Form and Symmetry: Art and Science
Buenos Aires Congress, 2007

PLANE TRANSFORMATIONS ON THE OTTOMAN TILES

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Abstract: *In this article, first affine transformations and the behavioral features of a plane are reviewed in general terms with suitable examples, and the similarities- symmetries formed by them are dwelled upon. Then the affine transformations, which was found on faience tiles and tile borders, and frequently used in the Ottoman architecture, are examined.*

1 TRANSFORMATION

To carry a point to another point inside its set; to carry it to another set in the same space; to carry it to a space outside the space where it is; to carry it to another dimension outside the dimension where it is; if we call all these moving a transformation, then collinations, which are the main title of transformations, are the key name of the process of imaging / figuring / copying / mapping. The affine transformations of a plane, which we will try to analyze here, are the plane collinations under certain conditions.
Transformation can briefly be defined as the f function which maps set A to set B in the same plane, and preserves the features of keeping 1-1 and over. A plane has basically 5 different types of transformation: Translation (*T*), Rotation (*R*), Reflection (*M*), Glide reflection (*G*), Stretch. Along with these, if transformation is an isometry (this is also called Rigid Transformation), than it preserves distances. Again, if a transformation preserves angles, it is called similarity, and if it maps straight lines to straight lines, it is called affine transformation. Since isometry and affine transformations are anaylzed in this study, the first four transformations above, which are isometry will be discussed. Stretch is a similarity.

Translation(T): It is a transformation which moves every point of a set within a plane by a fixed distance in the same direction to another set of points.
On the basis of T: $R^2 \to R^2$; $T(a,b)$: $(x,y) \to (x',y') = (x-a, y-b)$, the T(a,b) translation moves every P(x,y) point to the P′(x′,y′) point in the same plane. (PP Slide:1.T)
Rotation(R): It is a transformation that moves a set of points on a plane to another set of points on the same plane by rotating them around a fixed point up to a desired angle.
On the basis of R: $R^2 \to R^2$; $R(\phi)$: $(x,y) \to (x',y') = (x.\cos\phi + y.\sin\phi, -x.\sin\phi + y.\cos\phi)$, the R(ϕ) rotation moves every P(x,y) point of the plane to the P′(x′,y′) point. (PP Slide:1. R)
Reflection(M): It is a transformation which moves every point of a set on the plane to their mirror image on the same plane. On the basis of, M: $R^2 \to R^2$;
$M(\varphi)$: $(x,y) \to (x',y') = (x.\cos\varphi - y.\sin\varphi, x.\sin\varphi + y.\cos\varphi)$, the M(φ) reflection moves every P(x,y) point of the plane to the P′(x′,y′) point.(PP Slide:1.M)
Glide Reflection(G): It is a transformation which moves a set of points on a plane to another set of points on the same plane by applying a combination of a translation and a reflection through a symmetry axis. On the basis of G: $R^2 \to R^2$, M: $R^2 \to R^2$, T: $R^2 \to R^2$, and on the basis of $G(\delta)$: $(x,y) \to (x'',y'') = M(\varphi) \circ T(a,b)$ or $G(\delta)$: $(x,y) \to (x'',y'') = T(a,b) \circ M(\varphi)$,the G(δ) glide reflection moves every P(x,y) point of a plane to the P′(x′′,y′′) point.(PPSlide:1.G)
Affine Transformation (*AF*): In addition the brief description made above, an affine transformation can be explained as following: It moves straight lines to straight lines, intersecting lines to intersecting lines, parallel lines to parallel lines. In other words, an affine transformation is a combination of a translation, a rotation, a stretch and a shears (PP Slide: 1.AF). AF: $R^2 \to R^2$, AF: $(x,y) \to (ax'+by'+c, dx'+ey'+f)$, $a,b,c,d,e,f \in R$.
Symmetry can be described as the invariability of an object or a system against a transformation. It is also necessary to signify that an isometry is an important condition in order to be able to discuss the symmetry of a set.

2 TILING AND 2D TRANSFORMATIONS

The tile patterns which will be studied in this article were chosen from two significant Ottoman architectural structures. One of them is the Topkapı Palace (1460-78), the palace of the Ottoman Dynasty; particularly the tiles in the Harem section and the Circumcision Room were chosen. The other structure is Rüstem Paşa Mosque (1561) which has a very significant place in the Ottoman tile art. Although it is a small mosque, it is a very imposing structure with the tiles on its walls where all patterns to become a source for the tile art.

General Features

The geometric structures of faiences on the Ottoman wall tiles were composed of polygons. The most frequently used polygons here are triangle, square, hexagon and octagon forms. The most significant examples of especially hexagon and octagon forms are in Edirne and Konya respectively.
One of the most significant symmetric features of the Ottoman wall tiles is that they have a tessellating feature.
In the technical analyzes and the visual structures of the pattern networks, the projections of 2D isometrics and some affine transformation movements are quite clear.

