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Certain Laplace transforms of convolution type integrals involving product of two special ${}_pF_p$ functions

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Abstract: Recently the authors obtained several Laplace transforms of convolution type integrals involving Kummer's function ${}_1F_1$ [Appl. Anal. Discrete Math., 2018, 12(1), 257–272]. In this paper, the authors aim at presenting several new and interesting Laplace transforms of convolution type integrals involving product of two special generalized hypergeometric functions ${}_pF_p$ by employing classical summation theorems for the series ${}_2F_1$, ${}_3F_2$, ${}_4F_3$ and ${}_5F_4$ available in the literature.

Keywords: Gauss's summation theorem, Gauss's second summation theorem, Bailey's summation theorem, Kummer's summation theorem, Watson's summation theorem, Dixon's summation theorem, Whipple's first and second summation theorems, Dougall's theorem, Laplace transform, Convolution type integrals

MSC: Primary 33C20; Secondary 33C05, 33C90

1 Introduction and motivation

In the theory of hypergeometric and generalized hypergeometric functions, there exist a remarkably large number of hypergeometric summation formulas which can be expressed in terms of the Gamma functions. In particular, for specified values of the argument, usually, 1, −1 and 1/2, the hypergeometric function ${}_2F_1$ and the generalized hypergeometric function ${}_3F_2$ reduce to the the well-known classical summation theorems such as the Gauss, Gauss second, Bailey and Kummer ones for the ${}_2F_1$ series, as well as the Watson, Dixon and Whipple ones for the ${}_3F_2$ series, Whipple second for ${}_4F_3$ and Dougall for ${}_5F_4$ play an important role in the theory of generalized hypergeometric functions (cf. [1]).

Precisely, such theorems are given below so that the paper may be self-contained.

• Gauss's summation theorem

$$ {}_2F_1\left[\begin{matrix} a, b \\ c \end{matrix} \middle| 1\right] = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} = \Omega_1(a,b,c) \quad (\mathrm{Re}(c-a-b)>0); \tag{1.1} $$

• Gauss's second summation theorem

$$ {}_2F_1\left[\begin{matrix} a, b \\ \frac{1}{2}(a+b+1) \end{matrix} \middle| \frac{1}{2}\right] = \frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}(a+b+1))}{\Gamma(\frac{1}{2}a+\frac{1}{2})\Gamma(\frac{1}{2}b+\frac{1}{2})} = \Omega_2(a,b); \tag{1.2} $$

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- Bailey's summation theorem

$$ {}_2F_1\left[\begin{matrix} a, 1-a \\ b \end{matrix} \,\middle|\, \frac{1}{2}\right] = \frac{\Gamma(\frac{1}{2}b)\Gamma(\frac{1}{2}b+\frac{1}{2})}{\Gamma(\frac{1}{2}b+\frac{1}{2}a)\Gamma(\frac{1}{2}b-\frac{1}{2}a+\frac{1}{2})} = \Omega_3(a,b); \tag{1.3} $$

- Kummer's summation theorem

$$ {}_2F_1\left[\begin{matrix} a, b \\ 1+a-b \end{matrix} \,\middle|\, -1\right] = \frac{\Gamma(1+\frac{1}{2}a)\Gamma(1+a-b)}{\Gamma(1+a)\Gamma(1+\frac{1}{2}a-b)} = \Omega_4(a,b); \tag{1.4} $$

- Watson summation theorem

$$ {}_3F_2\left[\begin{matrix} a, b, c \\ \frac{1}{2}(a+b+1), 2c \end{matrix} \,\middle|\, 1\right] = \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(c+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}\right)\Gamma\left(c-\frac{1}{2}a-\frac{1}{2}b+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}a+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}b+\frac{1}{2}\right)\Gamma\left(c-\frac{1}{2}a+\frac{1}{2}\right)\Gamma\left(c-\frac{1}{2}b+\frac{1}{2}\right)} $$
$$ = \Omega_5(a,b,c) \qquad (\mathrm{Re}(2c-a-b) > -1); \tag{1.5} $$

- Dixon's summation theorem

$$ {}_3F_2\left[\begin{matrix} a, b, c \\ 1+a-b, 1+a-c \end{matrix} \,\middle|\, 1\right] $$
$$ = \frac{\Gamma(1+\frac{1}{2}a)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+\frac{1}{2}a-b-c)}{\Gamma(1+a)\Gamma(1+\frac{1}{2}a-b)\Gamma(1+\frac{1}{2}a-c)\Gamma(1+a-b-c)} $$
$$ = \Omega_6(a,b,c) \qquad (\mathrm{Re}(a-2b-2c) > -2); \tag{1.6} $$

- Whipple's summation theorem

$$ {}_3F_2\left[\begin{matrix} a, 1-a, c \\ e, 1+2c-e \end{matrix} \,\middle|\, 1\right] $$
$$ = \frac{2^{1-2c}\pi\,\Gamma(e)\Gamma(1+2c-e)}{\Gamma(\frac{1}{2}a+\frac{1}{2}e)\Gamma(\frac{1}{2}-\frac{1}{2}a+\frac{1}{2}e)\Gamma(c+\frac{1}{2}a-\frac{1}{2}e+\frac{1}{2})\Gamma(c-\frac{1}{2}a-\frac{1}{2}e+1)} $$
$$ = \Omega_7(a,c,e) \qquad (\mathrm{Re}(c) > 0); \tag{1.7} $$

- Second Whipple's summation theorem

$$ {}_4F_3\left[\begin{matrix} a, 1+\frac{1}{2}a, b, c \\ \frac{1}{2}a, a-b+1, a-c+1 \end{matrix} \,\middle|\, -1\right] = \frac{\Gamma(a-b+1)\,\Gamma(a-c+1)}{\Gamma(a+1)\,\Gamma(a-b-c+1)} $$
$$ = \Omega_8(a,b,c); \tag{1.8} $$

