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PERIODIC SPONGE POLYHEDRA-EXPANDING THE DOMAIN

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Abstract*: Sponge structures are the most abundant forms in nature, on all possible scales of material existence, while the amount of morphological insights into the phenomenon, as accumulated over the last millennia, is incredibly meager. Only in the last 200 years, and mostly in the second half of the 20th century, we witnessed the emergence of basic concepts, insights and imagery with a growing attention given by designers, architects and structural engineers. The author had participated conspicuously in expanding the domain with his publications on Periodic Hyperbolic Sponge Surfaces (1966); Uniform 'Infinite (Sponge) Polyhedra' (1974,2005) and 'The Periodic Table of the Polyhedral Universe' (1996). Lately, after confronting the prevalent definitions and allowing for polyhedral maps with curved edge-lines and face surfaces, the amount of uniform sponge polyhedra exploded, to reveal a multitude of new polyhedral sponge configurations, spherical, toroidal and hyperbolic, and their governing hierarchical order. The new sponge imagery might play a significant role in the morphological research of natural bio-forms and physical nano – structures, promote images and ideas of innovative space structures and influence the way we perceive our increasingly dense urban habitat.*

1 INTRODUCTION

Nature is saturated with sponge structures on every possible scale of physical-biological reality. The term was first adopted in biology:
"Sponge: any member of the Phylum Porifera, sessile aquatic animals with single cavity in the body, with numerous pores. The fibrous skeleton of such an animal. Remarkable for its power of sucking up water" ....Wordsworth dictionary.

With time the expression: Sponge, spongy, spongeous, Sponginess, was adopted in many languages to describe a physical phenomenon which is characterized by porosity and visual permeability and the condition of a lump of matter which, as a result of biological-chemical-physical processes of erosion-corrosion, growth and death, acquired its characteristic porosity. Numerous examples in the surrounding nature, on the microscopic and the macro-scopic scale, carry a testimony to the abundance of the phenomenon: microscopic radiolarya, bones, eroded rocks and gigantic cave labyrinths.
Slowly but steadily we are becoming aware that the **T** is basic and dominant on the nano-scale of the big molecules (protein +) and, according to Stephen Hauking, also on the macro-cosmic scale.
“The outer space of the cosmos lost it Euclidean rectiliniarity to the hyperbolic curvature of Riemannian geometry”,,,, (William Day- 1989). It transpires that the phenomenon is the most abundant in the physical and the organic material world of nature. With some extrapolation of the perceiving mind it is right to claim that the sponge phenomenon, with its porosity and permeability characteristics, is central to the physical morphological nature of the human habitat, on the urban and the building scale, and represents its defining imagery.
Historically speaking, the phenomenon was received with great consternation. It was associated in the human mind of its time with extreme, unresolved complexity. The labyrinth of Knossos became the mythological archetype of a complex phenomenon, beyond any descriptive discipline and capacity, geometric-mathematical or even literary-verbal. Of course it is beyond the reach of Euclidean geometry. The labyrinth myth came to represent impotence of the human intellect in face of a bewildering, puzzling mystery.

2 MORPHOLOGICAL APPROACH TO PERIODIC SPONGE STRUCTURES

“Our study of natural forms”, the essence of morphology, “is part of that wider science of form which deals with the forms assumed by nature under all aspects and conditions, and in a still wider sense, with forms which are theoretically imaginable”…(On Growth And Form…).
A sponge may be characterized by its envelope, which, if unbounded 2-d manifold (sponge surface), subdivides space into two complementary subspaces. On this 2-d manifold it is possible to draw infinite number of maps, thus giving rise to the notion of **‘sponge polyhedra’** and consequently to **‘sponge polyhedral structures’**.
Each of the complementary subspaces may be faithfully represented with **Tunnel Space Lattices** (T.S.L), thus justifying the observation that each pair of complementary (dual, reciprocal) 3-D Space Lattices gives rise to a topologically specific sponge surface partition, subdividing space between the two.
Within the random myriads of possible sponge surface envelopes, it is quite natural that the human mind is inclined to concentrate and explore first the causal and the periodic members of the evolving imagery and the related array of forms. Within this context it should be noted that each periodicity and symmetry feature of the **T.S.L Pair** is shared with the related Sponge Surface, and vice versa.
It should be stated that by **‘sponge polyhedron’** it is implied: any 3-D polyhedron which complies with the Euler’s Formula of $V-E+F=2(1-g)$, with V; E; F&g corresponding to

