A TALE OF TWO POLYTOPES RELATED TO GEODESIC FLOWS ON SPHERES

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Abstract. Separation of variables for the geodesic flows on round spheres leads to a large family of integrable systems whose integrals are defined through the separation constants. Reduction by the periodic geodesic flow leads to integrable systems on Grassmanians. Specifically for the geodesic flown on the round S^3 the reduced system defines a family of integrable systems on $S^2 \times S^2$. We show that the image of these systems under a continuous momentum map defined through the action variables has a triangle as its image. The image is rigid and does not change when the integrable system is changed within the family. Each member of the family can be identified with a point inside a Stasheff polytope. Corners of the polytope correspond to toric systems (possibly with degenerations), edges correspond semi-toric systems (in various meanings of the word), and the face corresponds to "generic" integrable systems. A fundamental difference of this momentum map to that of a toric or semi-toric system is that the number of tori in the preimage of a non-critical point may be 1, 2, or 4. The momentum map is continuous but not smooth along the images of hyperbolic singularities. The corresponding quantum problem and generalisations to higher dimensional spheres will be discussed.

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