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A FAMILY OF 3D CLOSED CURVES AND NEW REGIME OF THE STEREOGRAM THROUGH A SET OF THREE PICTURES

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Fields of interest: Mathematical science mainly geometrical and studies of wide rage including culture, education, society, peace, etc. without any boundary in principle, a kind of intellectual gourmet..

Abstract: *It is the following basic questions that guided to the present work:*
1. What is the uniform curve proper to each dimensionality?
2. What is the most reasonable attitude to relate a uniform 4D curve to a 3D curve?
3. Is it possible to improve stereogram towards simplicity?
The relating facts concerning to the 1st question above are summarized in a table below.

Dimensionality	*Family of uniform curves proper to the dimensionality*	*What is the new constant in the dimension.*	*Global feature*
1D	*Straight lines (Inevitably)*	*(Only a direction)*	*Open*
2D	*Circles.*	*curvature*	*Close*
3D	*Helixes.*	*Constant torsion.*	*Open*
Odd dimensionality	*Open curve*	*Indescribabe*	*Open*
Even dimensionality			*Close*

1 INTRODUCTION

The main related facts are summarized in the Table in Abstract. One of the main purposes of this paper is to fix general view about the uniform curves for any dimension. The second purpose is to construct a new reasonable scheme for 3D mankind to understand 4D geometrical object. In this paper, a family of 3D curves is obtained from a family of uniform 4D close curves. Of course, the way is not unique but there are many.
Next task is to show the 3D curve as sets of stereogram. A new devise is introduced.

2 UNIFORM CURVES PROPER TO DIMENSIONS

Now, a curve that can be specified with a single parameter, may be regarded as motion. The simplest motion is uniform one, straight and with constant velocity. It is one-dimensional. The second simplest is uniform circular motion. It is two-dimensional and cab be regarded as the combined one of two oscillations of mutually orthogonal directions with the same frequency and with phase difference of a quarter period. These two modes of motion is the most elementary or most basic in Scientific comprehension of physical world. The uniform curve proper to 3D is helixes that can be regarded as any combination of uniform linear motion in 1D and uniform circular motion in 2D. The combination yields a constant torsion including third order derivative.
It is natural to guess the uniform curve in 2D-dimension consists of D circular motions in the present terminology. It is also natural to guess that uniform curve in (2D+1)-dimension consists of D circular motions and a linear motion.
Now, I conclude that the uniform curve proper to 4D consists of two circular motions in mutually orthogonal 2D spaces. The uniform curve in 4D lies on a *hyperspherical surface*. It is noted that two frequencies and two radii are arbitrary. If two frequencies are rational, then the curve closes in finite and if they are irrational the curve covers the whole *hyperspherical surface* in dense.

3 FOR THE FUN AND CULTURE OF 3D HOMO SAPIENCE

The facts mentioned up to the previous section is logically enough to understand. For us, *symmetry and katachi* people, there remain many things to treat and to enjoy still now. First of all, we are all homo sapience living in 3D space and how can we feel the 4D curve? There are infinitely many ways for that. Here the author likes to introduce new idea about the relationship between 3D and 4D.
Needless to say that it is impossible to realize a set of four directions mutually orthogonal in 3D space. But it is possible to arrange four directions evenly enough by neglecting orthogonality if a set of four is used as in regular tetrahedron. The following set of four unit vectors, for example,
Now, the equation of the 3D closed curve is given by

$$\begin{pmatrix} a \\ a \\ a \end{pmatrix} \quad \begin{pmatrix} a \\ -a \\ -a \end{pmatrix} \quad \begin{pmatrix} -a \\ a \\ -a \end{pmatrix} \quad \begin{pmatrix} -a \\ -a \\ a \end{pmatrix} \quad \text{where} \quad = \frac{2}{\sqrt{3}}$$

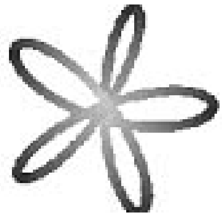
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos t + \sin f - \cos kt - \sin kt \\ \cos t - \sin f + \cos kt - \sin kt \\ \cos t - \sin f - \cos kt + \sin kt \end{pmatrix} = 2\sqrt{2} \begin{pmatrix} \cos[(k+1)t/2+\pi/4]\sin[(1-k)2/2] \\ \cos[(k+1)t/2+\pi/4]\cos[(1-k)t/2] \\ -\sin[(k+1)t/2+\pi/4]\sin[(1-k)t/2] \end{pmatrix}$$

where k is rational for the curve to be close as mentioned in the previous section. The simplest case not too trivial is $k = 3/2$ and $k = 13/8$.

