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CONTROL OF RESONANT OSCILLATIONS OF VISCOELASTIC SYSTEMS

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Abstract. Structures in the form of cylindrical ribbed shells and panels are widely used in engineering and construction. The problem of the action of moving loads on an infinitely long cylindrical shell, reinforced along the outer surface with longitudinal stiffeners and containing a viscoelastic inertial filler, is considered. The moving load is transferred to the shell only through the ribs, and there is no load outside the ribs. The discreteness of the location of the ribs is taken into account by writing the equations of motion of the beams, followed by the satisfaction of the conjugation conditions. The influence of the number and stiffness of ribs on the nature of the distribution of shell displacements and contact pressure at the boundary of a viscoelastic filler is shown. The movement of the shell is described by classic equations based on the Kirchhoff–Love hypothesis; for the filler, dynamic equations of the theory of visco-elasticity are used. It has been established that the reinforcement of shells with longitudinal ribs (oscillations of a cantilevered cylindrical shell) leads to a decrease in natural frequencies and damping coefficients in some shells, an increase in the density of the spectrum of natural frequencies, and the appearance of intermediate forms and forms with the same wave numbers, but with different frequencies. External forces increase natural frequencies and damping coefficients. It is found that the frequencies for the inner edges are lower than for the outer edges. In the high-frequency zone, any efforts reduce the natural frequencies and the damping coefficient. This means that additional mass plays a more significant role than additional rigidity. Consequently, the longitudinal strengthening of the shell worsens its dynamic properties.

1. Introduction

Structural elements in the form of cylindrical ribbed shells and panels are widely used in engineering and construction. Free vibrations of cylindrical panels have been considered previously [1–3], as have forced oscillations [3–5]; There is also an article devoted to the issues of optimal design of reinforced cylindrical panels [6]. Studies on the dynamics of cylindrical shells have been presented in previous papers [7, 8]. In addition, in a number of studies, the objects of studies are ribbed cylindrical

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shells with filler [9]. Some papers are devoted to the study of the dynamics of ribbed shells of revolution [10–12]. The limited volume of the present article does not allow even a brief description of all the papers in this area; therefore, below is a review of the studies on the dynamics of only circular cylindrical shells, supported by stringers and frames. A qualitative analysis of the frequency spectrum of circular cylindrical shells supported by stringers is given in [13, 14]. These papers show that a shell reinforced with longitudinal ribs has a discrete spectrum of natural frequencies. The natural frequencies of the shell, at which the middle surface of the shell and the stringer axis are divided into ï half-waves in the longitudinal direction, are located strictly individually between the frequencies of the mixed spectrum. In [15, 16], the modes of oscillations revealed experimentally are close to sinusoidal. It has been noticed that during the transition to a larger number of waves along the generatrix and the circle, intermediate forms of vibrations may appear. This is shown in [17] for the example of vibrations of a cantilevered cylindrical shell supported by four stringers. The conclusions of those papers are refuted in [18] and [19] on the band frequency spectrum of cylindrical shells supported by stringers. In this regard, new dynamic problems have arisen, and the requirements have increased in the direction of refining the mathematical model-a cylindrical shell, plus a moving body. Not only is the shell model specified (refined equations of motion and boundary conditions, geometric and physical non-linearity, heterogeneity, etc.) but models of contiguous objects (contact conditions, inertia of moving bodies, refinement of the equations of motion of the medium, nonlinearity, inhomogeneity, dissipation, thermal phenomena, etc.) are also specified [20, 21]. In [21], an axisymmetric oscillation of a cylindrical shell of finite length is considered under the action of a pressure wave uniformly distributed over the surface, moving at a constant speed. The solution is sought in the form of trigonometric angles in radians from the eigenfunctions of the cylinder oscillations. It was found that taking into account the transverse shear and rotational inertia reduced the value of the bending moment by 34%.

