# DISCRETE SYMMETRIES OF DYNAMICAL EQUATIONS WITH POLYNOMIAL INTEGRALS OF HIGHER DEGREES

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Abstract. An analysis is made of systems with toral configuration space and the kinetic energy in the form of a "flat" Riemannian metric on a torus. The potential energy $V$ is a smooth function on the configuration torus. The dynamics of such systems is described by "natural" Hamiltonian systems of differential equations. If one replaces $V$ with $\varepsilon V$, where $\varepsilon$ is a small parameter, then the investigation of such Hamiltonian systems at small values of $\varepsilon$ is the "main problem of dynamics" in the sense of Poincaré. A discussion is given of the hypothesis of single-valued polynomial (in momenta) integrals of the equations of motion: if there is an integral of degree $m$ polynomial in momenta, then there is necessarily a first integral linear or quadratic in momenta. This hypothesis is completely proved for $m = 3$ and $m = 4$. Also discussed are cases of "higher" degrees where $m = 5$ and $m = 6$. Following the theory of perturbation of Hamiltonian systems, resonant straight lines on the plane of momenta are introduced. If the system admits a polynomial integral, then the number of these lines is finite. Symmetries of the set of resonant lines are found, which gives, in particular, necessary conditions for integrability. Some new criteria for the existence of single-valued polynomial integrals are obtained.

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