FREE AND CONTROLLED ROLLING MOTION OF A BALL ON A VIBRATING PLANE

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Abstract. In this work we consider two problems on the motion of spherical bodies on a vibrating plane. The first problem is related to the motion of a Chaplygin sphere rolling without slipping on a plane performing horizontal periodic oscillations. For this problem we show that in the system under consideration the projections of the angular momentum onto the axes of the fixed coordinate system remain unchanged. The investigation of the reduced system on a fixed level set of first integrals reduces to analyzing a three-dimensional period advance map on SO(3). The analysis of this map suggests that in the general case the problem considered is nonintegrable. We find partial solutions to the system which are a generalization of permanent rotations and correspond to nonuniform rotations about a body- and space-fixed axis. We also find a particular integrable case which, after time is rescaled, reduces to the classical Chaplygin sphere rolling problem on the zero level set of the area integral. The second part of the work addresses the problem of a spherical robot having an axisymmetric pendulum drive and rolling without slipping on a vertically vibrating plane. It is shown that this system admits partial solutions (steady rotations) for which the pendulum rotates about its vertical symmetry axis. Special attention is given to problems of stability and stabilization of these solutions. An analysis of the constraint reaction is performed, and parameter regions are identified in which a stabilization of the spherical robot is possible without it losing contact with the plane. It is shown that the partial solutions can be stabilized by varying the angular velocity of rotation of the pendulum about its symmetry axis, and that the rotation of the pendulum is a necessary condition for stabilization without the robot losing contact with the plane.

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