LIFE AND INEQUALITIES: D. S. MITRINOVIĆ (1908–1995)

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1. Biographical Data

Professor Dragoslav S. Mitrinović, the famous scientist, a modest man, teacher and a model of many generations, died on April 2, 1995. He was born in Smederevo, Serbia, on June 23, 1908, as the first child of Svetislav and Marija Mitrinović. His sister Ružica (1909–1993) was the second and the last child in the Mitrinović family. Their father, a known judge, died when Dragoslav was seven, so that he was forced to fight for his living himself. He received elementary and secondary education in Priština and Vranje. In 1932 he graduated mathematics at the Faculty of Philosophy, University of Belgrade. The next year, as a student of Professor Mihailo Petrović – Alas, he defended his Ph.D. thesis in the field of Differential equations entitled *"Investigations of an important differential equation of the first order"*.

FIG. 1. Dragoslav with his sister Ružica (from 1913)

FIG. 2. D. S. Mitrinović as a student (from 1929)

1

In 1933 he got married to Olga Sretenović (1910–1996). Olga was also a mathematician and she worked as a secondary school teacher. Their sons, Svetislav (1934) and Mihailo (1945), are the university professors.

FIG. 3. Prof. Mihailo Petrović – Alas
(1868–1943)

FIG. 4. Dragoslav and Olga
(from 1933)

Until 1946 D. S. Mitrinović worked as a secondary school teacher. He spent some time as a researcher at the Paris University. His ID-cards from that period are shown in Figures 5 and 6.

FIG. 5. University immatriculation card

FIG. 6. Card for the National Library

During this period Mitrinović published about 50 scientific papers, mainly on differential equations.

2. Professional Career

Mitrinović started his university career in Skoplje, Macedonia, as an Associate Professor at the Philosophical Faculty. It took him only five years (1946–1951) to found the Skoplje School of Mathematics. At the Philosophical Faculty he founded

the Department of Mathematics and two mathematical journals (*"Zbornik radova Filozofskog fakulteta u Skoplju"* in 1948 and *"Bilten društva matematičara i fizičara Makedonije"* in 1950). The first mathematical research papers in Macedonia were done by Professor Mitrinović. His persistent work resulted in the foundation a rich professional mathematical library there and in a wide exchange of scientific publications with foreign countries. At the beginning, all the lecturing in Skoplje was performed by two mathematicians only. It was at that time that a core of scientific workers was formed in Skoplje, which is today one of the recognised scientific centers. A number of Ph.D. theses were defended, mainly under the supervision of Professor Mitrinović. Thanks to his scientific contribution he was elected the member of the *Macedonian Academy of Science and Art*.

From 1951 to his retirement in 1978 Professor Mitrinović taught at the Faculty of Electrical Engineering, University of Belgrade, and in 1953 he was elected the Head of the Department of Mathematics. During his long period of teaching he supported young and talented mathematicians, students of his faculty (to whom mathematics would be their future profession), gave them instructions for their scientific research, made them get to know the scientific references he knew so well and helped them publish their results in the country and abroad. He made his collaborators work as hard as he practised himself. He encouraged the progress and success of all his assistants. He founded the well-known Belgrade School of Functional Equations, Differential Equations and Inequalities. He was also the founder of the *Publications of the Faculty of Electrical Engineering, Series: Mathematics and Physics*, which soon became the worldwide renown journal. Numerous world well-known and outstanding mathematicians published their papers in the *Publications*. This journal is available in many university libraries all over the world.

Soon after foundation of the first faculties in Niš in 1960, Professor Mitrinović founded another school of mathematics. In the period between 1965 and 1975 he was the Head of the Department of Mathematics at the Faculty of Electronic Engineering, University of Niš. He supported the development of any field in mathematics, encouraged his collaborators and assistants, introduced them into new fields he himself didn't work in and was in touch with developed centers all over the world. His collaborators appreciated and accepted such approach of his. Thanks to all this, the Niš School of Mathematics soon grew into a powerful center of Approximation Theory, Inequalities and Numerical Mathematics, without any problems and separations which are characteristic for this country.