Vertices Edges; Faces and genus of the 2-d manifold, respectively.
Of critical importance, when dealing with the polyhedral universe and especially with its 'sponge domain', is the determination of their primary parameters, namely: $\sum\alpha$ av.; Valav. and-g (average sum of the polygonal angles in a vertex; average valency-the number of edges meeting in a vertex; and genus of the 2d manifold, respectively), on the basis of which, as coordinates of a Cartesian space, the' Periodic Table of the Polyhedral Universe' is constructed.
The 'Periodic Table', which provides a domain in which every conceivable polyhedron has a unique point representation and discloses patterns of polyhedra sharing various geometric-topological characteristics, related to the a.m. primary parameters. It constitutes a powerful research tool of the polyhedral universe, with its myriads of components and of its totality, much of it in visual terms.
While $\sum\alpha$av. and Val.av. Can be 'extracted' from Descartes formula: $V(2л - \sum\alpha \text{ av.}) = 4л(1-g)$ and the a.m. Euler's formula, determination of g, especially when in-the range of high spatial complexity, was proven to be more elusive and required mobilization of some basic insights from graph theory.
It was resolved, to a great extent, with the author's theorem, stating that:

The genus-g of the sponge polyhedron's 2d manifold (in 3D space), which subdivides between two complementary Tunnel Space Lattices A&B, each with L_A & L_B. Lines (axes) and NA. & NB. Nodes, respectively, is:
$g = L_A - N_A + 1 = L_B - N_B + 1$, and if periodic and infinite in extent, the polyhedron's geometry and its g – value may be represented by its Translation Unit (TU.),
Thus: $g_{(TU.)} = L_{A(TU.)} - N_{A(TU.)} + 1 = L_{B(TU.)} - N_{B(TU.)} + 1$.

First to receive mathematical attention were the Periodic Sponge Surfaces **P.S.S** (Schwartz; Mobius – 19-th century), followed by regular skew polyhedra (Petrie – Coxeter, early 20-th century).
Sponge polyhedra started to feature in molecular illustrations of chemistry books (Wells – Structural Inorganic chemistry – 1960). Of note was the author's doctoral thesis (1966), dealing with P.S.Ss and P.S.P.s, and the extrapolated 'Infinite Polyhedra' (in collaboration with Wachman and Kleinman – 1974). The topic was expanded by the author in his 'Periodic Table of the Polyhedral Universe' (1996). Few more colleagues joined in the exploration (Mackay, Lalvani, Stewart, Grunbaum, Miyazaki, and Korren), but with the self imposed constraints of the geometric – Symmetric nature (and especially the planarity of faces), the subject was bound to miss its potential scope and fail to reach beyond its already perceived horizon. It was from these premises that the author decided to venture again into the field, with the objective of generating periodic (symmetric) uniform sponge polyhedra, while accepting non-planarity and curvature of edges and faces.

