Form and Symmetry: Art and Science
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AN ART-IFICIAL HISTORY OF MATHEMATICS

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Abstract: *Through a series of papers, the author embarks on an alternative history of mathematics. It does not aim to be a historic overview of the achievements in the exact sciences, but an attempt to re-write its history as if it were an art form, like painting or music. The present paper further summarizes the contents of a previous paper in a single one-page table: from the earliest mathematics, visual arts and music to present. Together with a list of references, it shows how the main idea can be expanded, as will be illustrated in another paper in the present volume.*

1 PREREQUITES: FROM DAWN TO MINIMALISM

Douglas Hofstadter is best known for his book *Gödel, Escher, Bach, an Eternal Golden Braid*. At other moments in time it is possible to observe similar relations between mathematics, the visual arts and music. An open mind is required to accept this artist's view – it is not a scientific report. "Symmetry", or "συν-μετρον", that is "*with measure*", is an important guideline: sometimes it is translated as "*in proportion*", but this translation may be too restrictive.

Admittedly, the paper is very schematic, but this was the audacious goal. First, a previous paper (Huylebrouck 2007) is summarized to a few lines, a considerable effort.

	Mathematics *Gödel*	Visual art (painting) *Escher*	Music *Bach*
Hofstadter	Mathematics of ma-thematical	Drawings of drawings	Bach's name writen on a music staff
Prerequisites	35000 BC: Lebom-bo mountains S-A-frica: tally stick (?)	75000 BC: Blombos cave S-Africa: design pebble (?)	43000–82000 BC: Neanderthal cave Slovenia: flute (?)
Dawn	20000 BC: Ishango rods: base 6-10	25000 BC: Mezin Ukraine: patterns	7000 BC: playable flutes, Jiahu China
Greece: integer proportions	Pythagorean triangle: sides 3-4-5	Parthenon: 4 - 9, i. e. 2^2 - 3^2 proportions	Pythagorean scale: do-do' = 2:1; do-sol = 3:2; do-fa= 4:3 etc.
End Middle Ages: partial rediscovery Greece	1202: Fibonacci Liber Abaci: recalling some simple rules	Integer multiples of local foot explain proportions in art	Guillaume de Machaut: 1:2 - 2:3 proportions, sections, melodic cells …
Renaissance: Greece and beyond	1495: Luca Pacioli, (De) Divina Propor-tione	Brunelleschi, Alberti: perspective	Just intonation ratios: re=9:8; mi=5:4; fa= 4:3; sol=3:2 …
17^{th} - … century	Newton, Leibnitz: calculus; math & physics	Vermeer: optics & art; composition	re = (√5)/2; mi = ((√5)/2)^2; Bach's well-tempered scale
…19^{th} mecha- nical device	Euler…Gauss: full understanding	painting = photo-graphically perfect	one number ex-plains all: re=$2^{2/12}$; mi=$2^{4/12}$
Roots well-establis-hed view?	Cantor: numbers = but elements of sets; ∞ proportions	Pointillism: pain-ting = but set of dots of primary colors	Music = but sound waves: do = 261.63 Hz; re = 293.66Hz
Inaccuracies emer-ge: role observer	Einstein: theory of relativity	Cubism: painter himself influences representation	Helmholtz: dis-so-nance: in ear listener
Paradoxes	Russell's paradox	Vasarely's illusions	Sound illusions
Probability	Quantum physics: Schödinger's equat.	Jackson Pollock: Brownian motion?	Xenakis: stochastic music Achorripsis
Discrete	Finite geometries	Minimal art	Minimalism (La-Monte Young, …)

Figure1: Table 1: Survey of "Gödel, Esher, Bach Similarities in other Eras".

Yet, one additional comparison was added, for the 17th-18th century, since jumping over this epoch turned out to be too abrupt. Isaac Newton and Gottfried Wilhelm Leibnitz led the basis of calculus, away from elementary geometrical computations, towards to mathematics of changes, as in many physical processes. In painting too, optical instruments

such as the “camera obscura” became a tool (for Johannes Vermeer?).
Johann Sebastian Bach introduced elaborate mathematical constructions in music, in “Das Wohltemperierte Klavier”, or in geometrical ideas in compositions.

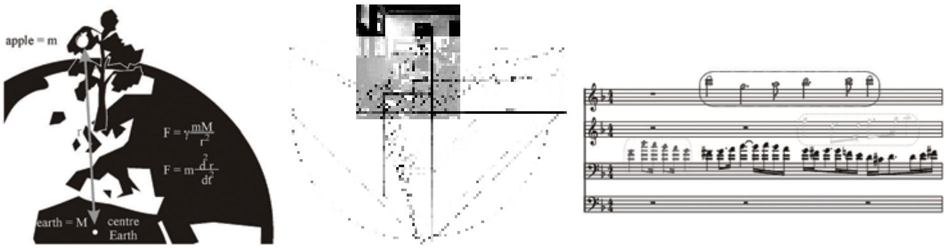

Figure 2: Newton’s gravitation with derivatives; 3-D reconstruction for Vermeer’s “Music lesson” and position of the camera; geometric patterns in a Bach composition.

2 EMOTIONAL MATHEMATICS AND BEYOND?

The above first part swiftly avoided art movements where emotions played a particular role, such as im- or expressionism. The envisaged different (modern) art currents lack a general consensus about their precise descriptions. Yet, mathematical parallels for these more subjective art movements will be proposed, in a second contribution in the present volume.

What is the benefit of such an exercice? Of course, it may be amusing for the non-mathematical artist (or scientist) to read how some mathematicians experience their exact field, emotionally. In this overview, the use of similar expressions as for art movements can be useful to improve its understanding. Future papers may extend these comparisons to, for instance, the contemporary competition between figurative and abstract painting, or between melodic and formal music, to the rivalry between Poincaré and Bourbaki adepts.

Another extension, with straightforward practical application to teaching, may be a comparison of the teaching of these fields (mathematics, painting and music) in different countries through times. Finally, there is no reason the parallels between the three fields should be restricted to the West. Similarities between the mathematics, the visual art and the music in other cultures may be another challenging goal. The traditional discussions in the erudite yet open-minded ISIS-group after a presentation during the meeting will hopefully provide good inspiration.

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