

SOME REMARKS TO THE CLASS OF HQC DIFFEOMORPHISMS OF THE UNIT DISK

MILJAN KNEŽEVIĆ

We analyze the properties of harmonic quasiconformal mappings and by comparing some suitably chosen conformal metrics defined in the unit disc we obtain some geometrically motivated inequalities for those mappings (see for instance [1, 2, 3, 4]). In particular, we obtain the answers to many questions concerning these classes of functions which are related to the determination of different properties that are of essential importance for validity of the results such as those that generalize famous inequalities of the Schwarz-Pick type. The approach used is geometrical in nature, via analyzing the properties of the Gaussian curvature of the conformal metrics we are dealing with. As a consequence of this approach we give a note to the co-Lipschicity of harmonic quasiconformal self mappings of the unit disc at the origin.

Theorem. *Let f be a harmonic k -quasiconformal mapping of the unit disc \mathbb{D} onto itself and let $f(0) = 0$. Then, for all $z \in \mathbb{D}$ we have*

$$\frac{1}{K}|z| \leq |f(z)| \leq K|z|,$$

where $K = \frac{1+k}{1-k}$.

In [2] we obtained a result, similar to the result stated in the previous theorem, for the class of k -quasiconformal hyperbolic harmonic self diffeomorphisms of the unit disc \mathbb{D} , which fix the point $z = 0$. In particular, for a mapping f that belongs to such a class we get $2|z|/(K+1) \leq |f(z)| \leq \sqrt{K}|z|$, for all $z \in \mathbb{D}$, where $K = (1+k)/(1-k)$.

REFERENCES

- [1] M. Knežević, Some Properties of Harmonic Quasi-Conformal Mappings, Springer Proceedings in Mathematics and Statistics (LTAPH) 36 (2013) 531–539.
- [2] M. Knežević, On the Theorem of Wan for K -Quasiconformal Hyperbolic Harmonic Self Mappings of the Unit Disk, *Mathematica Moravica*, Vol 19.1 (2015) 81–85.
- [3] M. Knežević, M. Mateljević, On the quasi-isometries of harmonic quasiconformal mappings, *Journal Math. Anal. Appl.* 334/1 (2007) 404–413.
- [4] M. Knežević, A Note on Harmonic Quasiconformal Diffeomorphisms of the Unit Disc, *Filomat* 29/2 (2015) 335–341.
- [5] M. Mateljević, V. Božin, M. Knežević, Quasiconformality of Harmonic mappings between Jordan domains, *Filomat* 24:3 (2010) 111–124.

UNIVERSITY OF BELGRADE, FACULTY OF MATHEMATICS, STUDENTSKI TRG 16, 11000 BELGRADE, SERBIA
E-mail address: kmiljan@matf.bg.ac.rs