Paulus Gerdes [1]

Analysing symmetric, mat weaving designs made by Makwe women in the Northeast of Mozambique: The example of the chicken's eye pattern

Abstract

Makwe speaking women in the Palma District in the Cabo Delgado Province in the Northeast of Mozambique have been producing, for centuries, mats by sewing together bright monochrome and decorated two-colour strips. Earlier the author analysed symmetry classes and colour inversion of decorated strips where natural and dark coloured strands alternate. The present paper analyses a different kind of decorated strip. A particularly woven band displays on both faces the same strip pattern albeit with its orientation inverted. The pattern is called the chicken's eye and has a horizontal axis of symmetry. The paper explains that the inversion of the orientation of the pattern results from the particular weaving texture and that the symmetry of design is a consequence of the chosen colouring code in association with the selected weaving texture. To analyse the woven band, the author constructs underlying number friezes that present interesting symmetry and anti-symmetry properties. The inventor(s) of the chicken's eye pattern constructed consciously and carefully the weaving texture. This texture did not result from mere experimentation, but some type of calculation and of geometric-symmetry considerations was involved in one way or another. The author illustrates how changing the colour code, the weaving texture or the dimensions produces other patterns with the same properties as the chicken's eye pattern. The paper invites to further mathematical study of particular number friezes and their generalization.

Introduction

Ethnomathematics as a field of research tries – among other objectives – to analyse mathematical ideas as embedded in their cultural context. The following paper presents an example of an ethnomathematical study in progress. It analyses mathematical ideas involved in the invention and reproduction of a particular design in the Makwe culture.

The fine mats made by the Makwe and neighbouring peoples in the coastal zones of Northeast Mozambique have been famous for a long time. In the 18th century they were among their most important products traded at Mozambique Island and other ports further to the south. Women make the mats from brightly dyed palm fibre by sewing long plaited bands together. The mats are laid down on the floor to eat, talk, sleep or rest upon, or hung on the walls as decoration. One-colour plain strips separate the decorated two-colour strips from each other. Photograph 1 presents a detail of a rectangular mat.

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Front side
Photograph 1

In an earlier study (Gerdes 2007a, b), the author analysed symmetry classes and colour inversion of decorated strips where natural and dark coloured strands alternate, as in the case of the decorated strip in Photograph 1. This context was one of the sources of inspiration for the discovery / invention of cycle matrices (Gerdes 2008). These square matrices present interesting visual properties and can be classified as positive and negative cycle matrices. The design in Photograph 1 corresponds to the cycle structure of a negative cycle matrix, whereas the design on its backside corresponds to the cycle structure of a positive cycle matrix (see Photograph 2).

Backside
Photograph 2

Chicken's eye pattern

In the present study, I analyse a decorated band where natural and pairs of dark coloured strands alternate. Photograph 3 displays a Makwe mat acquired in 2007 with a pattern called chicken's eye. As Idaía Amade, coordinator of the weaving cooperative 'Vomerijaupate' in Palma told me on September 3, 2009, the pattern is an old pattern. She and her colleagues learned ways to memorise and reproduce the pattern, but are not its inventors. The same pattern also appears in a catalogue of woven strip patterns from Zanzibar Island in Tanzania, where it refers to the python or to the heart in plaids (Förstle 2006, p. 48).

Part of a Makwe mat with the chicken's eye pattern
Photograph 3

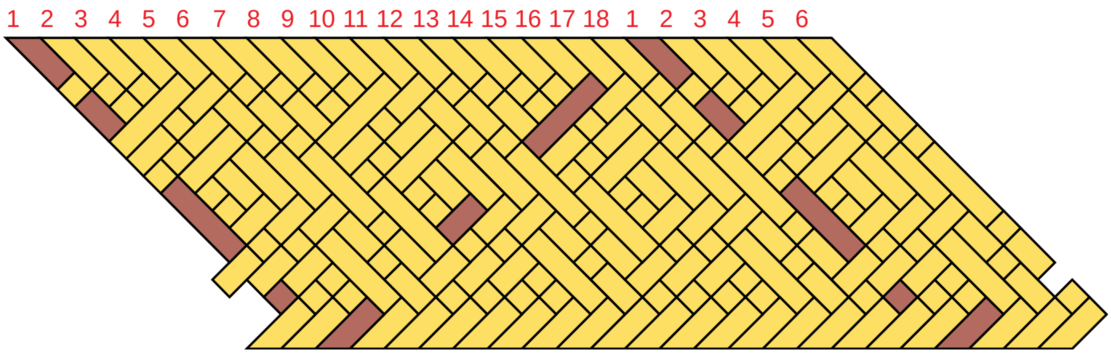
The visual image of the woven pattern belongs to symmetry class *p1m1*, as it has a symmetry axis in the direction of the band. Photograph 4 displays both sides of a part of a chicken's eye pattern. On the backside the same pattern appears, however its orientation has been inversed. The central squares are slightly displaced in relationship to the front side of the woven band. To understand why the strip pattern is (almost) the same on both sides, let us analyse its weaving texture and colouring.

Front side
a

Backside
b
Photograph 4

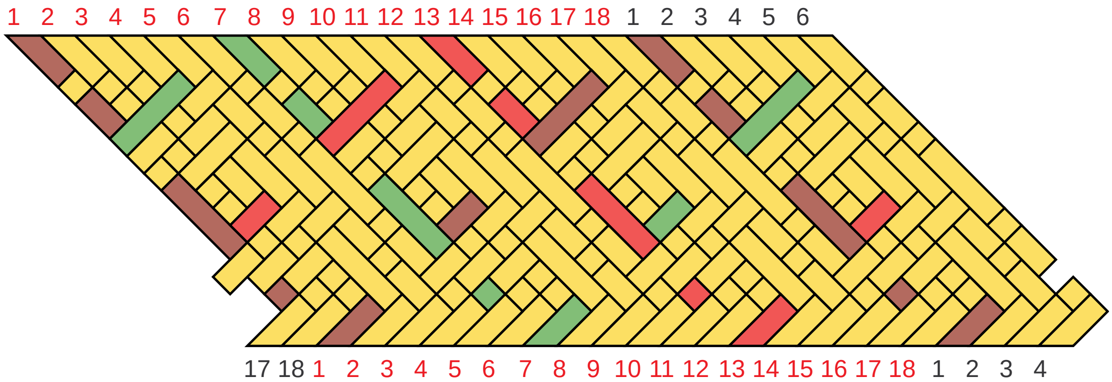
Analysing the weaving texture

The decorated band has a weaving width of 18, that is, it is composed of 18 zigzagging strands. The strands make angles of 45^o with the border of the band; when the end of a plaited line is reached, the strands are bent back, to be worked in the opposite direction (see Figure 1).



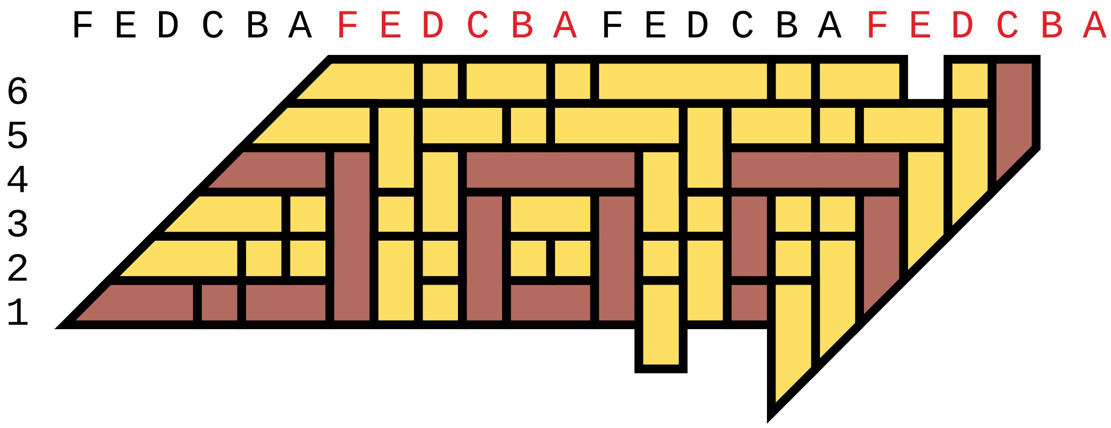
Strand 1 zigzagging along the woven band
Figure 1

Both the weaving texture and the visible design have period 6. This means that on the one hand the way the 7th and the 13th strand are interwoven is the same as that of the 1st strand, and on the other hand these three strand have the same colour. Similarly, the 2nd, 8th and 14th strand follow the same weaving rhythm of going over and under and have the same colour, etc. In this sense, we may say ‘1’ = ‘7’= ‘13’, ‘2’ = ‘8’ = ‘14’, … In other words, there are six sets of three equivalent strands (see Figure 2).



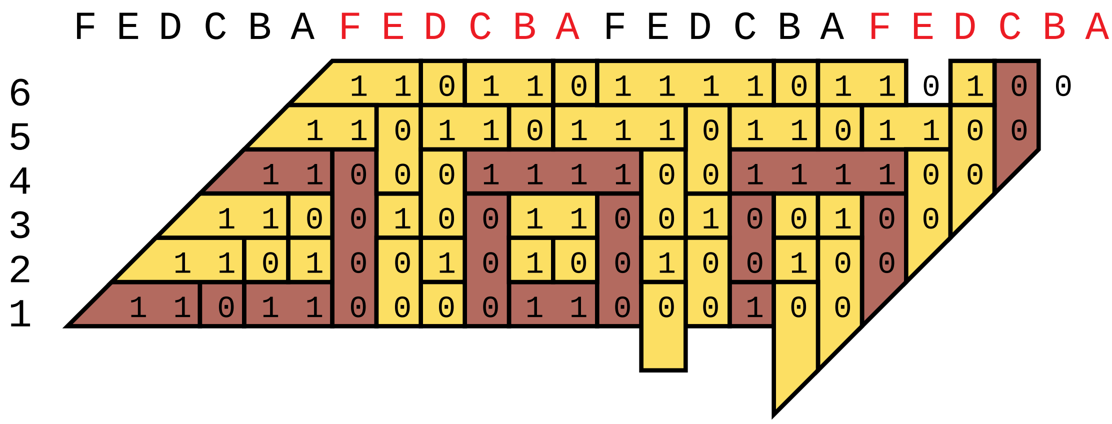
A set of equivalent strands (1, 7, and 13)
Figure 2

The pattern has period 6. Consider one element of the pattern in the position displayed in Figure 3. We may number the six strands in one direction (horizontal in the representation) 1 through 6, and the strands in the second (vertical) direction A through F.



Weaving texture of the chicken’s eye pattern collected in 2008
Figure 3

The unit squares where the horizontal and vertical strips cross each other may be numbered: 1 if the horizontal strip passes over the vertical strip; 0 if the horizontal strip passes under the vertical strip at the unit square under consideration. See Figure 4.



