UNIVERSAL SPACE OF PARAMETERS $\mathcal{F}_n$ AND THE MODULI SPACE $\bar{\mathcal{M}}(0,n)$

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ABSTRACT. The canonical action of the compact torus T^n on the complex Grassmann manifold $G_{n,2}$ of two-dimensional complex subspaces in $\mathbb{C}^n$ is an important and widely known example of a Hamiltonian action on a symplectic manifold. The study of this action naturally leads to problems of dynamical systems theory and the study of the orbit space $G_{n,2}/T^n$ is closely related to the problems on integrals of the correspinding dynamical systems. In the focus of our talk is going to be a space of parameters for T^n-action on $G_{n,2}$ and the structure of the orbit space of this action.
The moduli space $\bar{\mathcal{M}}(0,n)$ of stable n-pointed genus zero curves is the Deligne–Mumford–Grothendieck–Knudsen compactification of the moduli space $\mathcal{M}(0,n)$ of n-pointed genus zero curves. The space $\bar{\mathcal{M}}(0,n)$ is proved by Kapranov to coincide with the Chow quotient $G_{n,2}//(\mathbb{C}^*)^n$.
In this talk we present an another approach to the compactification of $\mathcal{M}_{0,n}$ from the point of view of T^n-equivariant topology of $G_{n,2}$ for the canonical compact torus T^n-action. In describing the orbit space $G_{n,2}/T^n$, the main stratum $W_n \subset G_{n,2}$ given by those points from $G_{n,2}$ whose all Plücker coordinates are non-zero, plays a crucial role, as it is an open dense set in $G_{n,2}$ and it belongs to any Plücker chart for $G_{n,2}$. In addition, we earlier proved that $W_n \cong \overset{\circ}{\Delta}_{n,2} \times F_n$, where $\Delta_{n,2}$ is the hypersimplex and $F_n = W_n/(\mathbb{C}^*)^n \subset \mathbb{C}P^N, N = \binom{n-2}{2}$ is an open algebraic manifold.
In order to construct a model for the orbit space $G_{n,2}/T^n$ it turns out to be important to look for a compactification $\mathcal{F}_n$ for F_n such that any automorphism of F_n induced by the transition maps between the Plücker charts extends to the automorphism of $\mathcal{F}_n$. Such compactification $\mathcal{F}_n$ we call the universal space of parameters. We obtain the space $\mathcal{F}_n$ by resolving singularities arising in the context of required extensions and by making use of the construction from algebraic geometry known as the wonderful compactification. Finally, we prove that the space $\mathcal{F}_n$ coincides with $\bar{\mathcal{M}}(0,n)$.

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