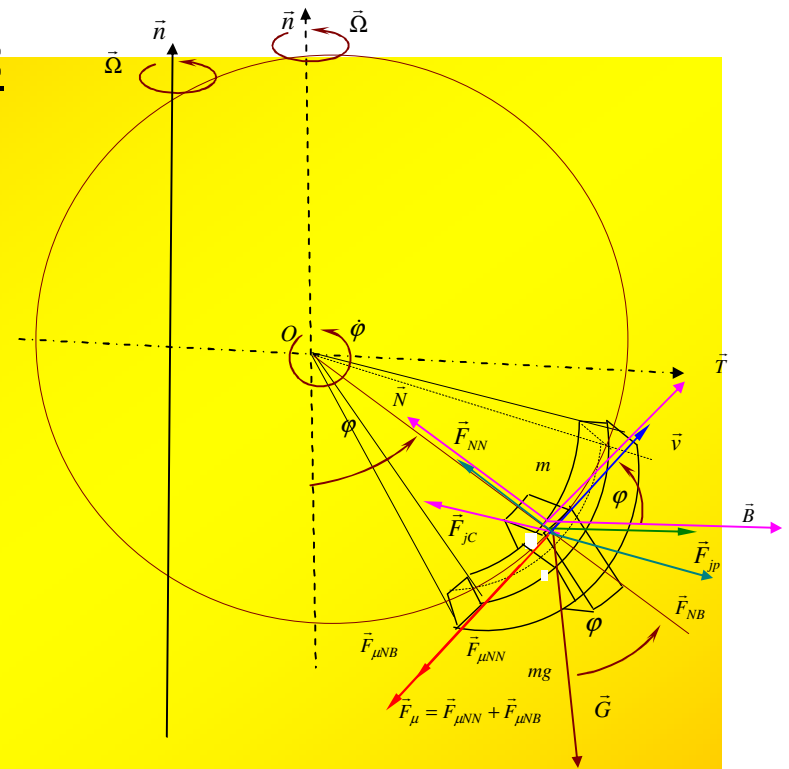


- Систем са коначним бројем крутих тела налазе се на осама које се не секу у односу једна на другу.
- Изводи по времену количине кретања и момента количине кретања, коришћењем вектора момента масе и вектор ротаторе за пол и осу су одређени.
- Дефинисан је број теорема.

**Marija B. Stamenković , Marija D. Mikić**

**4th International Congress of Serbian  
Society of Mechanics, IconSSM 2013**

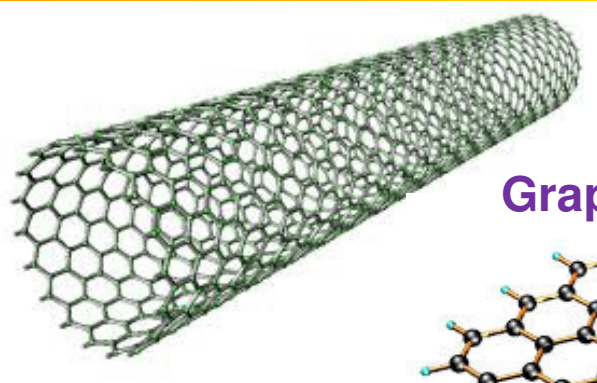
**Testing of singularity and position of non-linear dynamics relative equilibrium of heavy material particle on eccentrically rotating rough circle line, with constant angular velocity**



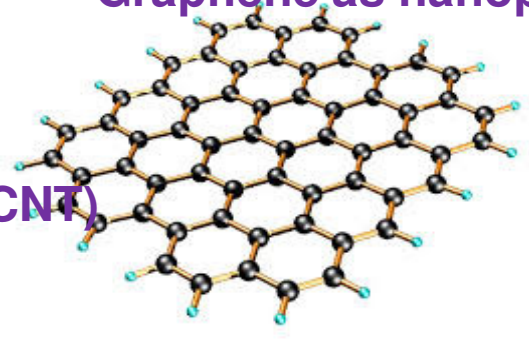
- Рад садржи аналитичко описивање материјалне честице која се креће по ротирајућој кружној храпавој линији, која ротира око вертикалне осе ексцентрично постављене у односу на центар кружне линије на растојању  $e$ , угаоне брзине  $\Omega$ .
- У раду се користи софтвер GeoGebra и врши се тестирање сингуларитета и положаја нелинеарне динамике релативне равнотеже тешке материјалне тачке на ексцентрично ротирајућој храпавој кружној линији.

