The Geometry of the Cordovan Polygons

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Abstract

This article is focused mainly in the study of the Cordovan proportion and its geometric properties. We present some general results linked with the regular octagon and with several polygons not considered before. From this proportion we find new polygonal shapes. The combination of these shapes will allow us to generate beautiful patterns and tilings by non regular polygons.

1. Introduction

The most known proportion in the regular octagon is the *Silver Mean* $\theta = \mathrm{D} : \mathrm{L} = 1+\sqrt{2}$ (see [24], [11], [16]). This dynamic proportion appears when in a regular octagon of side L, we draw the diagonal D, as in Figure 1, and we calculate the ratio D : L. Many examples of this number involved in the geometry of the octagon can be found in [5]. The *Silver Mean* is the positive solution of the quadratic equation $x^2-2x-1=0$, and this irrational number is one of the *Metallic Numbers* [24],[16], [13], [14].

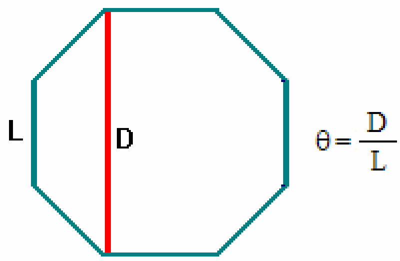


Figure 1: Silver Mean in octagon

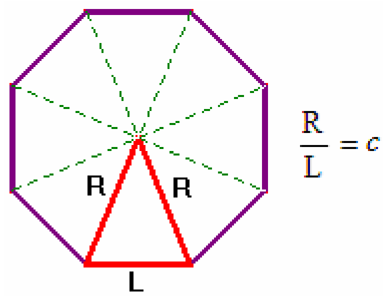


Figure 2: Cordovan proportion in octagon

The second important proportion in the regular octagon is the *Cordovan Proportion*. This proportion was introduced in 1973 by the Spanish architect Rafael de la Hoz Arderius, ([8], [9], [10]), as a result of his investigations into the proportions that are present in the architecture of the city of Cordoba, Spain. Since then, the Cordovan proportion has been considered in the analysis of art and architecture works.

The main objective of this work is to find this proportion in different shapes that have not been studied before. In fact, we have discovered a lot of new polygons where the Cordovan proportion is present and many shapes with which we can generate beautiful tiles, stars, rosettes, etc. Surprisingly, the presence of this proportion is very frequent, as we will see.

The Cordovan proportion is intimately linked with the 45º angle. In fact, if we calculate the ratio between the radius R of the circumscribed circle of the octagon and its side L, we have by means of the law of Cosines, in the marked triangle on Figure 2 with sides R, R and L, that R/L=1.306562964…

$$L^2 = 2R^2 - 2R^2 \cos 45° = R^2\left(2-\sqrt{2}\right) \Rightarrow \frac{R^2}{L^2} = \frac{1}{2-\sqrt{2}}$$

$$c = \frac{R}{L} = \frac{1}{\sqrt{2-\sqrt{2}}} = 1.306562964... = Cordovan\ Number$$

The relation R/L is known as the *Cordovan proportion* and c as the *Cordovan number*. This number is one of the solutions of the quartic equation $2x^4 - 4x^2 + 1 = 0$.
The Silver number θ and the Cordovan number c are related by the formula $c^2 = (1+\theta)/2$, which is equivalent to the expression $q = \sqrt{2}\,c^2$.
The geometric interpretation of this algebraic relation is given below in the property 1, where we are seeing the canonical dissection of a rectangle of ratio θ, or Silver rectangle. As it is well known, the proportion or ratio of a rectangle T whose sides are lengths a and b is defined by the quotient $p(T) = \max(a,b)/\min(a,b)$.

2. Cordovan polygons

2.1 Notable triangles on the regular octagon

The Cordovan proportion can be codified by means of the 45º angle and it is modelled in a natural way thanks to a notable isosceles triangle.

Definition 1 *: We will name "**Cordovan triangle**" the isosceles triangle which is similar to the one in Figure 2 of sides R, R and L.*
So, an isosceles triangle is a "Cordovan triangle" if its angles are π/4, 3π/8 and 3π/8 radians. (45º, 135º/2 and 135º/2). Figures 2 and 3 show as the Cordovan triangle can be located within the regular octagon.

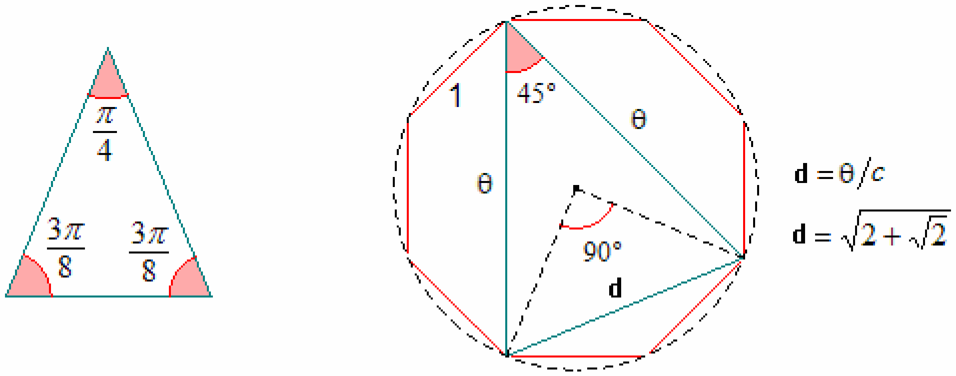


Figure 3: Cordovan triangles

Property 1. *A Silver rectangle can be divided into four isosceles triangles as in Figure 4. Two of them are Cordovan triangles and the other two triangles have angles 3p/4, p/8 and p/8.*

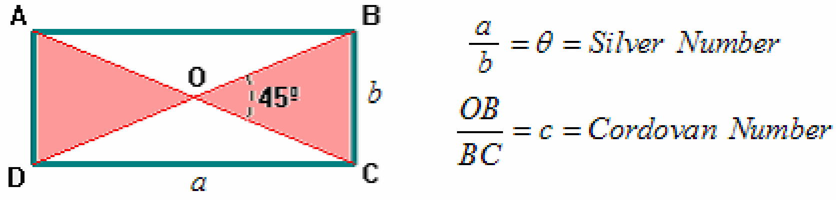


Figure 4: *Cordovan division of a Silver rectangle*

In fact, looking at Figure 4, we have $DB^2 = b^2 + b^2q^2 = b^2(2+2q) = 4b^2c^2$, so we give that

$$OB = \frac{1}{2}DB = bc \Rightarrow \frac{OB}{BC} = \frac{bc}{b} = c.$$

The second assertion of the property is a direct consequence of this equality

$$\tan(3p/8) = \sqrt{\frac{1-\cos(3p/4)}{1+\cos(3p/4)}} = \sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}} = 1+\sqrt{2} = q\ .$$

As it is well known, the *"gnomon"* is a figure that is juxtaposed to another figure obtaining a similar one. This concept is one of the relevant topics in the classical theory of the geometric proportion. Next, we will show the gnomon of the Cordovan triangle and some properties related to it. To simplify, the Cordovan triangle with sides *c*, *c* and 1 will be named *"unitary."*