- Dougall's summation theorem

$$ {}_5F_4\left[\begin{matrix} a, 1+\frac{1}{2}a, c, d, e \\ \frac{1}{2}a, a-c+1, a-d+1, a-e+1 \end{matrix} \,\middle|\, 1\right] $$
$$ = \frac{\Gamma(a-c+1)\,\Gamma(a-d+1)\,\Gamma(a-e+1)\,\Gamma(a-c-d-e+1)}{\Gamma(a+1)\,\Gamma(a-d-e+1)\,\Gamma(a-c-e+1)\,\Gamma(a-c-d+1)} $$
$$ = \Omega_9(a,c,d,e). \tag{1.9} $$

Very recently, the authors [2] derived various interesting Laplace transforms by making use of the following general product theorem [3, p. 43, Eq. 3.2.28]:

$$ g_1(t)\,g_2(t) = \int_0^\infty \mathrm{e}^{-st}\left\{\int_0^t f_1(\tau)f_2(t-\tau)\mathrm{d}\tau\right\}\mathrm{d}t, $$

for a pair of generalized hypergeometric functions [3, p. 43, Eq. 3.2.29]:

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{\mu-1}(t-\tau)^{\nu-1}\,{}_pF_q\left[\begin{matrix}(a)\\(b)\end{matrix}\,\middle|\,k\tau\right]\,{}_{p'}F_{q'}\left[\begin{matrix}(a')\\(b')\end{matrix}\,\middle|\,k'(t-\tau)\right]\mathrm{d}\tau\right\}\mathrm{d}t$$
$$= \Gamma(\mu)\Gamma(\nu)\,s^{-\mu-\nu}\,{}_{p+1}F_q\left[\begin{matrix}(a),\mu\\(b)\end{matrix}\,\middle|\,\frac{k}{s}\right]\,{}_{p'+1}F_{q'}\left[\begin{matrix}(a'),\nu\\(b')\end{matrix}\,\middle|\,\frac{k'}{s}\right], \tag{1.10}$$

where $\mathrm{Re}(\mu) > 0$, $\mathrm{Re}(\nu) > 0$, $\mathrm{Re}(s) > 0$ when $p < q$, $p' < q'$ or $\mathrm{Re}(\mu) > 0$, $\mathrm{Re}(\nu) > 0$, $\mathrm{Re}(s) > \mathrm{Re}(k)$, $\mathrm{Re}(s) > \mathrm{Re}(k')$ when $p = q$, $p' = q'$ for the case $p = q = 1 = p' = q'$ [3, p. 43, Eq. 3.2.30] *viz*:

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{\mu-1}(t-\tau)^{\nu-1}\,{}_1F_1\left[\begin{matrix}a\\c\end{matrix}\,\middle|\,k\tau\right]\,{}_1F_1\left[\begin{matrix}a'\\c'\end{matrix}\,\middle|\,k'(t-\tau)\right]\mathrm{d}\tau\right\}\mathrm{d}t$$
$$= \Gamma(\mu)\Gamma(\nu)\,s^{-\mu-\nu}\,{}_2F_1\left[\begin{matrix}a,\mu\\c\end{matrix}\,\middle|\,\frac{k}{s}\right]\,{}_2F_1\left[\begin{matrix}a',\nu\\c'\end{matrix}\,\middle|\,\frac{k'}{s}\right] \tag{1.11}$$

for $\mathrm{Re}(c) > 0$, $\mathrm{Re}(c') > 0$, $\mathrm{Re}(s) > \mathrm{Re}(k)$, $\mathrm{Re}(s) > \mathrm{Re}(k')$, $|s| > |k|$ and $|s| > |k'|$.

In this paper, by using product theorem for Laplace transform for a generalized hypergeometric functions, we employ the classical summation formulas (1.1) to (1.9) in order to derive several new Laplace transforms of convolution type integrals involving ${}_pF_p$, where $p = 1, 2, 3, 4$.

2 Laplace transforms of convolution type integrals involving product of two special ${}_pF_p(x)$ functions

In this section, we present various new Laplace type integrals by using product theorem of the Laplace transforms for a pair of two special generalized hypergeometric functions ${}_pF_p$.

For this, if we set $p = q = 1$, $p' = q' = 2$ and $p = q = p' = q' = 2$, respectively in (1.10), we obtain the formula

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{\mu-1}(t-\tau)^{\nu-1}\,{}_1F_1\left[\begin{matrix}a\\c\end{matrix}\,\middle|\,k\tau\right]\,{}_2F_2\left[\begin{matrix}a',b'\\d',e'\end{matrix}\,\middle|\,k'(t-\tau)\right]\mathrm{d}\tau\right\}\mathrm{d}t$$
$$= \Gamma(\mu)\Gamma(\nu)\,s^{-\mu-\nu}\,{}_2F_1\left[\begin{matrix}a,\mu\\c\end{matrix}\,\middle|\,\frac{k}{s}\right]\,{}_3F_2\left[\begin{matrix}a',b',\nu\\d',e'\end{matrix}\,\middle|\,\frac{k'}{s}\right], \tag{2.1}$$

where $\mathrm{Re}(\mu) > 0$, $\mathrm{Re}(\nu) > 0$, $\mathrm{Re}(s) > \mathrm{Re}(k)$, $\mathrm{Re}(s) > \mathrm{Re}(k')$, $|s| > |k|$ and $|s| > |k'|$ and

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{\mu-1}(t-\tau)^{\nu-1}\,{}_2F_2\left[\begin{matrix}a,b\\d,e\end{matrix}\,\middle|\,k\tau\right]\,{}_2F_2\left[\begin{matrix}a',b'\\d',e'\end{matrix}\,\middle|\,k'(t-\tau)\right]\mathrm{d}\tau\right\}\mathrm{d}t$$
$$= \Gamma(\mu)\Gamma(\nu)\,s^{-\mu-\nu}\,{}_3F_2\left[\begin{matrix}a,b,\mu\\d,e\end{matrix}\,\middle|\,\frac{k}{s}\right]\,{}_3F_2\left[\begin{matrix}a',b',\nu\\d',e'\end{matrix}\,\middle|\,\frac{k'}{s}\right], \tag{2.2}$$

where $\big(\mathrm{Re}(\mu) > 0$, $\mathrm{Re}(\nu) > 0$, $\mathrm{Re}(s) > \mathrm{Re}(k)$, $\mathrm{Re}(s) > \mathrm{Re}(k')$,$|s| > |k|$ and $|s| > |k'|$.