Numbering of the weaving structure
Figure 4

Let us compare the numbers in the rows and columns (see Figure 5). Something remarkable happens. Column ‘A’ (starting at the most leftward A and going downwards through successive elements of the pattern) is equal to row ‘5’ going from the left to the right. Shortly: ‘A’ = ‘5’. In the same sense, ‘B’ = ‘6’, ‘D’ = ‘2’, and ‘E’ = ‘3’.

A	1	1	0	1	1	0	1	1	1	0	1	1	0	1	1	0	0	5
B	1	1	0	1	1	0	1	1	1	1	0	1	1	0	1	0	0	6
C	1	1	0	1	1	1	0	0	0	1	1	1	0	0	1	0	0	?
D	1	1	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	2
E	1	1	0	0	1	0	0	1	1	0	0	1	0	0	1	0	0	3
F	1	1	0	0	0	0	1	1	1	0	0	0	1	1	1	0	0	?

Comparison of the numbers in the rows and columns
Figure 5

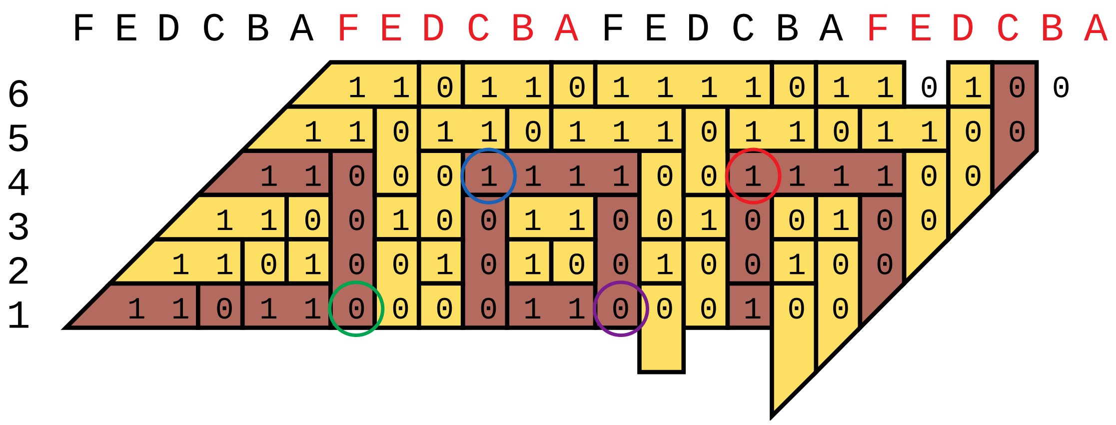
Column C is almost equal to row 1. At two places the numbers are different (see Figure 6). Similarly, column F is almost the same as row 4. Also at two places the numbers are different.

C	1	1	0	1	1	1	0	0	0	1	1	1	0	0	1	0	0	C
1	1	1	0	1	1	0	0	0	0	1	1	0	0	0	1	0	0	1

F	1	1	0	0	0	0	1	1	1	0	0	0	1	1	1	0	0	F
4	1	1	0	0	0	1	1	1	1	0	0	1	1	1	1	0	0	4

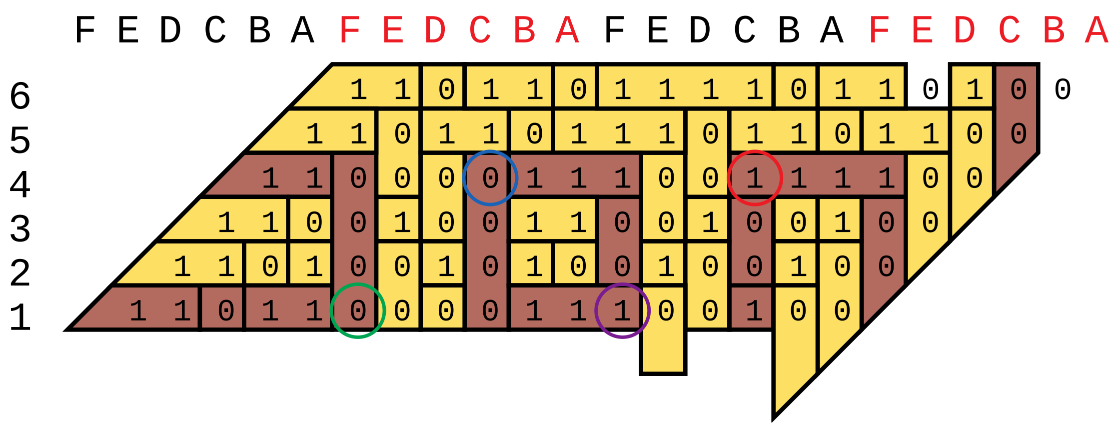
Differences
Figure 6

Let us see in the pattern where are those unit squares that correspond to different numbers (see Figure 7). They correspond to places where in both weaving directions strands of the same colour cross. In other words, changing which strand goes over at those particular unit squares does not change the colours of the respective unit squares on both faces of the woven band.



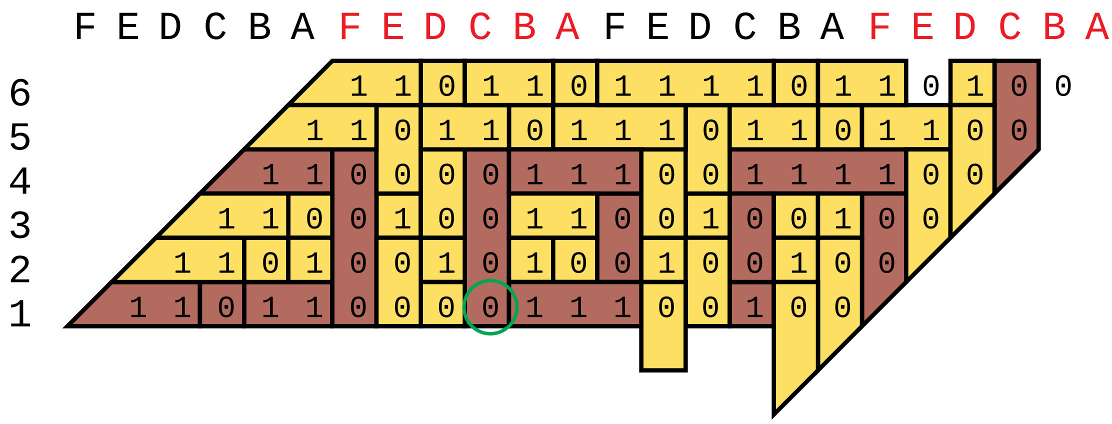
Particular unit squares
Figure 7

So let us change the over and under at two of these crossings so that ‘C’ = ‘1’ and ‘F’ = ‘4’ (See Figure 8).



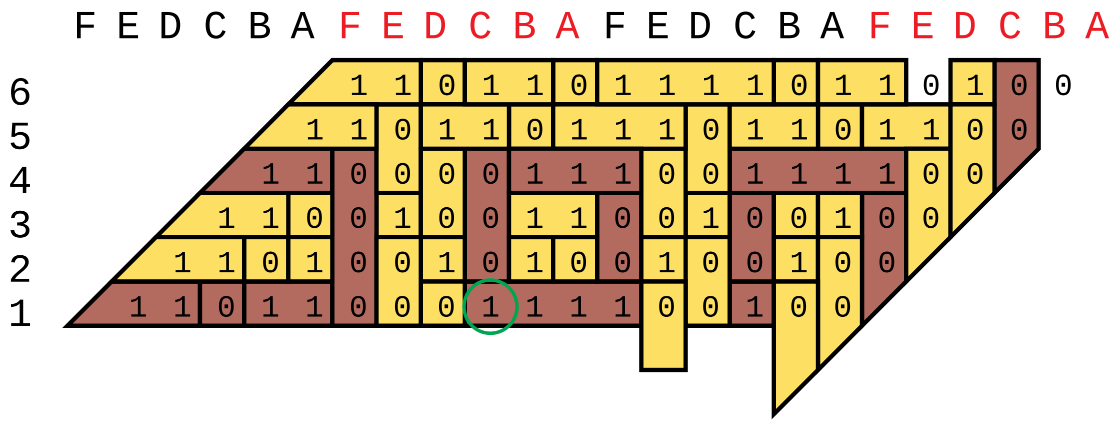
Introducing changes
Figure 8

It may be observed that on the central axis of the pattern the horizontal strands always pass over the vertical strands, being the only exception at the first row (see Figure 9).



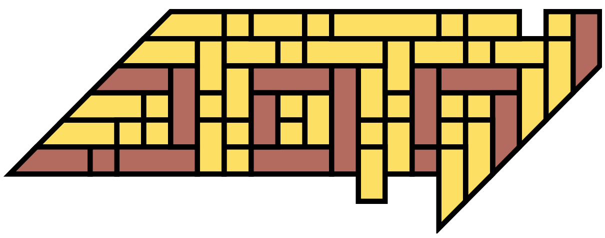
Exception on the first row
Figure 9

When we let the first horizontal strand pass, at its centre, over the vertical strand, the colour of this unit square does not change. So let us change the position of the first horizontal strand to make the central axis always crossed over, that is, only 1's at the central axis (in diagonal position in Figure 10).



One more change
Figure 10

Considering the other versions (see the example in Figure 11) of the chicken's eye pattern in our collection, I noted that although their images are the same, small differences in the weaving texture occur.



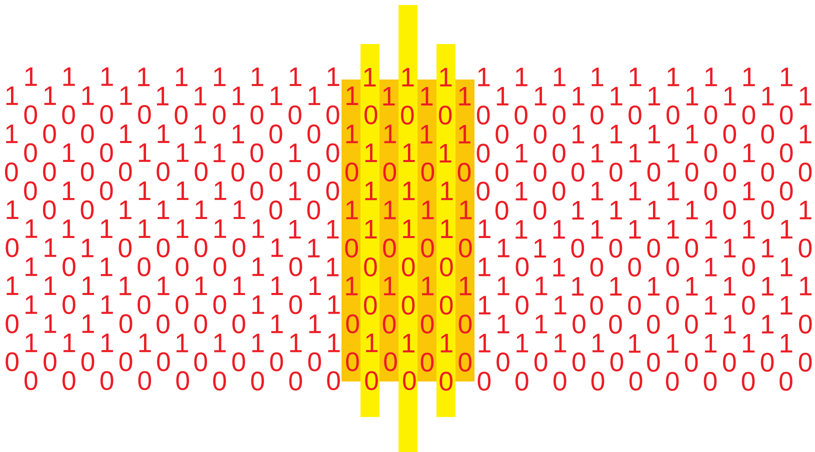
Weaving texture of the chicken's eye pattern collected in 2007
Figure 11

In all cases, an analysis of the type presented leads to the weaving structure in Figure 10. Taking also into account the consistency and regularity of this weaving texture with its equalities 'A' = '5', 'B' = '6', 'C' = 1, 'D' = '2', 'E' = '3', and 'F' = '4', I am led to suppose, that the inventor of the chicken's eye pattern used the weaving texture in Figure 10, or a very similar one.