Property 2. *The gnomon of a unitary Cordovan Triangle is the triangle whose sides have lengths c, c^2 and $\sqrt{2}/2$. This scalene triangle has angles p/8, 2p/8 and 5p/8. (See Figure 5).*

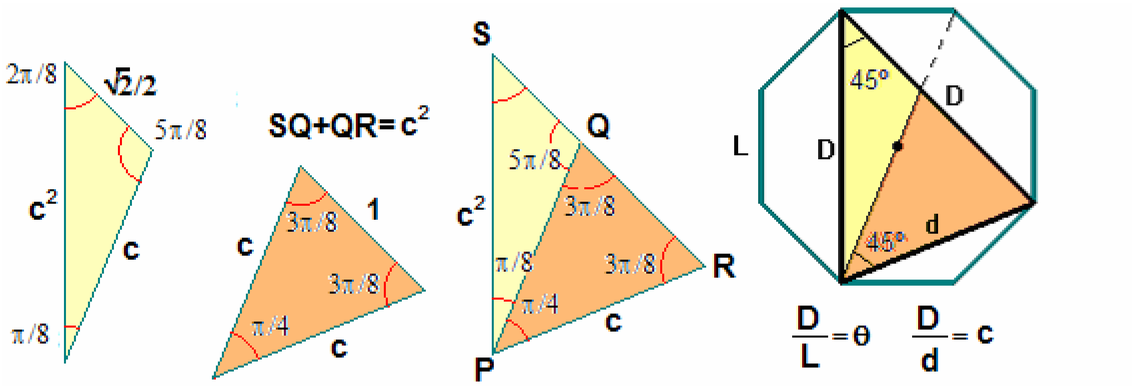


Figure 5: Cordovan triangle in orange and its gnomon in yellow

Beginning with the unitary Cordovan triangle PQR, if we juxtapose the scalene triangle with angles π/8, 2π/8 and 5π/8, then the result is the isosceles triangle SPR (Figure 5). On the other hand, SR=SQ+QR=(√2/2)+1=(2+√2)/2=c^2. Therefore, the triangle SPR is also Cordovan.

In fact, looking at the octagon in the same Figure, we can observe that if we draw a segment from the left vertex of the big triangle passing through its centre, the initial Cordovan triangle whose sides are D, D and d, results divided into two triangles. One of them (orange) also has an acute angle π/4. So, the other one (yellow), has angles π/8, 2π/8 and 5π/8. Therefore, the orange triangle has angles π/4, 3π/8 and 3π/8, and is a Cordovan triangle as well. It is obvious that the yellow scalene triangle becomes the *gnomon of a Cordovan triangle*.

Note 1. Figure 6 shows a Cordovan triangle inscribed in its regular octagon. When we join three vertices of the triangle with the centre of the octagon, the triangle is divided into other three more triangles. One of them is a right triangle (the inscribed angle is a half of its intercepted arc). The other two triangles are congruent and similar to the triangle determined by the minor diagonal d and two consecutive sides of the octagon. The three mentioned triangles have angles 3π/4, π/8 and π/8. Observe that they are similar to the white triangle obtained in the canonical dissection of the Silver rectangle, (Figure 4). Figure 7 explains this fact. Finally, this isosceles triangle has the curious property of being the gnomon of the cordovan triangle's gnomon (see Figure 8).

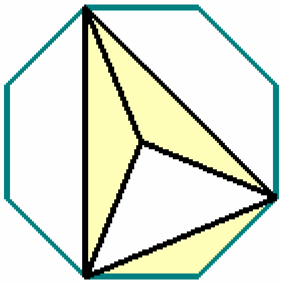


Figure 6: Triangles 3π/4, π/8 and π/8

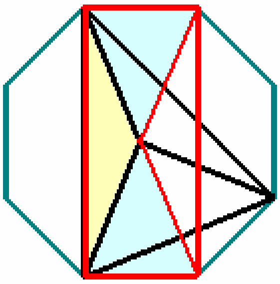


Figure 7: Silver rectangle

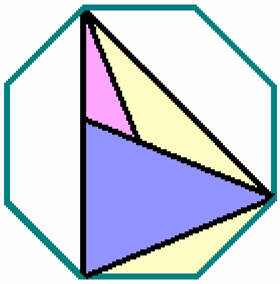


Figure 8: Gnomons

Figure 9 shows a rectangle of ratio $\sqrt{2}$ divided into a square with side 1 and a rectangle PQRS with sides $RQ = 1$ and $PQ = \sqrt{2} - 1$. Consequently, the ratio of this rectangle is $RQ\ /\ PQ\ = q$ and its diagonal is $d = \sqrt{2} \cdot \sqrt{2 - \sqrt{2}}$. From this, the ratio D/d is the Cordovan Number.

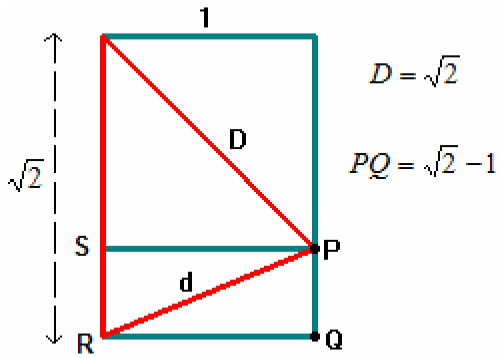


Figure 9: Cordovan triangle in an A4-sheet

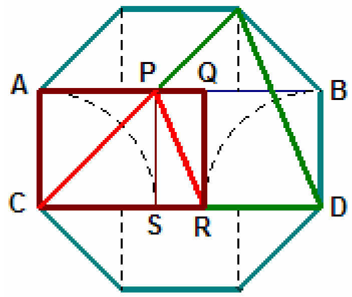


Figure 10: A4 in the regular octagon

The above figure also shows that it is very easy to construct the *Cordovan triangle* with an A4 sheet of paper. The procedure is as follows:

1. Take an A4 sheet. (It has proportion $\sqrt{2}$).
2. Make a square with side the smallest of the sheet and divide it diagonally.
3. Fold the paper joining the point **P** with the opposite vertex **R** of the sheet.

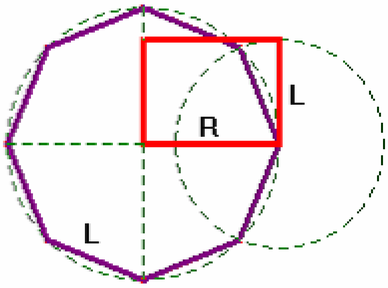
In Figure 10, inside the regular octagon, we can see the same A4 rectangle (maroon) placed in the Silver rectangle ABCD. The similarity of the red and green triangles becomes evident.

2.2 Quadrilaterals

In this section we begin the research of the Cordovan proportion in the polygons with more than three sides. Obviously, the first polygon considered is the rectangle.
Next definition extends the notions of the well known *Golden rectangle* and the aforementioned *Silver rectangle*.

Definition 2. *We will name "**Cordovan rectangle**" to a rectangle whose sides are in ratio c.*

For example, the red rectangle of sides R and L in Figure 11.



$$c = \frac{R}{L} = \frac{1}{\sqrt{2 - \sqrt{2}}}$$

Figure 11: Cordovan rectangle in a regular octagon

The mentioned architect *Rafael de la Hoz Arderius* found this rectangle in the plan and the elevation of the Mosque of Cordova (Spain), [1] . Figure 12 represents an original drawing of this architect of the plan of the Mosque, [18]. Figure 13 shows a pattern explaining three Cordovan rectangles found by de la Hoz.