Similar results can be written involving ${}_3F_3$ and ${}_4F_4$.

Now by employing Gauss's summation theorem, Gauss's second summation theorem, Bailey's summation theorem, Kummer's summation theorem, Watson's summation theorem, Dixon's summation theorem, Whipple's summation theorem, Whipple's second summation theorem and Dougall's summation theorems (1.2) to (1.9), we get the following thirty-nine interesting results asserted in the following statements. We remark in passing that in all there exists forty-five results of this type. Out of forty-five results, six results have already been recorded in [2].

Theorem 2.1. *For* Re$(s) > 0$, Re$(b) > 0$, Re$(b') > 0$, Re$(c - a - b) > 0$ *and* Re$(c' - a' - b') > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{b'-1}{}_1F_1\left[\begin{matrix} a \\ c \end{matrix}\,\middle|\,\tau s\right]{}_1F_1\left[\begin{matrix} a' \\ c' \end{matrix}\,\middle|\,(t-\tau)s\right]d\tau\right\}dt$$
$$= \Gamma(b)\,\Gamma(b')\,s^{-b-b'}\,\Omega_1(a,b,c)\,\Omega_1(a',b',c').$$

where $\Omega_1(a,b,c)$ *is given in* (1.2).

Proof. The proof of this theorem is quite straight-forward. In order to prove this result, setting $k = s$, $k' = s$, $\mu = b$ and $\nu = b'$ in (1.11), we have

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{b'-1}{}_1F_1\left[\begin{matrix} a \\ c \end{matrix}\,\middle|\,\tau s\right]{}_1F_1\left[\begin{matrix} a' \\ c' \end{matrix}\,\middle|\,(t-\tau)s\right]d\tau\right\}dt$$
$$= \Gamma(b)\,\Gamma(b')\,s^{-b-b'}\,{}_2F_1\left[\begin{matrix} a, b \\ c \end{matrix}\,\middle|\,1\right]{}_2F_1\left[\begin{matrix} a', b' \\ c' \end{matrix}\,\middle|\,1\right]. \tag{2.3}$$

We now observe that the ${}_2F_1$ twice appearing on the right-hand side of (2.3) can be evaluated with the help of Gauss's summation theorem (1.1), which yields at once the desired formula in Theorem 2.1. □

The remaining results, which are given in the following theorems, can also be proven in a similar lines by applying appropriate summation theorems (1.1) to (1.9) in (1.10). So we prefer to omit the details.

Theorem 2.2. *For* Re$(s) > 0$, Re$(b) > 0$, Re$(b') > 0$ *and* Re$(c - a - b) > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{b'-1}{}_1F_1\left[\begin{matrix} a \\ c \end{matrix}\,\middle|\,\tau s\right]{}_1F_1\left[\begin{matrix} a' \\ \frac{1}{2}(a'+b'+1) \end{matrix}\,\middle|\,\frac{1}{2}(t-\tau)s\right]d\tau\right\}dt$$
$$= \Gamma(b)\,\Gamma(b')\,s^{-b-b'}\,\Omega_1(a,b,c)\,\Omega_2(a',b'),$$

where $\Omega_1(a,b,c)$ *and* $\Omega_2(a,b)$ *are given in* (1.1) *and* (1.2), *respectively.*

Theorem 2.3. *For* Re$(s) > 0$, Re$(b) > 0$, Re$(1 - a') > 0$ *and* Re$(c - a - b) > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{-a'}{}_1F_1\left[\begin{matrix} a \\ c \end{matrix}\,\middle|\,\tau s\right]{}_1F_1\left[\begin{matrix} a' \\ b' \end{matrix}\,\middle|\,\frac{1}{2}(t-\tau)s\right]d\tau\right\}dt$$
$$= \Gamma(b)\,\Gamma(1-a')\,s^{a'-b-1}\,\Omega_1(a,b,c)\,\Omega_3(a',b'),$$

where $\Omega_1(a,b,c)$ *and* $\Omega_3(a,b)$ *are given in* (1.1) *and* (1.3), *respectively.*

Theorem 2.4. *For* Re$(s) > 0$, Re$(b) > 0$, Re$(b') > 0$ *and* Re$(c - a - b) > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{b'-1}{}_1F_1\left[\begin{matrix} a \\ c \end{matrix}\,\middle|\,\tau s\right]{}_1F_1\left[\begin{matrix} a' \\ 1+a'-b' \end{matrix}\,\middle|\,-(t-\tau)s\right]d\tau\right\}dt$$
$$= \Gamma(b)\,\Gamma(b')\,s^{-b-b'}\,\Omega_1(a,b,c)\,\Omega_4(a',b'),$$

where $\Omega_1(a,b,c)$ *and* $\Omega_4(a,b)$ *are given in* (1.1) *and* (1.4), *respectively.*

Theorem 2.5. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(b) > 0$, $\mathrm{Re}(c') > 0$ *and* $\mathrm{Re}(c-a-b) > 0$, $\mathrm{Re}(2c'-a'-b') > -1$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ c \end{matrix}\,\middle|\,\tau s\right]{}_2F_2\left[\begin{matrix} a', b' \\ \frac{1}{2}(a'+b'+1), 2c' \end{matrix}\,\middle|\,(t-\tau)s\right]d\tau\right\}dt = \frac{\Gamma(b)\,\Gamma(c')}{s^{b+c'}}\,\Omega_1(a,b,c)\,\Omega_5(a',b',c'),$$

where $\Omega_1(a,b,c)$ *and* $\Omega_5(a,b,c)$ *are given in* (1.1) *and* (1.5), *respectively.*

