THE 3-BODY PROBLEM IN R^4: STABLE RELATIVE EQUILIBRIA, CRITICAL POINTS AT INFINITY

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Abstract. The Newtonian 3-body problem is essentially considered in a space of dimension 3. The simplest motions are the relative equilibria. They are equilibria of the reduced dynamics with a given angular momentum. These equilibria are expected to be all Lyapunov unstable, even if for example the Lagrange relative equilibrium with a dominant mass is linearly stable. Indeed, its shape remains nearly equilateral for very long times.
The same equations define motions in dimension 4, and this is the highest possible dimension for 3 bodies in the Galilean frame of their center of mass. We prove that for any choice of angular momentum corresponding to truly 4-dimensional motions, there is a relative equilibrium which is Lyapunov stable. This is the minimum of the energy (a related result is published in [**2**]).
Richard Montgomery informed us of a gap in our published article [**1**]. We will present an interesting lemma which finishes the proof: a critical point at infinity of the energy restricted to a level set of the angular momentum cannot correspond to an infimum (with Holger Dullin).

References

1. A. Albouy, H. R. Dullin, *Relative equilibria of the* 3*-body problem in* R^4, J. Geom. Mech. **12** (2020), 323–341.
2. H. R. Dullin, J. Scheurle, *Symmetry reduction of the* 3*-body problem in* R^4, J. Geom. Mech. **12** (2020), 377–394

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