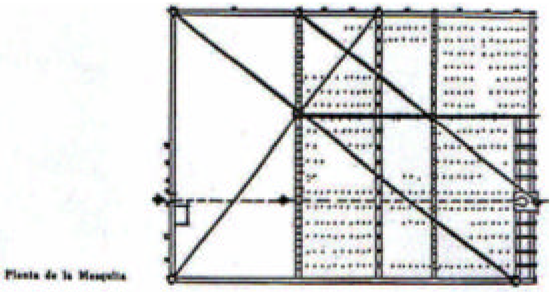


Figure 12: Plant of the Mosque (Cordova)

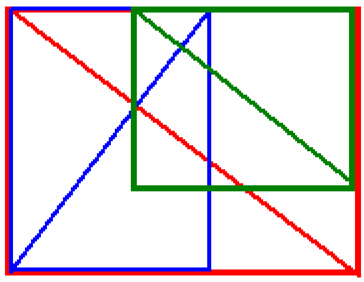


Figure 13: Cordovan rectangles

It is important to remark that *until now the Cordovan rectangle is the only Cordovan polygon* considered in the consulted references. ([9], [1] ,[3], [4]).

Looking at Figure 9, when a Cordovan triangle is built from an A4-sheet, we can observe that the two remaining pieces can be rearranged to form another Cordovan triangle as in Figure 14.

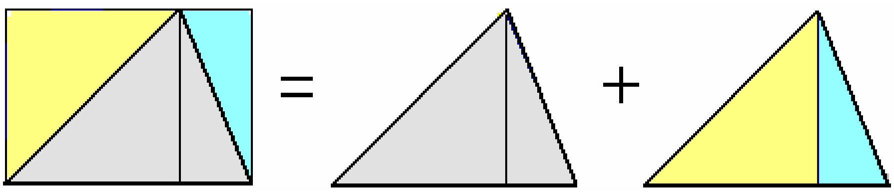


Figure 14: Two Cordovan triangles from the A4-sheet

If we join both triangles by the longest side we get a parallelogram with angles $3\pi/8$ and $5\pi/8$, and if we join them by their minor side, we obtain the parallelogram considered in the following definition (Figure 15).

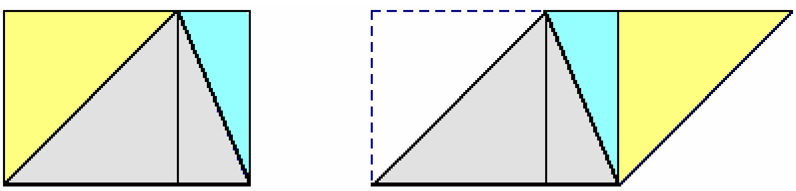


Figure 15: Rhombus of 45º and 135º from the A4-sheet

Definition 3 We call *"**Cordovan diamond**" to a rhombus whose angles are 45 and 135 degrees.*

Obviously, the area of the Cordovan diamond obtained from a A4-sheet is equal to the area of the A-4 sheet, and the area of a Cordovan triangle is a half of it.

This diamond also appears by the intersection of two regular octagons. In fact, four octagons intersected as on Figure 16 (left) produce an inner star formed by four diamonds. Conversely, the intersection of two diamonds determines a four point star and an inner octagon, Figure 16 (right). This four point star can be obtained by intersection of five octagons, as in Figure 17, where also we show a nice tiling with this pattern in the Granada's Alhambra (Spain).

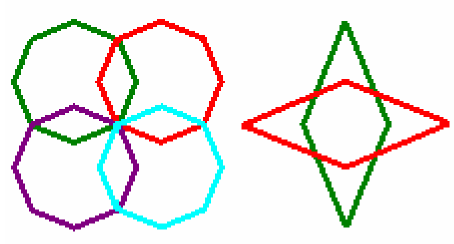


Figure 16: Octagons and diamonds

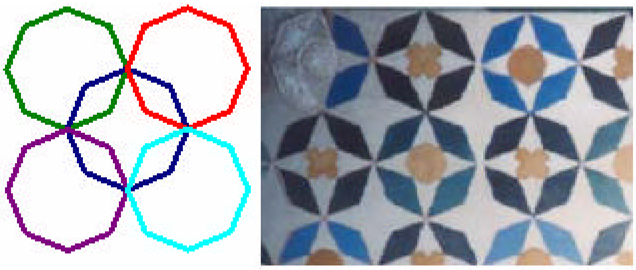


Figure 17: Octagons and stars in Granada's Alhambra

The Cordovan diamond is very frequent in the design of buildings, pavements, quilts, etc. from ancient and modern cultures [15], [18], [19], [20]. Indeed, this notable rhombus provides an easy way to develop a lot of geometry.

The tiling of Figure 18 is formed by square tiles. We find a diamond over the diagonal of each square and four scalene triangles around it. Four contiguous squares determine a regular octagon. Inside, we can see a four point star surrounded by four diamonds.

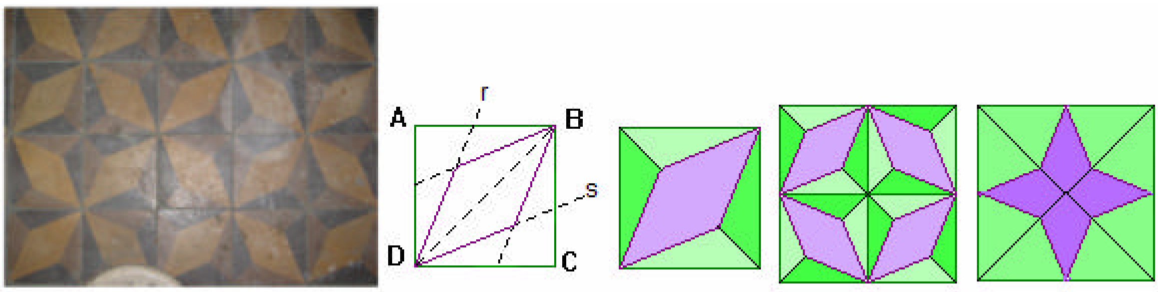


Figure 18: *Diamond, octagon and star in a Lisbon's tiling*

The four point star can be divided from its centre in four quadrilaterals, Figure 18 (right). If we place them in the corners of a square as in Figure 19 (left), a perfect Dudeney's Maltese Cross is determined, ([2]). The combination of these concave polygons drives in a natural way to a beautiful mosaic formed by four point stars and Maltese Crosses.