2. Methods

2.1. Problem Statements and Solution Methods. Consider an infinitely long circular cylindrical shell reinforced on the outer surface with longitudinal stiffeners and containing a viscoelastic inertial filler inside. The moving load is transferred to the shell only through the ribs; there is no load outside the ribs. The discreteness of the location of the ribs is taken into account by writing the equations of motion of the beams, followed by the satisfaction of the conjugation conditions. The effect of the number and stiffness of ribs on the nature of the distribution of shell displacements and contact pressure at the filler boundary is shown. The radius of the outer surface of the filler R_1 and internal R_0 in a linear-viscoelastic, homogeneous and isotropic mechanical system referred to a cylindrical r, θ, z coordinate system, the z-axis of which coincides with the axis of the cylindrical mechanical system. Due to the small thickness of the layers that make up the skin, it is assumed that they are in contact with the filler and the surrounding mass along their middle surfaces with radii R_1. The contact between the layers of the shell is assumed to be rigid or sliding. The non-axisymmetric motion of the shell is described by the equations. Here this sentence should be corrected as follows: The non-axisymmetric motion of the shell is described by the equations shell is described by the equations

$$\vec{U}_{ok} - \int_0^t R_{Ek} L_{ij}^{(k)} \vec{U}_{ok}(r,\theta,z,\tau)d\tau = \frac{(1-v_{ok}^2)}{G_{ok}h_{ok}}\vec{P}_r + \rho_{ok}\frac{(1-v_{ok}^2)}{G_{ok}}\frac{\partial^2 \vec{U}_{ok}}{\partial t^2}, \qquad (k=1,2). \tag{2.1}$$

Where

$$L^{(k)} = \begin{pmatrix} L_{11}^{(k)} & L_{12}^{(k)} & L_{13}^{(k)} \\ L_{21}^{(k)} & L_{22}^{(k)} & L_{23}^{(k)} \\ L_{31}^{(k)} & L_{32}^{(k)} & L_{33}^{(k)} \end{pmatrix}$$

for shells satisfying the Kirchhoff–Love hypotheses

$$\begin{aligned} &L_{11}^{(k)} = \frac{\partial^2}{\partial z^2} + \frac{1-v_{ok}}{R_k}\frac{\partial^2}{\partial t^2}; && L_{12}^{(k)} = L_{21}^{(k)} = \frac{1-v_{ok}}{2R_k}\frac{\partial^2}{\partial z\partial\theta}; \\ &L_{13}^{(k)} = L_{31}^{(k)} = \frac{v_{ok}}{R_k}\frac{\partial}{\partial z}; && L_{22}^{(k)} = \frac{1-v_{ok}}{R_k}\frac{\partial^2}{\partial z^2} + \frac{1}{R_k^2}\frac{\partial^2}{\partial\theta^2} - \rho_k\frac{1-v_{ok}}{R_k}\frac{\partial^2}{\partial t^2} \\ &L_{32}^{(k)} = L_{23}^{(k)} = -\frac{1}{R_k}\frac{\partial}{\partial\theta}; && L_{33}^{(k)} = \frac{1}{R_k} - \beta\cdot\nabla^4; \\ &\nabla^4 = \Big(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial\theta^2}\Big)^2, && \beta = \frac{H_k^2}{12R_k^2}. \end{aligned} \tag{2.2}$$

Here h_k, a_k are thickness and radius of the middle surface of a cylindrical shell, respectively; $R_{ek}(t-\tau)$ is the relaxation core; and v_{ok} is the instant modulus of elasticity. Index $k=1$ refers to the inner shell (or cylinder), $k=2$ refers to the outer shell, and $\vec{U}_{ok}$ is the displacement vector of points of the middle surface of the carrier layer. For Kirchhoff–Love shells, the displacement vector has a dimension equal to three. Here, P_k is the vector of dynamic (or static) loads acting on the shell. The dimension of the external load also depends on the chosen approximate theory (Kirchhoff–Love) of shells. The components of the vector of external loads for a cylindrical shell obeying the hypotheses of Kirchhoff–Love have the form:

$$\{P_{1k}, P_{2k}, P_{3k}\} = -\{p_{zk}+q_{zk}, p_{\theta k}+q_{\theta k}, p_{rk}+q_{rk}\}$$

where the minus sign depends on the choice of coordinate axes. In the present paper, for the inner shell, the load is taken with a minus sign, and for the outer shell, with a plus sign. Here $q_{zk}, q_{\theta k}, q_{rk}$ are the intensities of the external load, respectively, in directions z, θ, r.