Theorem 2.6. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(b) > 0$, $\mathrm{Re}(c') > 0$, $\mathrm{Re}(c-a-b) > 0$ *and* $\mathrm{Re}(a'-2b'-2c') > -2$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ c \end{matrix}\,\middle|\,\tau s\right]\right.$$
$$\left.\times\,{}_2F_2\left[\begin{matrix} a', b' \\ 1+a'-b', 1+a'-c \end{matrix}\,\middle|\,(t-\tau)s\right]d\tau\right\}dt$$
$$= \frac{\Gamma(b)\,\Gamma(c')}{s^{b+c'}}\,\Omega_1(a,b,c)\,\Omega_6(a',b',c'),$$

where $\Omega_1(a,b,c)$ *and* $\Omega_6(a,b,c)$ *are given in* (1.1) *and* (1.6), *respectively.*

Theorem 2.7. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(b) > 0$, $\mathrm{Re}(c') > 0$ *and* $\mathrm{Re}(c-a-b) > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ c \end{matrix}\,\middle|\,\tau s\right]{}_2F_2\left[\begin{matrix} a', 1-a' \\ e', 1+2c'-e' \end{matrix}\,\middle|\,(t-\tau)s\right]d\tau\right\}dt = \frac{\Gamma(b)\,\Gamma(c')}{s^{b+c'}}\,\Omega_1(a,b,c)\,\Omega_7(a',c',e'),$$

where $\Omega_1(a,b,c)$ *and* $\Omega_7(a,c,e)$ *are given in* (1.1) *and* (1.7), *respectively.*

Theorem 2.8. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(b) > 0$, $\mathrm{Re}(c') > 0$ *and* $\mathrm{Re}(2c'-a'-b') > -1$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ \frac{1}{2}(a+b+1) \end{matrix}\,\middle|\,\frac{1}{2}\tau s\right]\right.$$
$$\left.\times\,{}_2F_2\left[\begin{matrix} a', b' \\ \frac{1}{2}(a'+b'+1), 2c' \end{matrix}\,\middle|\,(t-\tau)s\right]d\tau\right\}dt$$
$$= \frac{\Gamma(b)\,\Gamma(c')}{s^{b+c'}}\,\Omega_2(a,b)\,\Omega_5(a',b',c'),$$

where $\Omega_2(a,b)$ *and* $\Omega_5(a,b,c)$ *are given in* (1.2) *and* (1.5), *respectively.*

Theorem 2.9. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(b) > 0$, $\mathrm{Re}(c') > 0$ *and* $\mathrm{Re}(a'-2b'-2c') > -2$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}\,(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ \frac{1}{2}(a+b+1) \end{matrix}\,\middle|\,\frac{1}{2}\tau s\right]\right.$$
$$\left.\times\,{}_2F_2\left[\begin{matrix} a', b' \\ 1+a'-b', 1+a'-c \end{matrix}\,\middle|\,(t-\tau)s\right]d\tau\right\}dt$$
$$= \frac{\Gamma(b)\,\Gamma(c')}{s^{b+c'}}\,\Omega_2(a,b)\,\Omega_6(a',b',c'),$$

where $\Omega_2(a,b)$ *and* $\Omega_6(a,b,c)$ *are given in* (1.2) *and* (1.6), *respectively.*

Theorem 2.10. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(b) > 0$ *and* $\mathrm{Re}(c') > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ \frac{1}{2}(a+b+1)\end{matrix}\,\Big|\,\frac{1}{2}\tau s\right] \times {}_2F_2\left[\begin{matrix} a', 1-a' \\ e', 1+2c'-e'\end{matrix}\,\Big|\,(t-\tau)s\right]d\tau\right\}dt = \frac{\Gamma(b)\Gamma(c')}{s^{b+c'}}\,\Omega_2(a,b)\,\Omega_7(a',c',e'),$$

where $\Omega_2(a,b)$ *and* $\Omega_7(a,c,e)$ *are given in* (1.2) *and* (1.7), *respectively.*

Theorem 2.11. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c') > 0$, $\mathrm{Re}(1-a) > 0$ *and* $\mathrm{Re}(2c'-a'-b') > -1$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{-a}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ b\end{matrix}\,\Big|\,\frac{1}{2}\tau s\right] \times {}_2F_2\left[\begin{matrix} a', b' \\ \frac{1}{2}(a'+b'+1), 2c'\end{matrix}\,\Big|\,(t-\tau)s\right]d\tau\right\}dt = \frac{\Gamma(1-a)\Gamma(c')}{s^{1-a+c'}}\,\Omega_3(a,b)\,\Omega_5(a',b',c'),$$

where $\Omega_3(a,b)$ *and* $\Omega_5(a,b,c)$ *are given in* (1.3) *and* (1.5), *respectively.*

Theorem 2.12. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c') > 0$, $\mathrm{Re}(1-a) > 0$ *and* $\mathrm{Re}(a'-2b'-c') > -2$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{-a}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ b\end{matrix}\,\Big|\,\frac{1}{2}\tau s\right] \times {}_2F_2\left[\begin{matrix} a', b' \\ 1+a'-b', 1+a'-c\end{matrix}\,\Big|\,(t-\tau)s\right]d\tau\right\}dt = \frac{\Gamma(1-a)\Gamma(c')}{s^{1-a+c'}}\,\Omega_3(a,b)\,\Omega_6(a',b',c'),$$

where $\Omega_3(a,b)$ *and* $\Omega_6(a,b,c)$ *are given in* (1.3) *and* (1.6), *respectively.*

Theorem 2.13. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(1-a) > 0$ *and* $\mathrm{Re}(c') > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{-a}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ b\end{matrix}\,\Big|\,\frac{1}{2}\tau s\right] \times {}_2F_2\left[\begin{matrix} a', 1-a' \\ e', 1+2c'-e'\end{matrix}\,\Big|\,(t-\tau)s\right]d\tau\right\}dt = \frac{\Gamma(1-a)\Gamma(c')}{s^{1-a+c'}}\,\Omega_3(a,b)\,\Omega_7(a',c',e'),$$

where $\Omega_3(a,b)$ *and* $\Omega_7(a,c,e)$ *are given in* (1.3) *and* (1.7), *respectively.*