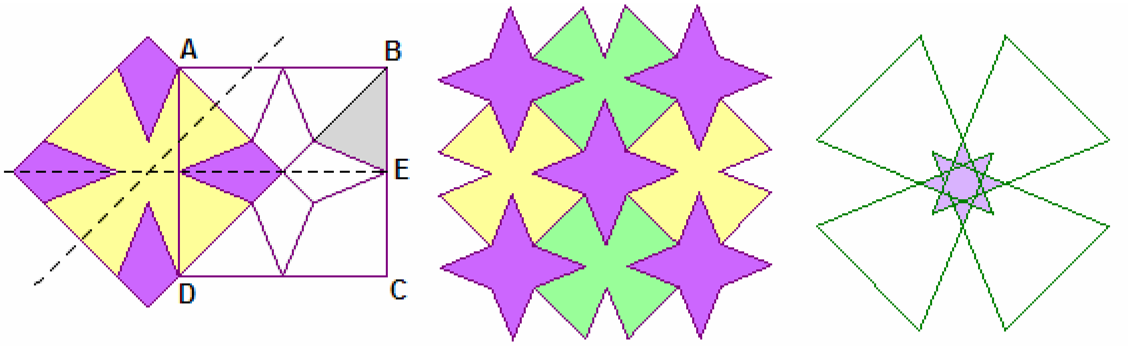


Figure 19: Dudeney's Maltese Crosses and stars

The same Cross can be constructed by the intersection of four Cordovan triangles. In this process, from the four vertex of 45º, a star polygon 8/3 is obtained, as well.

Remark 2. When the side of the square ABCD is equal to 1, the perimeter of the four point star is equal to $4/c$. In fact, the grey isosceles triangle is Cordovan, and $BE = 1/2$, therefore the length of each side of the star is $1/(2c)$ and its perimeter is $4/c$. Consequently, the Dudeney's Maltese Cross achieved has perimeter equal to $4 \cdot (1/2) + 4/c = 2 + 4/c$.
Figure 20 shows a traditional labour of Spanish crochet handicraft. It is easy to recognize the Maltese Cross, the four point star, the octagon, and other star formed by four diamonds.

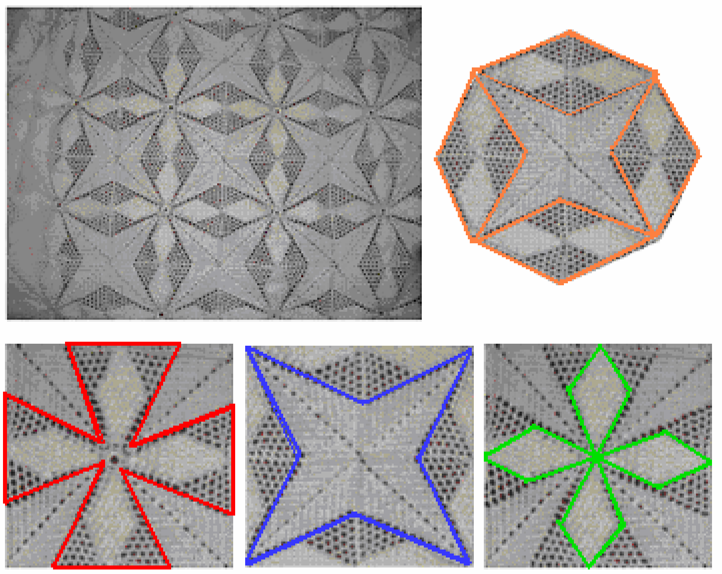


Figure 20: The Cordovan proportion in the crochet

Following the division of the octagon of the crochet, it is possible to obtain an expansive sequence of four point stars, diamonds and octagons whose sides are in the Silver number proportion.

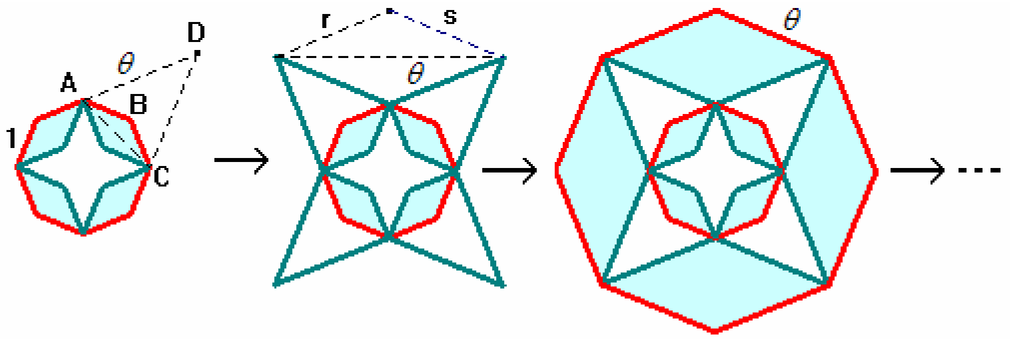


Figure 21: Sequence of stars and octagons

We begin with a regular octagon divided in a four point star rounded by four diamonds, and the process is the following:

1. From the longest diagonal of a diamond we construct a Cordovan triangle.
2. Four of these triangles determine a four point star.
3. We draw a regular octagon circumscribed to the above star and the process continues.

In Figure 21, the triangle ABC has sides $AB = BC = 1$ and angle $\angle ABC = 3p/4$. Then, by means of the Law of Cosines, easily we get $AC^2 = 2+\sqrt{2}$ and $AC = \sqrt{2}c$. On the other hand, the angle $\angle ADC = p/4$ shows that ADC is a Cordovan triangle. So, $AD = AC \cdot c = \sqrt{2}c^2 = q$.

Now, we are going to devote to trapezoids. In Figure 3, around the *Cordovan triangle* θ, θ, *d* we get an isosceles triangle with sides 1, 1, *d* and two isosceles trapezoids with three sides 1 and the biggest one θ. Their angles are 3π/4 and π/4. This new silvered shape should also be considered as a Cordovan shape. The reason is that it can be divided into three Cordovan triangles with sides 1, 1, and 1/c (see Figure 22) and another scalene triangle with sides 1, 1/c and θ-2. The angles of this triangle are π/8, 2π/8 and 5π/8, therefore it is a gnomon of the Cordovan triangle.

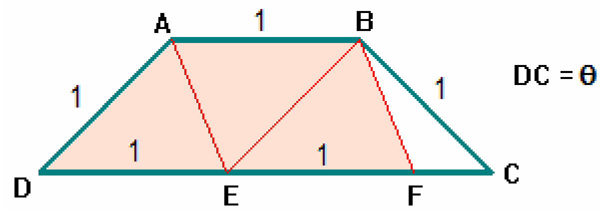


Figure 22: Silvered Cordovan trapezoid

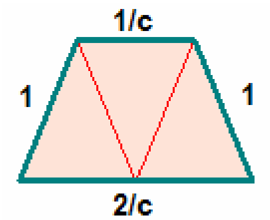


Figure 23: A Cordovan trapezoid

But this one is not the only trapezoid that deserves to be called Cordovan, since we have found another two. In fact, if we rearrange the three Cordovan triangles as in the Figure 23, we obtain an isosceles trapezoid with sides 1/c, 1, 2/c, 1, and angles 3π/8 and 5π/8. This shape is similar to the isosceles trapezoid of sides 1, c, 2 and c, so we will name it ***c-Cordovan trapezoid***.

As we have shown before, the Cordovan triangle ABE (Figure 24) can be constructed from a rectangle ABCD whose ratio is √2. Then, if we reflect the pattern across the side BC, the isosceles trapezoid AA'E'E is obtained. The rectangle AA'D'D is formed by two squares with sides 1 and two Silver rectangles whose sides are q^{-1} and 1.