Number frieze
Figure 12

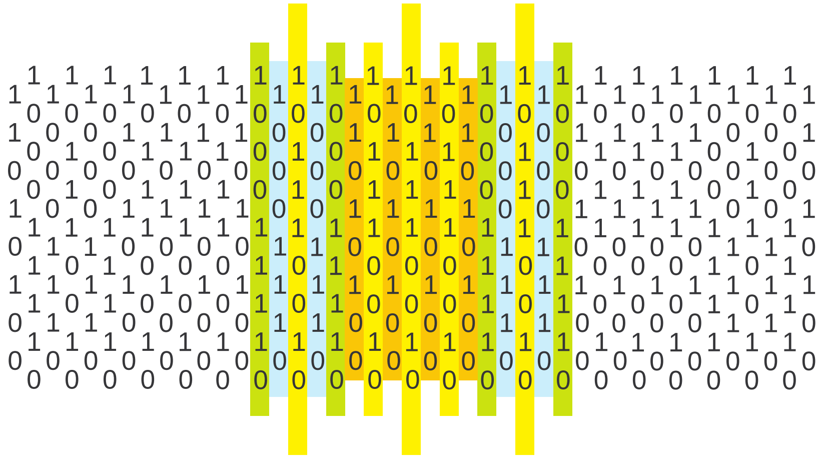
An interesting number frieze

The number frieze (Figure 12) that corresponds to the reconstructed original weaving texture has interesting and attractive properties. Figure 13 displays a part of the number frieze that can be thought of as continuing infinitely to the left and to the right. The number frieze has a vertical axis of symmetry; the number columns of the same colour are equal.



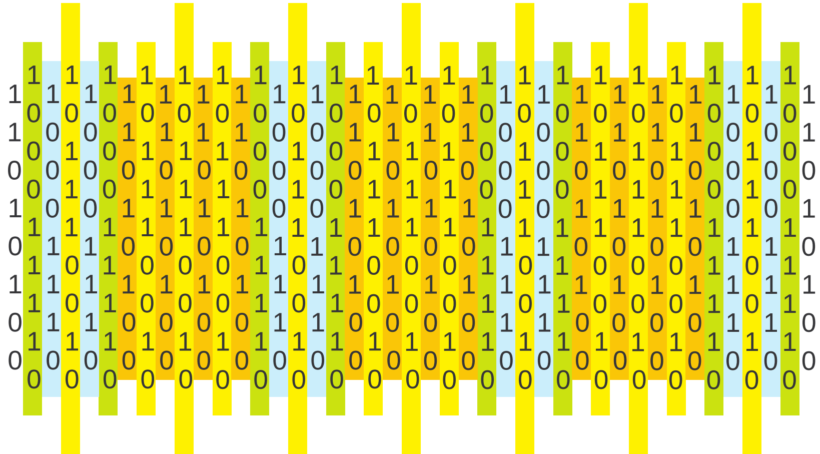
Vertical axis of symmetry
Figure 13

Left and right of the vertical central band of seven number columns, there lie two bands of five number columns, of which several are the same, and consequently marked by the same colour (See Figure 14).



Sets of equal columns
Figure 14

To the left and right of the marked number columns, the central band is repeated, and so forth (period 6) (See Figure 15).



Vertical axes of symmetry
Figure 15

The horizontal axis of the number frieze is an anti-symmetry axis: any two number rows at the same distance of it, above and below, are inverted: there were in one number row is a 1 there is a 0 in the other. In other words, the sum of any two such number rows is always the constant number row with only 1's (see Figure 16).

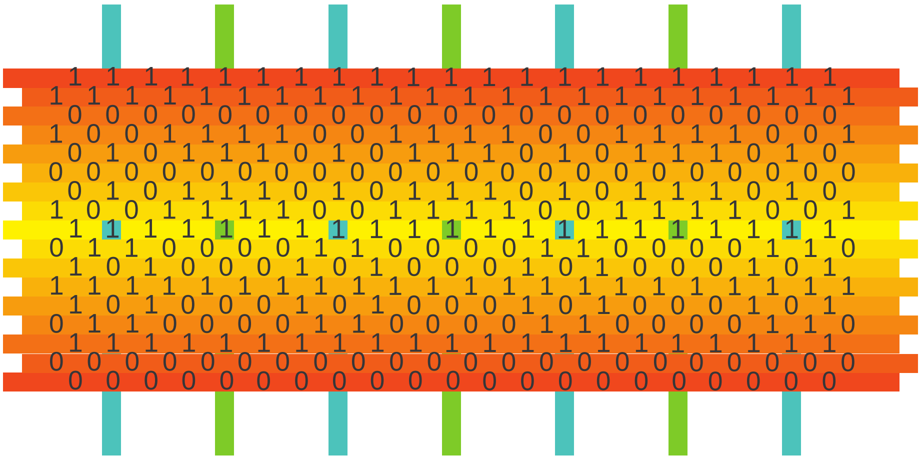
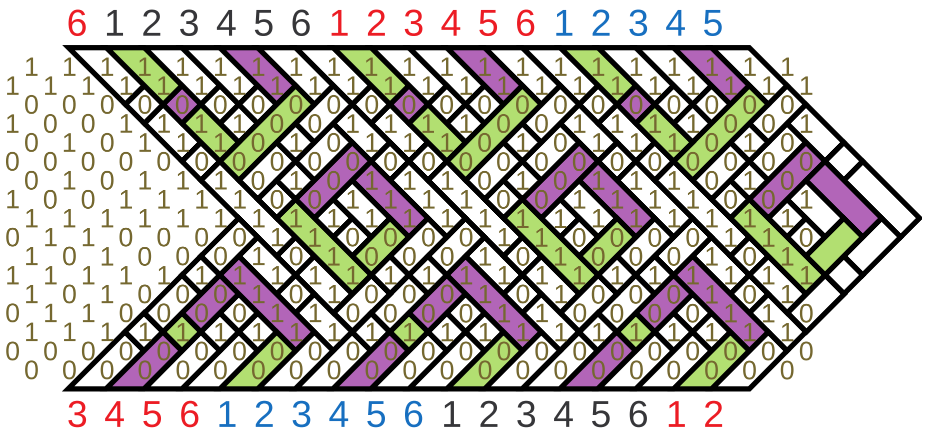


Figure 16

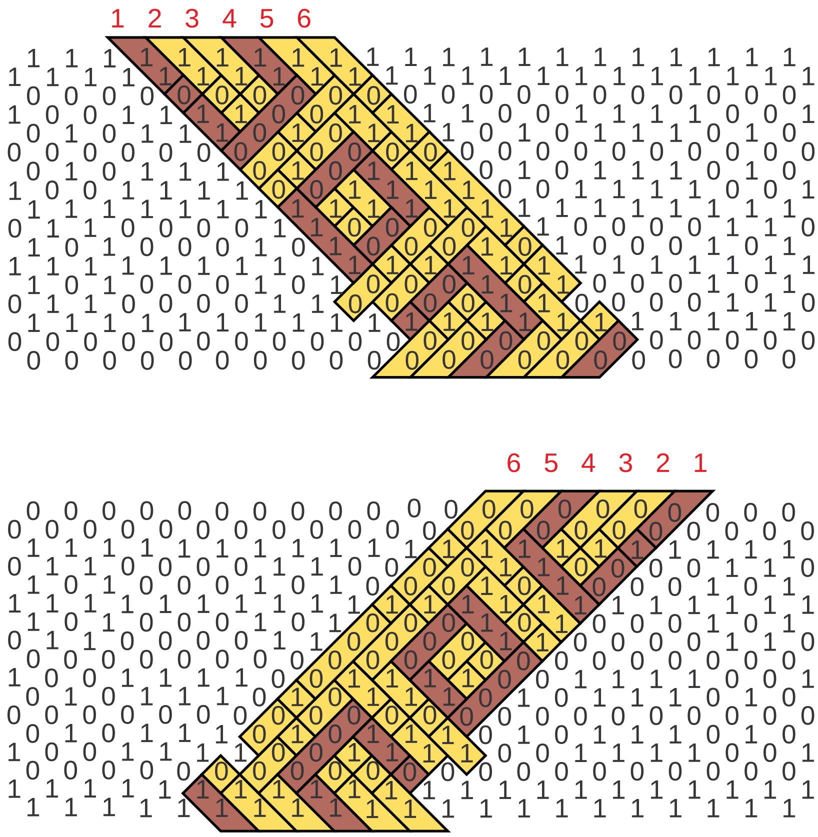
The anti-symmetry of the number frieze means that two parts of strands – one starting at a certain place the at top and going downwards from left to right and the other starting at the opposite place at the bottom and going upwards from left to right – follow exactly the same interweaving rhythm, with the exception of the unit square on the central, horizontal axis (see the example in Figure 17). In this way, strands 1 and 4 are symmetrical to each other, just like strands 2 and 5, and strands 3 and 6. This symmetry reduces by a half what the mat weaver has to memorize about the weaving rhythm (or weaving algorithm).



The pair of opposite strands 1 and 4
Figure 17

Front and back faces of the woven band

Let us return to the woven band itself. What happens if we flip over the woven band along its central horizontal axis? We will see the back face of the band pattern instead of the front face (see Photograph 4).
What will be the frieze pattern that corresponds to its back face?
We flip first the number frieze of the front face and then substitute the 1's on its horizontal axis by 0's (see the lower part of Figure 18).



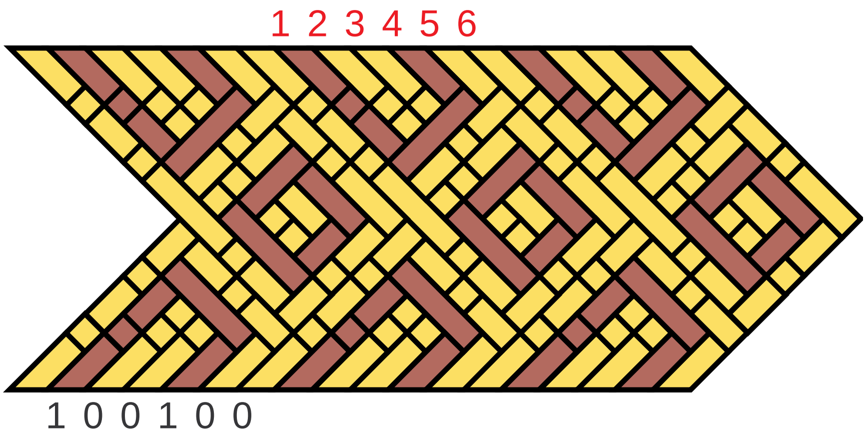
Number friezes corresponding to both faces
Figure 18

As a consequence of the vertical symmetry axes of the initial number frieze, on the backside the number columns 1, 2, 3, 4, 5, and 6 appear now from right to left (Figure 18 lower part), each going downwards from right to left. As the number columns are orthogonal to the initial number rows (Figure 18 upper part), their numbers 0's and 1's are inverted. This explains why on the backside the same chicken's eye design appears as on the front side, being its orientation inverted.