The linear equation of motion of the filler of the considered mechanical system in vector form in the absence of body forces takes the form:

$$\lambda_{ok}\,\mathrm{garaddiv}\,\vec{U}(\vec{r},t) + \mu_{ok}(\nabla^2\vec{u}(\vec{r},t)) - \mathrm{garaddiv}\,\vec{U}(\vec{r},t) - \lambda_{ok} - \int_0^t R_{\lambda k}(t-\tau)\,\mathrm{garaddiv}\,\vec{U}(\vec{r},t)d\tau$$

$$-\mu_{ok}\int_0^t R_{\mu k}(t-\tau)(\nabla^2\vec{u}(\vec{r},\tau)\,\mathrm{graddiv}(\vec{r},\tau))d\tau = \rho_k\frac{\partial^2\vec{u}}{\partial t^2}, \quad (k=1,2,3,\dots,N)$$

where $\vec{r}=\vec{r}(x,y,z,t)$; $R_{\lambda k}(t-\tau)$, $R_{\mu k}(t-\tau)$ are relaxation kernels; $\lambda_{k\theta}, \mu_{k\theta}$ are the instantaneous modulus of elasticity; $\vec{u}$ is a displacement vector; ρ_k is the medium density; is the ordinal number of layers, and v_k is Poisson's ratio, which we consider to be a non-relaxing quantity [22].

It is assumed that integral terms in (2.1) are small; then, the integrand can be represented as $f(t)=\psi(t)e^{-i\omega_R t}$, where $\psi(t)$ is a slowly changing function of time and ω_R is a real constant. Next, applying the freezing procedure [23], we replace the integral relations with approximate ones of the form

$$\lambda_k[f]=\lambda_{\theta k}\left[1-\int_0^\infty R_{\lambda k}(\tau)\cos\omega_R\tau d\tau - i\int_0^\infty R_{\lambda k}(\tau)\sin\omega_R\tau d\tau\right]f,$$
$$\mu_k[f]=\mu_{\theta k}\left[1-\int_0^\infty R_{\mu k}(\tau)\cos\omega_R\tau d\tau - i\int_0^\infty R_{\mu k}(\tau)\sin\omega_R\tau d\tau\right]$$

where $\omega=\omega_R+i\omega_i$ is the complex frequency and is an arbitrary function of time. For convenience, the following designations are introduced: $U_{1k}=u_k$; $U_{2k}=v_k$; $U_{3k}=\omega_k$ for Kirchhoff–Love shells. The motion of the stiffeners obeys the equations of the theory of beams

$$\tilde{E}_{\delta k}I_k\frac{\partial^4 y_k}{\partial z^4}+\rho_{\delta k}F_k\frac{\partial^2 y_k}{\partial t^2}=p_{\delta k}(z,t)-q_{\delta k}(z,t), \tag{2.3}$$
$$\tilde{E}_{\delta k}[f]=E_{o\delta k}\left[1-\int_0^\infty R_{Ek}(\tau)\cos\omega_R\tau d\tau - i\int_0^\infty R_{Ek}(\tau)\sin\omega_R\tau d\tau\right]f,$$

where q_{ok} is the pressure from the side of the shell per linear unit of length k-the edge, $p_{\delta k}$ is the intensity of the normal load on the k-the beam, and l is the number of stiffeners.