Professor Mitrinović was a very communicative person. He maintained epistolary relationship with numerous world respectable mathematicians. He was a long-time member of the *American Mathematical Society, Société Mathématique de France* and one of the founders of the *Serbian Scientific Society*. His social activity on the professional plan is also noteworthy. He was the founder of the *Mathematical documentation center of the Society of mathematicians and physicists of Serbia, the Vice-president of the Union of societies of mathematicians and physicists of Yugoslavia, the President of the Society of mathematicians and physicists of Macedonia, the President of the Commission for mathematics of the Federal*

Council for the coordination of scientific research, a Member and the President of the corresponding commission in Serbia, the Vice-president of the Commission for text-books, not to mention several other duties within the framework of the University. For a long time, Professor Mitrinović was a member of the Editorial Board of East European Series *"Mathematics and Its Applications"* in the Kluwer Academic Publishers.
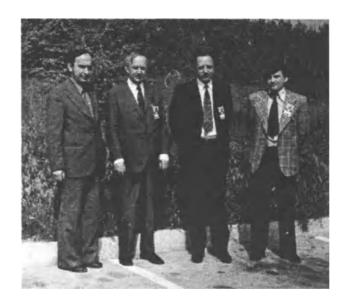

FIG. 7. S. Milojković, D. S. Mitrinović, R. Ž. Djordjević, and G. V. Milovanović
(Poreč, 1975)

Mitrinović was a prolific writer of many university books as well as significant monographs of high scientific level, published by the world's most famous publishing houses. His monograph *Analytic Inequalities* (with P. M. Vasić) published in 1970 by Springer Verlag, had a very powerful influence on the development of this field in Yugoslavia and abroad. Many generations of students and mathematicians studied from Professor Mitrinović's books. His name on the covers always signified high standards and a rigorous mathematical style.

3. Scientific Work in Inequalities

The scientific work of Professor Mitrinović and his contributions in mathematics can be classified into the following areas:

1. Differential equations;

2. Functional equations;

3. Inequalities;

4. Other fields.

His work in the first two areas (Differential and Functional equations) has been described in [10] (see also [1–4] and [6–7]). Beside more than one hundred papers on differential equations and more than thirty papers on functional equations, he

published three text-books on differential equations. His starting papers on functional equations from fifties were important for developing a well-known Belgrade School of Functional Equations as well as the appearing of his *"Mathematics Problem Book", Vol. III* (1960), with several interesting open problems related to the classical functional equations. These problems were a "glue" for young mathematicians and for the most talented students.

We mention that 7 mathematicians took their Ph. D. theses in differential equations with Professor Mitrinović: B. S. Popov (1952), I. Bandić (1958), D. Perčinkova (1963), I. Šapkarev (1964), J. D. Kečkić (1970), P. R. Lazov (1977), and B. Piperevski (1982). Also, Professor Mitrinović gave seven Ph. D. theses in functional equations: D.Ž. Djoković (1963), K. Milošević-Rakočević (1963), P. M. Vasić (1963), R. Ž. Djordjević (1966), R. R. Janić (1968), I. Stamate (1971), and B. Zarić (1975). A nice review on these theses has just been written by Professor B. D. Crstici (see [3]).

The last and the greatest Mitrinović's passion in mathematics was the one called – *Inequalities*. He was involved in all kinds of inequalities. He often used to say: *"There are no equalities, even in the human life, the inequalities are always met"*. Until early sixties only the classical work *Inequalities* by Hardy, Littlewood, and Pólya, appeared in 1934, intended to transform the field of inequalities from a collection of isolated formulas into a systematic discipline. Professor A. M. Fink (Iowa State University) even said: *"I had not considered inequalities as a research subject, even though I owned a copy of Hardy, Littlewood, and Pólya's "Inequalities". Inequalities were a sidelight to my research in differential equations. But through Professor Mitrinović's book "Analytic Inequalities" from 1970 and his correspondence with me, I saw the richness of the subject of inequalities, the care he took to ascribe intellectual ideas to their real sources, and his personal integrity in writing about the subject."*

Mitrinović's interest in inequalities started very early considering some inequalities for elementary symmetric functions (1959). His work can be classified into the following areas:

1. Elementary inequalities;
2. Geometric inequalities;
3. Means and their inequalities;
4. Analytic inequalities;
5. Inequalities and extremal problems with polynomials;
6. Various particular inequalities;
7. Inequalities in number theory.