Theorem 2.14. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(b) > 0$, $\mathrm{Re}(c') > 0$ *and* $\mathrm{Re}(2c'-a'-b') > -2$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ 1+a-b\end{matrix}\,\Big|\,-\tau s\right] \times {}_2F_2\left[\begin{matrix} a', b' \\ \frac{1}{2}(a'+b'+1), 2c'\end{matrix}\,\Big|\,(t-\tau)s\right]d\tau\right\}dt = \frac{\Gamma(b)\Gamma(c')}{s^{b+c'}}\,\Omega_4(a,b)\,\Omega_5(a',b',c'),$$

where $\Omega_4(a,b)$ *and* $\Omega_5(a,b,c)$ *are given in* (1.4) *and* (1.5), *respectively.*

Theorem 2.15. *For* $\mathrm{Re}(s)>0$, $\mathrm{Re}(b)>0$, $\mathrm{Re}(c')>0$ *and* $\mathrm{Re}(a'-2b'-c')>-2$, *the following result holds true:*

$$\int_0^\infty e^{-st}\bigg\{\int_0^t \tau^{b-1}(t-\tau)^{c'-1}{}_1F_1\bigg[\begin{matrix}a\\1+a-b\end{matrix}\bigg|-\tau s\bigg] \times {}_2F_2\bigg[\begin{matrix}a',b'\\1+a'-b',1+a'-c\end{matrix}\bigg|(t-\tau)s\bigg]\,d\tau\bigg\}dt = \frac{\Gamma(b)\,\Gamma(c')}{s^{b+c'}}\,\Omega_4(a,b)\,\Omega_6(a',b',c'),$$

where $\Omega_4(a,b)$ *and* $\Omega_6(a,b,c)$ *are given in* (1.4) *and* (1.6), *respectively.*

Theorem 2.16. *For* $\mathrm{Re}(s)>0$, $\mathrm{Re}(b)>0$ *and* $\mathrm{Re}(c')>0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\bigg\{\int_0^t \tau^{b-1}(t-\tau)^{c'-1}{}_1F_1\bigg[\begin{matrix}a\\1+a-b\end{matrix}\bigg|-\tau s\bigg] \times {}_2F_2\bigg[\begin{matrix}a',1-a'\\e',1+2c'-e'\end{matrix}\bigg|(t-\tau)s\bigg]\,d\tau\bigg\}dt = \frac{\Gamma(b)\,\Gamma(c')}{s^{b+c'}}\,\Omega_4(a,b)\,\Omega_7(a',c',e'),$$

where $\Omega_4(a,b)$ *and* $\Omega_7(a,c,e)$ *are given in* (1.4) *and* (1.7), *respectively.*

Theorem 2.17. *For* $\mathrm{Re}(s)>0$, $\mathrm{Re}(c)>0$, $\mathrm{Re}(c')>0$, $\mathrm{Re}(2c-a-b)>-1$ *and* $\mathrm{Re}(2c'-a'-b')>-1$, *the following result holds true:*

$$\int_0^\infty e^{-st}\bigg\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}{}_2F_2\bigg[\begin{matrix}a,b\\\frac{1}{2}(a+b+1),2c\end{matrix}\bigg|\tau s\bigg] \times {}_2F_2\bigg[\begin{matrix}a',b'\\\frac{1}{2}(a'+b'+1),2c'\end{matrix}\bigg|(t-\tau)s\bigg]\,d\tau\bigg\}dt = \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_5(a,b,c)\,\Omega_5(a',b',c'),$$

where $\Omega_5(a,b,c)$ *is given in* (1.5).

Theorem 2.18. *For* $\mathrm{Re}(s)>0$, $\mathrm{Re}(c)>0$, $\mathrm{Re}(c')>0$, $\mathrm{Re}(2c-a-b)>-1$ *and* $\mathrm{Re}(a'-2b'-c')>-2$, *the following result holds true:*

$$\int_0^\infty e^{-st}\bigg\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}{}_2F_2\bigg[\begin{matrix}a,b\\\frac{1}{2}(a+b+1),2c\end{matrix}\bigg|\tau s\bigg] \times {}_2F_2\bigg[\begin{matrix}a',b'\\1+a'-b',1+a'-c\end{matrix}\bigg|(t-\tau)s\bigg]\,d\tau\bigg\}dt = \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_5(a,b,c)\,\Omega_6(a',b',c'),$$

where $\Omega_5(a,b,c)$ *and* $\Omega_6(a,b,c)$ *are given in* (1.5) *and* (1.6), *respectively.*

Theorem 2.19. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c) > 0$, $\mathrm{Re}(c') > 0$ *and* $\mathrm{Re}(2c' - a' - b') > -1$, *the following result holds true:*

$$\int_0^\infty \mathrm{e}^{-st}\left\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}{}_2F_2\left[\begin{matrix} a, b \\ \frac{1}{2}(a+b+1), 2c \end{matrix}\,\middle|\,\tau s\right] \times {}_2F_2\left[\begin{matrix} a', 1-a' \\ e', 1+2c'-e' \end{matrix}\,\middle|\,(t-\tau)s\right]\mathrm{d}\tau\right\}\mathrm{d}t = \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_5(a,b,c)\,\Omega_7(a',c',e'),$$

where $\Omega_5(a,b,c)$ *and* $\Omega_7(a,c,e)$ *are given in* (1.5) *and* (1.7), *respectively.*

Theorem 2.20. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c) > 0$, $\mathrm{Re}(c') > 0$, $\mathrm{Re}(a - 2b - 2c) > -2$ *and* $\mathrm{Re}(a' - 2b' - c') > -2$, *the following result holds true:*

$$\int_0^\infty \mathrm{e}^{-st}\left\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}{}_2F_2\left[\begin{matrix} a, b \\ 1+a-b, 1+a-c \end{matrix}\,\middle|\,\tau s\right] \times {}_2F_2\left[\begin{matrix} a', b' \\ 1+a'-b', 1+a'-c \end{matrix}\,\middle|\,(t-\tau)s\right]\mathrm{d}\tau\right\}\mathrm{d}t = \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_6(a,b,c)\,\Omega_6(a',b',c').$$

where $\Omega_6(a,b,c)$ *is given in* (1.6).