# Истраживачи-сарадници Данило Карличић, Милан Цајић

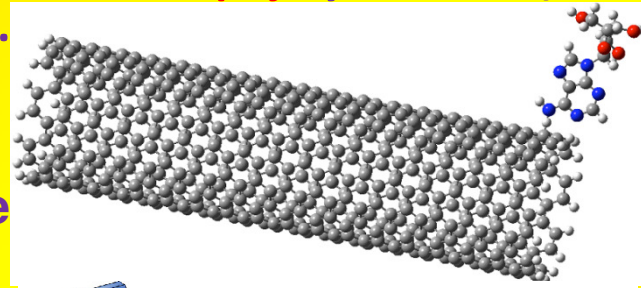
•Истраживања у области примене аналитичких метода линеарне и нелинеарне механике и Ерингенове нелокалне теорије у механици нано-структура.



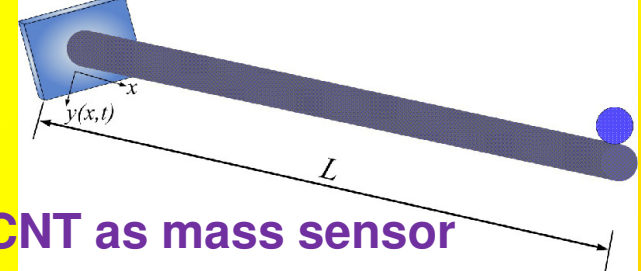
Carbon nanotube (CNT)  
as nanobeam



Graphene as nanoplate



CNT as mass sensor



•Прихваћена два коауторска рада за презентацију на [8th European Nonlinear Dynamics Conference](#), 6-11 jula, Beč.

•У првом раду под насловом „**Nonlocal axial vibration of a fractional order viscoelastic nanorod**“ примењује се фракциони модел вискоеластичности и нелокална теорија за испитивање слободних лонгитудиналних осцилација једнозидне угљеничне наноцеви (SWCNT)

•У другом раду „**Nonlinear vibration of nonlocal Kelvin-Voigt viscoelastic nanobeam embedded in elastic medium**“ испитују се слободне и принудне нелинеарне осцилације модела **наногреде** у медијуму са нелинеарним еластичним својствима.



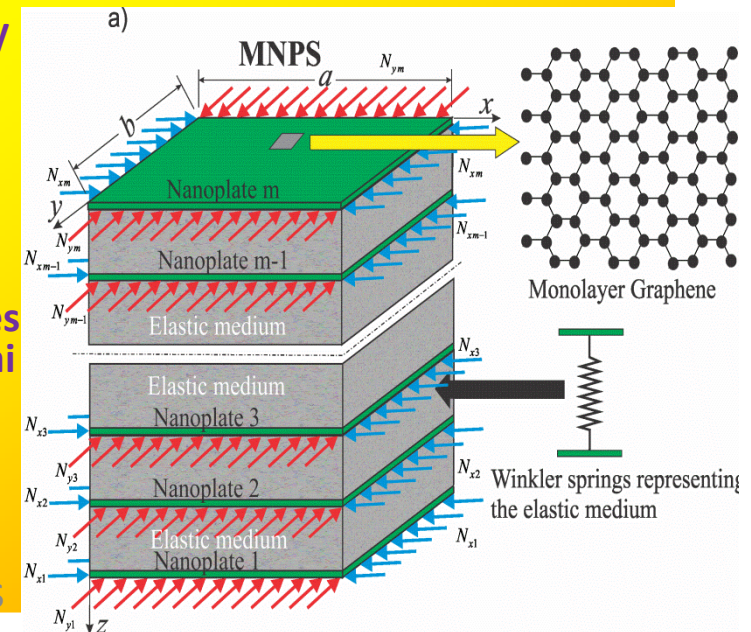
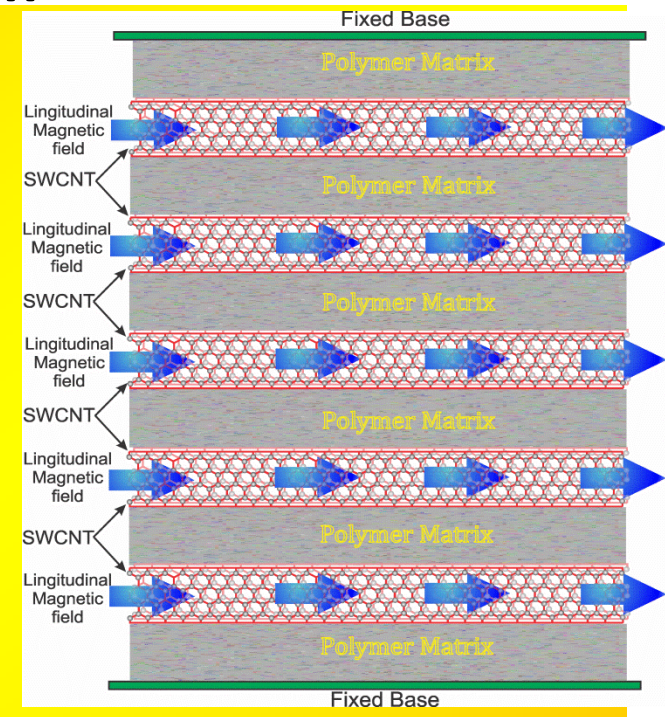
# Данило Карличић, Милан Цајић