To each of these areas Mitrinović devoted at least one monograph. At this point we could cite Professor Dick Askey, who told: *"He was a collector of interesting and important older mathematical results. This resulted in a number of books which have few if any rivals. When an inequality arises, as it often does in my work or in letters from others asking about one, the first place I look is in the books of Mitrinović. There are few with his dedication to preserving interesting*

mathematics. Fortunately, he did not write all of his books alone, so he helped train others to follow in his footsteps. May they carry on his legacy of service to the community of mathematicians around the world."

1. Mitrinović started with elementary inequalities in 1959. Very soon in 1964 he published (in cooperation with E. S. Barnes, D. C. B. Marsh and J. R. M. Radok) the book entitled *"Elementary Inequalities"* (P. Noordhoff, Groningen). This tutorial text and problem collection is designed to introduce the student, at undergraduate or senior high school level, to the elementary properties of inequalities. Considerably enlarged version of this book appeared in Polish in 1972, with P. M. Vasić and R. R. Janić as co-authors. Among many elementary inequalities treated by Professor Mitrinović we mention only those with elementary symmetric functions $\sigma_k = \sigma_k(x_1, \dots, x_n)$. If $1 \le k \le n-1$ and $0 \le \nu \le k-1$, Mitrinović proved that

$$(\Delta^\nu \sigma_{k-\nu})^2 - (\Delta^\nu \sigma_{k-\nu+1})(\Delta^\nu \sigma_{k-\nu-1}) \ge 0,$$

where Δ is the standard forward difference operator. Also he proved the following implication for $1 \le p \le \nu$,

$$(-1)^p \Delta^p \sigma_{k-\nu+1} > 0 \quad \Longrightarrow \quad (-1)^\nu \Delta^\nu \sigma_{k-\nu} > 0.$$

2. Several papers Mitrinović devoted to the geometric inequalities. In 1969 the book *"Geometric Inequalities"* (Groningen), written by O. Bottema, R. Ž. Djordjević, R. R. Janić, D. S. Mitrinović, and P. M. Vasić, was appeared. The book is very appreciated and has been much quoted in the mathematical literature. It contains about 400 varied geometric inequalities related to the elements of figures in the plane (triangles, quadrilaterals, n-gons, circles) and 225 authors are cited in it. After the appearance of this book (called "Bible of Bottema" in the Canadian journal Crux Mathematicorum), during the period from 1969–1986 a large number of papers and problems concerning geometric inequalities were published in mathematical journals and this inspired Professor Mitrinović to compile an encyclopedic work *"Recent Advances in Geometric Inequalities"* (Kluwer, 1989) jointly with J. E. Pečarić and V. Volenec. This book contains several thousands of inequalities, not only for elements of figures in the plane, but also for elements of figures in space and hyperspace (tetrahedra, polyhedra, simplices, polytopes, spheres). This book is a good base for the various synthesis of apparently unconnected results about geometric inequalities, and also represents a rich source book for obtaining some deeper and essential generalisations.

3. Mitrinović also devoted several papers to the means and their inequalities. His main collaborator in this field was P. M. Vasić (1934–1996). Unifying the results proved by W. N. Everitt [Amer. Math. Monthly **70** (1963), 251–255] and Mitrinović and Vasić [Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz. **No 159 – No 170** (1966), 1–8], H. W. McLaughlin and F. T. Metcalf [Pacific J. Math. **22** (1967), 303–311] obtained some interesting inequalities for means of order r. Later, Mitrinović and Vasić (1968) proved even more general results which contain inequalities of McLaughlin and Metcalf. In 1966 Mitrinović and Vasić introduced one method, so-called λ-method, for getting inequalities. This method can be summarised as follows:

(1) Start with an inequality which can be proved by the theory of maxima and minima;

(2) In a convenient manner introduce one or more parameters into the function from which that inequality was obtained;

(3) Find the extreme values of such a function, treating the parameters as fixed.