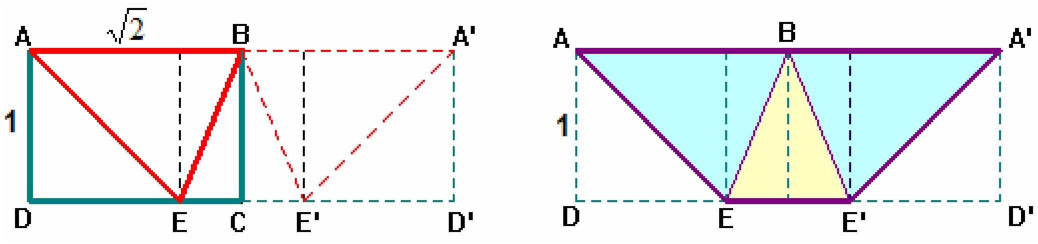


Figure 24: Cordovan trapezoid from a $\sqrt{2}$ rectangle

The trapezoid AA'E'E is formed by the union of an isosceles triangle BEE' and two congruent triangles, ABE and A'BE', with sides AE=AB=BA'=A'E'=√2. Both triangles are Cordovan, so $BE = BE' = \sqrt{2}c^{-1}$. On the other hand, $EE' = 2\sqrt{2} - 2 = \sqrt{2}\left(2 - \sqrt{2}\right) = \sqrt{2}c^{-2}$, and the ratio of the sides of BEE' is *c*. Therefore BEE' is a Cordovan triangle, and so $\mathrm{BE}/\mathrm{EE'} = \mathrm{c}$.

In this way, we have constructed a trapezoid with angles π/4 and 5π/8 and sides $2\sqrt{2}$, $\sqrt{2}$, $2\sqrt{2} - 2 = 2\theta^{-1}$ and $\sqrt{2}$, (Figure 25). This shape is similar to the isosceles trapezoid of sides $2c^2$, c^2, 1 and c^2, so we will name it ***c^2-Cordovan trapezoid***.

This shape is related to the preceding trapezoid through the canonical dissection of the two Silver rectangles contained in it.

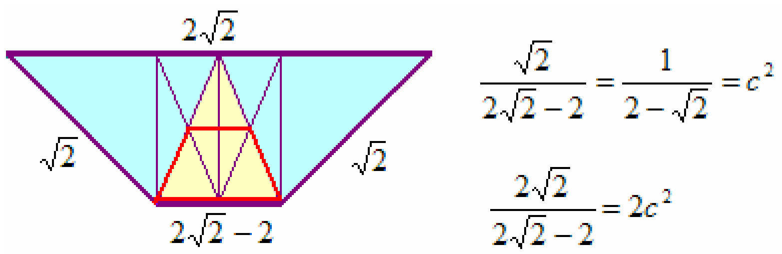


Figure 25: Two Cordovan trapezoids

In Figure 24, the prolongation of the segments AE and A'E' determines the point O, which is the centre of the square of side AA'. If we rotate 90º the trapezoid around the point O, (Figure 26), the figure generated is the star of Figure 18. Also, the rotation of 90º around M, midpoint of the biggest side of the trapezoid, gives a regular octagon inscribed in a square, Figure 27.

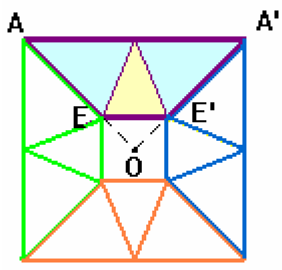


Figure 26: Star from the trapezoid

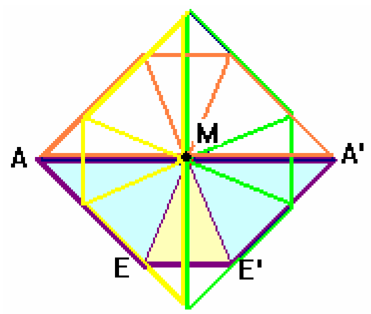


Figure 27: Regular octagon from the trapezoid

If we draw two ***c***-Cordovan trapezoids inside of the ***c^2***-trapezoid, the preceding process produces two new tiles which generate two beautiful patterns (Figure 28).

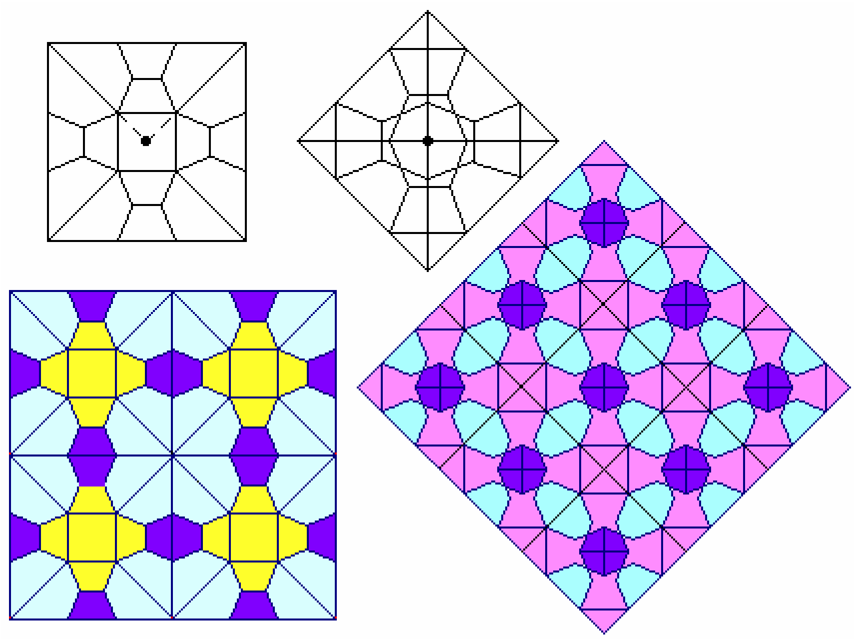


Figure 28: Two tilings from the Cordovan trapezoids

Next, we introduce two new quadrilaterals. These elemental shapes are very simple but they play an important role later on.
In fact, in an easy way, the regular octagon can be divided into four congruent quadrilaterals, which we name ***"c-kites"***. A *c*-kite is formed by two Cordovan triangles. This quadrilateral has angles $\pi/2$, $3\pi/8$, $3\pi/4$, and $3\pi/8$. If the side of the octagon is 1, the *c*-kite has sides 1, 1, c, and c, Figure 29.

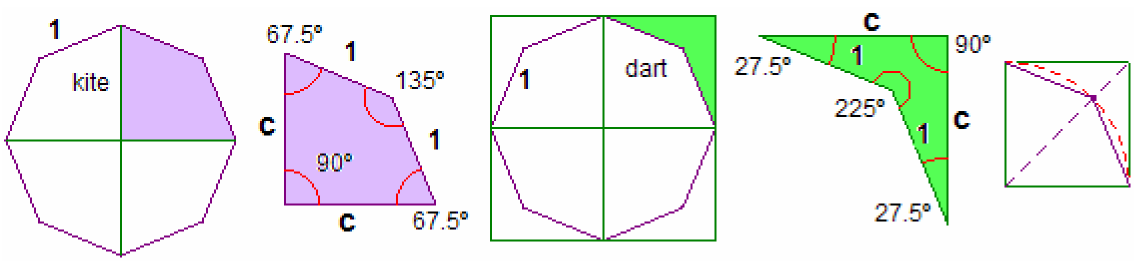


Figure 29: Cordovan kite, Cordovan dart and its construction from the square

If we inscribe the octagon in a square another four congruent concave quadrilaterals (Figure 29) are obtained. We agree to name this quadrilateral as ***"c-dart".*** A *c*-dart has angles $\pi/8$, $\pi/2$, $5\pi/4$ and $\pi/8$, and sides 1, 1, c, and c.
In Figure 29 (right), we can see the easy construction of the *c*-kite and the *c*-dart from the square. The similarity with the Penrose's kite and dart is evident, but our quadrilaterals are obtained from the regular octagon. So, in order to clarify, we will say ***"Cordovan kite"*** and *"**Cordovan dart**"*, or, shortly, *c-kite* and *c-dart* as before.

