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On certain normalizable natural deduction formulations of some propositional intermediate logics.


In a previous, quite interesting, paper [J. Philos. Logic **14** (1985), no. 4, 359–377; MR0816241 (87e:03148)] the author introduced a type of natural deduction systems, which he calls “sequence-conclusion natural deduction”, where deductions are trees having in each node a finite set or sequence of formulae rather than a single formula. There is a clear correspondence between Gentzen’s multiple-conclusion sequent system for classical predicate logic and a sequence-conclusion natural deduction system for classical predicate logic. Cut-elimination for the sequent system implies a normal form theorem for the natural deduction system.

In the present paper the author transforms cut-free sequent formulations given for some families of intermediate logics by T. Hosoi [J. Tsuda College No. 6 (1974), 23–38; MR0536634 (58 #27352)] and O. Sonobe [ibid. No. 7 (1975), 7–13; MR0366630 (51 #2877)] into sequence-conclusion natural deduction systems. It is stated that a normal form, as well as a normalization theorem, follows for the resulting natural deduction systems. Hosoi’s intermediate logics are obtained by taking particular instances of Peirce’s law, whereas the intermediate logics studied by Sonobe are those which are characterized by the linearly ordered Heyting algebras of Gödel and Dummett: Gödel introduced the logics of the finite algebras of this kind, whereas Dummett’s logic corresponds to the infinite case. Gödel’s logics were axiomatized with the help of Nagata’s sequence of particular instances of Peirce’s law by Hosoi [Proc. Japan Acad. **42** (1966), 1001–1006; MR0216939 (36 #34)].

Reviewed by Kosta Došen

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