Symmetry: Art and Science
Buenos Aires Congress, 2007

VISUAL PATTERN OF A MATHEMATICAL SYSTEM

GEORGE LUGOSI

Name: George Lugosi, Chem.Eng., Elect. Eng, IP Consultant, Dir. of R&D&I. (b. Budapest, Hungary, 1936).
Address: 2 Union Street, Kew 3101, Victoria, Australia.
E-mail: g.lugosi@hfi.unimelb.edu.au
Fields of interest: Geometry, mathematical crystallography, (sailing, gliding, mythology).
Publications: George Lugosi, Benapozás vizsgálata, [Investigating the Sun-path in a room, in Hungarian]
Művészet [Art], 1977, 3
Szabadág = semmibe vett szükségszerüség? [Is freedom = ignored necessity? in Hungarian]
Művészet [Art], 1978, 6

***Abstract**: This mathematical pattern based on a simple rule: we fill a square matrix with numbers, created as the sum of the elements of a column- and a row vectors. These elements are integers from n to 1 in the column, and from 1 to n in the row vectors. The change comes with the permutation of the elements. If in the n x n grid the sum of the corresponding elements of the column and row vectors are > n, then this square will be marked with a colour, and they form a "sign". (See Figure 1. I gave more details about this in my abstract, written to the Hiroshima Congress in 1992.) Compare to this formal area, it is much more interesting that how it exists and used in the folk art, and how it gives endless opportunities for stimulating analogues.*

1. THE NATURE OF THIS SYSTEM

The first feature, which we can observe is the enormous "speed" as with the growing *n* the number of the "signs" are multiplying. Let us see:

If the value of n =	1	2	3	4	5	6	7
then there are	1	4	36	576	14400	518400	25401600 signs.

How can we see them? We have to use some "aid", preferably a computer, where we can bring up only that part of it, which momentarily we want to see. With human eyes it is impossible to see all the generated signs on one print if *n* > 5.

We can arrange these signs in a big square, having *n!* rows and columns. If we use 2mm grid-cell size, (having the same space between the signs) then in the case *n* = 6 we would need more than 10 x 10 metre space for the print. Standing in the middle, we can not see the signs 5 metres from us.

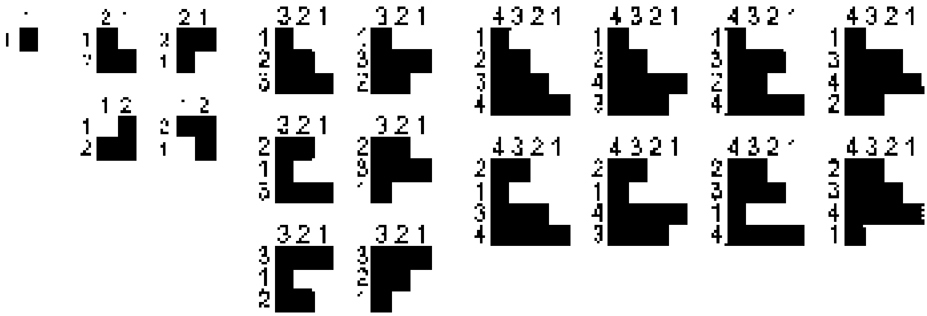

Figure 1

We can use different arrangement also. Instead of one big square, using so called “blocks”. For instance it is visible on Figure 1. the first block of n = 3. In one block the column-vector is the same (3,2,1) only the row-vector is changing according to the rules of permutation. The full set, in case of n = 3, consists of six blocks, having 36 signs. At the next one the column-vector will be 3,1,2 and again the row-vector sweeps through from 1,2,3 to 3,2,1.

If n = 4, then one block consists of 4 x 6 signs, and the full set has 4 x 6 blocks, all together 24 x 24 = 576 signs. The Figure 1. shows the beginning of the first two rows of the first block in case of n = 4. Now it is obvious, that the number of a full set of signs is $n!$ x $n!$, and a block consists of n lines and (n-1)! columns.

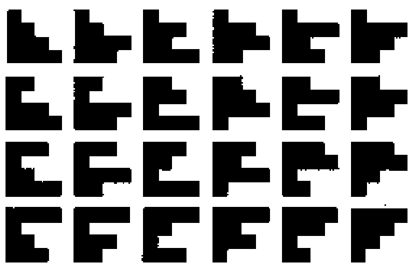

Figure 2

Figure 2. shows the full first block of n = 4. It is visible that in this block the column-vector is 4,3,2,1 and only the row-vector changing, according to permutation, from 1,2,3,4 to 4,3,2,1.

