

**INTEGRABLE EULER EQUATIONS RELATED
TO CHAINS OF SUBALGEBRAS**

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ABSTRACT. In 1983 Bogoyavlenski conjectured that if the Euler equations on a Lie algebra \mathfrak{g}_0 are integrable, then their certain extensions to semisimple Lie algebras \mathfrak{g} related to the filtrations of Lie algebras

$$\mathfrak{g}_0 \subset \mathfrak{g}_1 \subset \mathfrak{g}_2 \cdots \subset \mathfrak{g}_{n-1} \subset \mathfrak{g}_n = \mathfrak{g}$$

are integrable as well. In particular, by taking $\mathfrak{g}_0 = \{0\}$ and natural filtrations of $so(n)$ and $u(n)$, we have Gel'fand–Cetlin integrable systems. We proved the conjecture for filtrations of compact Lie algebras \mathfrak{g} : the system are integrable in a noncommutative sense by means of polynomial integrals. Various constructions of complete commutative polynomial integrals for the system are also given.

In addition, related to commutative polynomial integrability, we classify almost multiplicity free subgroups of compact simple Lie groups.

References

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