Међународна сарадња са проф. др. **Sondipon Adhikari**, Swansea University, др. **Tony Murmu**, University of the West of Scotland у 2013. и 2014. години и **Xiao-Jun Yang**, China University of Mining and Technology, Xuzhou у 2013. години.

Коауторски радови на рецензији:

- Longitudinal vibration of a nonlocal viscoelastic double nanorod system coupled by a light viscoelastic layer
- Nonlocal longitudinal vibration of a complex multi-nanorod system
- Exact closed-form solution for non-local vibration and buckling of bonded multi-nanoplate-system
- Dynamics of multiple viscoelastic carbon nanotube based nanocomposites with axial magnetic field
- Fractional order spring-damper/actuator element in a multibody system: application of an expansion formula
- Free transverse vibration of nonlocal viscoelastic orthotropic multi-nanoplate system embedded in a viscoelastic medium
- Axial vibrations of a nonlocal viscoelastic coupled multi-nanorod system
- Flexural vibration and buckling of single-walled carbon nanotubes using different gradient elasticity theories for Reddy and Huu-Thai beam theories

**Часописи:** European Journal of Mechanics, Journal of Applied Physics, Composites Part B, Composite Structures, Journal of Vibration and Control, International Journal of Engineering Science, Mechanics Research Communications





## Публиковани радови и планови за будућа истраживања

Радови публиковани у **2013.** години у часописима **међународног значаја:**

- Yang, X. J., Baleanu, D., Lazarević, M. P., & **Čajić, M. S.** (2013). Fractal boundary value problems for integral and differential equations with local fractional operators. Thermal Science, (00), 103-103. **M23**
- Kozić, P., Pavlović, R., & **Karličić, D.** (2014). The flexural vibration and buckling of the elastically connected parallel-beams with a Kerr-type layer in between. Mechanics Research Communications, 56, 83-89. **M22**

Радови публиковани у **2013.** години у часописима **националног значаја:**

- **Čajić, M. S.**, D., Lazarević, M. P. (2014). Determination of joint reaction forces in rigid multibody system, two different approaches. Accepted for publication in FME Transactions, 42, 141-150. **M51**

### Планови за даља истраживања:

Примена аналитичних и нумеричких метода у истраживању нелинеарних (осцилација) и мултифизичких феномена (утицај магнетног поља, провођење топлоте, лом итд.) и механичких својстава нано-структура. Примена фракционог рачуна и нелокалне теорије у моделирању комплексних нано-структура. Нумеричка симулација стохастичких процеса и молекуларне динамике нано-структура.



8<sup>th</sup> European Nonlinear Dynamics Conference  
July 6 – 11, 2014, Vienna, Austria



TECHNISCHE  
UNIVERSITÄT  
WIEN  
Vienna University of Technology

EUROPEAN  
MECHANICS  
SOCIETY



EUROPEAN  
MECHANICS  
SOCIETY



## Spisak radova prihvacenih za ENOC 2014

### ID: 326 Plenary session

Title: **Elements of mathematical phenomenology and qualitative /mathematical analogies on the basis of generalized Lissajous curves**

Katica (Stevanović) Hedrih, Julijana Simonović, Ana Ivanović-Šešić, Ljiljana Kolar Anić, Željko Cupić and Andjelka N Hedrih

