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GOLDEN SECTION AND NON-EUCLIDEAN GEOMETRY IN SCIENCE AND ART

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Abstract: *Gold section and Non-Euclidean geometry are overviewed as notions, forming of the classic and new conceptions of space in science of art and native sciences are bind with which. Historical scientific experience shows that these notions are independent and mathematical connection is absent between them. As a result of natural phenomenon (phillotaxis) researching is dedicated opposite: the fundamental mathematical connection between gold section and Non-Euclidean geometry (Minkovsky's geometry) These results allow to make important conclusions according to geometrical special imagination and to regularity of formshaping in the nature and art.*

The subject of study that demonstrates the intention of interrelated examination of the phenomena of the golden section and non-Euclidean geometry at first look may seem rather unexpected. The reasons for such opinion are easy to understand. The subjects of the golden section and non-Euclidean geometry have been elucidated in numerous scientific and scientific-popular publications, as a rule, in different ways. In general conscience the golden section and non-Euclidean geometry are perceived as non-heterogeneous notions, symbolising different historical epochs, different levels of world outlook, reflecting various aspects of properties of the objective world and, thus, are attributed to different domains of cognitive activity. So, what is common in them, is there a real connection

between these phenomena, to what extent of significance does scientific importance of such connection stretch? The proposed study sheds the light on these questions. The task set forth is considered through the prism of the scientific problem of the most general nature – the problem of concordance of geometrical spatial notions in art and in science, that have been formed differently in these different domains of cognitive activity. Geometrical ideas and spatial concept of the "old type" prevail in art – the concept of the so-called Euclidean geometry, and here the idea of the golden section stands out as an exemplifying conductor of the classical doctrine and the system of the world outlook connected therewith. The origins of the geometrical trend in the "art" and the idea of the golden section within it go back to the hoary antiquity, the time of the conception of the professional methodology of the form creation. As it is known, the main historical experience of mastering geometrical ideas in art is connected with the theory of proportions. Geometry established its role in art as the fundamental factor in the form creation, as a rational way of improvement of the methods of proportioning and aesthetic harmonisation in the earliest historical periods – in ancient Egypt, in ancient Greece. At the same time its role was manifested also in the formation of theoretical and cognitive problems of art, in stimulation of tendencies there towards generalisation of knowledge, expansion of the world outlook notions. Geometry has retained such its importance in the future as well. Owing to geometrical studies in art (theory of proportions) the development of rational thinking, the ability to understand achievements of science, readiness to synthesise any new knowledge for the purpose of modernisation of the form creation and correlation of world outlook principles have been maintained on the common level with natural sciences for many centuries. As a matter of fact, the slogan of the theory of proportions remains unchanged in the conscience of its modern adherents – art theoreticians of the 20th century. I. Zholtovsky (Zoltowski), D. Hambidge, Le Corbusier – the well known theoreticians of proportions, like many other authors, declare the principles of universalization of knowledge, and in order to substantiate their concepts apply information drawn from the fields of mathematics, physics, biology. And still the real integrating role of the theory of proportion as of today has been lost. The matter is, that during a prolonged historical period due to the specific nature of the professional problems the development of geometrical knowledge in art moved towards a narrow specialisation. Due to pragmatic objectives the perception of the theory of proportion on the part of "outside sciences" the pace of its scientific progress and advancement in geometrical studies were hindered and its specific share in the system of philosophical world outlook decreased. Mathematical art, advancing during a certain period in parallel with pure mathematics, with the passage of time has fell behind noticeably both, in the development of methodological apparatus and in its ability to catch up with the perception of the idea meaning of mathematical (geometrical, natural science) achievements. One of the main indices that define the character of falling behind of the theory of proportion today is, as a matter of fact, the crisis of "spatial notions" on which this theory is based. The "classical concept of space" – the idea of the so-called Euclidean geometry (Euclidean space), that is, those spatial notions that had been produced by Pythagorean-Plato school of geometry and on which the entire classical science and art have been based later on for millennia, lies at the base of its theoretical-notional system, analytical methods and world outlook logic and continues to retain its dominating positions therein. Meantime, a dramatic transformation of the notion of space has taken place today in the domain of natural sciences, and the entire system of notions of the surrounding world connected with it has changed as well. The development of such

sciences as mathematics, physics, philosophy in the 20th century has produced a new concept of space – the concept "space-time". The new in principle idea – the idea of the so-called non-Euclidean geometry – is placed at the foundation of modern spatial notions. As it is known, for the first time this idea (in purely mathematical form) was proposed by the Russian scientist M. I. Lobachevski as early as in the beginning of the 19th century. This idea ruined traditional views on the properties of space, touching on the very foundation the geometrical theory and, therefore, had not been easily accepted even in the circles of mathematicians. It was considered at first as an abstraction important only for the internal problems of mathematics. Yet, with the appearance of probability theory, particularly in the result of its geometrical interpretation, there have emerged quite unexpectedly the true sense of the ideas of non-Euclidean geometry, their application to the properties of real space (space-time). These results, as it is known, have had a dramatic effect on the development of physics. However, it has become clear, that the sphere of physics is only one of the fields of application of the latest geometrical ideas. It was ascertained, that geometry as a science of space faced the broadest scientific and philosophical problems. The process of the development of non-traditional geometrical notions involved, first of all, biology. Here the orientation of searches is connected with the study of symmetry of living nature forms; V. I. Vernadsky outlines the main program paths of research.

