THE PAINLEVÉ AND PETROVIĆ THEOREMS AND PUISEUX SERIES SOLUTIONS OF ALGEBRAIC ODEs OF THE FIRST ORDER

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Abstract. According to the celebrated Painlevé theorem every solution of an ordinary differential equation (ODE) of the first order can only have a singularity of an algebraic type at any point $x = x_0$, with the exception of points of some fixed finite set Σ at most, which is determined by the equation. In other words, non-algebraic singular points of the solutions of a first order algebraic ODE cannot fill domains in $\mathbb{C}$. This theorem completely describes a possible behavior of solutions near each point x_0 of the set $\mathbb{C} \setminus \Sigma$: solutions are presented by convergent Puiseux series in $(x - x_0)$ with a finite principal part. On other hand, Petrović's theorems allow to determine the leading term of such series.
These results partially help to study the question of convergence of formal Puiseux series solutions of an algebraic ODE. We also study this question of convergence paying attention to the cases where the Painlevé theorem cannot be applied explicitly.

The talk will be based on the joint work with Vladimir Dragović and Irina Goryuchkina.

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