Presenting Author: Katica (Stevanović) Hedrih

### ID: 325 MS-09 Nonlinear Dynamics of Structural and Machine Elements

Title: **Petrović's theory of elements of mathematical phenomenology and phenomenological mapping applied to system nonlinear dynamics**

Authors: Katica (Stevanović) Hedrih and Andjelka N Hedrih, Presenting Author: Andjelka N Hedrih

### ID: 237 MS-13 Nonlinear Dynamics in Biological Systems

Title: **Synchronization in oscillatory model of embryo's ZP molecules in context of polyspermy block**

Author(s) : Simonović Julijana; Hedrih, Andjelka , Presenting Author : Simonović Julijana

### Contribution ID: 438 Type: MS-09 Nonlinear Dynamics of Structural and Machine Elements

Title: **Rigid Body Coupled Rotation around Axes without Intersection**

Author(s): Veljović, Ljiljana , Presenting Author : Veljović, Ljiljana

### Contribution ID: 223 Type : GT General Track Contribution

Title: **Nonlinear vibration of nonlocal Kelvin-Voigt viscoelastic nanobeam embedded in elastic medium**

Author(s): Karličić, Danilo Z.; Cajić, Milan S.; Stamenković, Marija, Presenting Author: Karličić Danilo

### Contribution ID: 271 Type: MS-06 Fractional Derivatives

Title: **"Nonlocal axial vibration of a fractional order viscoelastic nanorod"**

Authors: Milan S. Cajić, Danilo Z. Karličić and Mihailo P. Lazarević , Presenting Author: Milan S. Cajić,

### ID: 199

Title: **The complex motion of Cable-suspended parallel robot under the influence of the disturbance**

Author(s): Kevac :jubomlp and Mprjana Filipović

Presenting Author: Kevac Ljubinko

### ID: 386 Poster

Title: **Three parametric testing of singularity and position of non-linear dynamics relative equilibrium of heavy material particle on eccentrically rotating rough circle line, with constant angular velocity**

Authors: Marija Mikić and Marija Stamenković







8<sup>th</sup> European Nonlinear Dynamics Conference  
July 6 – 11, 2014, Vienna, Austria



TECHNISCHE  
UNIVERSITÄT  
WIEN  
Vienna University of Technology

## Spisak radova prihvacenih za ENOC 2014

### ID: 326 Plenary session

Title: Elements of mathematical phenomenology and qualitative /mathematical analogies on the basis of generalized Lissajous curves

Katica (Stevanović) Hedrih, Juliajna Simonović, Ana Ivanvić-Šesić, Ljiljana Kolar Anić, Željko Cupić and Andjelka N Hedrih

Presenting Author: Katica (Stevanović) Hedrih

### ID: 325 MS-09 Nonlinear Dynamics of Structural and Machine Elements

Title: Petrović's theory of elements of mathematical phenomenology and

phenomenological mapping applied to system nonlinear dynamics

Authors: Katica (Stevanović) Hedrih and Andjelka N Hedrih

Presenting Author: Andjelka N Hedrih

## Spisak radova prihvacenih za ENOC 2014

### **ID: 237 MS-13 Nonlinear Dynamics in Biological Systems**

Title: Synchronization in oscillatory model of embryo's ZP molecules in context of polyspermy block

Author(s) : Simonović Julijana; Hedrih, Andjelka

Presenting Author : Simonović Julijana

### **Contribution ID: 438 Type: MS-09 Nonlinear Dynamics of Structural and Machine Elements**

Title: Rigid Body Coupled Rotation around Axes without Intersection

Author(s): Veljović, Ljiljana

Presenting Author : Veljović, Ljiljana

### **Contribution ID: 223 Type : GT General Track Contribution**

Title : Nonlinear vibration of nonlocal Kelvin-Voigt viscoelastic nanobeam embedded in elastic medium