With the time the spirit of the new geometrical ideas spreads to the domain of art, where an attempt of appropriate re-evaluation of the notions of space, form, perception are made. However, in their philosophical considerations art theoreticians do not touch on the essential aspects of the problem, its "geometrical details". The research boils down, mainly, to the interpretation of the idea of four-dimensionality of space (space-time), or to re-evaluation of the notions of space and time, bearing in mind their continuous nature. At the same time and in a similar manner changes in the notions of these fundamental categories of art take place on the level of pragmatic thinking as well, particularly in connection with the development of the theory and practice of form creation, while here these changes are not subjected to the immediate influence of "outside irritants". They are determined by the natural course of internal creative searches, that in their turn are caused by the need to satisfy current demands of industrial technology. The dominant role in this process was played by the ideas of the so-called technological form creation, demands to enrich the palette, means and techniques of form creation, adherence to which has brought the creative thought to contact with the new chapters of modern geometry. Mastering of certain knowledge in such fields as combinatorics, projective geometry, crystallographic geometry, etc., aside from resolving practical tasks, has had an impact on the level of "geometrical consciousness" in art. Particularly, one can say, that re-evaluation has taken place of such fundamental notions as "symmetry", "dynamics" and "dynamism", that an essential in the sense of progress of spatial notions awareness of inseparable connection of the notions of space and time has emerged. However, in its main part – in understanding the essential meaning of the concept "space-time", decoding geometrical regularities lying at its foundation, modern trends of mathematical art criticism also appeared to be futile. And the matter here, obviously, is not in the mathematical complexity of the problem. Far from obvious is the very sense of going deep into its geometrical nuances. With popularisation of the relativity theory the idea has spread of unreality of the laws of non-Euclidean geometry in the ordinary, "non-cosmic" conditions. A conviction has grown, that the idea of non-Euclidean geometry is of no value in the sense of theoretical-cognitive and, moreover, pragmatic tasks of art. It seems that the traditional and conventional experience

of geometrical studies have established reasonable boundaries for mathematical needs of art. Neither theoreticians of proportions, nor authors of the non-traditional concepts of form creation do actually go beyond these imaginary boundaries. It has to be stated now, that neither in the system of classical approach, nor within the lines of the new trends of the theory of form creation the task of study and interpretation of geometrical aspects of the concept "space-time" has never been set forth. Here in the long run we speak of the fact of inertia of the theory of art with respect to the idea, whose role is marked by the greatest in the last centuries transformations of scientific consciousness and which due to its fundamental character could cause in art a cardinal blow on the world outlook foundations and even on the methodological principles. All of this is favourable now for the formation of the idea, that science and art build up their world outlook on various sources and that for this reason different trends of general cognitive development are being developed in connection with this in the said domains, that do not anticipate a merger and obtaining in the end of some general, maximally all-embracing understanding of the world. In this case, naturally, priority is given to scientific (based on proofs) variant of vision of the world and, first of all, in the meaning of its geometrical substantiation.This is the way how the entire depth is established of the intrigue arising from the comparison of the terms "golden section" and "non-Euclidean geometry". It is obvious, how important and essential would be the fact of proving the interrelation and unity of mathematical nature of these two ideas. An attempt of such proof made in this research is based on the use of the well-known in science (biology, mathematics) and in art information on realisation of the golden section in natural form creation. The ideas of V. I. Vernadsky pertaining to "non-Euclidean character of geometrical laws of living nature" [39] have played the general guiding role in determining the approach and the sphere of searches for the author. The main research boiled down to the study of the botanical phenomenon – phyllotaxis, characterised by the spiral symmetry and, let us make a stress on this, by the presence in its structure of the golden section and Fibonacci numbers. It was this study, that has led to the key result – the proof of realisation in nature, particularly in phyllotaxis forms, of the regularities of non-Euclidean geometry (Minkowski geometry) and the discovery of common character of this geometry and the law of golden section. Golden section plays in geometry of phyllotaxis the same role as the Napier number plays in "classical" hyperbolic geometry. Geometry of phyllotaxis is a specific "case" of Minkowski geometry. This is, in fact, geometry of living nature. Till recently only physics (special relativity theory) was regarded as the sphere of realisation of Minkowski geometry. The study of phyllotaxis has shown the universal nature of this geometry. It was from this study that the answer came to the question put in the beginning about the connection between the golden section and non-Euclidean geometry. Besides, it has become possible to show, that mathematics of phyllotaxis can be applied for the description of the system of proportioning based on the golden section, particularly Modulor system. It has become possible to describe dynamic symmetry of Hambidge in terms of mathematics of phyllotaxis. In the result of this, taking into consideration the role of Minkowski geometry in relativity theory, it was possible to come to the conclusion on the universal character of this geometry, that manifests itself together with golden section as the fundamental organising regularity of nature and art. The obtained result becomes fundamental for the development of philosophy of art towards understanding of the concept of space. It has an important synthesising meaning, as the concrete points at the connection of geometrical space notions in natural science and art.