Rearranging four of these *c*-darts, the four point star is formed (Figure 30), that it is really a concave octagon. Obviously, four *c*-kites form a regular octagon and the square can be divided into a four point star rounded by four *c*-kites. This combination of octagons and stars underlies the pattern of the tilings in Figures 17 and 18.

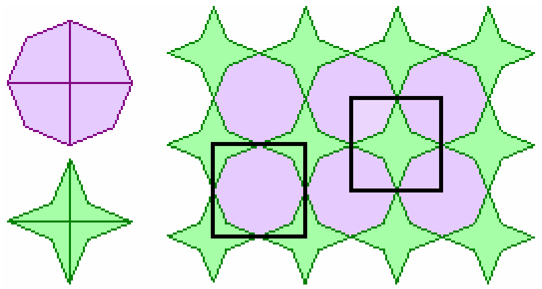


Figure 30: Octagons and four point stars

Starting with a square, by means of the canonical dissection of the Silver rectangle, (Figure 4), we also obtain the regular octagon and the star. The process is very simple:

1) Draw a Silver rectangle on each side of the square as in Figure 31.
2) Trace the diagonals of four Silver rectangles.
3) Prolong the eight diagonals.

After the step 2) the regular octagon is achieved and finally, we find two four point stars which form a star polygon 8/3. In addition, we obtain an inner star polygon 8/2.

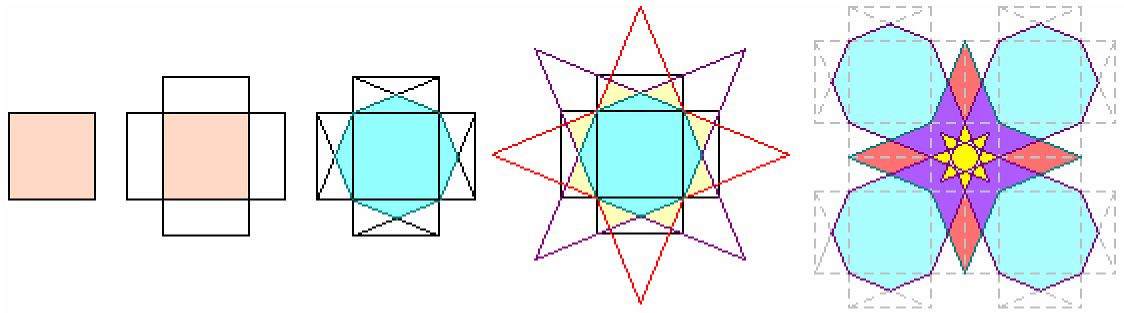


Figure 31: Octagon and stars from the square and the Silver rectangle

In Figure 31 (right) a nice rosette is constructed from the pattern obtained after the step 2). Here we can also observe an original Maltese Cross rounded by four diamonds and four regular octagons. In its centre two other polygon stars 8/3 and 8/2 appear.

2.3 Pentagons and heptagons

It is possible to find several pentagons related to the Cordovan proportion. The first is formed by three consecutive Cordovan triangles. Its sides are 1, 1, 1, c and c. The same pentagon is obtained with an adjacent *c*-kite to a Cordovan triangle. The second is the union of four consecutive Cordovan triangles. This pentagon has sides 1, 1, 1, 1 and 2c and it is also obtained by addition of two *c*-kites.
These pentagons are trivially contained in the regular octagon and they can be divided into Cordovan shapes in several ways (Figure 32). Many artistic shapes can be generated by rotating these pentagons. Some examples are shown in Figure 33.

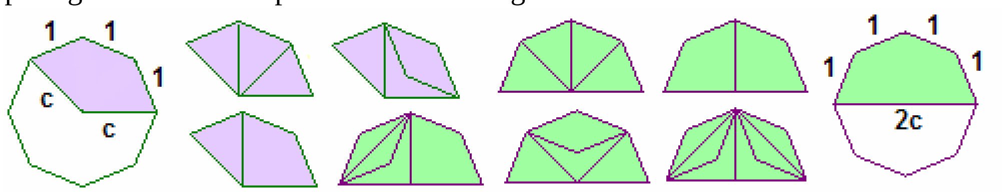


Figure 32: First and second Cordovan pentagons

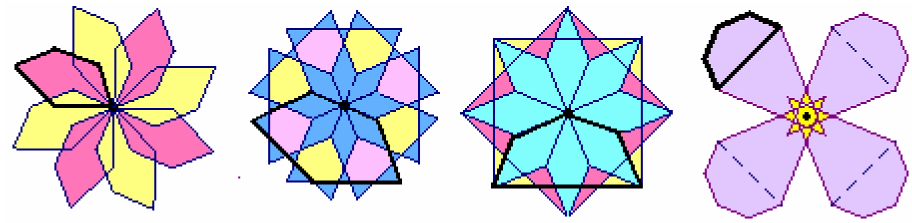


Figure 33: Rosettes from the Cordovan pentagons

Note that the flower of Figure 33 (right) is the same that appears in the construction of Figure 31 (right), excluding the red diamonds. It is created with a Dudeney's Maltese Cross by addition of four Cordovan pentagons.

The third Cordovan pentagon is the most relevant. It is formed by juxtaposing two right triangles to the equal sides of the Cordovan triangle. This pentagon has four sides equal to **c/Ö2** and the remaining side is 1. Its angles are $5\pi/8$, $\pi/2$, $3\pi/4$, $\pi/2$ and $5\pi/8$. We obtain the same pentagon by adding two *c*-kites and a right triangle, Figure 34.

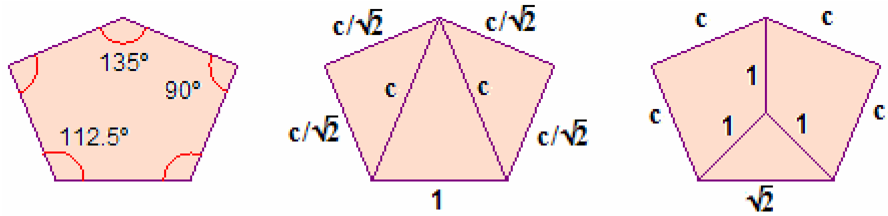


Figure 34: The third Cordovan pentagon

This Cordovan pentagon tiles the plane. Indeed, it is one of the fourteen covering pentagons of the plane. (Specifically, the number 9 of the reference [21], see also [21]). By rotating this pentagon 90º as in Figure 35, a par-hexagon formed by four pentagons is drawn. A par-polygon is defined as a polygon with opposite sides parallel and the same length. One tessellation of the plane generated by a par-hexagon can be visualized. As it is well known, a par-hexagon always tiles the plane.