Author(s): Karličić, Danilo Z.; Cajić, Milan S.; Stamenković, Marija

Presenting Author: Karličić Danilo

## Spisak radova prihvacenih za ENOC 2014

### **Contribution ID: 271 Type: MS-06 Fractional Derivatives**

**Title:** "Nonlocal axial vibration of a fractional order viscoelastic nanorod"

**Authors:** Milan S. Cajić, Danilo Z. Karličić and Mihailo P. Lazarević

**Presenting Author:** Milan S. Cajić,

### **ID: 199**

**Title:** The complex motion of Cable-suspended parallel robot under the influence of the disturbance

**Author(s):** Kevac :jubomlp and Mprjana Filipović

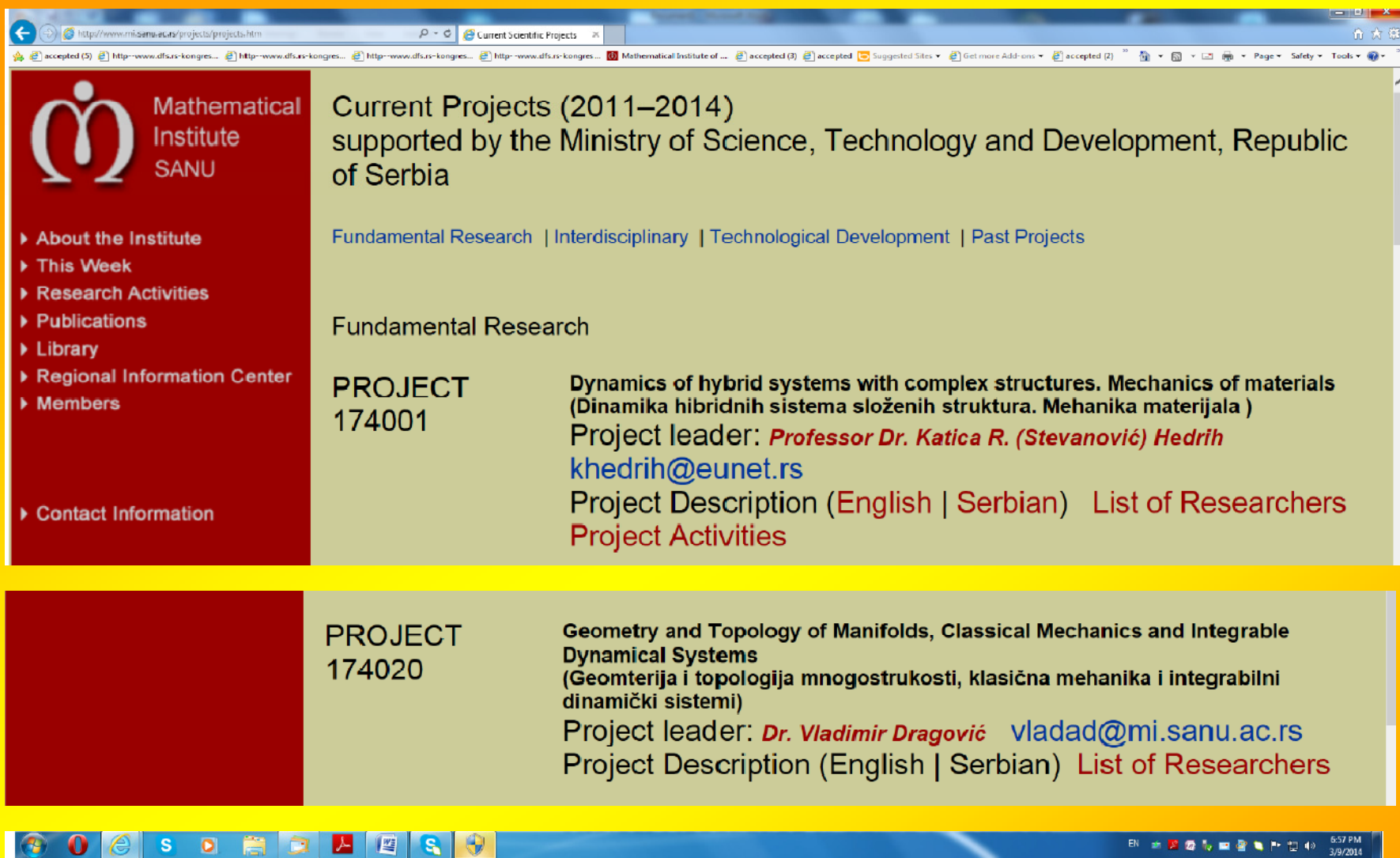
**Presenting Author:** Kevac Ljubinko

### **ID: 386 Poster**

**Title:** Three parametric testing of singularity and position of non-linear dynamics relative equilibrium of heavy material particle on eccentrically rotating rough circle line, with constant angular velocity