The boundary conditions for the filler at $r=R_1$ are sliding contact between the filler and the shell, taking the following form:

$$\sigma_{rz}=\sigma_{r\theta}=0, \quad \sigma_{rr}=-q_c, u_r=w. \tag{2.4}$$

The inner surface of the filler is assumed to be stress-free; thus,

$$r=R_0; \quad \sigma_{rz}=\sigma_{r\theta}=\sigma_{rr}=0. \tag{2.5}$$

Assuming that the contact between the ribs and the shell occurs along straight lines (the axes of the beams), the conjugation conditions can be written as:

a) the external load on the shell is equal to the sum of the pressures transmitted through each rib:

$$p(z,\theta,t)=\sum_{k=1}^{\infty}q_{ok}(z,t)\delta(\theta-\theta_0); \tag{2.6}$$

b) at the points of contact $\theta-\theta_k$, the displacements of the shell are equal to the deflections of the beams, and since the displacements directed towards the convexity of the shell are considered positive, these conditions have the form:

$$\omega(z\theta_k, t) = -y_k(z,t) \qquad (k = 1, \dots, l). \tag{2.7}$$

For a constant speed of movement of loads and a steady process, the solution of the problem is sought in a moving coordinate system: $\eta = (z - ct)/H$, where H is the length dimension parameter. Applying the Fourier transform on p and expanding all given desired quantities in the Fourier series in terms of Equations (2.2) and (2.3) provide potential functions satisfying the wave equations. The solutions of these equations, at the speeds of movement of loads, are lower than the speeds of propagation of shear waves in the filler. Placeholder displacements are written using special Bessel and Neumann functions in the form:

$$\begin{aligned}
u^0_{xn}(r,\xi) &= i\frac{\xi}{R_1}K_n(m_1,r)A_n + i\frac{\xi}{R_1}I_n(m_1,r)B_n \\
&\quad + m^2_{s1}K_n(m_{s1}r)C_n + m^2_{s1}I_n(m_{s1}r)D_n, \\
u^0_{\theta n}(r,\xi) &= i\frac{\xi}{R_1}K_n(m_1,r)A_n + i\frac{\xi}{R_1}I_n(m_1,r)B_n + m^2_{s1}K_n(m_{s1}r)C_n \\
&+m^2_{s1}I_n(m_{s1}r)D_n\Big[\frac{n}{r}K_n(m_{s1}r) - m_{s1}K_{n+1}(m_{s1}r)\Big]E_n + \Big[\frac{n}{r}I_n(m_{s1}r) - m_{s1}I_{n+1}(m_{s1r})\Big]S_n, \\
u^0_{rn}(r,\xi) &= \Big[\frac{n}{r}K_n(m_1r) - m_1K_{n+1}(m_1r)\Big]A_n + \Big[\frac{n}{r}I_n(m_1r) - m_1I_{n+1}(m_1r)\Big]B_n \\
&- -i\frac{\xi}{a}\Big[\frac{n}{r}K_n(m_{s1}r) - m_{s1}K_{n+1}(m_{s1}r)\Big]C_n - i\frac{\xi}{R_1}\Big[\frac{n}{r}I_n(m_{s1}r) - m_{s1}I_{n+1}(m_{s1r})\Big]D_n \\
&\quad + \frac{n}{r}K_n(m_{s1}r)E_n + \frac{n}{r}I_n(m_{s1}r)S_n; \\
&\quad m_1 = \frac{m\xi}{R_1}m_{s1} = \frac{m_s\xi}{R_1}m = \Big(1 - \frac{c^2}{c_p^2}\Big)^{\frac{1}{2}}; \quad m_s = \Big(1 - \frac{c^2}{c_s^2}\Big)^{\frac{1}{2}}.
\end{aligned} \tag{2.8}$$

Using equation (2.8), the harmonics of the voltage conversion in the filler are found. Satisfying the transformed boundary conditions, the functions $A'_n(\xi), \dots, S'_n(\xi)$ are expressed in terms of the Fourier coefficients of the radial displacement of the shell:

$$\{A_n, \dots, S_n\} = \frac{w^0_n}{\det \|a_{ij}\|}\{A'_n, \dots, S'_n\}. \tag{2.9}$$

Furthermore, the functions $\{A'_n, \dots, S'_n\}$ are calculated through the boundary conditions (2.4) and (2.5). The qualifier elements $\det \|a_{ij}\|$ are found if we apply the formulas $t_3 = 0$ to elements a_{ij} elements a_{6j} are calculated by formulas

$$\begin{aligned}
&a_{61} = ns_3 - m\xi, && a_{62} = n + m\xi s_6 && a_{63} = ns_9 - m_s\xi, \\
&a_{62} = -n - m_s\xi s_{12}, && a_{65} = n, && a_{66} = n.
\end{aligned}$$