In this way an inequality involving one or more parameters is obtained. Assigning conveniently chosen values to those parameters, one may obtain various inequalities whose forms bear no similarity to the original. This method often unifies isolated inequalities and yields known inequalities as special cases. Using this method Mitrinović and Vasić obtained many interesting inequalities with means.

As a top in this field is certainly the monograph *"Means and Their Inequalities"* written on 459 pages by D. S. Mitrinović, P. S. Bullen and P. M. Vasić and published by Kluwer in 1988.

4. The most important Mitrinović's work on inequalities inequalities appeared in the Mathematical Analysis. He considered many important classical inequalities including their variations and generalisations. Especially, we mention his work on the Steffensen inequality from 1969, as well as a joint paper with P. M. Vasić on an integral inequality ascribed to Wirtinger. In 1974 Mitrinović and Vasić published one important paper on history, variations and generalisations of the Chebyshev inequality and the question of some priorities.

In 1965 Mitrinović published the book "Nejednakosti" in Serbian on 240 pages. Five years later, a grandiose work appeared by Springer Verlag – "Analytic Inequalities." Talking on Mitrinović's contribution in mathematics, Professor P. S. Bullen says: *"During his long and active life Professor Mitrinović not only did much original work in various fields, although mainly in inequalities. In addition he became famous for research into the obscure origins of many famous results. However his most abiding contribution are three. The famous book, done with the collaboration of Professor Vasić, "Analytic Inequalities". It is, after the classic "Inequalities" by Hardy, Littlewood and Pólya, the most referred to book in the field of inequalities."*[1]

The complete material of this book is divided into three parts. In the first part ("Introduction") an approach to inequalities is given, while the main attention is devoted to convex functions. The second and main part ("General Inequalities") consists of 27 sections, each of which is dedicated to a class of inequalities of

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[1] Further, Bullen says: "I have called the Publikacije Elektrotehničkog Fakulteta Univerziteta u Beogradu, serija Matematika i Fizika "his journal" and it was so in a very real sense. It is an essential tool for working in the field of inequalities, and the almost complete run that I have is one of my most valuable possessions in the my mathematical library. I only wish that it were complete. Finally there are the many students Professor Mitrinović brought along and who are now carrying on his work all over the world. I mention Professors Vasić, Pečarić as being the ones that I know best, but there are many others as any perusal of "his journal" will show. I think it is no exaggeration to say that they are keeping him alive, and will continue to do so for many years to come."

importance in Analysis. Finally, the third part ("Particular Inequalities") gives a collection of various inequalities.

5. The monograph *"Topics in Polynomials: Extremal Problems, Inequalities, Zeros"*, written by G. V. Milovanović, D. S. Mitrinović, and Th.M. Rassias, and published by World Scientific, contains some of the most important results on the analysis of polynomials and their derivatives. Besides the fundamental results, which are treated with their proofs, the book also provides an account of the most recent developments concerning extremal properties of polynomials and their derivatives in various metrics with an extensive analysis of inequalities for trigonometric sums and algebraic polynomials, as well as their zeros. Many extremal problems of Markov, Bernstein, Nikolskiĭ, and Turán type were considered. The inequalities are given for various domains, various norms and for various subclasses of polynomials, both algebraic and trigonometric. Some 1200 references have been cited, including preprints. Professor T. Erdélyi in his review on this book in the *Journal of Approximation Theory* (Vol. **82** (1995), 471–472) says: *"The topics are tastefully selected and the results are easy to find. Although this book is not really planned as a textbook to teach from, it is excellent for self-study or seminars. This is a very useful reference book with many results which have not appeared in a book form yet. It is an important addition to the literature."* Professor E. W. Cheney in *Mathematics of Computation* (Vol. **65** (1996), 438–439) concludes his review by words: *"The book is written in a gentle style: one can open it anywhere and begin to understand, without encountering unfamiliar notation and terminology. It is strongly recommended to individuals and to libraries."* (see also the reviews written by Professor N. K. Govil in *Mathematical Reviews* (95m: 30009) and by H. M. Srivastava in *Zentralblatt für Mathematik* (848–147)).