**Authors:** Marija Mikić and Marija Stamenković





**Mathematical Institute SANU**

- ▶ About the Institute
- ▶ This Week
- ▶ Research Activities
- ▶ Publications
- ▶ Library
- ▶ Regional Information Center
- ▶ Members
- ▶ Contact Information

## Current Projects (2011–2014)

supported by the Ministry of Science, Technology and Development, Republic of Serbia

[Fundamental Research](#) | [Interdisciplinary](#) | [Technological Development](#) | [Past Projects](#)

### Fundamental Research

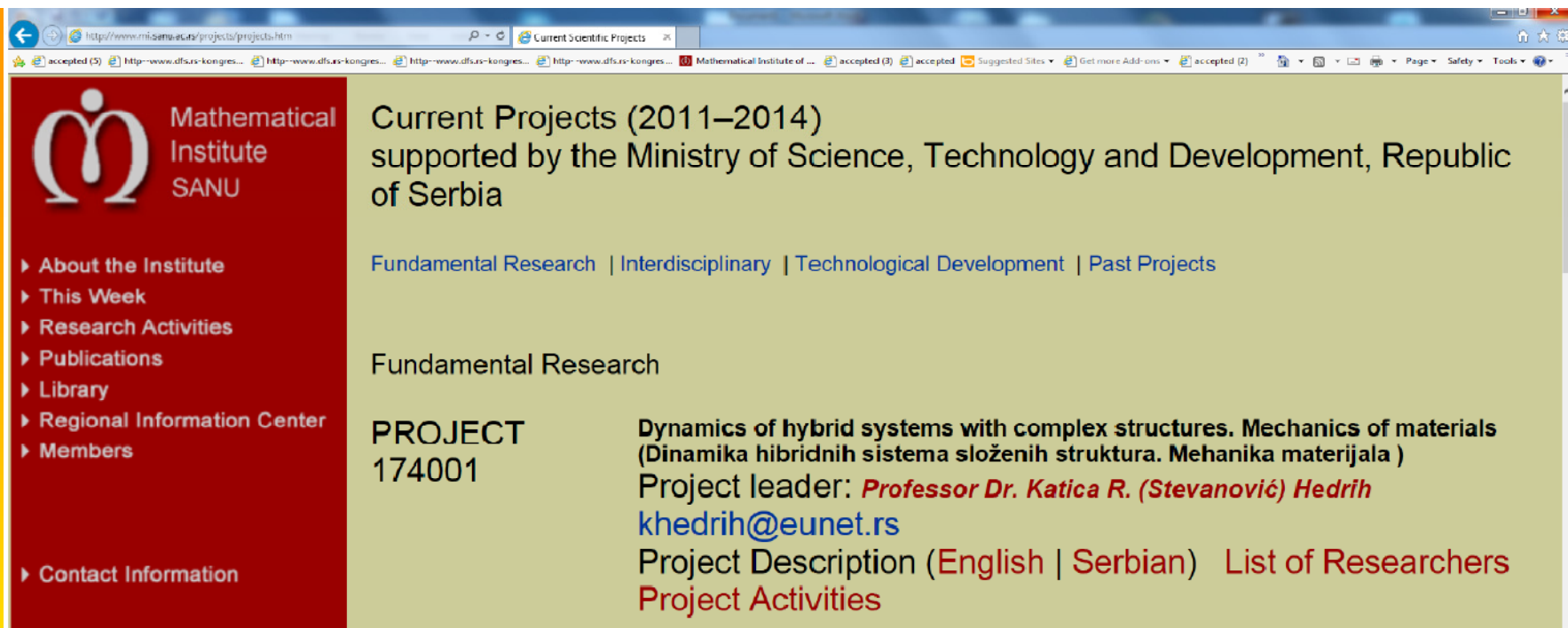
|                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>PROJECT 174001</b></p> | <p><b>Dynamics of hybrid systems with complex structures. Mechanics of materials (Dinamika hibridnih sistema slozenih struktura. Mehanika materijala)</b></p> <p>Project leader: <b>Professor Dr. Katica R. (Stevanović) Hedrih</b><br/> <a href="mailto:khedrih@eunet.rs">khedrih@eunet.rs</a></p> <p><a href="#">Project Description (English   Serbian)</a> <a href="#">List of Researchers</a><br/> <a href="#">Project Activities</a></p> |
| <p><b>PROJECT 174020</b></p> | <p><b>Geometry and Topology of Manifolds, Classical Mechanics and Integrable Dynamical Systems (Geomterija i topologija mnogostrukosti, klasična mehanika i integrabilni dinamički sistemi)</b></p> <p>Project leader: <b>Dr. Vladimir Dragović</b> <a href="mailto:vladad@mi.sanu.ac.rs">vladad@mi.sanu.ac.rs</a></p> <p><a href="#">Project Description (English   Serbian)</a> <a href="#">List of Researchers</a></p>                      |