Theorem 2.21. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c) > 0$, $\mathrm{Re}(c') > 0$ *and* $\mathrm{Re}(a - 2b - 2c) > -2$, *the following result holds true:*

$$\int_0^\infty \mathrm{e}^{-st}\left\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}{}_2F_2\left[\begin{matrix} a, b \\ 1+a-b, 1+a-c \end{matrix}\,\middle|\,\tau s\right] \times {}_2F_2\left[\begin{matrix} a', 1-a' \\ e', 1+2c'-e' \end{matrix}\,\middle|\,(t-\tau)s\right]\mathrm{d}\tau\right\}\mathrm{d}t = \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_6(a,b,c)\,\Omega_7(a',c',e'),$$

where $\Omega_6(a,b,c)$ *and* $\Omega_7(a,c,e)$ *are given in* (1.6) *and* (1.7), *respectively.*

Theorem 2.22. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c) > 0$ *and* $\mathrm{Re}(c') > 0$, *the following result holds true:*

$$\int_0^\infty \mathrm{e}^{-st}\left\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}{}_2F_2\left[\begin{matrix} a, 1-a \\ e, 1+2c-e \end{matrix}\,\middle|\,\tau s\right] \times {}_2F_2\left[\begin{matrix} a', 1-a' \\ e', 1+2c'-e' \end{matrix}\,\middle|\,(t-\tau)s\right]\mathrm{d}\tau\right\}\mathrm{d}t = \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_7(a,c,e)\,\Omega_7(a',c',e'),$$

where $\Omega_7(a,c,e)$ *is given in* (1.7).

Theorem 2.23. *For* $\operatorname{Re}(s) > 0$, $\operatorname{Re}(b) > 0$, $\operatorname{Re}(c') > 0$ *and* $\operatorname{Re}(c-a-b) > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ c \end{matrix}\,\middle|\,\tau s\right] \times {}_3F_3\left[\begin{matrix} a', 1+\frac{1}{2}a', b' \\ \frac{1}{2}a', a'-b'+1, a'-c'+1 \end{matrix}\,\middle|\,-(t-\tau)s\right]\mathrm{d}\tau\right\}\mathrm{d}t = \frac{\Gamma(b)\,\Gamma(c')}{s^{b+c'}}\,\Omega_1(a,b,c)\,\Omega_8(a',b',c'),$$

where $\Omega_1(a,b,c)$ *and* $\Omega_8(a,b,c)$ *are given in* (1.2) *and* (1.8), *respectively.*

Theorem 2.24. *For* $\operatorname{Re}(s) > 0$, $\operatorname{Re}(b) > 0$, $\operatorname{Re}(c') > 0$, $\operatorname{Re}(c-a-b) > 0$ *and* $\operatorname{Re}(a'-c'-d'-e'+1) > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ c \end{matrix}\,\middle|\,\tau s\right] \times {}_4F_4\left[\begin{matrix} a', 1+\frac{1}{2}a', d', e' \\ \frac{1}{2}a', a'-c'+1, a'-d'+1, a'-e'+1 \end{matrix}\,\middle|\,(t-\tau)s\right]\mathrm{d}\tau\right\}\mathrm{d}t = \frac{\Gamma(b)\,\Gamma(c')}{s^{b+c'}}\,\Omega_1(a,b,c)\,\Omega_9(a',c',d',e'),$$

where $\Omega_1(a,b,c)$ *and* $\Omega_9(a,c,d,e)$ *are given in* (1.2) *and* (1.9), *respectively.*

Theorem 2.25. *For* $\operatorname{Re}(s) > 0$, $\operatorname{Re}(b) > 0$ *and* $\operatorname{Re}(c') > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ \frac{1}{2}(a+b+1) \end{matrix}\,\middle|\,\frac{1}{2}\tau s\right] \times {}_3F_3\left[\begin{matrix} a', 1+\frac{1}{2}a', b' \\ \frac{1}{2}a', a'-b'+1, a'-c'+1 \end{matrix}\,\middle|\,-(t-\tau)s\right]\mathrm{d}\tau\right\}\mathrm{d}t = \frac{\Gamma(b)\,\Gamma(c')}{s^{b+c'}}\,\Omega_2(a,b)\,\Omega_8(a',b',c'),$$

where $\Omega_2(a,b)$ *and* $\Omega_8(a,b,c)$ *are given in* (1.1) *and* (1.8), *respectively.*

Theorem 2.26. *For* $\operatorname{Re}(s) > 0$, $\operatorname{Re}(b) > 0$, $\operatorname{Re}(c') > 0$ *and* $\operatorname{Re}(a'-c'-d'-e'+1) > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ \frac{1}{2}(a+b+1) \end{matrix}\,\middle|\,\frac{1}{2}\tau s\right] \times {}_4F_4\left[\begin{matrix} a', 1+\frac{1}{2}a', d', e' \\ \frac{1}{2}a', a'-c'+1, a'-d'+1, a'-e'+1 \end{matrix}\,\middle|\,(t-\tau)s\right]\mathrm{d}\tau\right\}\mathrm{d}t = \frac{\Gamma(b)\,\Gamma(c')}{s^{b+c'}}\,\Omega_2(a,b)\,\Omega_9(a',c',d',e'),$$

where $\Omega_2(a,b)$ *and* $\Omega_9(a,c,d,e)$ *are given in* (1.1) *and* (1.9), *respectively.*