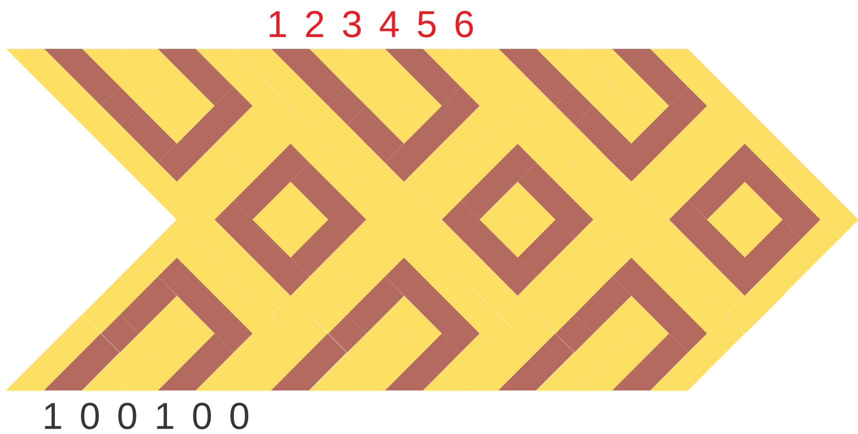
The inversion of the orientation of the design is a direct consequence of the weaving texture under consideration. Whatever the design that is produced by any colouring of the six (sets of) strands, an inversion of orientation of the design on both faces of the woven band will occur.

Different colourings

The chicken's eye design corresponds to the 100100-colouring. The first and the fourth strands have one colour (indicated by 1), whereas the other four strands have another colour (indicated by 0), as Figures 19a and b illustrate.



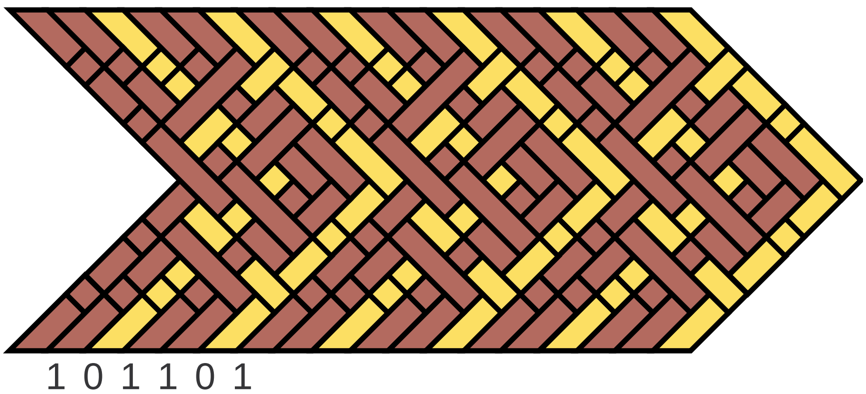
a: 100100-Colouring of the weaving texture



b: Corresponding visual image

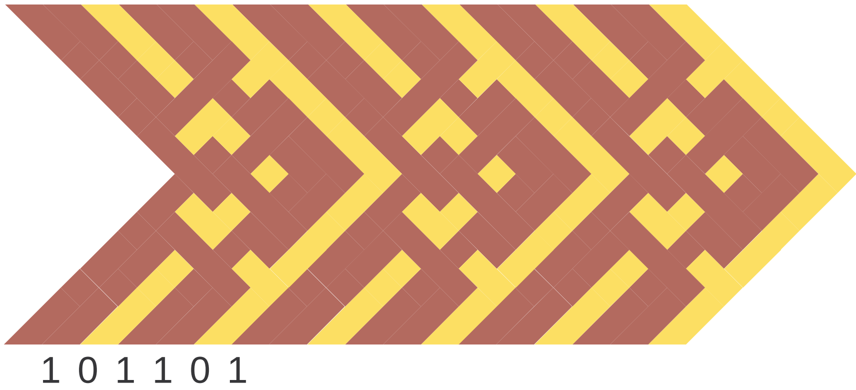
Figure 19

Any colouring that attributes the same colour to the symmetrically opposed strands 1 and 4, 2 and 5, and 3 and 6, leads to a pattern that has a horizontal axis of symmetry. Besides the chicken's eye 100100 colouring code, two other possibilities to obtain such a symmetric pattern with two colours exist: colouring codes 101101 and 110110, as displayed in Figures 20 and 21.

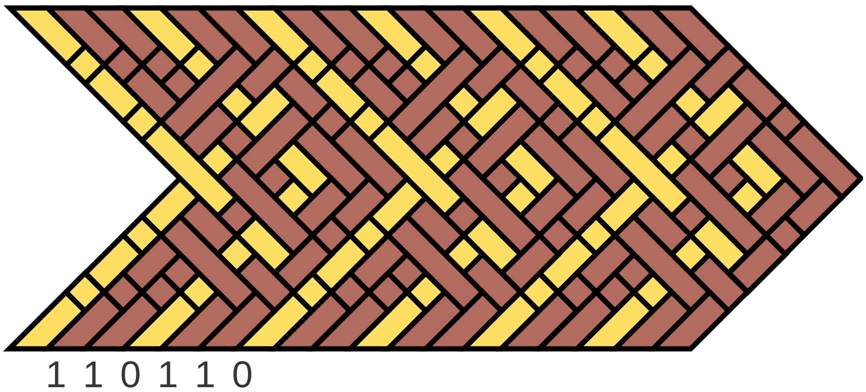


a: 101101-Colouring of the weaving texture

Figure 20 (First part)



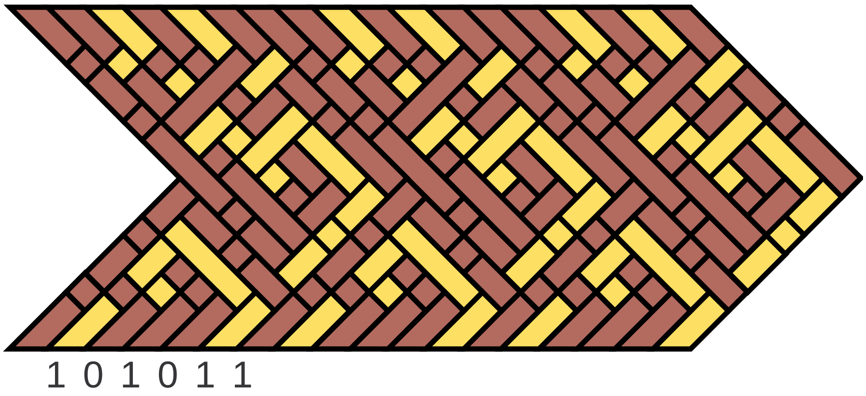
b: Visual image of the 101101-colouring
Figure 20 (Conclusion)



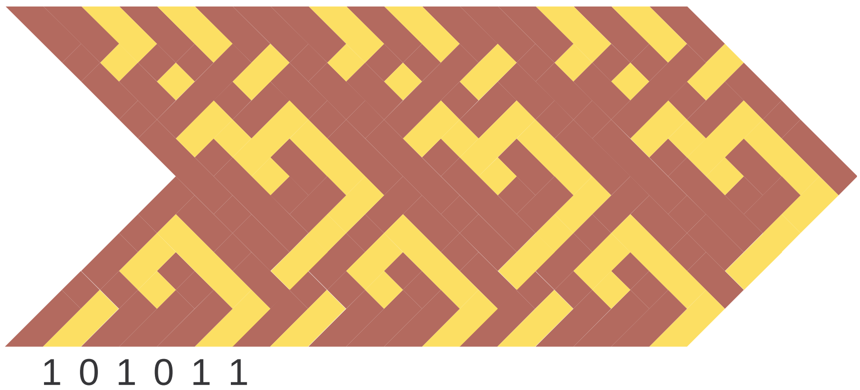
a: 110110-Colouring of the weaving texture

b: Visual image corresponding to the 110110-Colouring
Figure 21

Any other colouring with two colours will not lead to a pattern with a horizontal axis of symmetry. Figure 22 illustrates the case of the 101011-colouring. Although the visual pattern does not have a symmetry axis, the same pattern appears on both faces of a correspondingly woven band with opposite orientations.



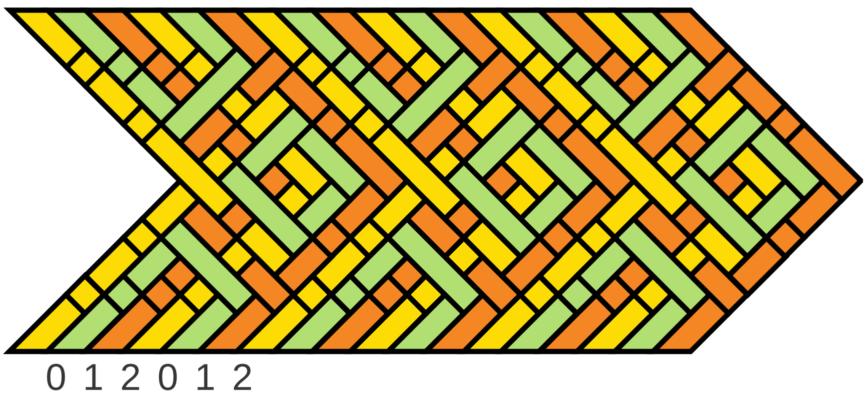
a: 101011-Colouring of the weaving texture



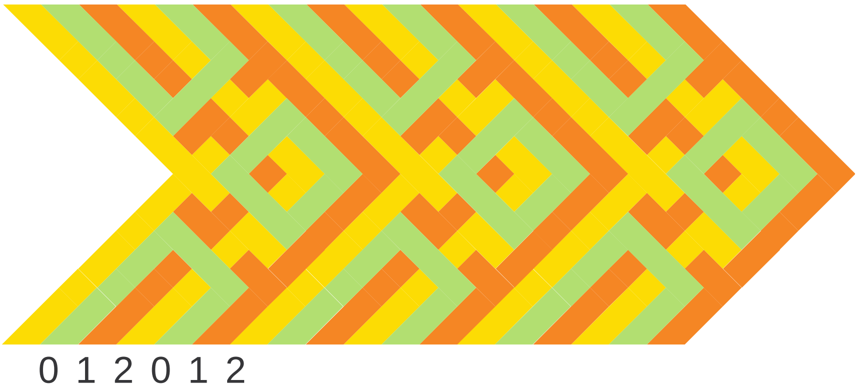
b: Visual image corresponding to the 101011-Colouring
Figure 22

Giving the first strand the first colour (1), there are 32 ways to colour the next five strands. Excluding the mono-colouring 111111, there are 3 patterns with a horizontal axis of symmetry.