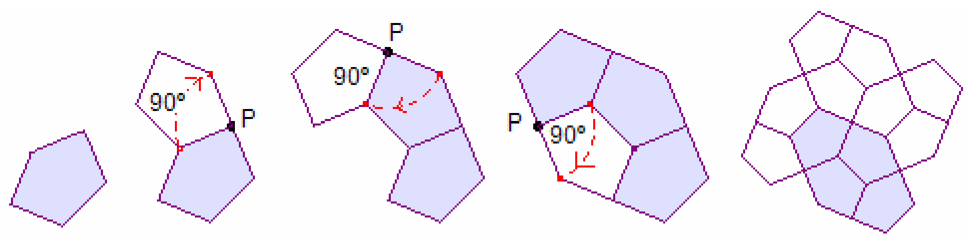


Figure 35: The Cordovan pentagon tiles the plane

This third Cordovan pentagon is involved in several dissections of the octagon. Surprisingly, we have discovered it in the famous dissection of Lindgren (Figure 36), [12]. As it is known, by fitting together the pieces of this dissection, an eight point star 8/3 can be achieved, [6],[17].

The authors present in Figure 37 some original dissections related to the Cordovan proportion. (Notice that the same pieces have the same colour).
In *a*), the regular octagon is divided into two pentagons and two diamonds. By rotating 90º the two purple pentagons around the centre of the octagon a four point star is obtained.
Also, by rotating 90º this pentagon around the vertex of 135º, we obtain an artistic cross which achieves a beautiful dissection of the regular octagon (see Figure 37 *b), c)*). The pieces for this dissection are four concave heptagons, four quadrilaterals and four isosceles triangles, which are

similar to the isosceles triangles of the Figure 6, that is with angles 3π/4, π/8 and π/8 (the gnomon of the Cordovan triangle's gnomon).

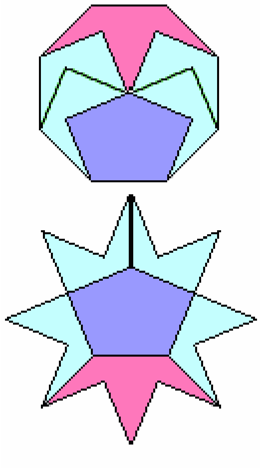


Figure 36: H. Lindgren's dissection

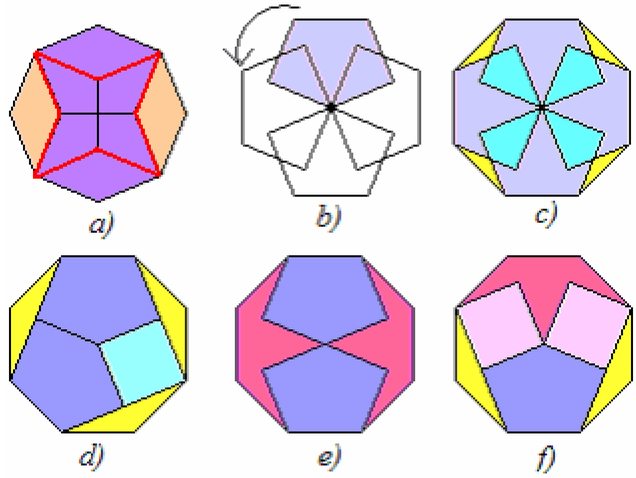


Figure 37: Authors' Cordovan dissections

Figure 37 shows in d), e) and f) three symmetric dissections using the square, the "gnomon of gnomon" and the concave heptagon, being the latter the same as in Lindgren's dissection. In Figure 38, two examples of beautiful patterns created from our dissections are presented.

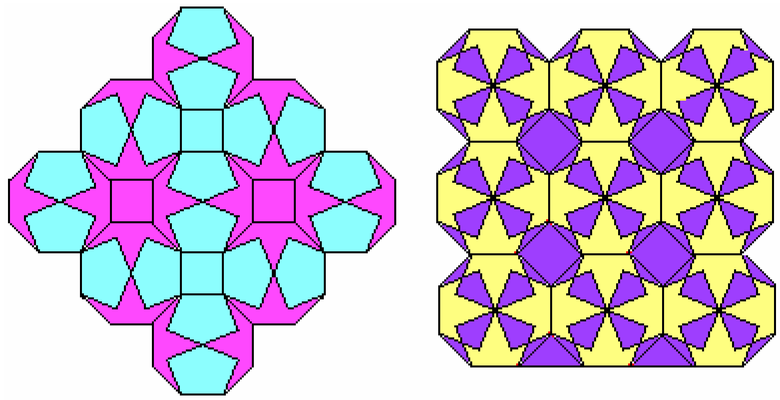
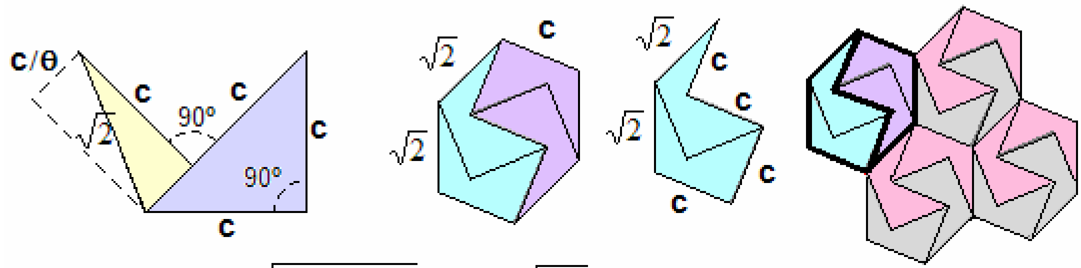


Figure 38: Designs with pentagons, heptagons and squares

Indeed, the four concave pentagons of Lindgren's dissection, (Figure 36), are Cordovan shapes. In fact, each of these pentagons is the addition of an isosceles right triangle with legs c, and a half of a Silver rectangle with sides c and c/θ, Figure 39. Therefore, it has a side equal to the diagonal $\sqrt{2}$ of the Silver rectangle, and the other four equal to c. Its angles are 5π/8, π/8, 3π/2, π/4 and π/2. Four pentagons can be rearranged to form a par-hexagon. So, this concave pentagon tiles the plane. (Figure 39). Observe that two pentagons produce a concave no regular hexagon which also tile the plane.



$$\sqrt{c^2+(c/\theta)^2}=(c/\theta)\sqrt{4c^2}=2c^2/\theta=\sqrt{2}$$

Figure 39: Tessellation with concave Cordovan polygons

2.4 Par-hexagons and par-octagons

A part of the par-hexagons mentioned before (Figures 35 and 39), other hexagons and octagons can be constructed by combination of *c*-kites and *c*-darts. Next, we present a wide collection. All of them generate monohedral or dihedral [7] tilings, which can be drawn on a squared grid. To abridge, we name them in their correspondent figures, where we show their sides' length. The angles are not specified since its values can be trivially calculated, and have already been presented. Also, we show the monohedral and dihedral tilings generated. Notice that our par-octagons tile the plane, which is not a trivial fact.