Theorem 2.27. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c') > 0$ *and* $\mathrm{Re}(1-a) > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{-a}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ b\end{matrix}\,\middle|\,\frac{1}{2}\tau s\right]\right.$$
$$\left.\times\ {}_3F_3\left[\begin{matrix} a', 1+\frac{1}{2}a', b' \\ \frac{1}{2}a', a'-b'+1, a'-c'+1\end{matrix}\,\middle|\,-(t-\tau)s\right]d\tau\right\}dt$$
$$= \frac{\Gamma(1-a)\Gamma(c')}{s^{1-a+c'}}\,\Omega_3(a,b)\ \Omega_8(a',b',c'),$$

where $\Omega_3(a,b)$ *and* $\Omega_8(a,b,c)$ *are given in* (1.3) *and* (1.8), *respectively.*

Theorem 2.28. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c') > 0$, $\mathrm{Re}(1-a) > 0$ *and* $\mathrm{Re}(a'-c'-d'-e'+1) > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{-a}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ b\end{matrix}\,\middle|\,\frac{1}{2}\tau s\right]\right.$$
$$\left.\times\ {}_4F_4\left[\begin{matrix} a', 1+\frac{1}{2}a', d', e' \\ \frac{1}{2}a', a'-c'+1, a'-d'+1, a'-e'+1\end{matrix}\,\middle|\,(t-\tau)s\right]d\tau\right\}dt$$
$$= \frac{\Gamma(1-a)\Gamma(c')}{s^{1-a+c'}}\,\Omega_3(a,b)\ \Omega_9(a',c',d',e'),$$

where $\Omega_3(a,b)$ *and* $\Omega_9(a,c,d,e)$ *are given in* (1.3) *and* (1.9), *respectively.*

Theorem 2.29. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(b) > 0$ *and* $\mathrm{Re}(c') > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ 1+a-b\end{matrix}\,\middle|\,-\tau s\right]\right.$$
$$\left.\times\ {}_3F_3\left[\begin{matrix} a', 1+\frac{1}{2}a', b' \\ \frac{1}{2}a', a'-b'+1, a'-c'+1\end{matrix}\,\middle|\,-(t-\tau)s\right]d\tau\right\}dt$$
$$= \frac{\Gamma(b)\,\Gamma(c')}{s^{b+c'}}\,\Omega_4(a,b)\ \Omega_8(a',b',c'),$$

where $\Omega_4(a,b)$ *and* $\Omega_8(a,b,c)$ *are given in* (1.4) *and* (1.8), *respectively.*

Theorem 2.30. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(b) > 0$, $\mathrm{Re}(c') > 0$ *and* $\mathrm{Re}(a'-c'-d'-e'+1) > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{b-1}(t-\tau)^{c'-1}{}_1F_1\left[\begin{matrix} a \\ 1+a-b\end{matrix}\,\middle|\,-\tau s\right]\right.$$
$$\left.\times\ {}_4F_4\left[\begin{matrix} a', 1+\frac{1}{2}a', d', e' \\ \frac{1}{2}a', a'-c'+1, a'-d'+1, a'-e'+1\end{matrix}\,\middle|\,(t-\tau)s\right]d\tau\right\}dt$$
$$= \frac{\Gamma(b)\,\Gamma(c')}{s^{b+c'}}\,\Omega_4(a,b)\ \Omega_9(a',c',d',e'),$$

where $\Omega_4(a,b)$ *and* $\Omega_9(a,c,d,e)$ *are given in* (1.4) *and* (1.9), *respectively.*

Theorem 2.31. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c) > 0$, $\mathrm{Re}(c') > 0$ *and* $\mathrm{Re}(2c - a - b) > -1$, *the following result holds true:*

$$\int_0^\infty \mathrm{e}^{-st}\left\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}{}_2F_2\left[\begin{matrix} a, b \\ \frac{1}{2}(a+b+1), 2c \end{matrix}\,\Big|\,\tau s\right] \times {}_3F_3\left[\begin{matrix} a', 1+\frac{1}{2}a', b' \\ \frac{1}{2}a', a'-b'+1, a'-c'+1 \end{matrix}\,\Big|\,-(t-\tau)s\right]\mathrm{d}\tau\right\}\mathrm{d}t = \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_5(a,b,c)\,\Omega_5(a',b',c'),$$

where $\Omega_5(a,b,c)$ *and* $\Omega_8(a,b,c)$ *are given in* (1.5) *and* (1.8), *respectively.*

Theorem 2.32. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c) > 0$, $\mathrm{Re}(c') > 0$, $\mathrm{Re}(2c - a - b) > -1$ *and and* $\mathrm{Re}(a' - c' - d' - e' + 1) > 0$, *the following result holds true:*

$$\int_0^\infty \mathrm{e}^{-st}\left\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}{}_2F_2\left[\begin{matrix} a, b \\ \frac{1}{2}(a+b+1), 2c \end{matrix}\,\Big|\,\tau s\right] \times {}_4F_4\left[\begin{matrix} a', 1+\frac{1}{2}a', d', e' \\ \frac{1}{2}a', a'-c'+1, a'-d'+1, a'-e'+1 \end{matrix}\,\Big|\,(t-\tau)s\right]\mathrm{d}\tau\right\}\mathrm{d}t = \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_5(a,b,c)\,\Omega_9(a',c',d',e'),$$

where $\Omega_5(a,b,c)$ *and* $\Omega_9(a,c,d,e)$ *are given in* (1.5) *and* (1.9), *respectively.*

Theorem 2.33. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c) > 0$, $\mathrm{Re}(c') > 0$ *and* $\mathrm{Re}(a - 2b - 2c) > -2$, *the following result holds true:*

$$\int_0^\infty \mathrm{e}^{-st}\left\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}{}_2F_2\left[\begin{matrix} a, b \\ 1+a-b, 1+a-c \end{matrix}\,\Big|\,\tau s\right] \times {}_3F_3\left[\begin{matrix} a', 1+\frac{1}{2}a', b' \\ \frac{1}{2}a', a'-b'+1, a'-c'+1 \end{matrix}\,\Big|\,-(t-\tau)s\right]\mathrm{d}\tau\right\}\mathrm{d}t = \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_6(a,b,c)\,\Omega_8(a',b',c'),$$

where $\Omega_6(a,b,c)$ *and* $\Omega_8(a,b,c)$ *are given in* (1.6) *and* (1.8), *respectively.*

