COMMENTS ON THE INTERPLAY BETWEEN VORTICES AND HARMONIC FIELDS

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Abstract. Let M a compact manifold without boundary with metric g. When on writes Arnold's geodesic equations [**1**] in $SDiff(M)$ in terms of 1-forms ν instead of vectorfields, it is well known that a Poisson structure results ie in the dual of the Lie algebra. The Hodge decomposition $\nu = df + \delta\psi + \eta$, with η harmonic will not contain the term df, because one wants $\nu^\sharp$ to be divergence free. Moreover, $\delta\phi$ can be uniquely recovered from the vorticity 2-form $\omega = d\delta\psi$, and one could write the equations of motion in terms of (ω, η) [**2**]. In the literature it is usually assumed that the ambient is simply connected. However, when $H_1(M) \neq \emptyset$, there is an interplay between ω and η. When $M = \Sigma$ is a Riemann surface, this coupling has been made explicit very recently by Björn Gustaffson [**3**] and checked by Clodoaldo Ragazzo [**4**] with additional results. In this talk I will comment on these developments and the impact that I imagine these works should provoke in the vorticitists community.

References

1. V. I. Arnold, *Sur la géométrie différentielle des groupes de Lie de dimension infinie et ses applications à l'hydrodynamique des fluides parfaits*, Annales de l'Institut Fourier **16**(1) (1966), 319–361.
2. J. Marsden, A. Weinstein, *Coadjoint orbits, vortices, and Clebsch variables for incompressible fluids*, Physica D: Nonlinear Phenomena **7**(1–3) (1983), 305–323.
3. B. Gustafsson, *Vortex pairs and dipoles on closed surfaces*, Arxiv4101583, Jan. 3, 2022.
4. C. Ragazzo, personal communication. *Surprising dynamics occurs even in genus 1. Discrete symmetries, such as when Σ is Bolza's surface allows detecting stationary solutions.*

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