Using three colours (0, 1, 2), there is one possibility to produce a pattern that has a horizontal axis of symmetry. Figure 23 displays the 012012-colouring.



a: 012012-Colouring of the weaving texture
Figure 23 (First part)



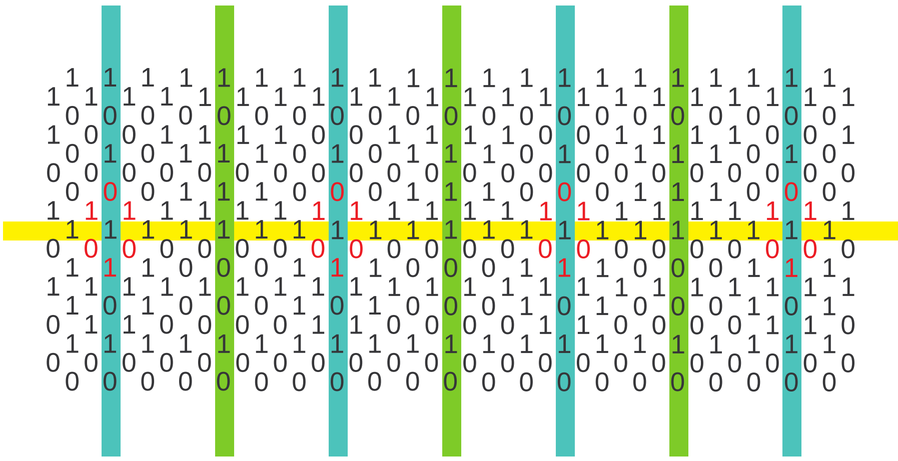
b: Visual image corresponding to the 012012-colouring
Figure 23 (Conclusion)

Changing the weaving texture

When we change the number frieze in such a way that its basic symmetry and anti-symmetry characteristics are preserved, a new weaving texture is produced that has the property of producing the same pattern on both faces of the woven band and of inverting the orientation of this pattern.

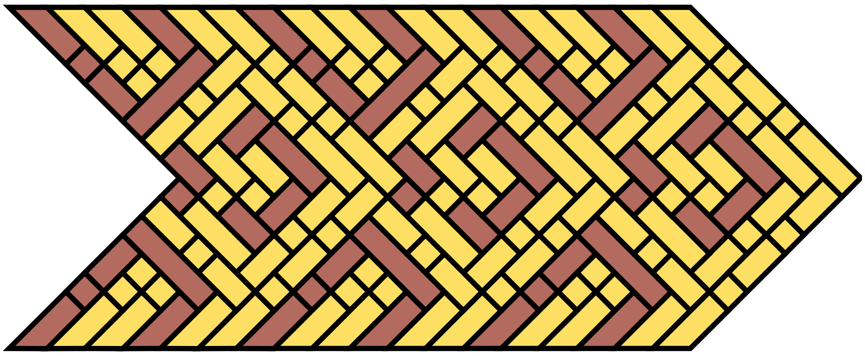
First example

Figure 24 presents the first example. The numbers in red are the only changes relatively to the chicken's eye number frieze: I interchanged 0's and 1's.



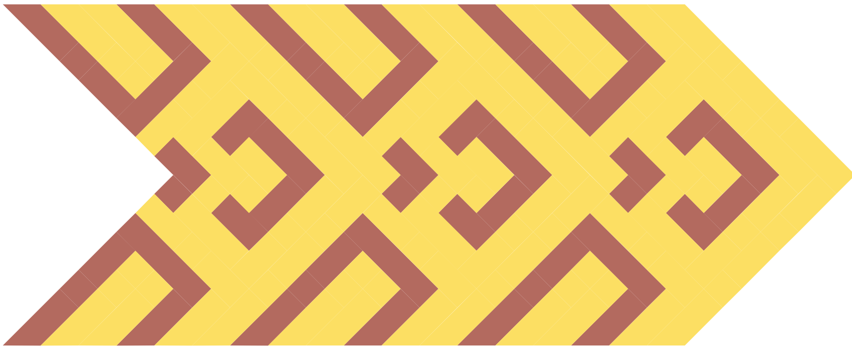
Changing some numbers
Figure 24

This number frieze generates a weaving texture, that once again, irrespective of the particular colouring of the six sets of strands, always reproduces the same pattern on both faces of the woven band and inverts the orientation of the pattern from one face to the other. The colourings corresponding to the codes 100100, 101101, and 110110 lead to designs with axes of symmetry (see Figures 25, 26, and 27).
Figure 28 presents an example of a pattern that does not have a horizontal axis of symmetry.



1 0 0 1 0 0

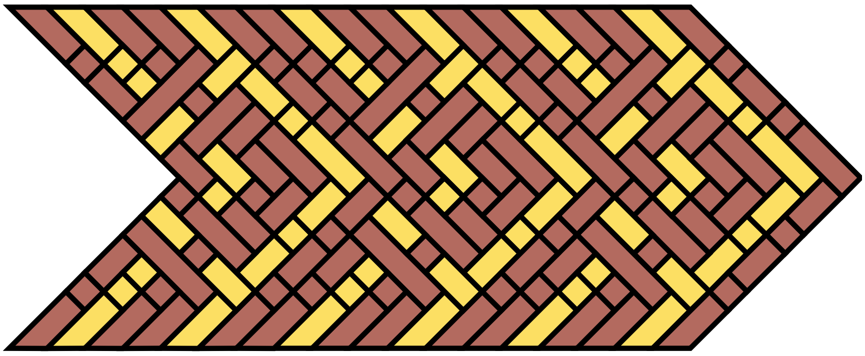
a: 100100-Colouring of the weaving texture generated by the number frieze in Fig. 24



1 0 0 1 0 0

b: Visual image

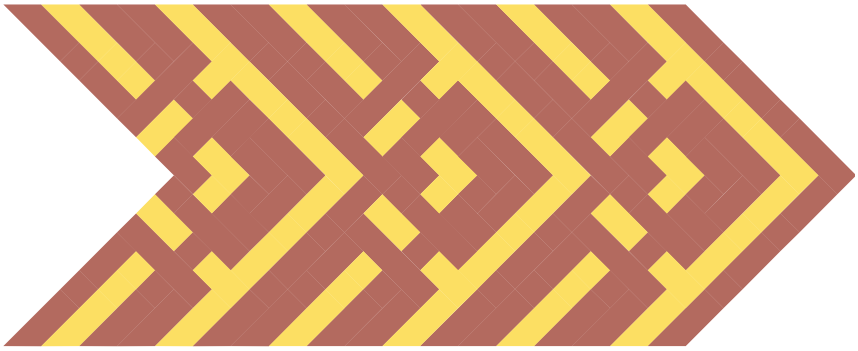
Figure 25



1 0 1 1 0 1

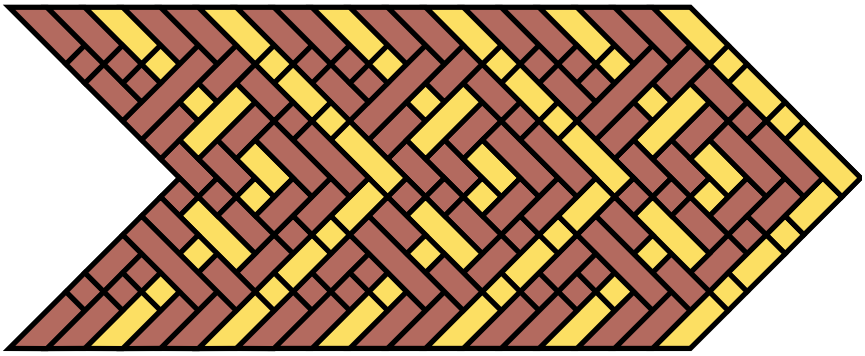
a: 101101-Colouring of the weaving texture generated by the number frieze in Fig. 24

Figure 26 (First part)



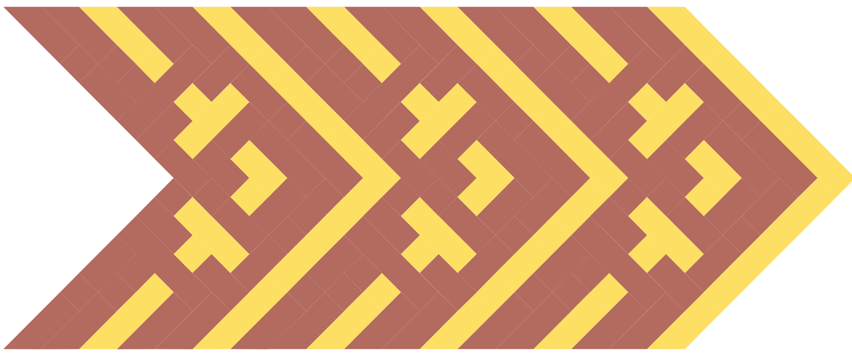
1 0 1 1 0 1

b: Corresponding visual image
Figure 26 (Conclusion)



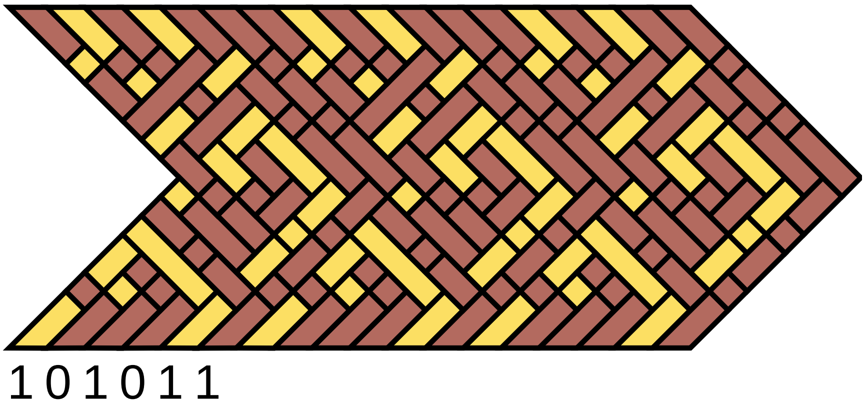
1 1 0 1 1 0

a: 110110-Colouring of the weaving texture generated by the number frieze in Fig. 24