**Projekat ON 174005**

**Viskoelasticnost frakcionog tipa I optimizacija oblika u teorji stapova**  
**Viscoelasticity fractional type and optimization in theory of rods**

**Project Leader Teodor Atanackovic in 2013**

**Project Leader in 2014 Dusan Zorica**



The screenshot shows a web browser window with the URL <http://www.mi.sanu.ac.rs/projects/projects.htm>. The page title is "Current Scientific Projects". The left sidebar contains a navigation menu for the Mathematical Institute SANU, including links to "About the Institute", "This Week", "Research Activities", "Publications", "Library", "Regional Information Center", "Members", and "Contact Information". The main content area is titled "Current Projects (2011–2014) supported by the Ministry of Science, Technology and Development, Republic of Serbia". It lists categories: "Fundamental Research", "Interdisciplinary", "Technological Development", and "Past Projects". Under "Fundamental Research", a specific project is highlighted: "PROJECT 174001" titled "Dynamics of hybrid systems with complex structures. Mechanics of materials (Dinamika hibridnih sistema slozenih struktura. Mehanika materijala)". The project leader is "Professor Dr. Katica R. (Stevanović) Hedrih" with email "khedrih@eunet.rs". Links for "Project Description (English | Serbian)", "List of Researchers", and "Project Activities" are provided.

## Radovi u 2013 vidljiva na KoBSON-u

Autori Hedrih (Stevanovic) Katica R

**44 rada**

M23=3 plus M24=3/2 Plus M21=8/2 Plus 3 rada u medjunarodnim casopisim koji nisu vidljivi na KOBSONU 2 troautorska I jedan I dva u PAMM

**Suma 14(8,5) u 2013 na KoBSON-u**

Plus chapter in Book , [;us drugi nevidljivi na Kobsonu

## Acknowledgement

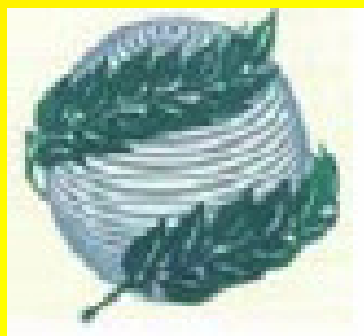
Parts of this research were supported by the Ministry of Sciences of Republic of Serbia through Mathematical Institute SANU Belgrade

*Grant OI174001 “Dynamics of hybrid systems with complex structures. Mechanics of Materials”*

and Faculty of Mechanical Engineering University of Niš.



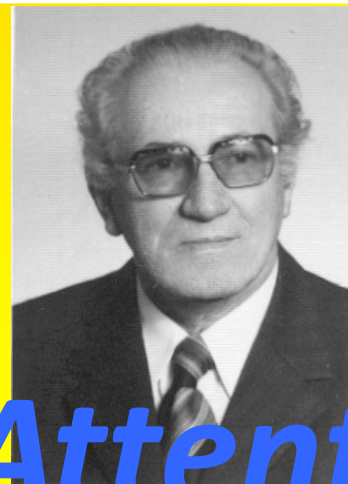
**Ukrainian Higher Education Academy of Sciences - Akademija nauka visokih skola i univerziteta Ukrajine**



[www.anvsu.org.ua](http://www.anvsu.org.ua)







Serbian Scientific Society

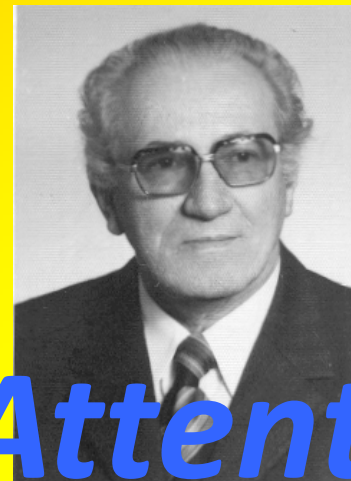
*Thank You for Attention!*



*Хвала на пажњи!*







*Thank you for Attention!*



*Хвала на пажњи!*