Theorem 2.34. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c) > 0$, $\mathrm{Re}(c') > 0$, $\mathrm{Re}(a - 2b - 2c) > -2$ *and* $\mathrm{Re}(a' - c' - d' - e' + 1) > 0$, *the following result holds true:*

$$\int_0^\infty \mathrm{e}^{-st}\left\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}{}_2F_2\left[\begin{matrix} a, b \\ 1+a-b, 1+a-c \end{matrix}\,\Big|\,\tau s\right] \times {}_4F_4\left[\begin{matrix} a', 1+\frac{1}{2}a', d', e' \\ \frac{1}{2}a', a'-c'+1, a'-d'+1, a'-e'+1 \end{matrix}\,\Big|\,(t-\tau)s\right]\mathrm{d}\tau\right\}\mathrm{d}t = \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_6(a,b,c)\,\Omega_9(a',c',d',e'),$$

where $\Omega_6(a,b,c)$ *and* $\Omega_9(a,c,d,e)$ *are given in* (1.6) *and* (1.9), *respectively.*

Theorem 2.35. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c) > 0$ *and* $\mathrm{Re}(c') > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}{}_2F_2\left[\begin{matrix}a, 1-a\\ e, 1+2c-e\end{matrix}\,\middle|\,\tau s\right]\right.$$
$$\left.\times\ {}_3F_3\left[\begin{matrix}a', 1+\frac{1}{2}a', b'\\ \frac{1}{2}a', a'-b'+1, a'-c'+1\end{matrix}\,\middle|\,-(t-\tau)s\right]d\tau\right\}dt$$
$$= \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_7(a, c, e)\ \Omega_8(a', b', c'),$$

where $\Omega_7(a, c, e)$ *and* $\Omega_8(a, b, c)$ *are given in* (1.7) *and* (1.8), *respectively.*

Theorem 2.36. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c) > 0$, $\mathrm{Re}(c') > 0$ *and* $\mathrm{Re}(a' - c' - d' - e' + 1) > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}{}_2F_2\left[\begin{matrix}a, 1-a\\ e, 1+2c-e\end{matrix}\,\middle|\,\tau s\right]\right.$$
$$\left.\times\ {}_4F_4\left[\begin{matrix}a', 1+\frac{1}{2}a', d', e'\\ \frac{1}{2}a', a'-c'+1, a'-d'+1, a'-e'+1\end{matrix}\,\middle|\,(t-\tau)s\right]d\tau\right\}dt$$
$$= \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_7(a, c, e)\ \Omega_9(a', c', d', e'),$$

where $\Omega_7(a, c, e)$ *and* $\Omega_9(a, c, d, e)$ *are given in* (1.7) *and* (1.9), *respectively.*

Theorem 2.37. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c) > 0$ *and* $\mathrm{Re}(c') > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}{}_3F_3\left[\begin{matrix}a, 1+\frac{1}{2}a, b\\ \frac{1}{2}a, a-b+1, a-c+1\end{matrix}\,\middle|\,-\tau s\right]\right.$$
$$\left.\times\ {}_3F_3\left[\begin{matrix}a', 1+\frac{1}{2}a', b'\\ \frac{1}{2}a', a'-b'+1, a'-c'+1\end{matrix}\,\middle|\,-(t-\tau)s\right]d\tau\right\}dt$$
$$= \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_7(a, c, e)\ \Omega_7(a', c', e'),$$

where $\Omega_7(a, c, e)$ *is given in* (1.7).

Theorem 2.38. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c) > 0$, $\mathrm{Re}(c') > 0$ *and* $\mathrm{Re}(a' - c' - d' - e' + 1) > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}{}_3F_3\left[\begin{matrix}a, 1+\frac{1}{2}a, b\\ \frac{1}{2}a, a-b+1, a-c+1\end{matrix}\,\middle|\,-\tau s\right]\right.$$
$$\left.\times\ {}_4F_4\left[\begin{matrix}a', 1+\frac{1}{2}a', d', e'\\ \frac{1}{2}a', a'-c'+1, a'-d'+1, a'-e'+1\end{matrix}\,\middle|\,(t-\tau)s\right]d\tau\right\}dt$$
$$= \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_8(a, b, c)\ \Omega_9(a', c', d', e'),$$

where $\Omega_8(a, b, c)$ *and* $\Omega_9(a, c, d, e)$ *are given in* (1.8) *and* (1.9), *respectively.*

Theorem 2.39. *For* $\mathrm{Re}(s) > 0$, $\mathrm{Re}(c) > 0$, $\mathrm{Re}(c') > 0$, $\mathrm{Re}(a - c - d - e + 1) > 0$ *and* $\mathrm{Re}(a' - c' - d' - e' + 1) > 0$, *the following result holds true:*

$$\int_0^\infty e^{-st}\left\{\int_0^t \tau^{c-1}(t-\tau)^{c'-1}\,{}_4F_4\left[\begin{matrix} a, 1+\frac{1}{2}a, d, e \\ \frac{1}{2}a, a-c+1, a-d+1, a-e+1\end{matrix}\,\middle|\,\tau s\right]\right.$$
$$\left.\times\ {}_4F_4\left[\begin{matrix} a', 1+\frac{1}{2}a', d', e' \\ \frac{1}{2}a', a'-c'+1, a'-d'+1, a'-e'+1\end{matrix}\,\middle|\,(t-\tau)s\right]\mathrm{d}\tau\right\}\mathrm{d}t$$
$$= \frac{\Gamma(c)\,\Gamma(c')}{s^{c+c'}}\,\Omega_9(a, c, d, e)\ \Omega_9(a', c', d', e'),$$

where $\Omega_9(a, c, d, e)$ *is given in* (1.9).

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