Substituting the functions found according to (2.9) into a formula for σ^0_{rrn} and then using the resulting expression in a condition $\sigma^0_{rrn} = -q^o_{cn}(r = a)$ has been found the relationship between the reaction of the filler and the radial displacements of the shell, where the function $f_1(n,\xi,c)$ is calculated by the formula

$$f_1(n,\xi,c) = \frac{\sigma_1(n,\xi,c)}{\det_n \|a_{ij}\|}. \tag{2.10}$$

Here,

$$\begin{aligned}\sigma_1(n,\xi,c) = \{&[(1+m_s^2)\xi^2 + 2n(n-1)]s_6 + 2m\xi\}A_{61}\\ &+ \{[(1+m_s^2)\xi^2 + 2n(n-1)]s_6 + 2m\xi\}A_{62}\\ &+ 2m_s^2\xi^2\left\{\left[1+\frac{n(n-1)}{m_s^2\xi^2}\right]s_{12} + \frac{1}{m_s\xi}\right\}A_{63}\\ &- 2m_s^2\xi^2\left\{\left[1+\frac{n(n-1)}{m_s^2\xi^2}\right]s_{12} + \frac{1}{m_s\xi}\right\}A_{64}\\ &- 2n[(n-1)s_9 - m_s\xi]A_{65} + 2n[(n-1)s_9 - m_s\xi]A_{66},\end{aligned}$$

A_{6j} is the algebraic complements of elements of determinants $\det_n \|a_{ij}\|$. In the case of a solid placeholder, the function $f_1(n,\xi,c)$ is calculated by equation (2.10). Substituting (2.7) into the tranformed equations of motion of the shell, we obtain a system of algebraic equations with respect to u_n^0, v_n^0, w_n^0 from which the sought quantities are expressed in terms of the Fourier coefficients of the transformant of the external pressure on the shell. In particular, for the transformant of radial displacement, we obtain

$$w^0(\xi,\theta) = \frac{1-v}{2k\tilde{G}}\sum_{n=1}^{\infty}\frac{p_n^0(\xi)}{f(\xi,n,c)}\cos(n\theta). \tag{2.11}$$

The Fourier coefficients of the transform of the external pressure on the shell are unknown and must be determined using the conditions for pairing the ribs with the shells (2.6) and (2.7).

To do this, the Fourier transform is used in a moving coordinate system. Then, in the image space

$$p^0(\xi,\theta) = \frac{1}{R_1}\sum_{n=1}^{\infty}\left(\sum_{k=1}^{l} q_{ok}^o(\xi)a_{nk}\right)\cos(n\theta), \tag{2.12}$$

where a_{nk} is the Fourier coefficients of the function $\delta(\theta-\theta_k)$.

Taking into account (2.12), equation (2.11) takes the form

$$w^0(\xi,\theta) = \frac{1-v}{2k\tilde{G}}\sum_{n=1}^{\infty}\left(\sum_{k=1}^{l} q_{ok}^o(\xi)a_{nk}\right)\frac{\cos(n\theta)}{f(\xi,n,c)}. \tag{2.13}$$

Substituting (2.13) into the transformed conditions (2.7), a system of algebraic equations is obtained

$$y_k^0(\xi) = -\frac{(1-v)}{5k\tilde{G}}\left\{\sum_{k=0}^{\infty} q_{ok}^0(\xi)\cdot\sum_{n=1}^{\infty}\frac{a_{nk}\cos(n\theta_k)}{f(\xi,n,c)}\right\} \qquad (k=1,\ldots,l). \tag{2.14}$$

Resolving this system with respect to q_{ok}^o transformants, they are expressed as

$$q_{ok}^o = f_k(y_1^0,\ldots,y_l^0) \qquad (k=1,\ldots,l). \tag{2.15}$$

The specific form of this dependence is determined by the number of edges. Substituting (2.15) into the transformed equations of motion (2.7), a system of equations for determining the conversion coefficient of beam deflections is obtained

$$\xi^2\Big[\xi^2 - \frac{c_{1k}^2}{3(1-v_c)}\frac{a^2F_k}{l_k}\Big]y_k^* = \frac{a_1^4}{\tilde{E}_{\delta k}I_k h}[p_{0k}^0 f_k(y_1^*,\dots,y_l^*)] \qquad (k=1,\dots,l),$$

where $y_k^* = y_k^0/h$, $c_{1k}^2 = 3c^2 p_{\delta k}/2/\tilde{G}_{\delta k}$.