If we go from the simple to the complex, we can see that the simplest of the tessellating movement is a pattern network which is obtained by a unit form constantly repeating itself. Following it are respectively double pattern networks produced by different unit forms on two edges or corners, and triple pattern networks produced by different unit forms over three edges or three corners. One of the richest pattern networks is quadrangles. The pattern networks are obtained by using different unit forms on their four edges (or four corners). Here basically three different unit forms are used. While the pattern network produced with these unit forms generally consists of two conjugate edges (or corners) and two different motifs, the remaining two conjugate edges (or corners) are produced by using various colour combinations where a third motif preserves the general symmetry.

Faiences are composed of edges and angles. A few round edged faiences can be seen on the tile walls, but they are used as a single piece according to the needs of the pattern.

The symmetric plane transformations which are used in accordance with the features of pattern networks form a general classification. If the produced pattern network is composed of round structures, the most basic plane transformations playing a role in this formation are reflections and rotations. If the produced network is composed of angled structures, translations and glide reflections can be seen more distinctly. It should be noted that these moves may relocate themselves if they provide certain dimensions. Among these dimensions, especially the points, where a unit circle intersects the axis, such as 0˚, 90˚, 180˚, 270˚, 360˚ are the dimensions that come to mind in a flash.

Borders are generally composed of quadrangles or squares.

The main plane transformations on the borders can be identified as following:

Translation symmetry is obtained horizontally or vertically, and it generally appears as a movement of faience.

Rotational symmetry is obtained with a rotation of 180˚ from the mid point of the intersection of straight lines horizontally and vertically.

Reflection symmetry can only be obtained on horizontal / vertical axles or the mid points on these axles.

A possible Glide reflection symmetry appears as the combination of reflection and translation symmetries on the borders where these two symmetric movements are seen.

The symmetric movements, which are formed by an affine transformation, overlap the combinations mentioned on the 4th article.

The 5 articles mentioned earlier for borders are valid for the formed pattern networks. In addition to these, the following articles can briefly be added:

The plane transformations on the pattern networks cover the affine plane on every necessary direction and angle in accordance with their basic descriptions.

On the networks produced with a unit form, form chains, which are composed of small symmetric focuses, are formed according to the variety and colours of the motifs. These form chains are generally two types, and three types of them can very rarely be seen.

Two, three, even four symmetric groups appear in the pattern networks formed by double, triple, quad unit forms different from each other in terms of both motif and colour.

The multiple symmetric groups, which appear on the pattern networks, form imposing networks giving an impression as if they have a strong visual effect and are not symmetric.

The significant feature on the imposing pattern networks is that both sharp lines and edge lines are no used in forming telescopic symmetric groups. The motifs have round lines; their transitions and completions with round-line harmonic lines.

The pattern networks, that have the features in the 4 and 5th articles, form a symmetric cyclic structure which is difficult to perceive where it starts and ends.
In addition to the 6th article, it is necessary to mention a dynamic vivacity on the pattern networks composed of symmetric groups. Apart from the analytic values of the pattern networks examined on a 2D affine plane, the visual impression, which it creates in our mind, is quite dynamic. Here the communication among the symmetric groups forming the pattern network can clearly be observed. The other groups, forming the pattern network according to a symmetry group that a viewer is focused, draw the general network elegantly by positioning themselves. This process shows that the same pattern network can be formed with different movements by image. At the same time, as stated above, this situation is an image indication of the exchanges among translation, reflection, rotation, glide reflection and affine transformations under certain conditions.

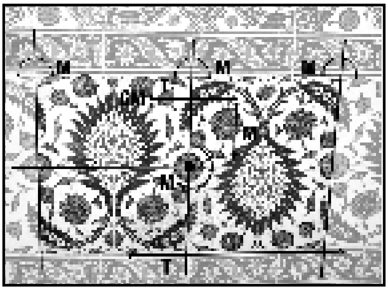

Figure 1:
The tile from Rüstem Paşa Mosque.

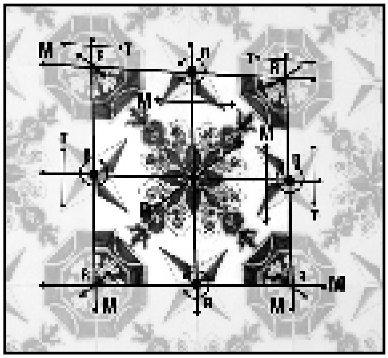

Figure 2:
The tile from Topkapı Palace, Harem section.

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