6. The third part of Mitrinović's monograph *"Analytic Inequalities"*, which is entitled "Particular Inequalities", represents a collection of various inequalities, more or less closely interconnected. This 200-pages part includes discrete inequalities, inequalities with algebraic and trigonometric functions and polynomials, inequalities with exponential, logarithmic and gamma functions, as well as integral inequalities and inequalities in the complex domain. Many of these results belong to Professor Mitrinović. Besides extensions and generalisations, Mitrinović always wanted to link various isolated inequalities and find their common source. Recently he published by Kluwer two monographs with such results: *"Inequalities Involving Functions and Their Integrals and Derivatives"* and *"Classical and New Inequalities in Analysis"* (jointly with J. E. Pečarić and A.M. Fink). In his recent papers, mainly written jointly with Pečarić, various particular inequalities were considered (Erdős-Mordell's and related inequalities of Gauss-Winckler, inequalities for polygons, some trigonometric inequalities, Neuberg-Pedoe and Oppenheim inequalities, Steffensen's inequality, some determinantal inequalities, inequalities of Godunova and Levin, Ozeki's inequalities, Lebed's inequality, inequalities of Hilbert and Widder, Masuyama's inequality, etc.).

7. The last Mitrinović's monograph was the *"Handbook of Number Theory"*, written jointly by J. Sándor and B. Crstici and published by Kluwer this year (1996). Unfortunately, after the manuscript was finished and during its preparation for

printing, Professor Mitrinović died, not having the chance to see his last work in libraries. The aim of this book was to systematise and to present in an easily accessible framework the most important results from some parts of Number Theory, which are expressed by inequalities or by estimates. The book focuses on the most important arithmetic functions in Number Theory, together with various generalisations, analogues and extensions of such functions, and also properties of some functions related to the distribution of the primes and of the quadratic residues and to partitions, etc. We note that the "yeast" for this Handbook was the previous book *"Inequalities in Number Theory"* published in 1978 by Mitrinović and M.S. Popadić (Naučni Podmladak, University of Niš).

We mention also that 4 mathematicians took their Ph. D. theses in inequalities with Professor Mitrinović: Lj. R. Stanković (1975), I. B. Lacković (1975), G. V. Milovanović (1976) and I. Ž. Milovanović (1980).

Mitrinović's scientific interest was also in the other fields as Bernoulli's and Stirling's numbers and polynomials (31 papers), as well as in complex analysis, special functions, orthogonal polynomials, summation formulas, abstract algebra, etc. Especially, we mention the monograph *"The Cauchy Method of Residues – Theory and Applications"* in two volumes, written jointly with J. D. Kečkić and published by Kluwer. The first volume, which appeared in 1984, is the only book that covers all known applications of the calculus of residues. They range from the theory of equations, theory of numbers, matrix analysis, evaluation of real definite integrals, summation of finite and infinite series, expansions of functions into infinite series and products, ordinary and partial differential equations, mathematical and theoretical physics, to the calculus of finite differences and difference equations. On the other hand, the second volume (appeared in 1993) is devoted to new results in this field. Also, it contains some special contributions written by various authors and they are based mainly on their own research work. They include topics as the generalised value of an improper integral, numerical evaluation of definite integrals, inclusive calculus of residues, polynomials orthogonal on a semicircle in the complex plane, and an interesting generalisation of the residue theorem to distribution.

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The total bibliography of Professor Mitrinović contains 372 units, including 278 scientific papers and 30 other papers, as well as 16 monographs, 35 text-books, and 13 other books (see [5]). There are over 40 scientists who received their doctoral degrees by Professor Mitrinović. He enabled his collaborators and doctorands to use his huge scientific documentation in which he kept old, rare and valuable papers, systematically collected over the past years, and pedantically arranged into fields. His collaborators were frequently surprised by his familiarity with references in topics that were not in his immediate circle of interest. In his last years, he used to give whole folders of precious papers to his visitors as a present, saying: *"I do not have any more time for that"*.

Professor Mitrinović devoted his whole life to mathematics. He led a modest life. His works will remain a long lasting value and will be cited in mathematical

literature for a long time. He will remain in the memory of his numerous associates and students as a truly exceptional man they could learn a lot of from.

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