From equation (2.13) the transformants of deflections of beams through loads on each of them and the stiffness parameters of the shell and filler ribs are found. Symbolically, these dependencies can be written as

$$y_k^* = \varphi_k(p_{\delta 1}^0,\dots,p_{\delta l}^0) \qquad (k=1,\dots,l).$$

Substituting (2.14) into (2.12), transformations of the pressure of each of the beams on the shell are found, after which the deflections of the shell are determined from (2.13). Then, using (2.4), the components of the stress-strain state of the filler are determined.

The algorithm for obtaining the final solution is greatly simplified if we assume that the material and geometric characteristics of the ribs are the same ($E_{0\delta} = E_\delta$, $p_{\delta k} = p_\delta$, $I_k = l$, $F_k = F$), the ribs themselves are located at the same distance from each other, and the loads moving along the ribs have the same intensity and a uniform Poisson's ratio v under the same length distribution law. Under these conditions, due to the symmetry of the problem, the deflection of all ribs and the pressure transmitted from each of the beams to the shell is the same. Then, $q_{0k}^0(\xi) = q_0^0(\xi)$, and instead of (2.12), the following can be written:

$$p^0(\xi) = R_1^{-1} q_0^0(\xi)\sum_{n=0}^{\infty} a_n \cos(n\theta),$$

where a_n is the Fourier coefficients of the function, which are defined by the following formulas:

$$\triangle_1 = \sum_{k=1}^{l}\delta(\theta-\theta_k),$$

$$y_k^0(\xi) = -\frac{(1-v)}{5kG}q_{0k}^0(\xi)\cdot\sum_{n=0}^{\infty}\frac{a_{nk}\cos(n\theta_k)}{f(\xi,n,c)} \qquad (k=1,\dots,l). \tag{2.16}$$

Since, due to the symmetry noted above, the deflections of all ribs are the same, then assuming in (2.16) $\theta_k = \theta_1 = \theta$ the following dependence is found, explicitly, without solving the system, the following dependence:

$$q_0^0(\xi) = -\frac{2k\tilde{G}y^0(\xi)}{1-v\sigma_1(\xi)}, \quad \sigma_1(\xi) = \sum_{n=0}^{\infty}\frac{a_n}{f(\xi,n,c)}. \tag{2.17}$$

Substituting (2.17) into the transformed equation of motion (2.16), the transformants of the beam deflection is founded

$$y^0(\xi) = \frac{a^4}{E_\sigma I}\sigma_1(\xi)\frac{p_\delta^0(\xi)}{d_1\sigma_1 - \frac{a^{4k\gamma_1}}{I(1-v^2)}} \quad d_1(\xi) = \xi^2\Big[\xi^2 - \frac{c_1^2}{3(1+vb)}\frac{a^2F}{I}\Big]. \tag{2.18}$$

Taking into account (2.18), $q_0^0(\xi)$ is founded, and then from (2.10) the transformants of the radial displacements of the shell:

$$q_0^0(\xi) = -\frac{\gamma_1}{1-\upsilon^2}\frac{a^4}{I}\frac{p_0^0(\xi)}{d_2(\xi)}, \qquad d_2(\xi) = d_1\sigma_1 - \frac{a^4}{I}\frac{k\gamma_1}{1-\upsilon^2},$$

$$\sigma_2(\xi,\theta) = \sum_{n=0}^{\infty}\frac{a_{nk}\cos(n\theta)}{f(\xi,n,c)}, \qquad w^0(\xi,\theta) = \frac{\gamma_1}{2k\tilde{G}(1+\upsilon)}\frac{a^4}{I}\sigma_2(\xi\theta)\frac{p_\delta^0(\xi)}{d_2(\xi)}.$$

The contact pressure transformant is determined by the formula

$$q_0^0(\xi) = \frac{\gamma_1}{2\gamma(1+\upsilon)}\frac{R_1^4}{I}\frac{p_o^o(\xi)}{d_2(\xi)}\cos(n\theta). \tag{2.19}$$

The final solution for specific types of loads moving along the beams $p_\delta(\xi)$ is found using the inverse Fourier transform.