It is important to observe an interesting situation. In every block there are two (and only two) diagonal-symmetric signs. In any block of the n = 4 (4 x 6 signs per block) as well as in a block of n = 8 (8 x 5040 signs per block). Naturally, there are as many blocks in a full system, as many signs in a block. These diagonal-symmetric signs are helping to

“find our way” within a block and also within the full table.
It is very remarkable that certain signs are widely used in folk art, in cross-stitch, as I discussed it in my earlier papers. There are even “connected in art” signs, as the stairs and the spiral, which are used together, many times in geographically far from each other cultures. I could find them in Hungarian cross stitch patterns, in Hopi drawings, in Aztec shield decorations and even on a clay drinking vessel from Peru that these two signs used together.

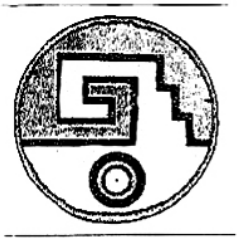

Figure 3

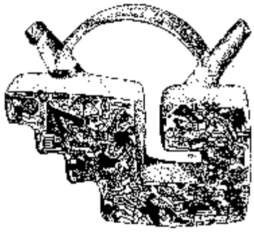

Figure 4

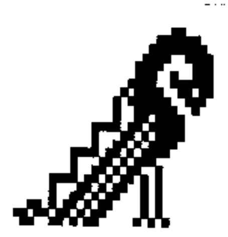

Figure 5

Aztec shield Clay drinking vessel (Peru) Cross stitch (Hungary)
(These pictures are copied from the Sydney Congress Journal.)

It would be necessary to make some research work on the field of ethno-mathematics to find out what is the reason that certain ethnic groups using very characteristic patterns to decorate their clothing, carpet or building, using for thousands of years the same pattern.

2 ASKING FOR HELP

I know it (the long practice convinced me), when we start to “play” with this simple but endless sign-system, then only new and new questions are popping up, unfortunately without answers. When I show different signs, with different “*n*” to people. they tell (without hesitation) that it is a chair, a bird, a swan, a dog. Especially, children are very inventive to “see” something in them. As much as I was able to ask in Melbourne, Australia, where there are people from different ethnic groups, there wasn’t big differences in their answers.

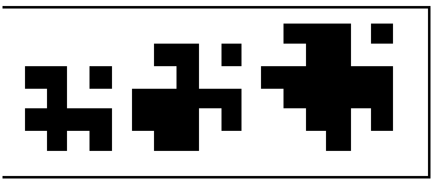

Figure 3

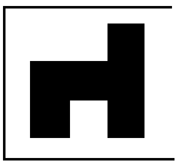

Figure 4

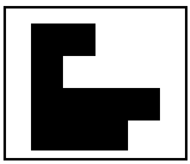

Figure 5

The three signs in Fig. 6., which are from the same so called logical place from the system of $n = 4$, 5 and 6, by almost all whom I tried to question, were named: bird! On the Fig. 7. some children even didn’t think, only answered: little chair, about Fig. 8. after some hesitation they said it is a swan.

For the computer the largest "bird" on Fig. 6. is 1,3,5,6,2,4 / 4,2,6,5,3,1; the "chair" is only 2,1,3 / 1,3,2; the "swan" is 4,3,2,1 / 2,1,4,3.

Naturally, the solution is not on the side of the engineers, but somewhere on the side of the hard to approach sciences, perhaps psychology, anthropology, art history, or (and it would be the best solution) the more and more developing ethno-mathematics. I ask everyone who read this paper try to give me some help: why do we see bird in Fig. 6., and how the folk art can find the motives, which can fit to the folk and practically all of them find it suitable.

References

Ditfurt, H. (1973), "A világegyetem gyermekei" [Children of the Universe, in Hungarian trans.] Budapest McIntyre, L. (1975) "Mystery of the Ancient Nazca Lines" *National Geographic*, May 1975

Lugosi, G. (1992) "Symmetries in permutation-generated patterns" *Symmetry: Art and Science*, Hiroshima Congress, Vol. 3, Nr. 2.

Lugosi, G. (1995) "More investigation on permutation-generated patterns" *Symmetry: Culture and Science*, Washington Congress, Vol. 6, Nr. 2

Lugosi, G. (2001) "Folk art and symmetries of permutation-generated signs" *Symmetry: Art and Science*, Sydney Congress, Vol. 1, Nr. 1-2

Lugosi, G. (2004) "Mathematical Tourism in Siberia" *The Mathematical Intelligencer*, vol. 6, No. 2, pp 34-36.

Lugosi, G. (2005) "Les maillages sibérians, Mathématiques exotiques" *Pour la Science*, Edition francaise de Scientific American, Dossier No. 47, Avril-Juin 2005, pp. 58-59.

Molnár, V. J., (1999), Világ – Virág, [World-Flower, in Hungarian] , Budapest

Purce, Jill, (1980), "The Mystic Spiral", *Thames and Hudson*, London

Spinden, H. J.,(1975), "A Study of Maya Art", *Dover Publications*, Inc., New York.