1 1 0 1 1 0

b: Corresponding visual image
Figure 27



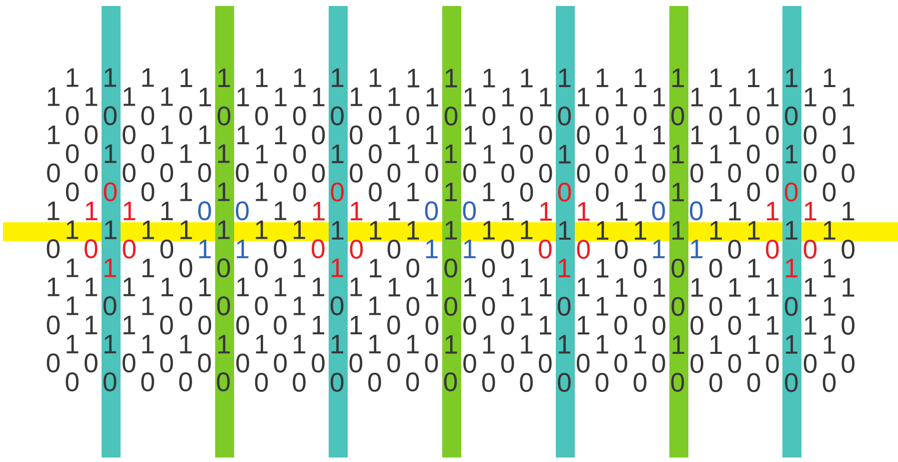
a: 101011-Colouring of the weaving texture generated by the number frieze in Fig. 24

b: Corresponding visual image

Figure 28

Second example

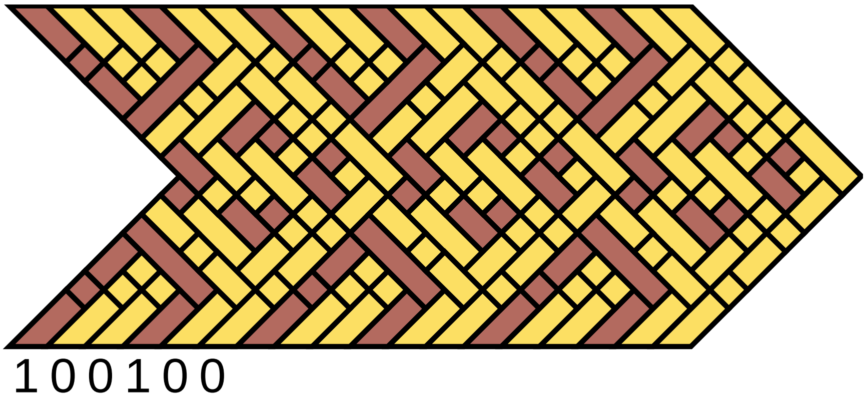
Figure 29 presents a second example of a number frieze with the same characteristics as the chicken's eye number frieze. The numbers in red mark the differences.



Different numbers marked red

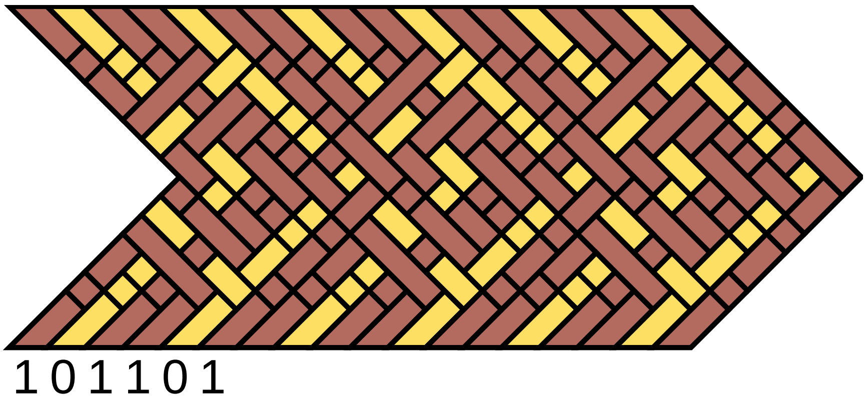
Figure 29

Figures 30, 31, and 32 present the 100100, 101101, and 110110-colourings of the weaving texture generated by the new number frieze. These three patterns have a horizontal axis of symmetry.

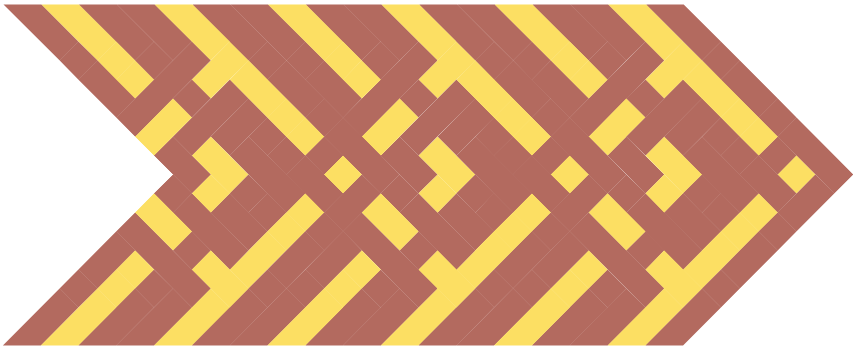


a: 100100-Colouring of the weaving texture generated by the number frieze in Fig. 29

b: Visual image
Figure 30

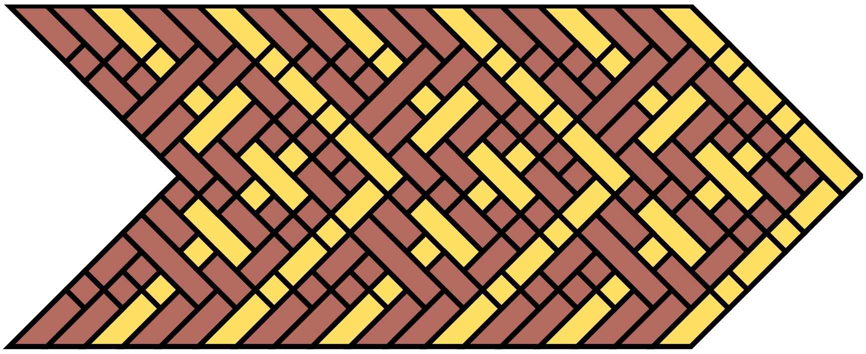


a: 101101-Colouring of the weaving texture generated by the number frieze in Fig. 29
Figure 31 (First part)



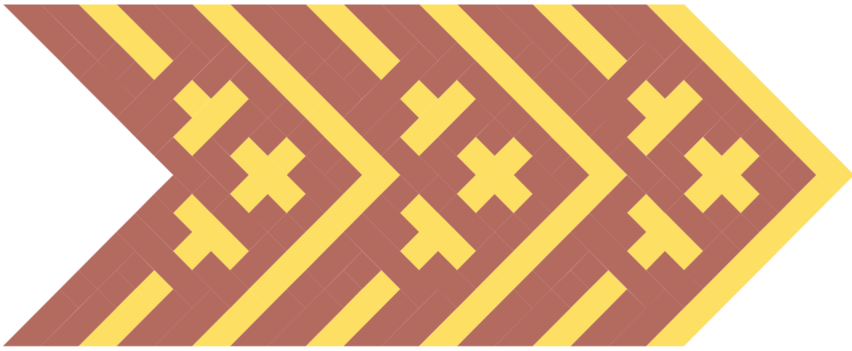
1 0 1 1 0 1

b: Visual image
Figure 31 (Conclusion)



1 1 0 1 1 0

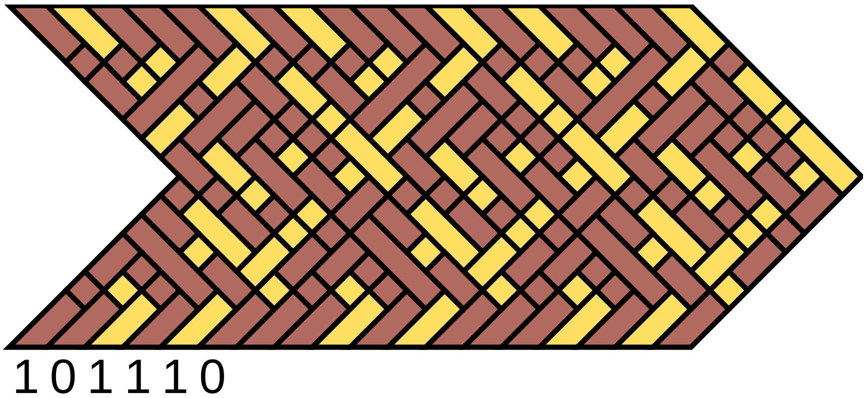
a: 110110-Colouring of the weaving texture generated by the number frieze in Fig. 29



1 1 0 1 1 0

b: Visual image
Figure 31

Figure 33 presents an example of a pattern generated by the number frieze under consideration. The asymmetric 101011-colouring implies that the pattern does not have a horizontal axis of symmetry.

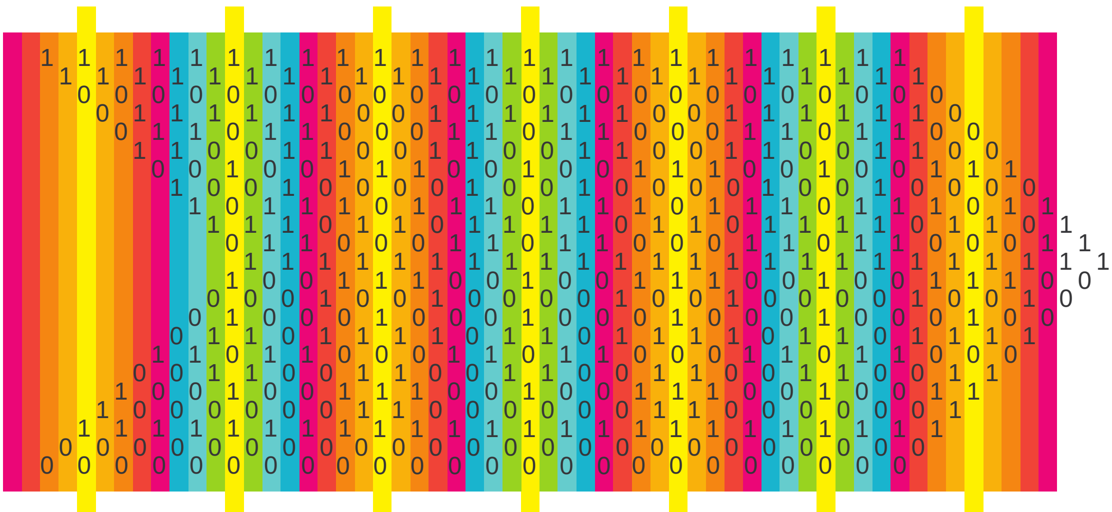


a: 101110-Colouring of the weaving texture generated by the number frieze in Fig. 29