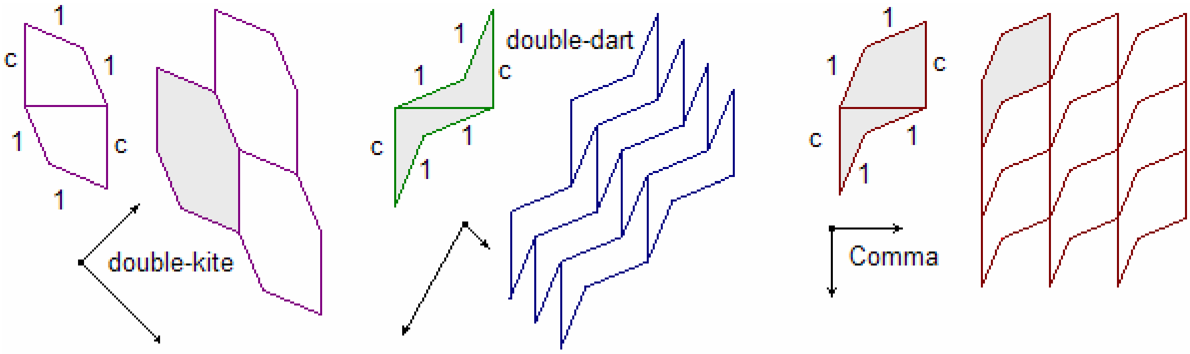


Figure 40: Tiling the plane with three Cordovan par-hexagons

The "***c-sun***" (regular octagon) is trivially a par-octagon. The four point "***star***" or (***c-star***) is another par-octagon which jointly with the regular octagon tiles the plane as we have seen before (Figure 30).

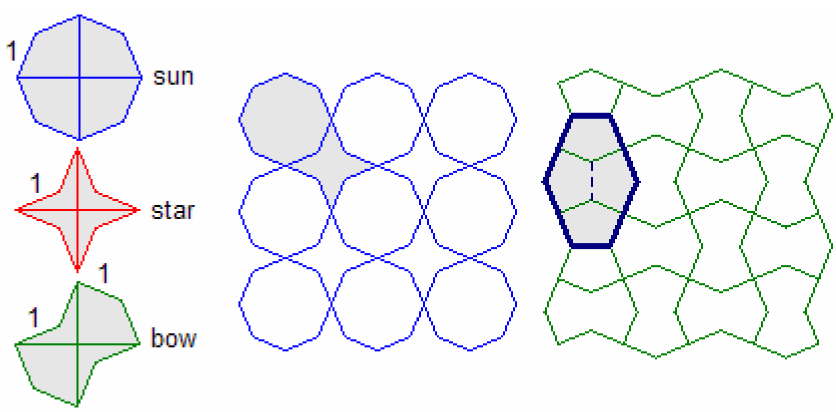


Figure 41: Tilings with *suns*, *stars* and *bows*

Two *c*-darts and two *c*-kites placed as in Figure 41, form a concave par-octagon named for us "***bow***" or "***c-bow***". The *bow* is the union of two Cordovan pentagons, the same as in Figure 34. So, this polygon also tiles the plane (Figure 41). Over the tessellation with bows, par-hexagons can be depicted in order to obtain the tessellation of Figure 35.

The *c*-sun, the *c*-star and the *c*-bow are the basis of the design of the Persian Carpet of the Figure 42, see [23].

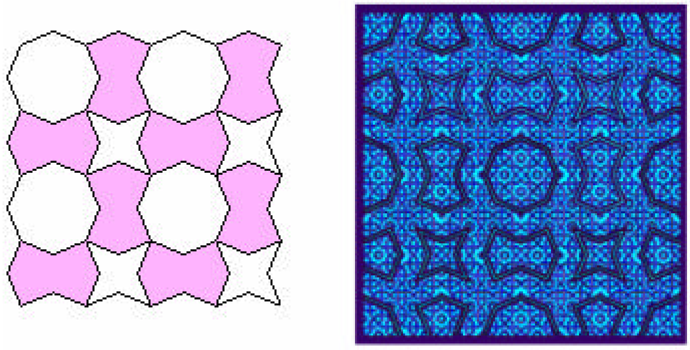


Figure 42: *c*-sun, *c*-stars and *c*-bows in a Persian carpet

Two *c*-darts and two *c*-kites also form the concave par-octagon showed in Figure 43. We name this shape "***umbrella***"

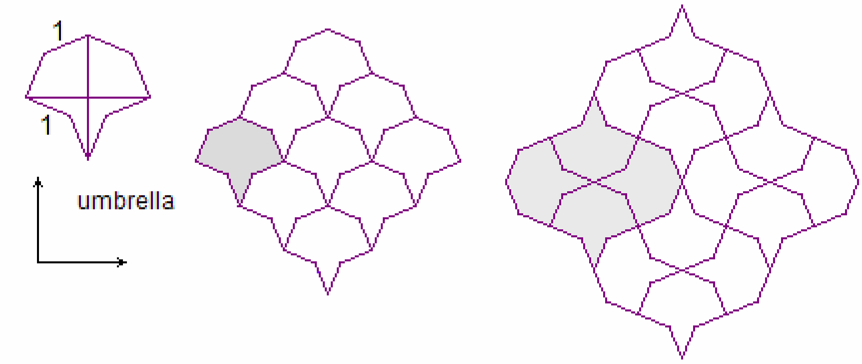


Figure 43: Two monohedral tilings with the *umbrella*

Combining umbrellas in two different ways, two tessellations are obtained.
In the first, we have used two translations in two independent directions, and the second is based on rotations.

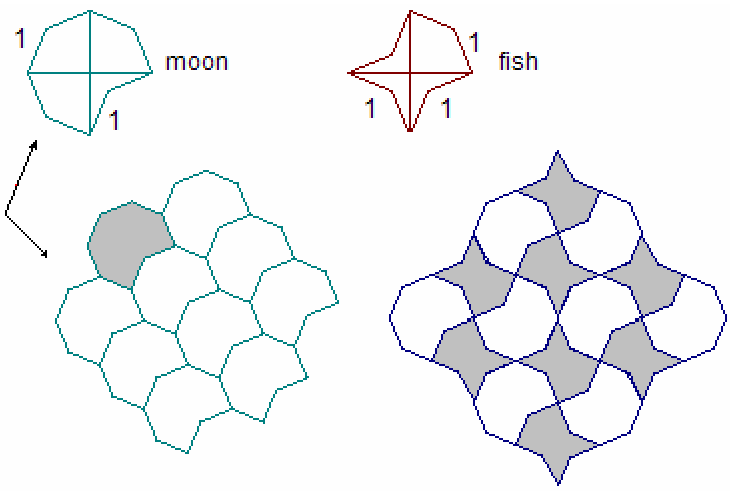


Figure 44: Tiles with moons and fishes

And, so we may continue….

3. Conclusions

In 1973, the Cordovan proportion has been introduced and named by the Architect de la Hoz. From this date on, this proportion has been mentioned and slightly studied by some authors, but only in rectangular shapes, and always through the common presence of the Cordovan number and the Silver Number in the regular octagon. In this paper, we have shown the geometric significance of this dynamic proportion, that is, the relevance of the angle of 45º degrees. Furthermore, we have found and studied many polygonal shapes related to the Cordovan proportion. In addition, the possibility of generating new patterns and original tiles is surprising. The number 45 is equal to the sum of all the single digits 0, 1, 2, ..., 9. For this reason Numerology asserts that this number symbolizes "all the possibilities". It can be accepted or not, but it is certain that the Cordovan Number "is almost everywhere".

4. References

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