TOPICS IN POLYNOMIALS:

EXTREMAL PROBLEMS, INEQUALITIES, ZEROS

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Preface

The theory of best approximation with respect to the supremum norm was established as a bona fide branch of mathematical analysis mainly by the work of P. L. Chebyshev (1821–1894), who in the 1850s studied some of the properties of polynomials with least deviation from a given continuous function. Since that time, the work of the celebrated Petersburg school of Mathematics – also called the Chebyshev school – has had a lasting impact in theoretical and applied mathematics and produced students such as A. A. Markov (1856–1922).

The present book contains some of the most important results on the analysis of polynomials and their derivatives. Besides the fundamental results which are treated with their proofs, the book also provides an account of the most recent developments concerning extremal properties of polynomials and their derivatives, as well as properties of their zeros. An attempt has been made to present the material in an integrated and a self-contained fashion. The book is intended, not only for the specialist mathematician, but also for those researchers in the applied and computational sciences who use polynomials as a tool.

The subject of polynomial inequalities is of course vast. We chose to restrict ourselves here only to a few directions. We present some striking results (to us at least), but also novel aspects as well as old and new results not normally found in book form. On the other hand, subjects such as orthogonal polynomials are not included here as such (for that subject see the excellent books of G. Szegő, G. Freud, T. S. Chihara, and P. K. Suetin).

Some 1200 references have been cited here, including preprints. As a rule, we have studied the original sources and in some cases have retrieved some forgotten but useful results. The references appear at the end of each chapter. At the end of the book we include a symbol index, as well as a name and subject index.

The first chapter reviews some of the classical results on polynomials of one and several variables. The second chapter provides an account of some selected inequalities involving algebraic polynomials as well as inequalities with trigonometric polynomials.

The third chapter studies zeros of polynomials, with emphasis on the distribution of zeros of the algebraic polynomials, the Sendov-Ilieff conjecture, as well as bounds for the zeros and their number in certain domains. We also consider the Eneström-Kakeya theorem and its various generalizations.

Chapter 4 treats inequalities connected with trigonometric sums. Besides of the classical results of L. Fejér, T. H. Gronwall, D. Jackson, W. H. Young, W. W. Rogosinski and G. Szegő, we give a special emphasis to the analysis of positivity and monotonicity of certain trigonometric sums and some related orthogonal polynomial sums. In particular, we point out an inequality of R. Askey and G. Gasper which was the final step in L. de Branges's remarkable proof (1984) of the Bieberbach conjecture (1916).

The fifth and sixth chapters are devoted to extremal problems for polynomials.

In Chapter 5 we investigate the extremal problems for polynomials and their coefficients, (which, as is known, are suggestive of results in a much more general context!) starting by the classical results of P. L. Chebyshev, A. A. Markov, E. J. Remez, S. N. Bernstein, A. N. Korkin and E. I. Zolotarev, which are basic to approximation theory. In particular, we study polynomials with minimal L^r norm, many generalizations of such polynomials, estimates for coefficients of polynomials in L^2 norm with prescribed zeros, extremal problems in mixed norms, as well as G. Szegő's and related extremal problems. Section 5.2 treats incomplete polynomials introduced in 1976 by G. G. Lorentz and weighted polynomial inequalities, including extremal problems with exponential weights and L^r inequalities for Freud weights. Section 5.3 is devoted to extremal problems and inequalities of Nikol'skiĭ type.

Inequalities of Markov and Bernstein-type are fundamental for the proof of many inverse theorems in polynomial approximation theory. Frequently further progress in inverse theorems has depended on first obtaining a corresponding generalization or analog of Markov's and Bernstein's inequalities. There are many results on Markov's and Bernstein's theorems and their generalizations. In Chapter 6 we consider such problems involving additional other classical results of Markov and Bernstein. In Section 6.2 we study the corresponding extremal problems on restricted classes of polynomials, and finally, in Section 6.3, we conclude with the extremal problems in a circle on the complex plane.

The final chapter provides some selected applications of polynomials. There are applications to least squares approximation with constraints, to vectorial and simultaneous approximations and to computer aided geometric design (CAGD) so much in vogue today. We also study the Bernstein conjecture of 1913, which was settled negatively in 1985 by R. S. Varga and A. J. Carpenter.

Finally, we wish to thank Professors R. Askey, B. Bojanov, R. Ž. Djordjević, G. Gasper, A. Guessab, Lj. M. Kocić, I. Ž. Milovanović, P. Nevai, M. Tomić, R. S. Varga, who read parts of the manuscript of the book and provided some very useful comments. Thanks also go to Professor A. W. Goodman who kindly donated to us his collection of papers on the theory of polynomials and to Professor Lj. M. Kocić who wrote Section 7.4 dealing with computer aided geometric design. We also wish to thank Professor D. Carmocolias for his assistance in proofreading the manuscript in English.

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