b: Visual image
Figure 33

Generalisation

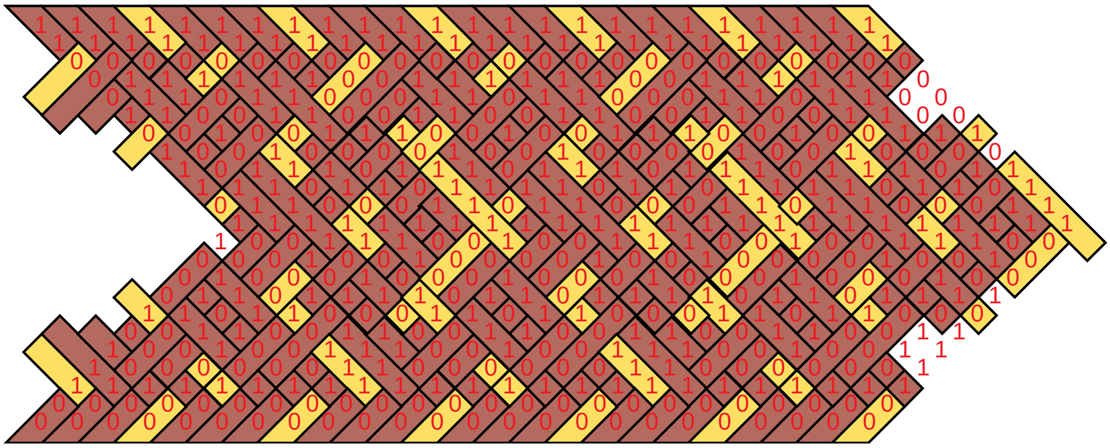
The number frieze (Figure 15) that underlies the chicken’s eye pattern may be generalised to other dimensions. Figure 34 displays an example. The period of the number frieze is 8 and it corresponds to a woven bandwidth of 24.



Example of a number frieze
Figure 34

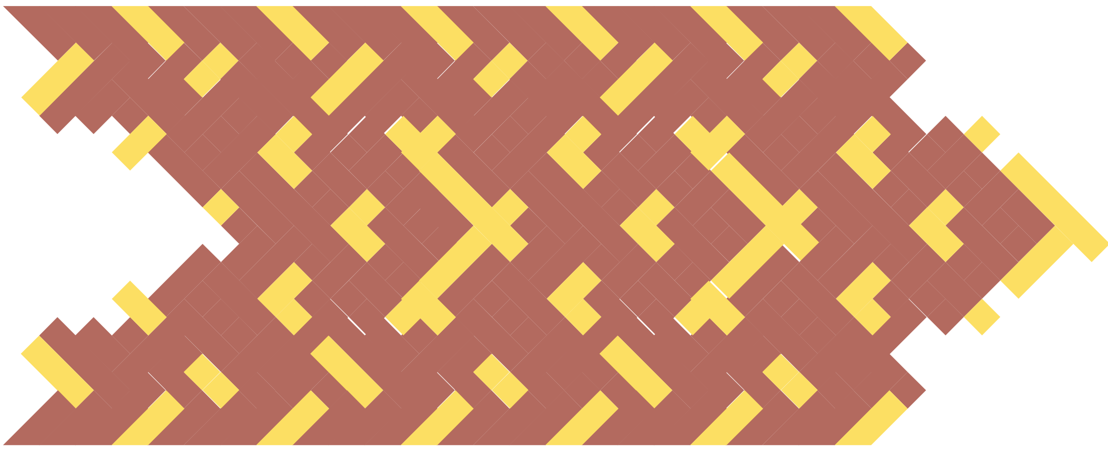
This number frieze generates a weaving texture, that whatever the colouring consistent with period 8, leads to a pattern that is the same on both sides of the woven band, albeit with its orientation inverted. Figures 35 to 38 present the 0001, 0010, 0100, and 0001-colourings, respectively.

0 0 0 1 0 0 0 1



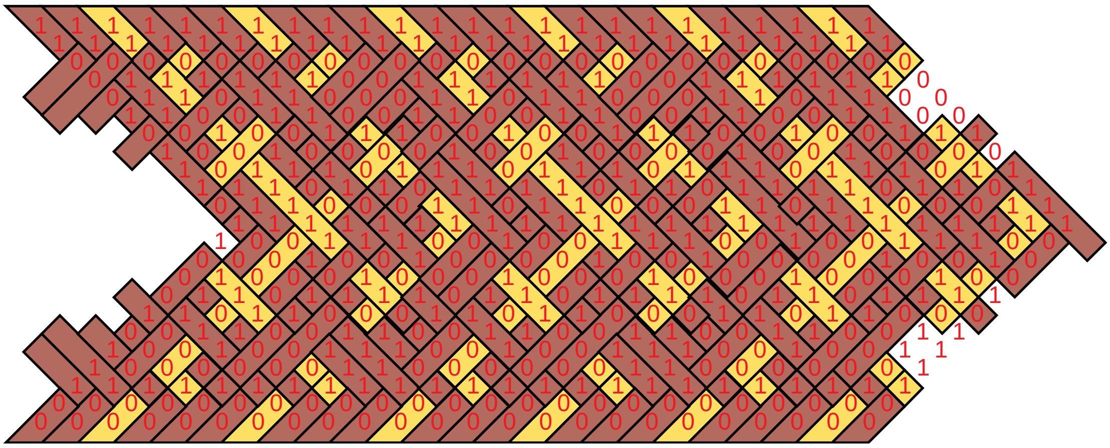
00010001-Colouring of the weaving texture
corresponding to the number frieze in Fig. 34
a

0 0 0 1 0 0 0 1



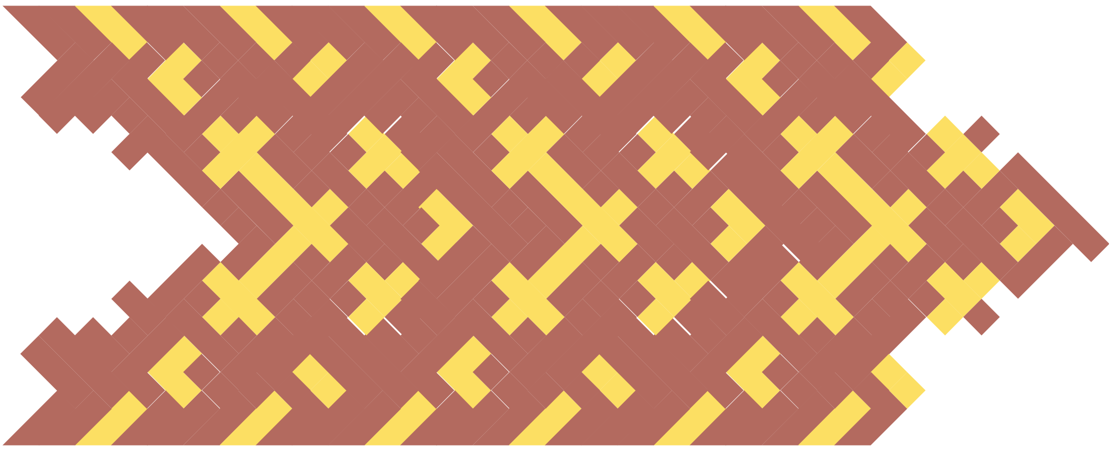
Visual image
b
Figure 35

0 0 1 0 0 0 1 0



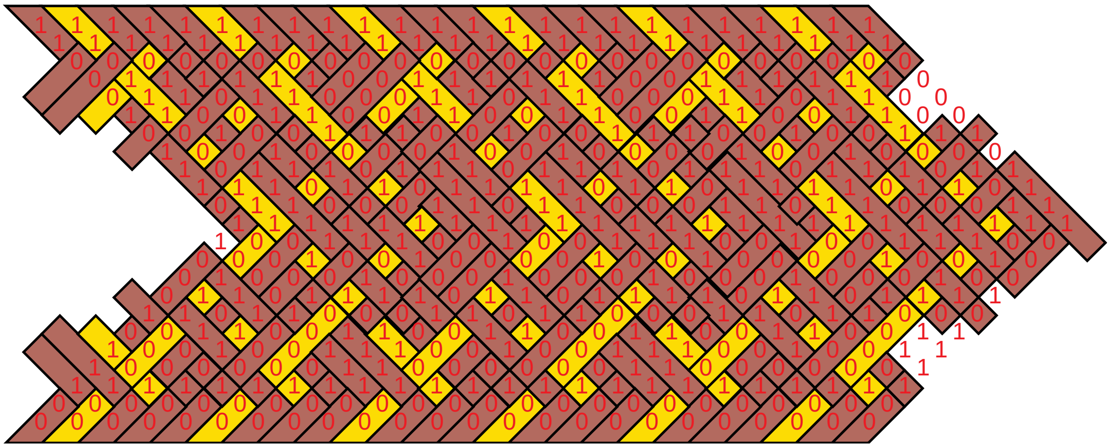
00100010-Colouring of the weaving texture
corresponding to the number frieze in Fig. 34
a

