BI-HAMILTONIAN STRUCTURES OF SPIN SUTHERLAND MODELS FROM POISSON REDUCTION

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Abstract. We review our results on bi-Hamiltonian structures of spin Sutherland models built on collective spin variables. Our basic observation was that the holomorphic cotangent bundle $T^*\mathrm{GL}(n,\mathbb{C})$ and its real form $T^*\mathrm{U}(n)$, as well as $T^*\mathrm{GL}(n,\mathbb{C})_{\mathbb{R}}$, carry a natural quadratic Poisson bracket, which is compatible with the canonical one. The quadratic bracket arises by change of variables and analytic continuation from an associated Heisenberg double. Then the reductions of $T^*\mathrm{GL}(n,\mathbb{C})$ and $T^*\mathrm{U}(n)$ by the conjugation actions of the corresponding groups lead to the holomorphic and real trigonometric spin Sutherland models, respectively, equipped with a bi-Hamiltonian structure. The reduction of $T^*\mathrm{GL}(n,\mathbb{C})_{\mathbb{R}}$ by the group $\mathrm{U}(n)\times\mathrm{U}(n)$ gives a generalized Sutherland model coupled to two $\mathfrak{u}(n)^*$-valued spins. We also show that a bi-Hamiltonian structure on the associative algebra $\mathfrak{gl}(n,\mathbb{R})$ that appeared in the context of Toda models can be interpreted as the quotient of compatible Poisson brackets on $T^*\mathrm{GL}(n,\mathbb{R})$. All these reductions were studied previously using the canonical Poisson structures of the cotangent bundles, without realizing the bi-Hamiltonian aspect.

References

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