As an example, a solution is obtained when moving concentrated forces along the beams, so that

$$p_\delta(\xi) = p_1\delta(\eta),$$

then $p_\delta = p_1$.

Assuming that for any number of beams, the total load remains the same, we set $p_i = p_0/l$. Then, from (2.18) and (2.19), the following is obtained:

$$w^* = \frac{2\tilde{G}w}{p_0} = -w_0\sum_{n=0}^{\infty}\left[\int_0^{\infty}\frac{\cos(\xi\eta)d\xi}{d_2(\xi)f(\xi,n,c)}\right]a_n\cos(n\theta), \tag{2.20}$$

$$q_c = \frac{q_cR_1}{p_0} = q_0\sum_{n=0}^{\infty}\left[\int_0^{\infty}\frac{f_1(\xi,n,c)\cos(\xi\eta)d\xi}{d_2(\xi)f(\xi,n,c)}\right]a_n\cos(n\theta), \tag{2.21}$$

$$w_0 = \frac{\gamma_1}{\pi l(1+\gamma)}\frac{R_1^4}{I}, \; q_0 = \frac{w_0}{2\gamma}.$$

Similar formulas can be written to determine the stress-strain state at the internal points of the filler. Using representations of the delta function by a finite Fourier series with improved convergence, the infinite series in (2.20) and (2.21) are replaced by finite ones. In addition, the coefficients a_n take the form $a_n = \frac{l}{2\pi}$ at $n = o(l = 2,4,6,\dots)$, $a_n = \frac{lN}{n\pi^2}\sin\frac{n\pi}{N}$ for $n = lj$ $(j = 1,2,3,\dots)$. If $n \neq lj$, then the coefficients $a_n = 0$

3. Results and analysis

The following parameters are entered: $E_1 = \frac{E_{\delta 1}}{E_0}$, $E_2 = \frac{E_{\delta 2}}{E_0}$, $E_3 = \frac{E_{\delta 3}}{E_0}$, $E_4 = \frac{E_{\delta 4}}{E_0}$, $E_z = \frac{E_c}{E_0}$, a cylindrical shell with filler, with four longitudinal ribs (vibrations of a cantilever cylindrical shell) is considered. The ribs are installed on the shell at the corners at 90°.

The calculations are carried out for the following values of dimensionless parameters:

$$R(t) = Ae^{-\beta t}/t^{1-\alpha}, \quad A = 0.048, \quad \beta = 0.05, \quad a = 0.1, \quad k_1 = k_2 = k = 0.002,$$

$$v_j = v_0 = v_c = 0.25, \quad E_j = 0.85, \quad E_z = 0.62, \quad (j = 1, \ldots, 4)$$
$$k = 0.02, \quad v = v_\delta = v_c = 0.3, \quad \gamma_1 = 0.1, \quad p_1^* = p/p_\delta = 0.5.$$

At given load rates the shell deflection along the contour is investigated. The calculation results are shown in Figure 1. With an increase in the number of ribs, the corresponding radial displacements decrease. The eigenfrequencies of the considered mechanical system are also investigated. It is established that natural frequencies and damping coefficients decrease with a decrease in the number of ribs.

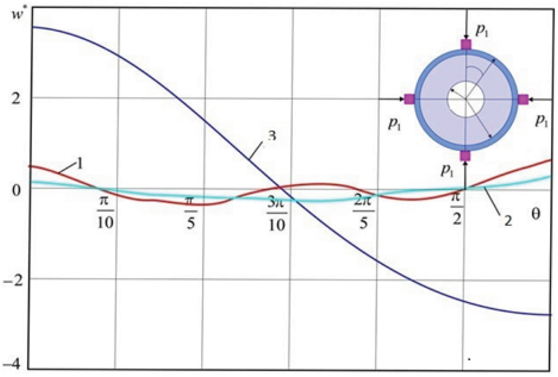

FIGURE 1. Changing the circumferential deflections of the shell for a different number of stiffeners (1-4 ribs, 2-3 ribs, 3-1 ribs).

