

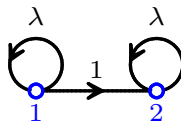
# 8. Jordan Canonical Form

# The Jordan Blocks

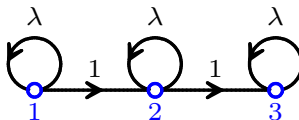
$$[\lambda]$$



$$\begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$$



$$\begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}$$

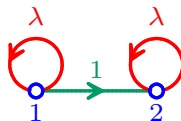


# The Jordan Blocks

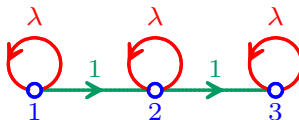
$$[\lambda]$$



$$\begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$$



$$\begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}$$

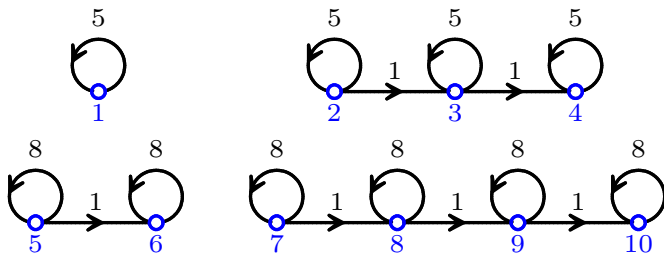


# Example 7.3.10 (p. 161)

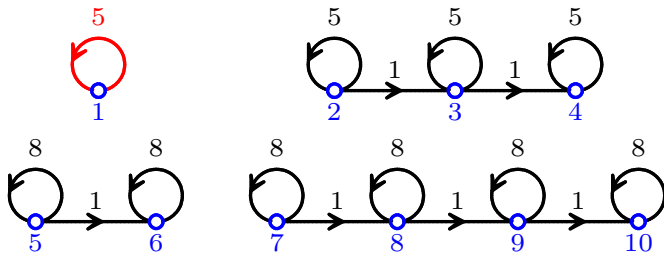
$$J = J_1(5) \oplus J_3(5) \oplus J_2(8) \oplus J_4(8)$$

$$J = \left[ \begin{array}{c|c|c|c} 5 & O & O & O \\ \hline O & \begin{array}{ccc} 5 & 1 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 5 \end{array} & O & O \\ \hline O & O & \begin{array}{cc} 8 & 1 \\ 0 & 8 \end{array} & O \\ \hline O & O & O & \begin{array}{cccc} 8 & 1 & 0 & 0 \\ 0 & 8 & 1 & 0 \\ 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 8 \end{array} \end{array} \right]$$

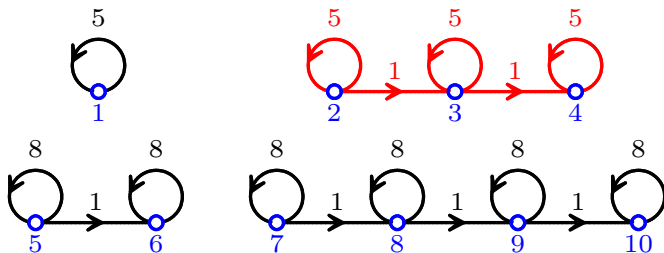
$$J = \left[ \begin{array}{c|c|c|c} 5 & O & O & O \\ \hline O & \begin{matrix} 5 & 1 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 5 \end{matrix} & O & O \\ \hline O & O & \begin{matrix} 8 & 1 \\ 0 & 8 \end{matrix} & O \\ \hline O & O & O & \begin{matrix} 8 & 1 & 0 & 0 \\ 0 & 8 & 1 & 0 \\ 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 8 \end{matrix} \end{array} \right]$$



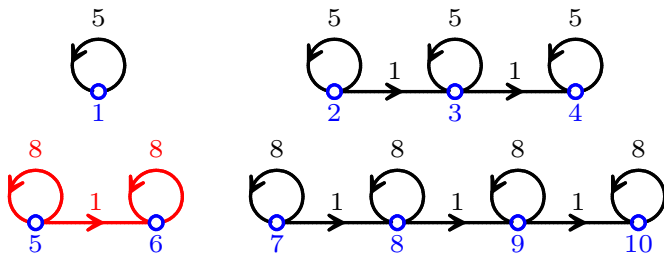
$$J = \begin{bmatrix} \overset{5}{\color{red}5} & O & O & O \\ \hline O & \begin{smallmatrix} 5 & 1 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 5 \end{smallmatrix} & O & O \\ \hline O & O & \begin{smallmatrix} 8 & 1 \\ 0 & 8 \end{smallmatrix} & O \\ \hline O & O & O & \begin{smallmatrix} 8 & 1 & 0 & 0 \\ 0 & 8 & 1 & 0 \\ 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 8 \end{smallmatrix} \end{bmatrix}$$



$$J = \left[ \begin{array}{c|c|c|c} 5 & O & O & O \\ \hline O & \begin{matrix} 5 & 1 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 5 \end{matrix} & O & O \\ \hline O & O & \begin{matrix} 8 & 1 \\ 0 & 8 \end{matrix} & O \\ \hline O & O & O & \begin{matrix} 8 & 1 & 0 & 0 \\ 0 & 8 & 1 & 0 \\ 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 8 \end{matrix} \end{array} \right]$$

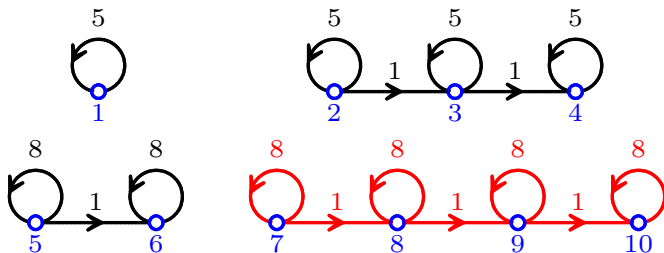


$$J = \left[ \begin{array}{c|cc|c|c} 5 & O & O & O \\ \hline & 5 & 1 & 0 \\ O & 0 & 5 & 1 \\ & 0 & 0 & 5 \\ \hline O & O & \begin{smallmatrix} 8 & 1 \\ 0 & 8 \end{smallmatrix} & O \\ \hline & & & \begin{smallmatrix} 8 & 1 & 0 & 0 \\ 0 & 8 & 1 & 0 \\ 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 8 \end{smallmatrix} \\ O & O & O & \end{array} \right]$$





$$J = \left[ \begin{array}{c|c|c|c} 5 & O & O & O \\ \hline & \begin{smallmatrix} 5 & 1 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 5 \end{smallmatrix} & O & O \\ \hline O & O & \begin{smallmatrix} 8 & 1 \\ 0 & 8 \end{smallmatrix} & O \\ \hline O & O & O & \begin{smallmatrix} 8 & 1 & 0 & 0 \\ 0 & 8 & 1 & 0 \\ 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 8 \end{smallmatrix} \end{array} \right]$$



The eigenvalues of  $J$  are  $5$  with algebraic multiplicity

$$1 + 3 = 4$$

and  $8$  with algebraic multiplicity

$$2 + 4 = 6.$$

The geometric multiplicity of  $5$  equals  $2$ , the number of Jordan blocks with  $5$  on the main diagonal;

the geometric multiplicity of  $8$  equals  $2$ , the number of Jordan blocks with  $8$  on the main diagonal.