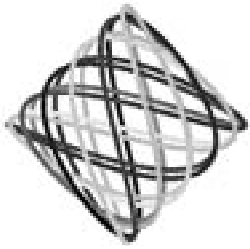
The view along x-axis The view along y-axis The view along z-axes

$k = 3/2$

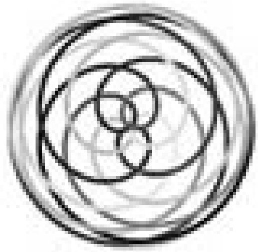
$k = 3/2$



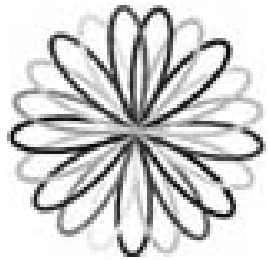
$k = 3/2$



$k = 13/8$



$k = 13/8$



$k = 13/8$

3 NEW REGEME OF STEREOGRAM WITH THREE PIECES

A usual stereogram consists of two pieces, for left eye and right eye. But, some people like parallel view and others cross view. Therefore, two pairs, altogether four pieces of pictures are necessary. Note that a set of three pieces is enough and both of front view and back view, though exactly speaking the latter is reversal. Two in both sides among three pieces can be the same. You can enjoy the stereograms successfully when three pieces can be seen as if they were four objects. The following examples are for the previous cases for and

k=3/2
x

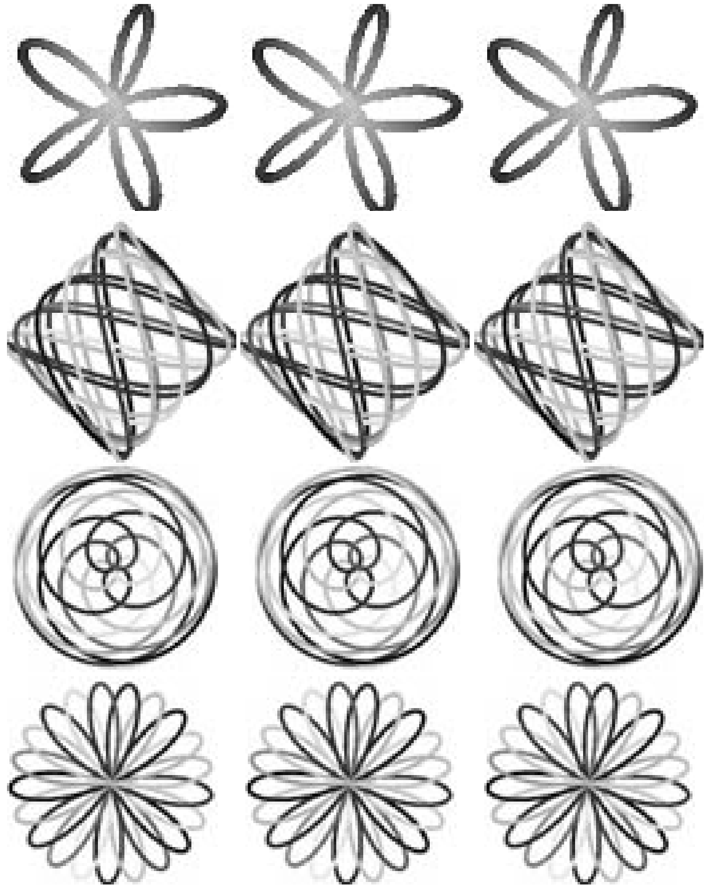
k=3/2
y

k=3/2
z

k=13/8
x

k=13/8
y

k=13/8
z



Now several short communications in Japanese are available about the work. A relating paper is now in preparation for FORMA, electronic journal for The Society for Science on Form. http://www.scipress.org/journals/forma/index.html