Additionally, the shell increases with an increase in the number of ribs, which leads to a rapid mitigation of the effect of discrete reinforcement and loading. For the case of two diametrically opposite ribs, the filler, in the case of sliding contact, has little effect on the nature of the distribution of deflections in areas remote from the loading point, which leads to a qualitative difference in the $l = 2$, $l = 4$, $l = 6$ Figure 2 illustrates the distribution of contact pressure at the boundary of the shell and filler for the same values of the parameters as in Figure 1. It can be seen that with an increase in the number of ribs, the contour contact stress in the filler increases.

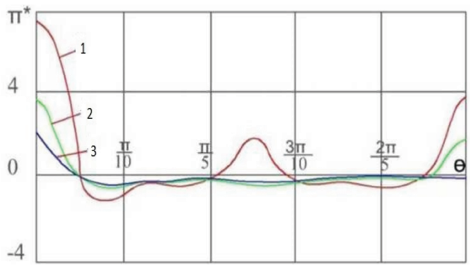

FIGURE 2. Contact stresses at the boundary of the filler and the ribbed shell (1-4 ribs, 2-3 ribs, 3-1 ribs)

As seen from the analysis of the results, the contact stresses change sign very quickly with increasing distance from the loading point, which in the case of a one-way connection leads to a lag of the shell from the filler.

4. Conclusions

It has been established that the reinforcement of shells with longitudinal ribs (oscillations of a cantilevered cylindrical shell) leads to a decrease in natural frequencies and coefficients of some shells, an increase in the density of the spectrum of natural frequencies, and the appearance of intermediate forms and forms with the same values of wavenumbers but with different frequencies. External reinforcement increases the natural frequencies and damping coefficients; with internal edges, the frequencies are much lower than those with external edges.

In the high-frequency region, any reinforcement of the ribs reduces the natural frequencies and damping coefficients. This means that in this area, the additional mass plays a more significant role than the additional stiffness of the reinforcement. Consequently, the longitudinal reinforcement worsens the dynamic properties of the shell.

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УПРАВЉАЊЕ РЕЗОНАНТНИМ ОСЦИЛАЦИЈАМА ВИСКОЕЛАСТИЧНИХ СИСТЕМА

Резиме. Конструкције у облику цилиндричних ребрастих љуски и панела се широко користе у инжењерству и грађевинарству. Разматран је проблем дејства покретних оптерећења на бесконачно дугу цилиндричну љуску, ојачану дуж спољне површине уздужним укрућењима и која садржи вискоеластичан инерцијални филтер. Покретно оптерећење се преноси на љуску само преко ребара, а изван ребара нема оптерећења. Једначине система се изводе у сагласности са дискретном позицијом ребара и условима коњугације. Приказан је утицај броја и крутости ребара на природу дистрибуције померања љуске и контактног притиска на граници вискоеластичног филтера. Кретање љуске је описано класичним једначинама заснованим на Кирхоф-Лав хипотези; за филтер се користе динамичке једначине теорије вискоеластичности. Утврђено је да ојачање љуски уздужним ребрима (осцилације цилиндричне љуске) доводи до смањења природних фреквенција и коефицијената пригушења у неким љускама, повећања густине спектра природних фреквенција и појаве форме са истим таласним бројевима, али са различитим фреквенцијама. Спољашње силе повећавају природне фреквенције и коефицијенте пригушења. Утврђено је да су фреквенције за унутрашње ивице ниже него за спољашње ивице. У зони високих фреквенција смањују се природне фреквенције и коефицијент пригушења. То значи да додатна маса игра значајнију улогу од додатне крутости. Сходно томе, уздужно јачање љуске погоршава њена динамичка својства.

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