PROJECTIVE AND AFFINE EQUIVALENCE OF SUB-RIEMANNIAN METRICS: INTEGRABILITY, GENERIC RIGIDITY, THE WEYL TYPE THEOREMS, AND SEPARATION OF VARIABLES CONJECTURE

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Abstract. Sub-Riemannian metrics are defined by a distribution (a subbundle of the tangent bundle) together with an Euclidean structure on each fiber. The Riemannian metrics correspond to the case when the distribution is the who;e tangent bundle. Two sub-Riemannian metrics are called projectively equivalent if they have the same geodesics up to a reparameterization and affinely equivalent if they have the same geodesics up to affine reparameterization. In the Riemannian case, both equivalence problems are classical: local classifications of projectively and affinely equivalent Riemannian metrics were established by Levi-Civita in 1898 and Eisenhart in 1923, respectively. In particular, a Riemannian metric admitting a nontrivial (i.e. non-constant proportional) affinely equivalent metric must be a product of two Riemannian metrics i.e. separation of variables (the de Rham decomposition) occur, while for the analogous property in the projectively equivalent case a more involved ("twisted") product structure is necessary. The latter is also related to the existence of sufficiently many commuting nontrivial integrals quadratic with respect to velocities for the corresponding geodesic flow. We will describe the recent progress toward the generalization of these classical results to sub-Riemannian metrics. In particular, we will discuss the genericity of metrics that do not admit non-constantly proportional affinely/projectively equivalent metrics and the separation of variables on the level of linearization of geodesic flows (i.e. on the level of the Jacobi equations) for metrics that admit non-constantly proportional affinely equivalent metrics. We also describe the sub-Riemannian analog of the Weyl theorem that all metrics that are simultaneously projectively equivalent and conformal are constantly proportional.

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