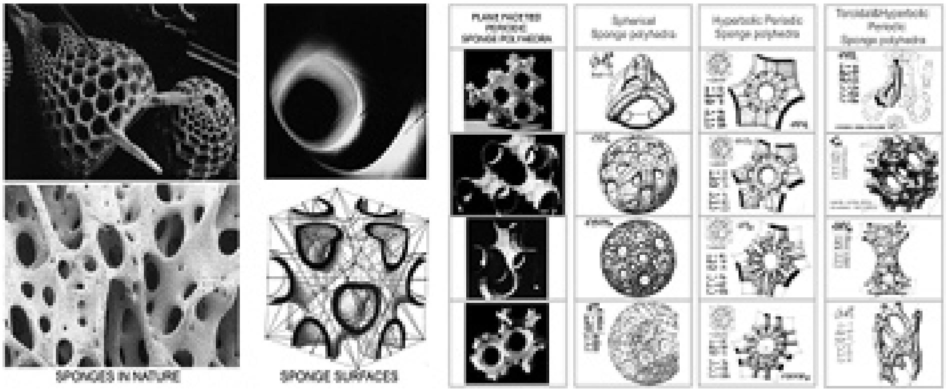
3 GENERATIVE CONCEPTS AND HIERARCHICAL CLASSIFICATION OF THE DOMAIN.

The employed exploratory process involved a certain fusion of symmetry and topological reasoning and combinatorics.
It was clear from the outset that the domain of the periodic sponge polyhedra is dominated

by certain topologically categorized classes: the **Spherical; Toroidal; Hyperbolical** and, what could be aptly categorized as **Primitive** (Cyclic; Stripe; Columnar and helicoidal) P.S.Ps. All these Periodic Surfaces and Polyhedra correspond to various symmetry groups which account for the uniformity of the vertex configurations.
Few generative conceptual features have to be introduced at this point:
• **Genetic Surface (GeS.)**: when a T.S.L of a specific sponge surface can be perceived as a polyhedral map, drawn on (embedded within) a 2d-manifold, this manifold should be considered as its **Genetic surface**. Any GeS, can sustain many polyhedral maps and therefore many sponge surfaces can be genetically related to one and the same GeS, (and when periodical) the symmetries of which will be shared by them all. All the genetically related sponges, sharing also the same symmetry groups represent **a genetic clan of sponge surfaces and related polyhedra.** The number of all g-levels of all the genetically related sponge surfaces derived from one and same GeS. and their corresponding sponge polyhedra are deducible from symmetry combinatorics.
• **Repeating GeS. Unit** is another ordering feature of P.S.Ps. It is represented by possible topological similarity of repeating (symmetrical) GeS. Units, leading to identical polyhedral notations (therefore equality of ∑αav. and Valav.) while differing in their g-values.
• **Multi – Layer GeS.** Spongeous arrangement is possible while still in conformity with the uniformity of vertex configurations, and its local and overall symmetry definition. The resulting n – layered sponge surfaces and polyhedra, where n can reach to infinity, and with their g, ∑αav. & Valav. values, as functions of the number n, raise considerably the attainable complexity of the polyhedral sponge configurations. The whole domain of P.S.P displays a hierarchical organization and its classification could be spread over five levels:
1. Topological categorization into phenotypes: **spherical, primitive, toroidal and hyperbolical;**
2. Symmetry groups, corresponding to particular phenotypes;
3. Genetically related **Clans**, corresponding to same Genetic Surfaces;
4. **Families**, corresponding to common T.S.Ls and g-levels;
5. Individual polyhedral - species with specific notations.

4 THE EXTENT OF THE P.S.P DOMAIN, WITHIN THE POLYHEDRAL UNIVERSE

When perceived through the coordinated space of the **'Periodic table of the Polyhedral Universe'**, it becomes clear that with the addition of the Sponge Polyhedra domain, the emerging polyhedral universe is exploding to cosmic dimensions, as compared with the **'Ptolemaic world picture'** of the polyhedral domain of just one decade ago.When all the horizon of sponge structures is taken in, it dawns on us that the number of P.S.Ps, with all their phenotypes, clans and families, many of which include infinite number of members, each, is overwhelming, much in excess of all the familiar polyhedra in the g = 0 (spherical) and the g = 1 (toroidal) domains. So, **it's not just in the natural** – physical, but also **in the theoretical and possibly imaginable world of geometry that the sponge configurations constitute the overwhelmingly greater majority of shapes and forms.**



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