0 0 1 0 0 0 1 0



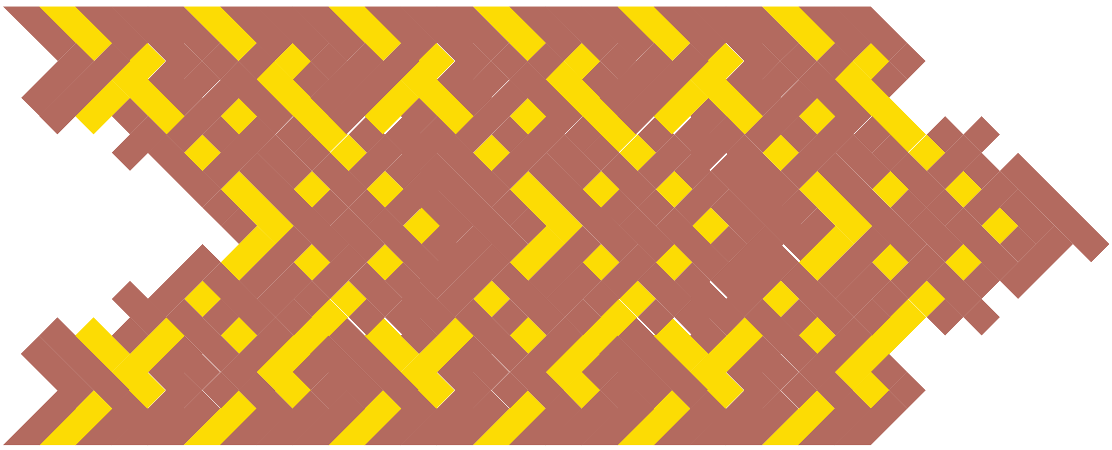
Visual image
b
Figure 36

0 1 0 0 0 1 0 0

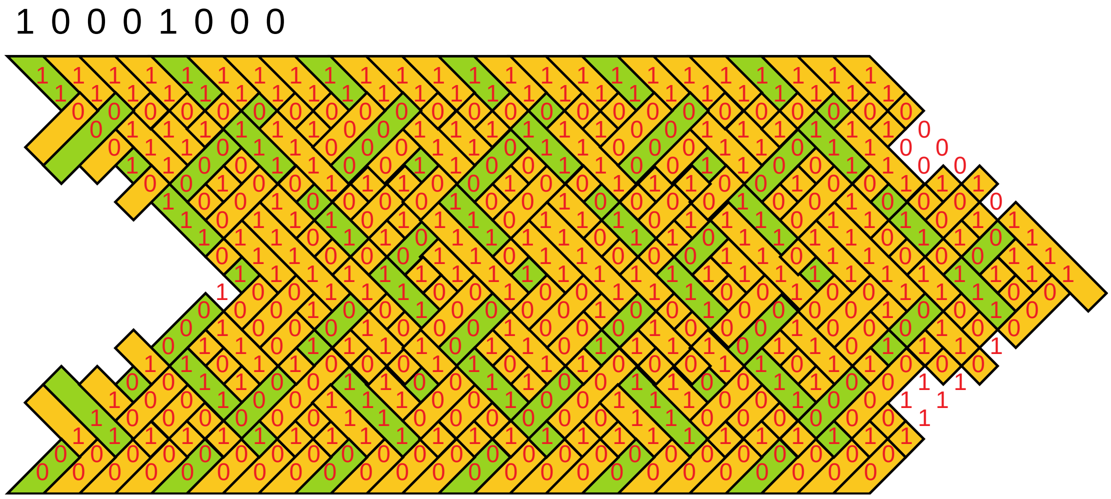


01000100-Colouring of the weaving texture
corresponding to the number frieze in Fig. 34
a

0 1 0 0 0 1 0 0



Visual image
b
Figure 37

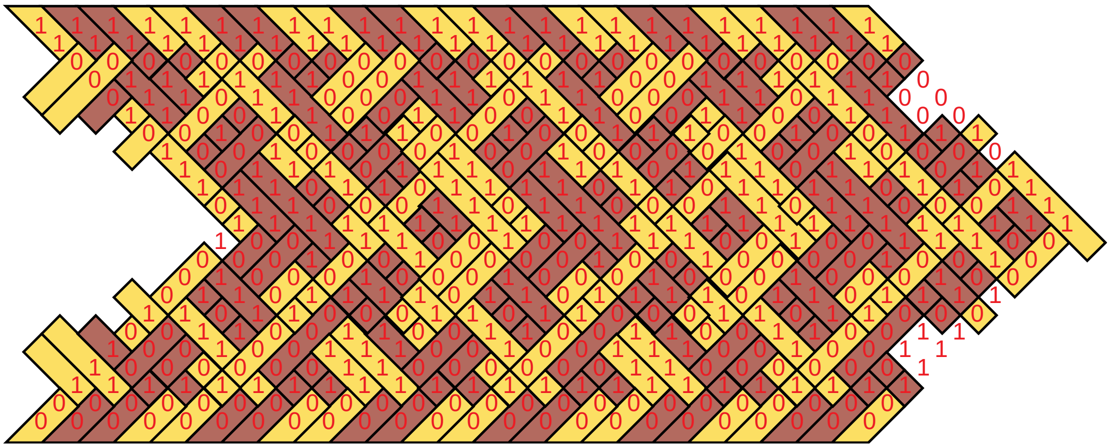


10001000-Colouring of the weaving texture
corresponding to the number frieze in Fig. 34
a

Visual image
b
Figure 38

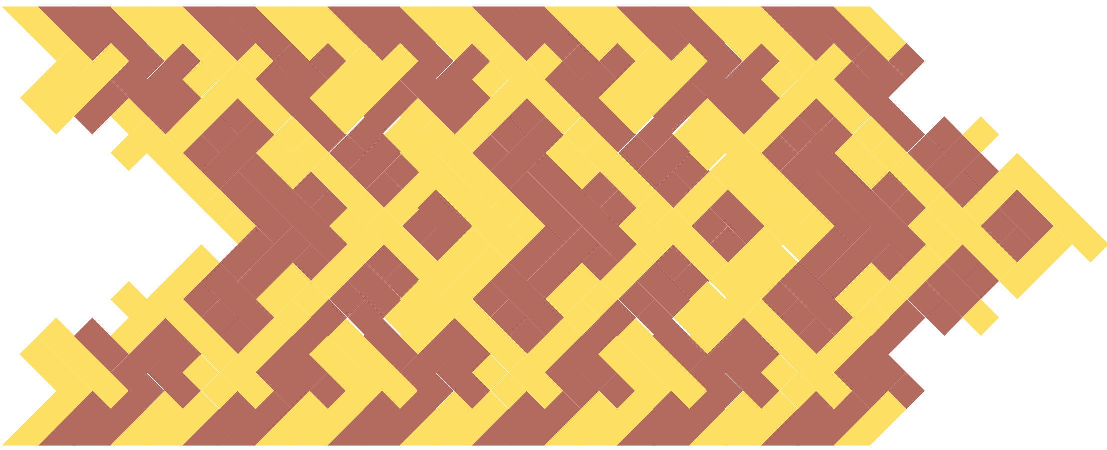
Figures 39 and 40 present the 01100110 and 11001100-colourings, respectively.

0 1 1 0 0 1 1 0

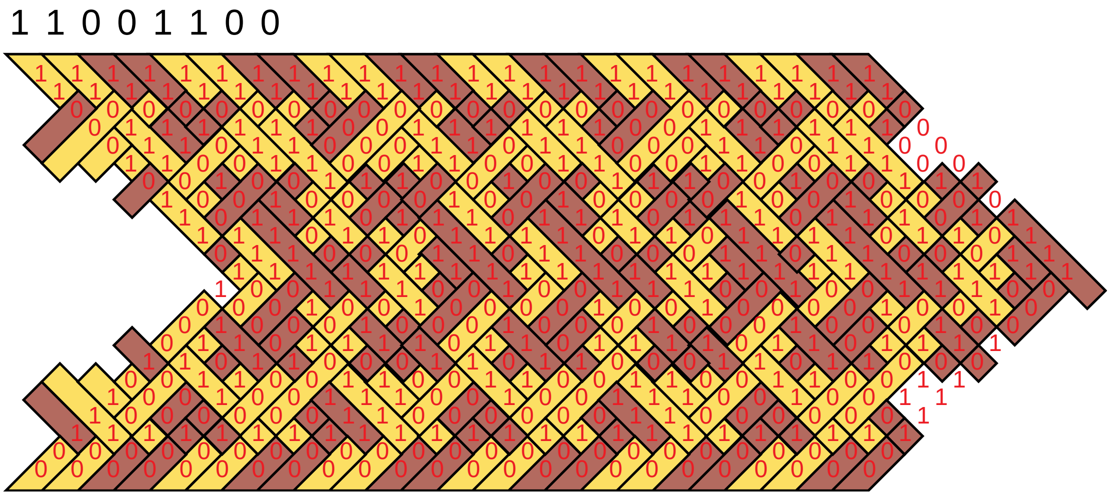


01100110-Colouring of the weaving texture
corresponding to the number frieze in Fig. 34
a

0 1 1 0 0 1 1 0



Visual image
b
Figure 39



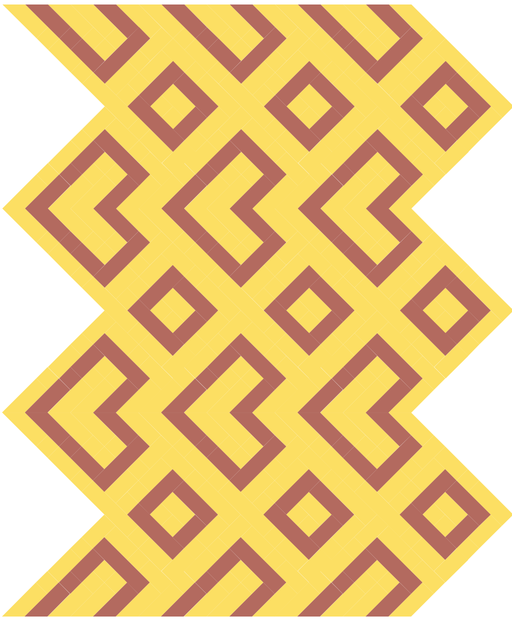
11001100-Colouring of the weaving texture
corresponding to the number frieze in Fig. 34
a

Visual image
b
Figure 40

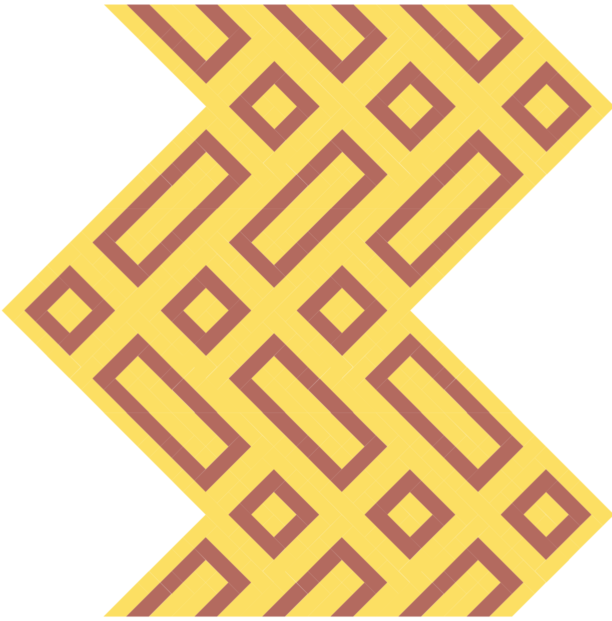
Plane patterns

The chicken's eye pattern and possible variations and generalisations like the ones presented in this paper may be explored to generate plane patterns. As the Makwe women sow together woven bands to produce rectangular mats and other objects, it is possible to generate plane patterns by joining strip patterns.

The chicken's eye pattern may generate the 'heart' plane pattern in Figure 40, as produced in this way by women on the island of Zanzibar (Tanzania) (see the bag presented in Förstle 2006, p. 108). Figure 42 illustrates a plane pattern that may be imagined as a combination of front and backside of the chicken's eye strip pattern.



Plane pattern
Figure 41



Alternative plane pattern
Figure 42

Concluding remarks

As the analysis of the chicken's eye pattern shows, its inventor(s) had constructed consciously and carefully the weaving texture. This texture did not result from mere experimentation, but some type of calculation and of geometry-symmetry considerations were involved in one way or another. Changing the colour code, the weaving texture, its underlying number frieze or the dimensions produces other patterns with the same properties as the chicken's eye pattern. The paper invites the readers to further mathematical study of particular number friezes and their generalization.

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Figures and Photographs

All figures were drawn and all photographs taken by the author.